

# Hw9

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## Function

```
In[*]:= (*Power Law functions*)
DegreeFreq[g_] := Map[{#[[1]], #[[2]] / VertexCount[g]} &, Tally[VertexDegree[g]]];
CumDigDist[g_, min_, max_] :=
Module[{df, cm, minpos, maxpos}, df = Sort[DegreeFreq[g], #1[[1]] < #2[[1]] &];
minpos = First@First@Position[df[[All, 1]], min];
maxpos = First@First@Position[df[[All, 1]], max];
cm = Reverse[Accumulate[Reverse[df[[All, 2]]]]];
Transpose[{df[[minpos ;; maxpos, 1]], cm[[minpos ;; maxpos]]}]]
(*Shift diagram functions*)
RenameGraphN[g_, sl_] :=
Module[{G = g, VrDg = sl, VrLs, VrLst, SVrLst, NewName, edges, NewEdges, ed},
VrLs = VertexList[G];
VrLst = Table[{VrDg[[i]], VrLs[[i]]}, {i, Length[VrDg]}];
SVrLst = Sort[VrLst, #1[[1]] > #2[[1]] &];
NewName = Table[SVrLst[[i]][[2]], {i, Length[VrDg]}];
edges = EdgeList[G];
If[DirectedGraphQ[G],
NewEdges = Table[Position[NewName, edges[[i][[1]][[1]][[1]]] ↔
Position[NewName, edges[[i][[2]][[1]][[1]], {i, Length[edges]}],
NewEdges = Table[Position[NewName, edges[[i][[1]][[1]][[1]]] ←
Position[NewName, edges[[i][[2]][[1]][[1]], {i, Length[edges]}]]];
Graph[Range[Length[VrLst]], NewEdges]]
SymmetryPoint[G_] :=
Module[{s, EE = {}, EP = {}, PE = {}, PP = {}, ee = 0, ep = 0, pe = 0, pp = 0, tc, bc, tr, br},
s = Length[VertexList[G]];
ee = AdjacencyMatrix[G][[1]][[1]];
pe = Total[Total[Take[AdjacencyMatrix[G], {2, s}, {1, 1}]]];
ep = Total[Total[Take[AdjacencyMatrix[G], {1, 1}, {2, s}]]];
pp = Total[Total[Take[AdjacencyMatrix[G], {2, s}, {2, s}]]];
EE = {0, ee / 2};
EP = {0, ep / 2};
PE = {0, pe / 2};
PP = {EdgeCount[G], pp / 2};
For[i = 1, i < s - 1, i++,
tc = Total[Total[Take[AdjacencyMatrix[G], {1, i - 1}, {i, i}]]];
bc = Total[Total[Take[AdjacencyMatrix[G], {i + 1, s}, {i, i}]]];
tr = Total[Total[Take[AdjacencyMatrix[G], {i, i}, {1, i - 1}]]];
br = Total[Total[Take[AdjacencyMatrix[G], {i, i}, {i + 1, s}]]];
ee = ee + tc + tr + AdjacencyMatrix[G][[i]][[i]];
ep = ep + br - tc;
pe = pe + bc - tr;
pp = pp - bc - br - AdjacencyMatrix[G][[i]][[i]]; EE = Join[EE, {ee / 2}];
```

```

EP = Join[EP, {ep / 2}];
PE = Join[PE, {pe / 2}];
PP = Join[PP, {pp / 2}];];
{EE, EP + PE, PP}
PlotShift[graph_, title_String] := Module[{a},
  a = ListLinePlot[SymmetryPoint[RenameGraphN[graph, VertexDegree[graph]]],
    PlotLegends → Placed[{"I( $\mathcal{E}, \mathcal{E}$ )", "I( $\mathcal{P}, \mathcal{E}$ )", "I( $\mathcal{P}, \mathcal{P}$ )"}, Bottom],
    PlotRange → All, AxesLabel → {"Core Size", "Number of Edges"},
    PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
    PlotLabel → Row[{"Shift diagram - ", title, " model"}]];
  Return[a];
]
(*Community detection function, using the Louvaion method from HW7*)
(*Need to install this packet:*)
SetDirectory[NotebookDirectory[]];
PacletUninstall["IGraphM"];
Get["https://raw.githubusercontent.com/szhorvat/IGraphM/master/IGInstaller.m"];
<< "IGraphM`";
IGVersion[];
CommunityDetectionIGM[graph_, title_String] := Module[{cl, clcl, len, tot, plot},
  cl = IGCommunitiesMultilevel[graph];
  clcl = cl["Communities"];
  len = Length[clcl];
  tot = Table[Length[clcl[[i]]], {i, 1, Length[clcl]}];
  plot = BarChart[tot, ChartLabels → Range[Length[clcl]],
    ChartStyle → "Pastel", LabelingFunction → (Placed[#, Center] &),
    PlotLabel → Row[{"Community detection - ", title, " model"}]];
  Return[plot];
]
The currently installed versions of IGraph/M are: {}

```

Installing IGraph/M is complete: PacletObject[ Name: IGraphM  
Version: 0.6.5].

It can now be loaded using the command << IGraphM`

---

1. Find a network with power law degree distribution. Plot the degree distribution and try to find power exponent  $\beta$ .

---

```

In[ ]:= net = ExampleData[{"NetworkGraph", "DiseaseGeneNetwork"}];
Print["Vertex Count for the network: ", VertexCount[net]]
maxD = Max[VertexDegree[net]];
meanD = Mean[VertexDegree[net]] // N;
Print["The mean of Vertex Count for the network: ", meanD]
Print["The max Degree of the network: ", maxD]
Vertex Count for the network: 1777
The mean of Vertex Count for the network: 8.43106
The max Degree of the network: 70

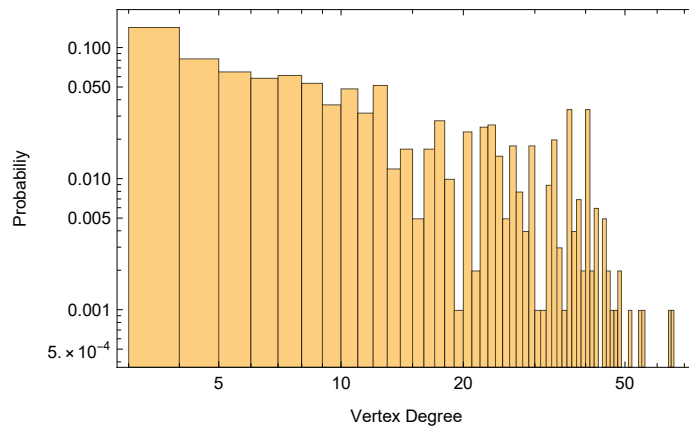
```

```

In[ ]:= Histogram[VertexDegree[net], {3, maxD, 1},
  "Probability", ScalingFunctions -> {"Log", "Log"},
  FrameLabel -> {"Vertex Degree", "Probability"}, Frame -> True]
Powerfit = Fit[Log[CumDigDist[net, 5, maxD]], {1, x}, x]
beta = -Powerfit[[2]][[1]];
Print[Style["The power exponent  $\beta$  is: ", 20], beta, "→  $2 \leq \beta \leq 3$ "]
Show[ListLogLogPlot[CumDigDist[net, 5, maxD]],
  Plot[Powerfit, {x, 1, 58}, PlotStyle -> Green]]

```

Out[ ]=

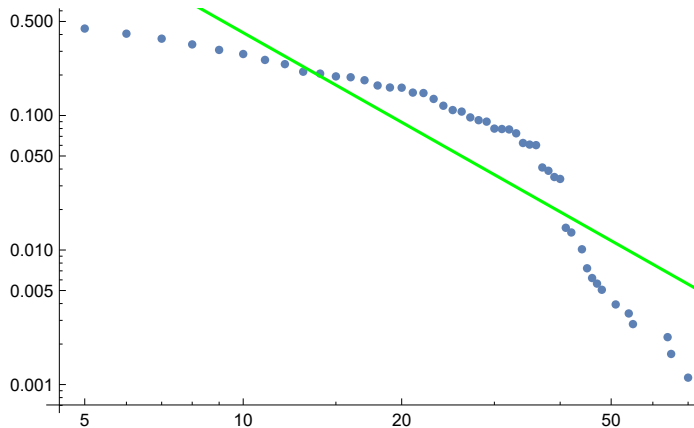


Out[ ]=

4.20769 - 2.2119 x

The power exponent  $\beta$  is: 2.2119 →  $2 \leq \beta \leq 3$

Out[ ]=




---

2. Generate the symmetry point graph for PA with several sizes (on what fraction it happens? what is the "core" fraction)

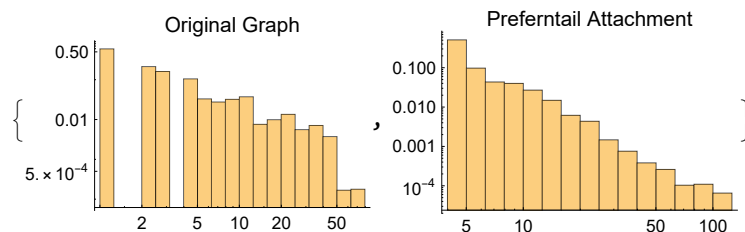
---

```

In[ ]:= Pa = RandomGraph[BarabasiAlbertGraphDistribution[
  VertexCount[net], Round[EdgeCount[net] / VertexCount[net]]]];
Print["Vertex Count for the PA: ", VertexCount[Pa]]
Print["The mean of Vertex Count for the PA: ", Mean[VertexDegree[Pa]] // N]
{Histogram[VertexDegree[net], {"Log", 15}, {"Log", "PDF"},
  PlotLabel → "Original Graph"], Histogram[VertexDegree[Pa],
  {"Log", 15}, {"Log", "PDF"}, PlotLabel → "Preferntail Attachment"]}
PowerfitPa = Fit[Log[CumDigDist[Pa, 5, Max[VertexDegree[Pa]]]], {1, x}, x]
Show[ListLogLogPlot[CumDigDist[Pa, 5, Max[VertexDegree[Pa]]],
  Plot[PowerfitPa, {x, 1, 58}, PlotStyle → Green]]
ListLogLogPlot[{CumDigDist[Pa, 5, Max[VertexDegree[Pa]]],
  CumDigDist[net, 5, Max[VertexDegree[net]]]},
  PlotLegends → {"Preferntail Attachemnet", "Original graph"}]
(*not work*)
(*spPA=PlotShift[Pa,"PA"]*)
Vertex Count for the PA: 1777
The mean of Vertex Count for the PA: 7.98875

```

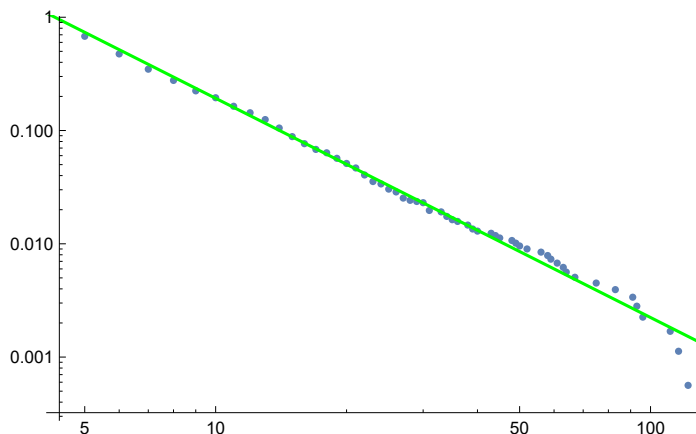
Out[ ]=



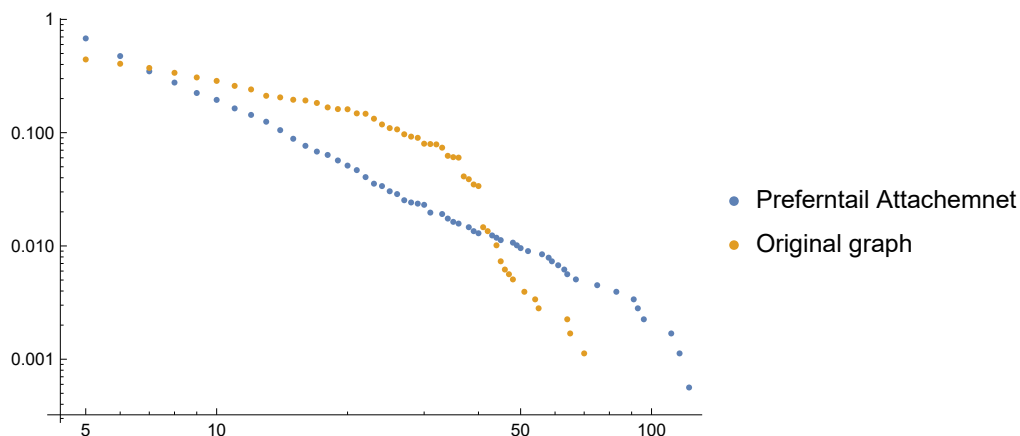
Out[ ]=

2.80841 - 1.93525 x

Out[ ]=



Out[ ]:=




---

3. Add Preferential Attachment to the models you check "Network of Throne" with . Like we did in home work 8

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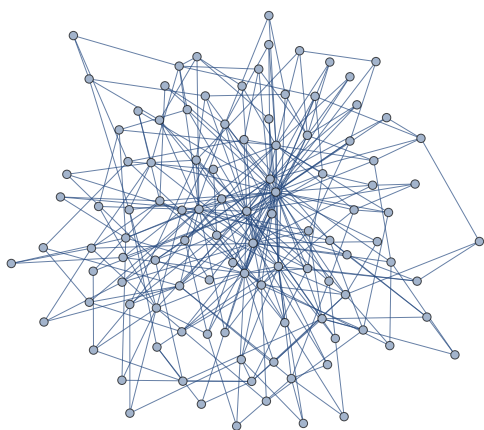
```
In[ ]:= (*Load the netork*)
SetDirectory[NotebookDirectory[]];
file = Rest[Import["stormofswords.csv"]];
tribes = Import["tribes.csv"];
nodes = Flatten[tribes[[All, 1]]];
nodesTribe = Flatten[tribes[[All, 2]]];
edges = #[[1]] ↔ #[[2]] & /@ file[[All, {1, 2}]];
G = Graph[nodes, edges];
```

```

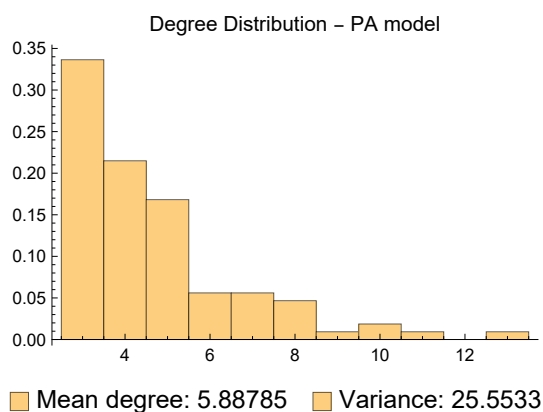
In[ ]:= (*Creat the PA - Graph*)
PaG = RandomGraph[BarabasiAlbertGraphDistribution[
  VertexCount[G], Round[EdgeCount[G] / VertexCount[G]]]]
(*Degree distribution*)
degDisPa = Histogram[VertexDegree[PaG], {1},
  "Probability", PlotLabel -> "Degree Distribution - PA model",
  ChartLegends -> Placed[{StringForm["Mean degree: `", Mean[VertexDegree[PaG]] // N],
    StringForm["Variance: `", Variance[VertexDegree[PaG]] // N]}, Bottom]]

```

Out[ ]=



Out[ ]=



```

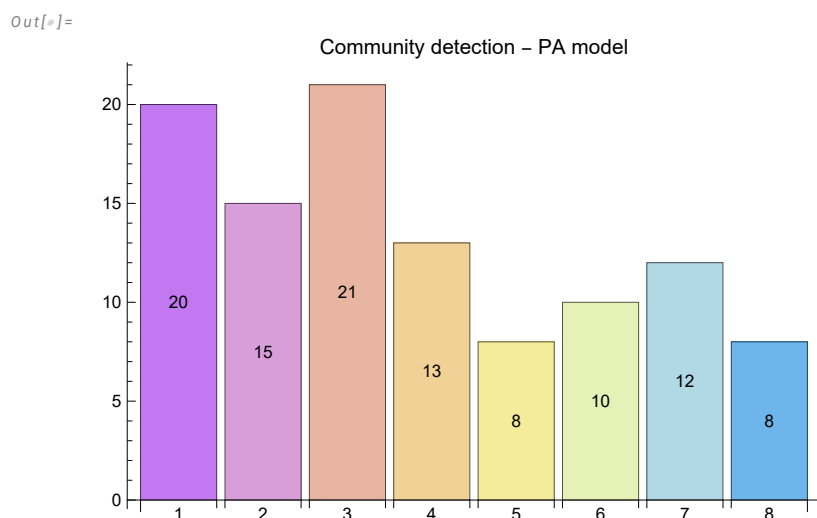
In[ ]:= (*Giant component*)
GiantComponentG = Length[First[ConnectedComponents[PaG]]];
Print["Giant component: ", GiantComponentG]
(*Average distance, to avoid ∞ in un-connected graph,
I took the largest connected components for that*)
AverageDistancePa = Mean[Flatten[
  GraphDistanceMatrix[Subgraph[PaG, First[ConnectedComponents[PaG]]]]] // N];
Print["Average distance: ", AverageDistancePa]

Giant component: 107
Average distance: 2.5866

(*Shift diagram, not work at all*)
(*sdPa=PlotShift[PaG,"PA"]*)

```

```
In[ ]:=
(*Community detection, for Throne model I plot the true communities by tribes*)
CommunityDetectionPa = CommunityDetectionIGM[PaG, "PA"]
```



## Discuss our finding

### For the graph DiseaseGeneNetwork network

for the discuss models,  $G(n, p)$  and  $G(n, m)$  models generate random graphs with varying edge configurations, while the block model and configuration model aim to simplify and analyze specific aspects of the original graph, such as community structure or degree distribution, respectively. The choice of model depends on the specific characteristics of the original graph that we want to preserve or explore.

The average distance and giant component are similarly present, and we can see that expected value (Degree) approximately the same for each model and the result is

$$N \cdot P = 1777 \cdot 0.0622465 \sim 110.612$$

```
In[ ]:= NP = 1777 * 0.0622465
```

```
Out[ ]:=
110.612
```

and for the Diameter for all model is the same again and the calculate  $D \sim \log(1777) = 3.249$ ,  $\log(110.612) = 2.043$ . We can see that is less than  $\log(107) = 4.672$  as we expected. In summary, the differences between the original network and the graph models arise due to the specific generation methods employed by each model. These differences are purposeful and serve different analytical or modeling objectives, such as studying random graphs, understanding community structures, or analyzing degree distributions. The choice of model depends on the research. The average distance is around  $\log(n)/\log(z) = (\log(1777)/\log(110.612)) \sim 1.59$ .

```
In[ ]:=
Log2[1777]
Log2[110.612]
```

```
Out[ ]:=
1.59002
```

## For the graph Throne network

$$N \cdot P = 107 \cdot 0.0622465 \sim 6.6$$

different analytical or modeling objectives, such as studying random graphs, understanding community structures, or analyzing degree distributions. The choice of model depends on the research.

The average distance is around  $\log(n)/\log(z) = \log(107)/\log(6.6) \sim 2.47624$

```
In[*]:= 
$$\frac{\text{Log2}[107]}{\text{Log2}[6.6]}$$

Out[*]= 2.47624
```