HW 8

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Function from the class

In[ ]:=
```
SetDirectory[NotebookDirectory[]];
PacletUninstall["IGraphM"];
Get["https://raw.githubusercontent.com/szhorvat/IGraphM/master/IGInstaller.m"];
<< "IGraphM`";
```

```
The currently installed versions of IGraph/M are: {}
```

```
Installing IGraph/M is complete: PacletObject[ Name: IGraphM Version: 0.6.5 ].
```

```
It can now be loaded using the command << IGraphM`
```

In[ ]:=
```
(*Need to install this packet:*)
<< "IGraphM`";
IGVersion[];
CommunityDetectionIGM[graph_, title_String] := Module[{cl, clcl, len, tot, plot},
  cl = IGCommunitiesMultilevel[graph];
clcl = cl["Communities"];
  len = Length[clcl];
tot = Table[Length[clcl⟦i⟧], {i, 1, Length[clcl]}];
  plot = BarChart[tot, ChartLabels → Range[Length[clcl]],
    ChartStyle → "Pastel", LabelingFunction → (Placed[#, Center] &),
    PlotLabel → Row[{"Community detection - ", title, " model"}]];
  Return[plot];]
```

In[ ]:=
```
Pmatrix[graph_, nodesTribe_] := Module[{numTribes, pMatrix, sizes, i, j},
   (*Let K(A,B) be the number of edges between
    tribe A to B, |A| and |B| the sizes of tribes A and B.
       Then the probability P_{A,B} for edge between node
       from tribe A to node from tribe B is
     P_{A,B}=K(A,B)/(|A||B|) , if A=B then:
       P_{A,A}=K(A,A)/Binom(|A|,2)*)
  numTribes = Length[DeleteDuplicates[nodesTribe]];
  pMatrix = ConstantArray[0, {numTribes, numTribes}];
  Scan[If[PropertyValue[{graph, First[#]}, "Tribe"] ≠ PropertyValue[{graph, Last[#]},
       "Tribe"], {pMatrix⟦PropertyValue[{graph, Last[#]}, "Tribe"],
        PropertyValue[{graph, First[#]}, "Tribe"]⟧++, pMatrix⟦PropertyValue[
          {graph, First[#]}, "Tribe"], PropertyValue[{graph, Last[#]}, "Tribe"]⟧++},
     pMatrix⟦PropertyValue[{graph, First[#]}, "Tribe"],
       PropertyValue[{graph, Last[#]}, "Tribe"]⟧++] &, EdgeList[graph]];
   (*calculate |tribe|*)
  sizes = Table[Count[nodesTribe, i], {i, 1, numTribes}];
   (*divide by n to get P_ij probability*)
```

```
  For[i = 1, i ≤ numTribes, i++,
   For[j = 1, j ≤ numTribes, j++,
    If[i == j, pMatrix[[i, j]] = pMatrix[[i, j]] / Binomial[sizes[[i]], 2] // N;,
      pMatrix[[i, j]] = pMatrix[[i, j]] / (sizes[[i]] * sizes[[j]]) // N];

   ]
  ];
pMatrix
 ]
(*create Random Block Model*)
BlockModel[graph_, pMat_] := Module[{B, adjMat, pEdge, n, i, j, tribeA, tribeB},
  (*Loop over all combination of 2 nodes,
  for each 2 node, the first from tribe A and the second from tribe B,
  I check the P_matrix for the probability for such edge
   and update in adjancency matrix*)
  n = VertexCount[graph];
  adjMat = ConstantArray[0, {n, n}];
  For[i = 1, i < n, i++,
   tribeA = PropertyValue[{graph, VertexList[graph][[i]]}, "Tribe"];
   For[j = i + 1, j ≤ n, j++,
    tribeB = PropertyValue[{graph, VertexList[graph][[j]]}, "Tribe"];
    pEdge = pMat[[tribeA, tribeB]];
    If[RandomReal[] ≤ pEdge,
     {adjMat[[i, j]] = 1;
      adjMat[[j, i]] = 1;}
    ]
   ]
  ];
  B = AdjacencyGraph[VertexList[graph], adjMat, GraphLayout → "CircularEmbedding"];
  Return[B]
 ]
(*Shift diagram functions*)
RenameGraphN[g_, sl_] :=
 Module[{G = g, VrDg = sl, VrLs, VrLst, SVrLst, NewName, edges, NewEdges, ed},
  VrLs = VertexList[G];
  VrLst = Table[{VrDg[[i]], VrLs[[i]]}, {i, Length[VrDg]}];
  SVrLst = Sort[VrLst, #1[[1]] > #2[[1]] &];
  NewName = Table[SVrLst[[i]][[2]], {i, Length[VrDg]}];
  edges = EdgeList[G];
  If[DirectedGraphQ[G],
   NewEdges = Table[Position[NewName, edges[[i]][[1]]][[1]][[1]] ⧴
      Position[NewName, edges[[i]][[2]]][[1]][[1]], {i, Length[edges]}],
   NewEdges = Table[Position[NewName, edges[[i]][[1]]][[1]][[1]] ⟷
      Position[NewName, edges[[i]][[2]]][[1]][[1]], {i, Length[edges]}]];
  Graph[Range[Length[VrLst]], NewEdges]]
SymmetryPoint[G_] := Module[
  {s, EE = {}, EP = {}, PE = {}, PP = {}, ee = 0, ep = 0, pe = 0, pp = 0, tc, bc, tr, br},
  s = Length[VertexList[G]];
  ee = AdjacencyMatrix[G][[1]][[1]];
  pe = Total[Total[Take[AdjacencyMatrix[G], {2, s}, {1, 1}]]];
```

```
  ep = Total[Total[Take[AdjacencyMatrix[G], {1, 1}, {2, s}]]];
  pp = Total[Total[Take[AdjacencyMatrix[G], {2, s}, {2, s}]]];
  EE = {0, ee / 2};
  EP = {0, ep / 2};
  PE = {0, pe / 2};
  PP = {EdgeCount[G], pp / 2};
  For[i = 1, i < s - 1, i++;
   tc = Total[Total[Take[AdjacencyMatrix[G], {1, i - 1}, {i, i}]]];
   bc = Total[Total[Take[AdjacencyMatrix[G], {i + 1, s}, {i, i}]]];
   tr = Total[Total[Take[AdjacencyMatrix[G], {i, i}, {1, i - 1}]]];
   br = Total[Total[Take[AdjacencyMatrix[G], {i, i}, {i + 1, s}]]];
   ee = ee + tc + tr + AdjacencyMatrix[G]〚i〛〚i〛;
   ep = ep + br - tc;
   pe = pe + bc - tr;
   pp = pp - bc - br - AdjacencyMatrix[G]〚i〛〚i〛;;
   EE = Join[EE, {ee / 2}];
   EP = Join[EP, {ep / 2}];
   PE = Join[PE, {pe / 2}];
   PP = Join[PP, {pp / 2}];];
  {EE, EP + PE, PP}]
PlotShift[graph_, title_String] := Module[{a},
  a = ListLinePlot[SymmetryPoint[RenameGraphN[graph, VertexDegree[graph]]],
    PlotLegends → Placed[{"I(ℰ,ℰ)", "I(𝒫,ℰ)", "I(𝒫,𝒫)"}, Bottom],
    PlotRange → All, AxesLabel → {"Core Size", "Number of Edges"},
    PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
    PlotLabel → Row[{"Shift diagram - ", title, " model"}]];
  Return[a];
 ]
```

Loading the network

```
In[ ]:= SetDirectory[NotebookDirectory[]];
file = Rest[Import["stormofswords.csv"]];
tribes = Import["tribes.csv"];
nodes = Flatten[tribes〚All, 1〛];
nodesTribe = Flatten[tribes〚All, 2〛];
edges = #〚1〛 ⟷ #〚2〛 & /@ file〚All, {1, 2}〛;
G = Graph[nodes, edges]
```

Out[ ]=

In[ ]:=
```
G = SetProperty[{G, VertexList[G]}, Thread["Tribe" → nodesTribe]];
PropertyValue[{G, "Daenerys"}, "Tribe"]
```

Out[ ]=
```
5
```

1. Using Network of Throne data generate G (n, p), G (n, m), Block model, configuration model

- first we creating Random Configuration Model

In[ ]:=
```
edgeDensity = EdgeCount[G] / Binomial[VertexCount[G], 2] // N
```

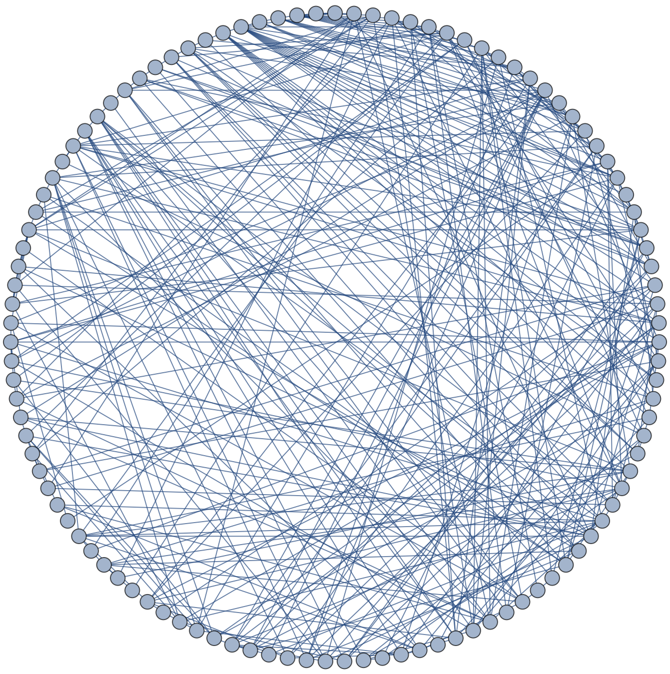
Out[ ]=
```
0.0622465
```

- Presentation of the models

In[◦]:=
```
ERCircular = Graph[RandomGraph[BernoulliGraphDistribution[VertexCount[G], edgeDensity],
    GraphLayout → "CircularEmbedding"]];
Labeled[ERCircular, "ER Circular", Top]
ER = Graph[RandomGraph[BernoulliGraphDistribution[VertexCount[G], edgeDensity]]];
Labeled[ER, "ER Standart", Top]
```

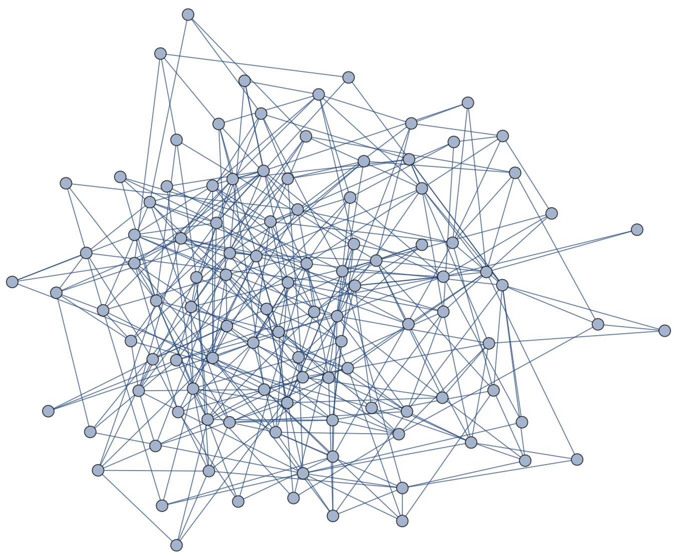
Out[◦]=

ER Circular



Out[◦]=

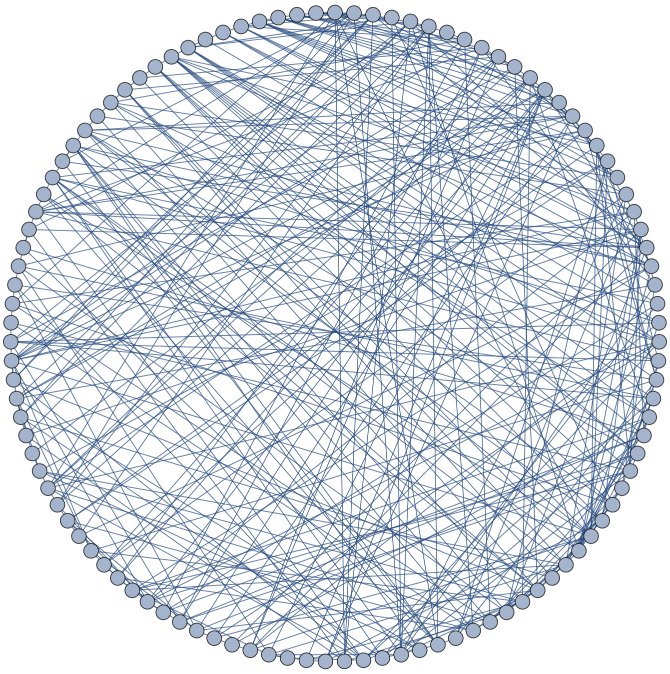
ER Standart



```
In[•]:= URCircular = RandomGraph[UniformGraphDistribution[VertexCount[G], EdgeCount[G]],
          GraphLayout → "CircularEmbedding"];
        Labeled[URCircular, "UR Circular", Top]
        UR = RandomGraph[UniformGraphDistribution[VertexCount[G], EdgeCount[G]]];
        Labeled[UR, "UR Standart", Top]
```

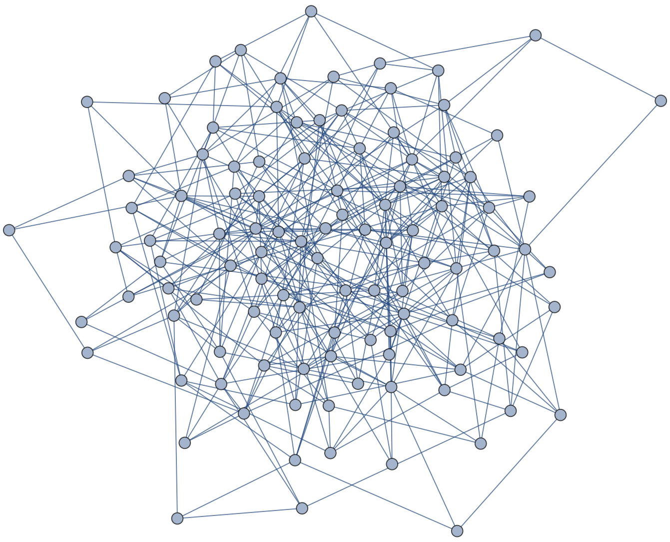
Out[•]=

UR Circular



Out[•]=

UR Standart



```
(*This functiuon from the class is'nt working *)
```

In[∘]:=
```
UR1 = RandoGraph[UniformGraphDistribution[VertexCount[G],
    Binomial[VertexCount[G], 2] * 1 / 5], GraphLayout → "CircularEmbedding"]
```

Out[∘]=

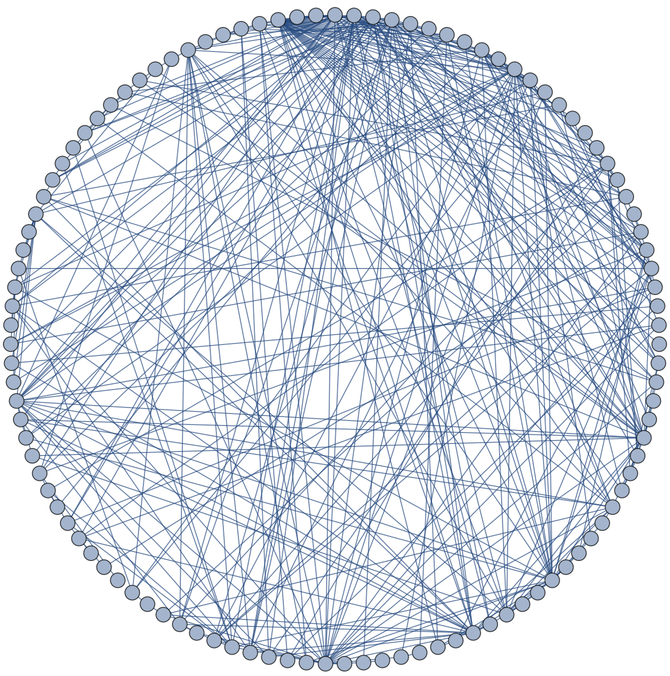
RandoGraph$\left[\text{UniformGraphDistribution}\left[107, \frac{5671}{5}\right], \text{GraphLayout} \to \text{CircularEmbedding}\right]$

- Degree Sequence

In[∘]:=
```
DSCircular = RandomGraph[
    DegreeGraphDistribution[VertexDegree[G]], GraphLayout → "CircularEmbedding"];
Labeled[DSCircular, "UR Circular", Top]
DS = RandomGraph[DegreeGraphDistribution[VertexDegree[G]]];
Labeled[DS, "UR Standart", Top]
```

Out[∘]=

UR Circular



Out[∘]=

UR Standart

- Block Model

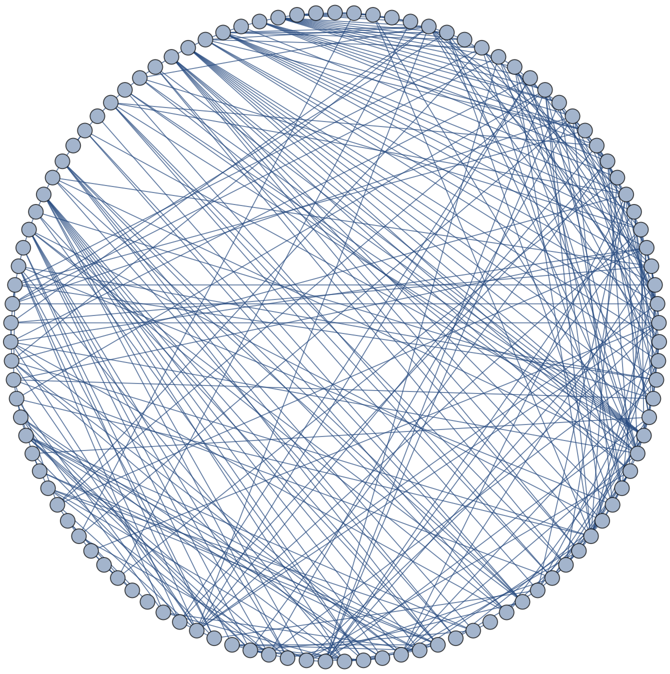
In[ ]:=
```
blockMatrix = Pmatrix[G, nodesTribe];
MatrixForm[blockMatrix]
BM = BlockModel[G, blockMatrix];
Labeled[BM, "Block Model Circular", Top]
```

Out[ ]//MatrixForm=

0.263158	0.0394737	0.00438596	0.00657895	0.	0.0438596	0.0150376
0.0394737	0.428571	0.0520833	0.00625	0.	0.	0.0714286
0.00438596	0.0520833	0.424242	0.04375	0.	0.0277778	0.0595238
0.00657895	0.00625	0.04375	0.166667	0.01	0.0291667	0.0714286
0.	0.	0.	0.01	0.247619	0.	0.
0.0438596	0.	0.0277778	0.0291667	0.	0.4	0.
0.0150376	0.0714286	0.0595238	0.0714286	0.	0.	0.714286

Out[ ]=

Block Model Circular

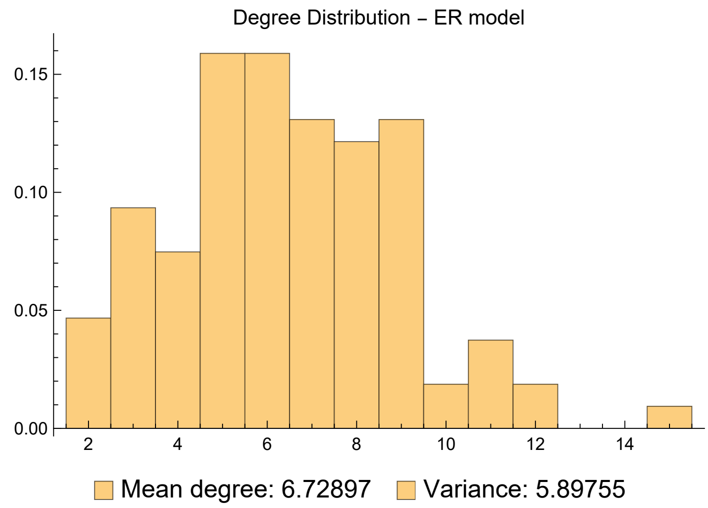


2. Compare for graphs : Degree Distribution, Giant component, Average distance, Shift Diagram for Core - Periphery, Community Detedction .

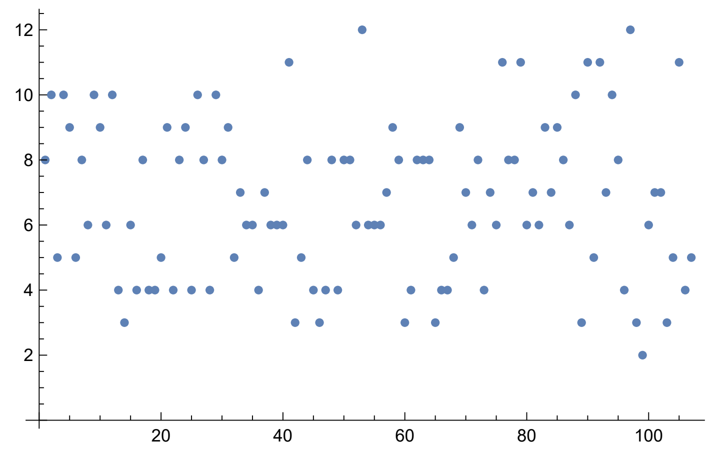
Degree distribution for G(n, p)

In[ ]:=
```
Histogram[VertexDegree[ERCircular], {1},
 "Probability", PlotLabel → "Degree Distribution – ER model",
 ChartLegends → Placed[{StringForm["Mean degree: ``", Mean[VertexDegree[ER]] // N],
    StringForm["Variance: ``", Variance[VertexDegree[ER]] // N]}, Bottom]]
ListPlot[Transpose[{Range[Length[VertexDegree[ER]]], VertexDegree[ER]}],
 PlotStyle → PointSize[Medium]]
```

Out[ ]=



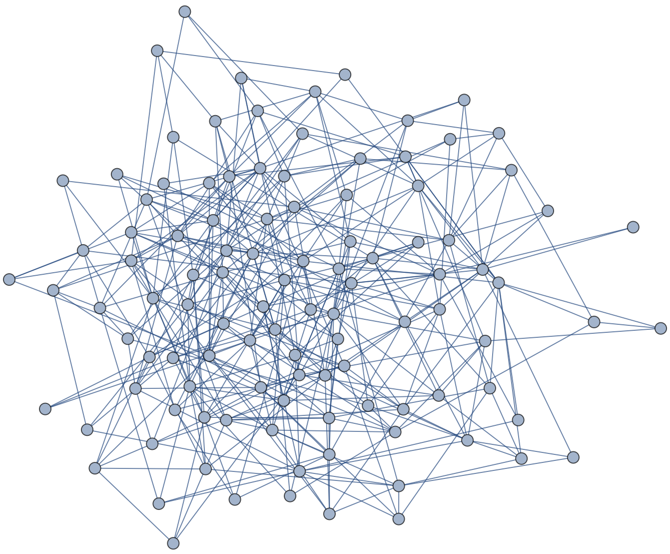
Out[ ]=



In[•]:=
```
adjM = AdjacencyMatrix[ER];
graph = AdjacencyGraph[adjM];
giantComponent = SortBy[ConnectedComponents[graph], Length][[-1]];
giantGraph = Subgraph[graph, giantComponent];
(*Giant component*)
GcER = Length[First[ConnectedComponents[ER]]];
Print["Giant component G(n, p) - ", GcER ]
Graph[giantGraph]
```

Giant component G(n, p) - 107

Out[•]=

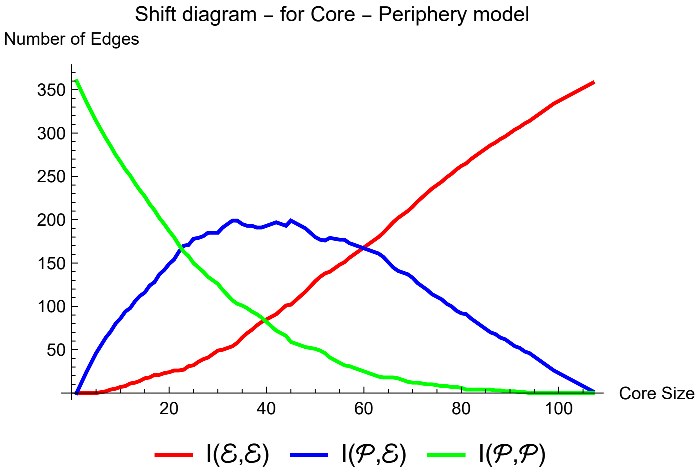


In[•]:=
```
distanceMatrix = GraphDistanceMatrix[ER];
n = VertexCount[ER];
totalDistance = Total[Flatten[distanceMatrix]];
averageDistance = totalDistance / (n * (n - 1));
Print["Average Distance:", averageDistance // N]
```

Average Distance:2.62529

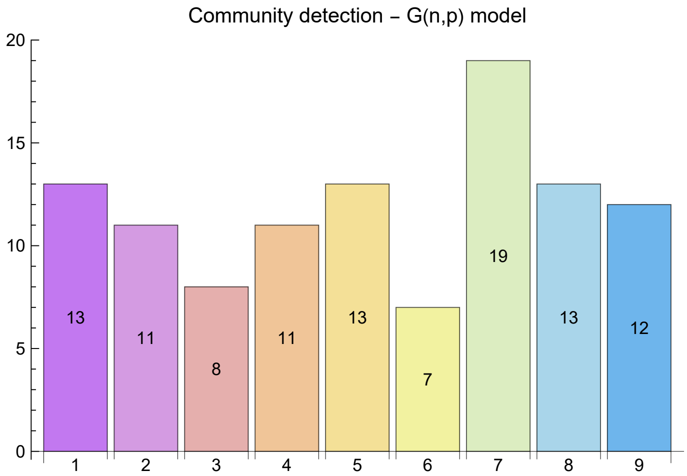
In[ ]:=
```
(*Shift diagram*)
UnWight = AdjacencyMatrix[ER];
degrees = Total /@ UnWight;
namelistcore = VertexList[ER];
res = RenameGraphN[ER, VertexDegree[ER]];
PlotShift[res, "for Core - Periphery"]
```

Out[ ]=

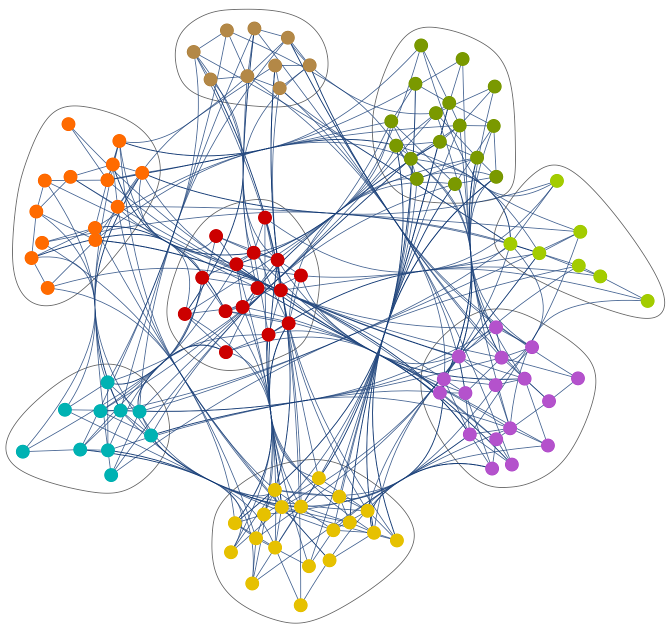


In[ ]:=
```
(*Community Detection*)
CommunityDetectionIGM[ER, "G(n,p)"]
cl = IGCommunitiesMultilevel[ER];
clcl = cl["Communities"];
CommunityGraphPlot[ER, clcl]
```

Out[ ]=



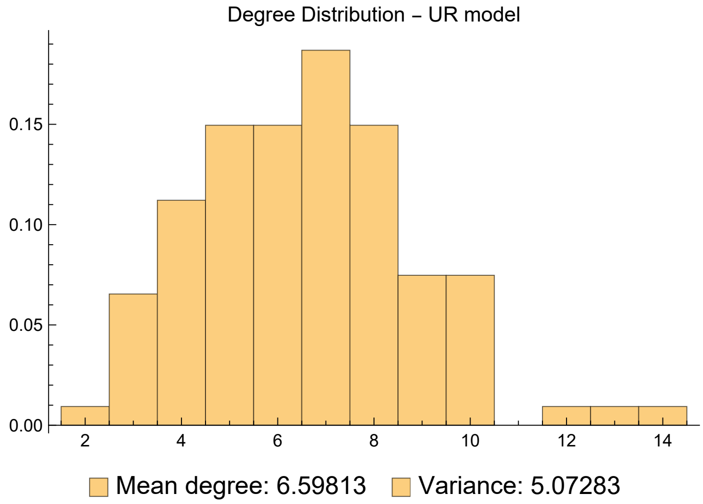
Out[ ]=



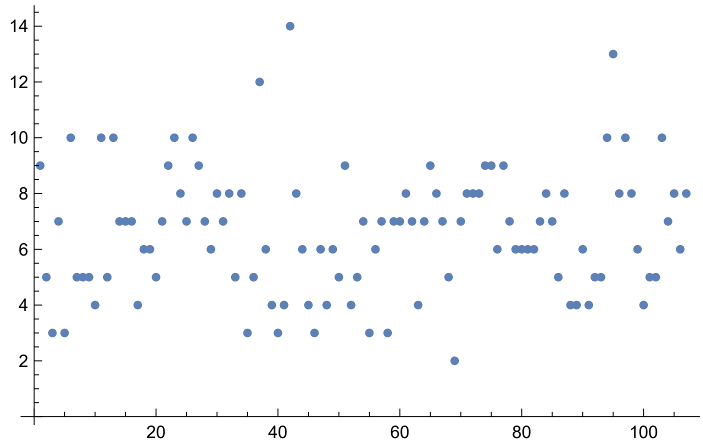
Degree distribution for G (n, m)

In[∘]:=
```
Histogram[VertexDegree[UR], {1}, "Probability",
 PlotLabel → "Degree Distribution – UR model",
 ChartLegends → Placed[{StringForm["Mean degree: ``", Mean[VertexDegree[UR]] // N],
    StringForm["Variance: ``", Variance[VertexDegree[UR]] // N]}, Bottom]]
ListPlot[Transpose[{Range[Length[VertexDegree[UR]]], VertexDegree[UR]}],
 PlotStyle → PointSize[Medium]]
```

Out[∘]=



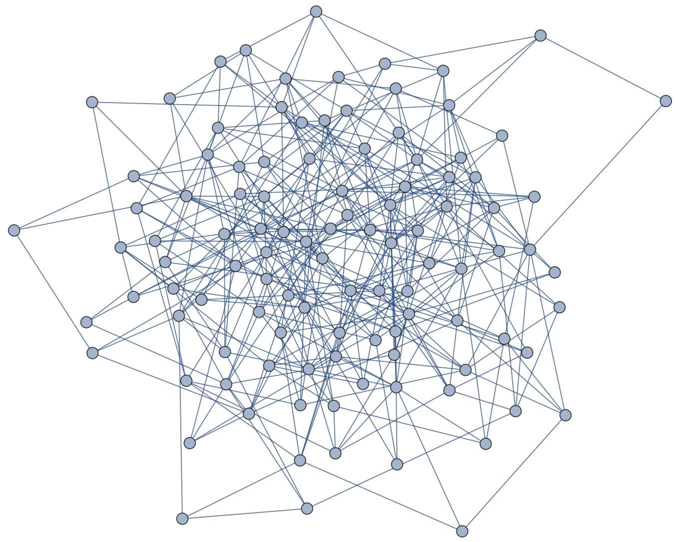
Out[∘]=



In[ ]:=
```
adjM = AdjacencyMatrix[UR];
graph = AdjacencyGraph[adjM];
giantComponent = SortBy[ConnectedComponents[graph], Length][[-1]];
giantGraph = Subgraph[graph, giantComponent];
(*Giant component*)
GcUR = Length[First[ConnectedComponents[UR]]];
Print["Degree distribution for G(n, m), Giant component G(n, m) - ", GcUR ]
Graph[giantGraph]
```

Degree distribution for G(n, m), Giant component G(n, m) - 107

Out[ ]=

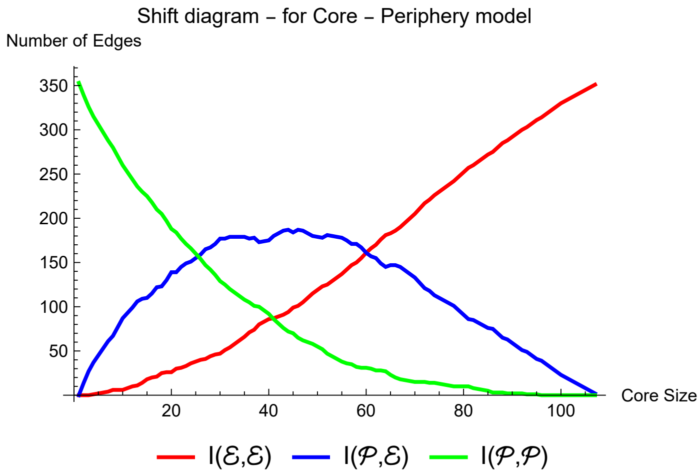


In[ ]:=
```
distanceMatrix = GraphDistanceMatrix[UR];
n = VertexCount[UR];
totalDistance = Total[Flatten[distanceMatrix]];
averageDistance = totalDistance / (n * (n - 1));
Print["Average Distance:", averageDistance // N]
```

Average Distance:2.64063

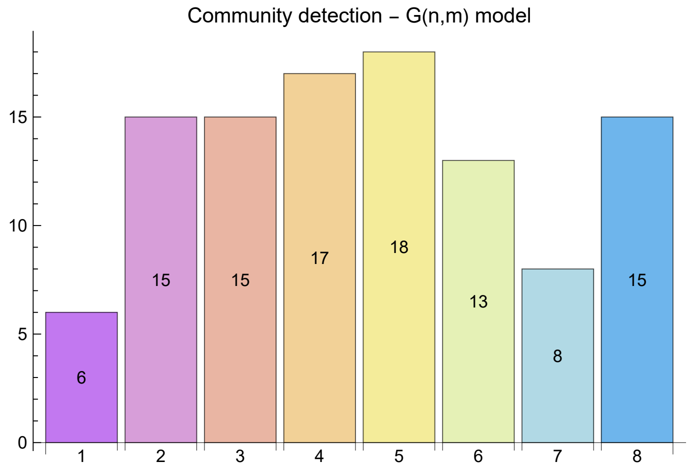
In[ ]:=
```
(*Shift diagram*)
UnWight = AdjacencyMatrix[UR];
degrees = Total /@ UnWight;
namelistcore = VertexList[UR];
res = RenameGraphN[UR, VertexDegree[UR]];
PlotShift[res, "for Core - Periphery"]
```

Out[ ]=

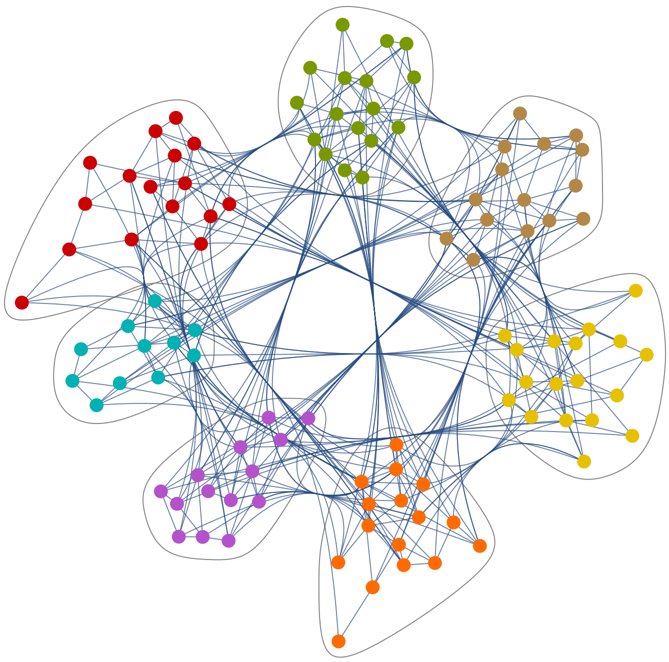


In[•]:=
```
(*Community Detection*)
CommunityDetectionIGM[ER, "G(n,m)"]
cl = IGCommunitiesMultilevel[UR];
clcl = cl["Communities"];
CommunityGraphPlot[UR, clcl]
```

Out[•]=



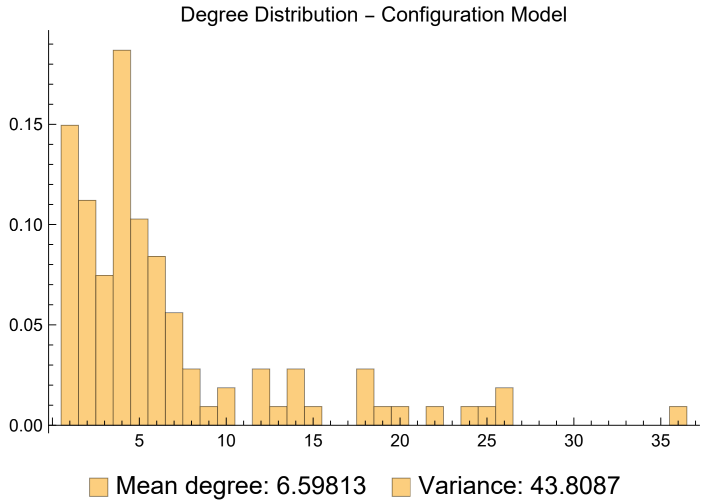
Out[•]=



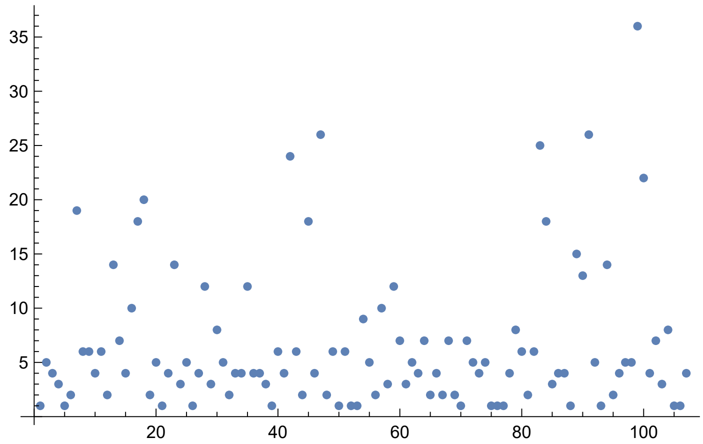
Configuration Model

In[ ]:=
```
Histogram[VertexDegree[DS], {1}, "Probability",
 PlotLabel → "Degree Distribution – Configuration Model",
 ChartLegends → Placed[{StringForm["Mean degree: ``", Mean[VertexDegree[DS]] // N],
    StringForm["Variance: ``", Variance[VertexDegree[DS]] // N]}, Bottom]]
ListPlot[Transpose[{Range[Length[VertexDegree[DS]]], VertexDegree[DS]}],
 PlotStyle → PointSize[Medium]]
```

Out[ ]=



Out[ ]=



In[ ]:=
```
adjM = AdjacencyMatrix[DS];
graph = AdjacencyGraph[adjM];
giantComponent = SortBy[ConnectedComponents[graph], Length][[-1]];
giantGraph = Subgraph[graph, giantComponent];
(*Giant component*)
GcDS = Length[First[ConnectedComponents[DS]]];
Print["Configuration Model, Giant component- ", GcDS ]
Graph[giantGraph]
```

```
Configuration Model, Giant component- 107
```

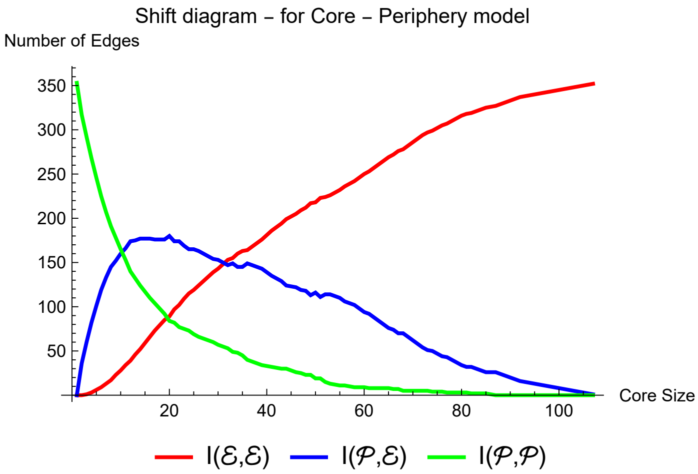
Out[ ]=

In[ ]:=
```
distanceMatrix = GraphDistanceMatrix[DS];
n = VertexCount[DS];
totalDistance = Total[Flatten[distanceMatrix]];
averageDistance = totalDistance / (n * (n - 1));
Print["Average Distance:", averageDistance // N]
```

```
Average Distance:2.69564
```

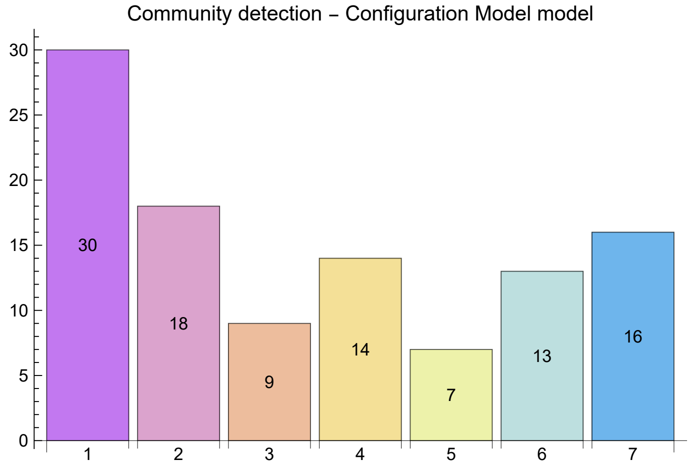
In[ ]:=
```
(*Shift diagram*)
UnWight = AdjacencyMatrix[DS];
degrees = Total /@ UnWight;
namelistcore = VertexList[DS];
res = RenameGraphN[DS, VertexDegree[DS]];
PlotShift[res, "for Core - Periphery"]
```

Out[ ]=

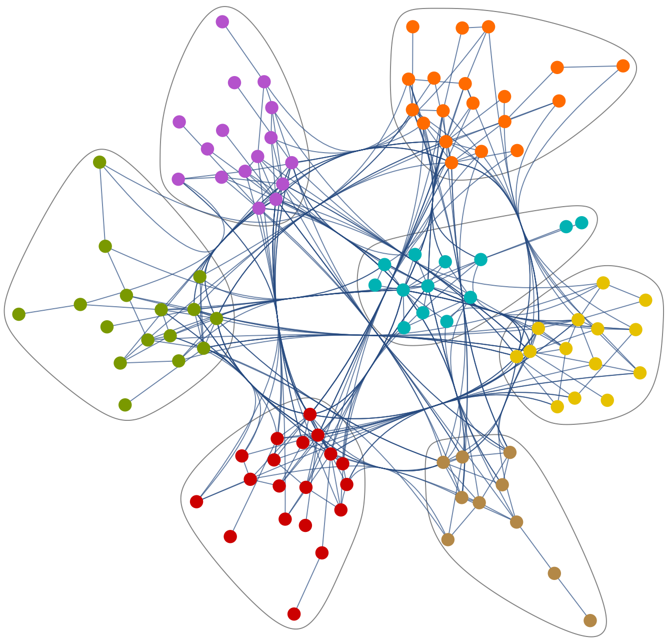


In[∘]:=
```
(*Community Detection*)
CommunityDetectionIGM[ER, "Configuration Model"]
cl = IGCommunitiesMultilevel[DS];
clcl = cl["Communities"];
CommunityGraphPlot[DS, clcl]
```

Out[∘]=



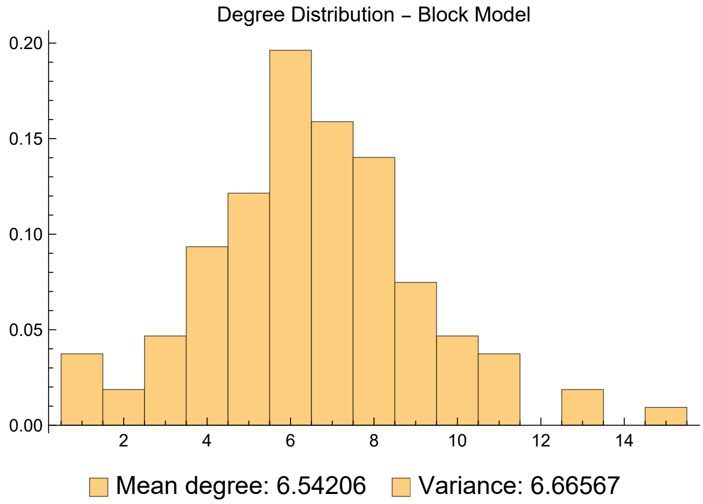
Out[∘]=



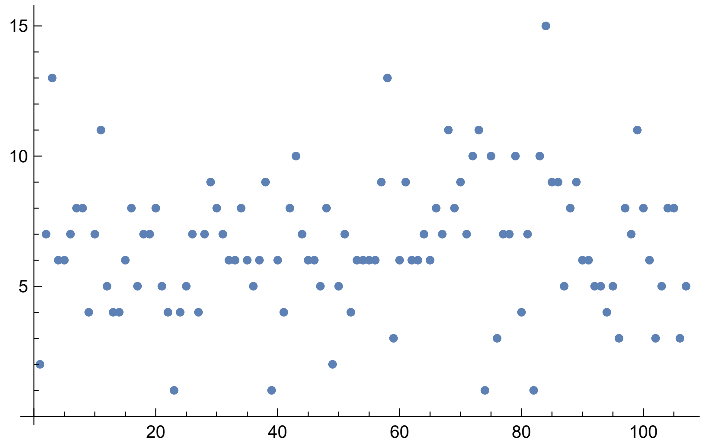
Block Model

In[∘]:=
```
Histogram[VertexDegree[BM], {1}, "Probability",
 PlotLabel → "Degree Distribution - Block Model",
 ChartLegends → Placed[{StringForm["Mean degree: ``", Mean[VertexDegree[BM]] // N],
    StringForm["Variance: ``", Variance[VertexDegree[BM]] // N]}, Bottom]]
ListPlot[Transpose[{Range[Length[VertexDegree[BM]]], VertexDegree[BM]}],
 PlotStyle → PointSize[Medium]]
```

Out[∘]=



Out[∘]=



In[ ]:=
```
adjM = AdjacencyMatrix[BM];
graph = AdjacencyGraph[adjM];
giantComponent = SortBy[ConnectedComponents[graph], Length][[-1]];
giantGraph = Subgraph[graph, giantComponent];
(*Giant component*)
GcBM = Length[First[ConnectedComponents[BM]]];
Print["Block Model, Giant component- ", GcBM]
Graph[giantGraph]
```

Block Model, Giant component- 107

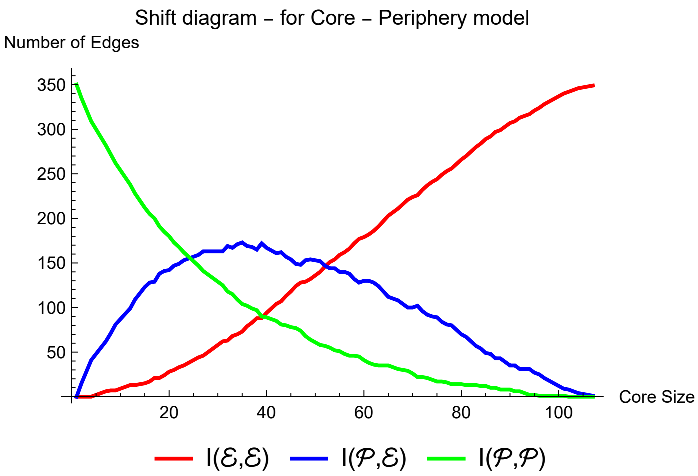
Out[ ]=

In[ ]:=
```
distanceMatrix = GraphDistanceMatrix[BM];
n = VertexCount[BM];
totalDistance = Total[Flatten[distanceMatrix]];
averageDistance = totalDistance / (n * (n - 1));
Print["Average Distance:", averageDistance // N]
```

Average Distance:2.99788

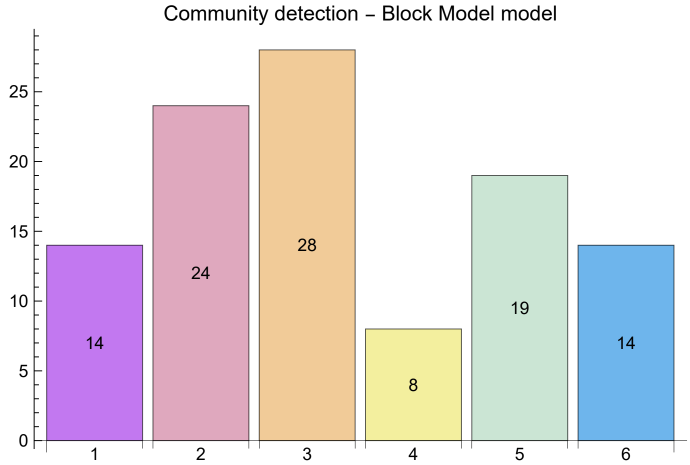
In[ ]:=
```
(*Shift diagram*)
UnWight = AdjacencyMatrix[BM];
degrees = Total /@ UnWight;
namelistcore = VertexList[BM];
res = RenameGraphN[BM, VertexDegree[BM]];
PlotShift[res, "for Core - Periphery"]
```

Out[ ]=

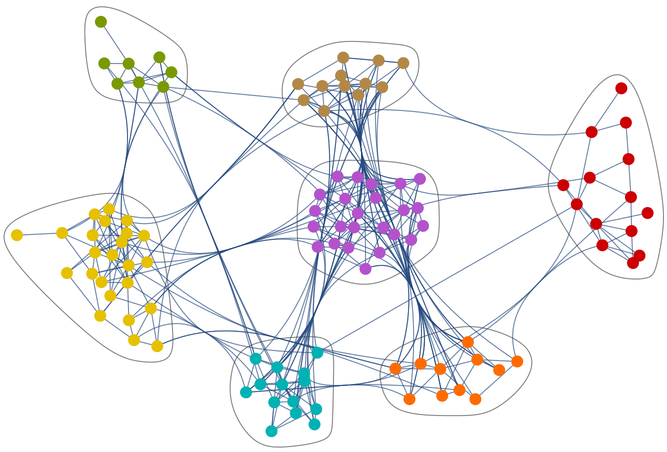


In[ ]:=
```
(*Community Detection*)
CommunityDetectionIGM[BM, "Block Model"]
cl = IGCommunitiesMultilevel[BM];
clclBM = cl["Communities"];
CommunityGraphPlot[BM, clclBM]
```

Out[ ]=



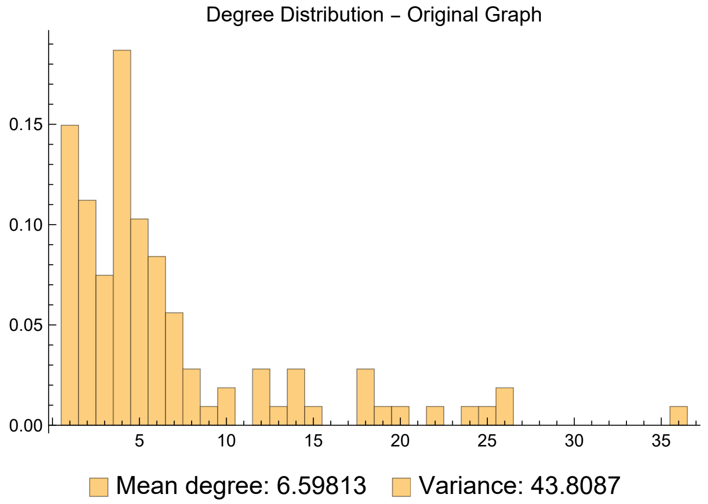
Out[ ]=



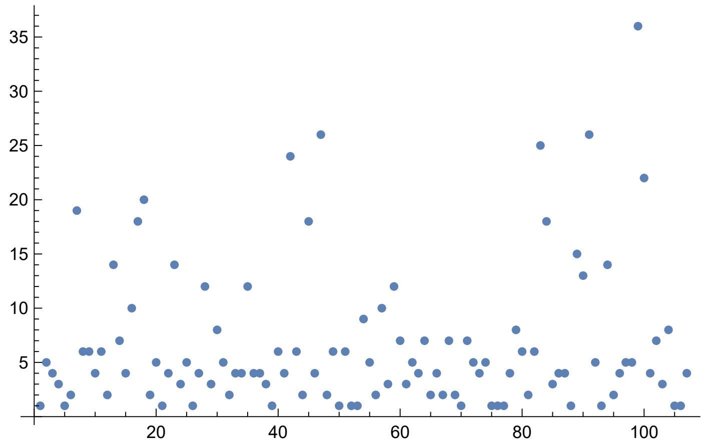
Original Graph Throne network

In[*]:=
```
Histogram[VertexDegree[G], {1}, "Probability",
 PlotLabel → "Degree Distribution - Original Graph",
 ChartLegends → Placed[{StringForm["Mean degree: ``", Mean[VertexDegree[G]] // N],
    StringForm["Variance: ``", Variance[VertexDegree[G]] // N]}, Bottom]]
ListPlot[Transpose[{Range[Length[VertexDegree[G]]], VertexDegree[G]}],
 PlotStyle → PointSize[Medium]]
```

Out[*]=



Out[*]=



In[∘]:=
```
adjM = AdjacencyMatrix[G];
graph = AdjacencyGraph[adjM];
giantComponent = SortBy[ConnectedComponents[graph], Length][[-1]];
giantGraph = Subgraph[graph, giantComponent];
(*Giant component*)
GcG = Length[First[ConnectedComponents[G]]];
Print["Configuration Model, Giant component- ", GcG ]
Graph[giantGraph]
```

Configuration Model, Giant component- 107

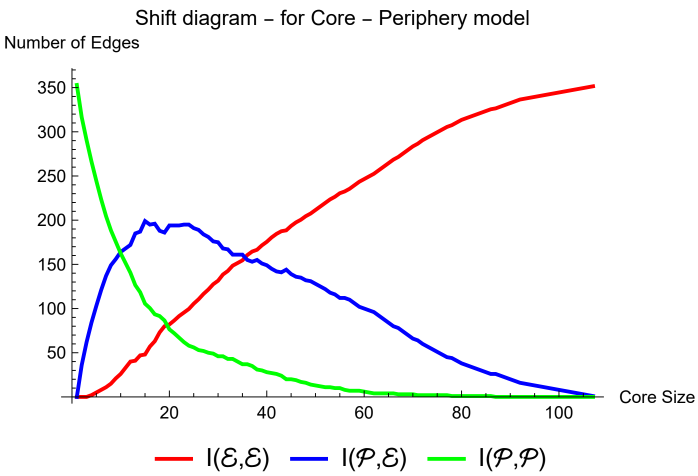
Out[∘]=

In[∘]:=
```
distanceMatrix = GraphDistanceMatrix[G];
n = VertexCount[G];
totalDistance = Total[Flatten[distanceMatrix]];
averageDistance = totalDistance / (n * (n - 1));
Print["Average Distance:", averageDistance // N]
```

Average Distance:2.9039

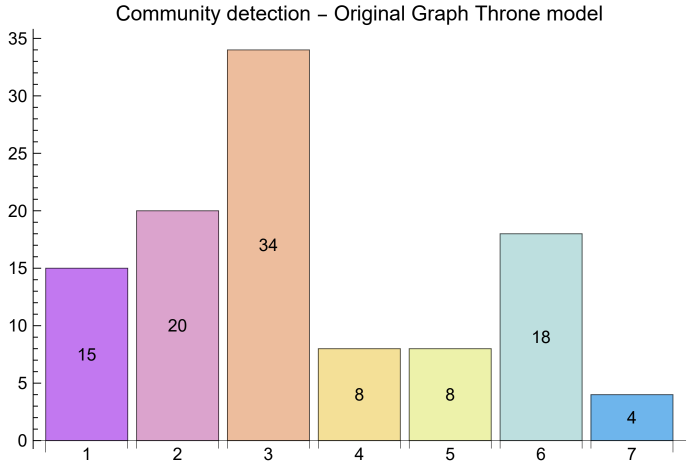
In[∘]:=
```
(*Shift diagram*)
UnWight = AdjacencyMatrix[G];
degrees = Total /@ UnWight;
namelistcore = VertexList[G];
res = RenameGraphN[G, VertexDegree[G]];
PlotShift[res, "for Core - Periphery"]
```

Out[∘]=

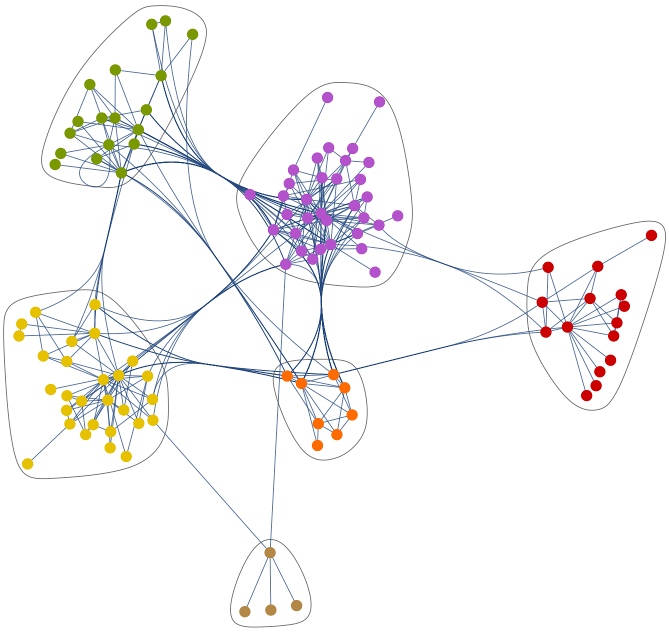


In[∘]:=
```
(*Community Detection*)
CommunityDetectionIGM[G, "Original Graph Throne"]
cl = IGCommunitiesMultilevel[G];
clclG = cl["Communities"];
CommunityGraphPlot[G, clclG]
```

Out[∘]=



Out[∘]=



3. Discuss your finding, what (and why) are the differences with the original network .

for the discuss models, G (n, p) and G (n, m) models generate random graphs with varying edge configurations, while the block model and configuration model aim to simplify and analyze specific aspects of the original graph, such as community structure or degree distribution, respectively . The choice of model depends on the specific characteristics of the original graph that we want to preserve or explore.

Now we explain the reasons for The differences for each model :

1. The Reasons for G (n, p): The randomness in edge formation reflects a scenario where connections between nodes are created independently with a given probability . This model is used to study random graphs and analyze their properties, such as connectivity, percolation, and phase transitions
2. The Reasons for G (n, m): This model allows for controlling the number of edges in the graph while maintaining randomness. It is useful in situations where the desired number of edges is known or needs to be controlled independently of the original network’s specific connectivity patterns.
3. The Reasons for Block model: The block model is used to uncover structural patterns, community structures, or functional modules within the network. It provides a more abstract view of the network’s organization and facilitates analysis and understanding of complex networks by grouping similar nodes together
4. The Reasons for configuration: The configuration model is useful when studying the role of degree distribution in a network while disregarding the original network’s specific connectivity patterns. It allows for generating random graphs with a prescribed degree sequence, which can be used for statistical analyses or comparison with other networks

The average distance and giant component are similarly present, and we can see that expected value(Degree) approximately the same for each model and the result is $N*P = 107\times 0.0622465 \sim 6.6$.
and for the Diameter for all model is the same again and the calculate $D \sim \frac{\log(107)}{\log(6.6)} = 2.476$.

We can see that is less then $\log(107) = 4.672$ as we expected.

In summary, the differences between the original network and the graph models arise due to the specific generation methods employed by each model . These differences are purposeful and serve different analytical or modeling objectives, such as studying random graphs, understanding community structures, or analyzing degree distributions . The choice of model depends on the research questions or objectives at hand.