HW6

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Assignment 6-Core-Periphery

- Compute the Core - Periphery partition "Network of Thrones"
- with and without weights (nodes degree is the sum of weights)
- You need to
  - 1. Generate the power shift diagram (according to degree (weighted and unweighted)
  - 2. Generate the Core graph.What is the size of the core?Who is in the core?
  - 3. Generate the Periphery graph.What is the size of the core?
  - 4. Generate the matrix and show the symmetry point
- Compare the results to a shift diagram of random order of nodes and to a shift diagram with random configuration model of the same degree sequence
- Bonus:Do the same but with your favorite centrality measure

Load the network with and without weigths

```
In[∘]:= SetDirectory[NotebookDirectory[]];
        file = Rest[Import["stormofswords.csv"]];
        tribes = Import["tribes.csv"];
        nodes = Flatten[tribes⟦All, 1⟧];
        nodesTribe = Flatten[tribes⟦All, 2⟧];
        edges = #⟦1⟧ ⟷ #⟦2⟧ & /@ file⟦All, {1, 2}⟧;
        GraphTWight = Graph[nodes, edges, EdgeWeight → file⟦All, 3⟧];
        GraphT = Graph[nodes, edges];
```

functions

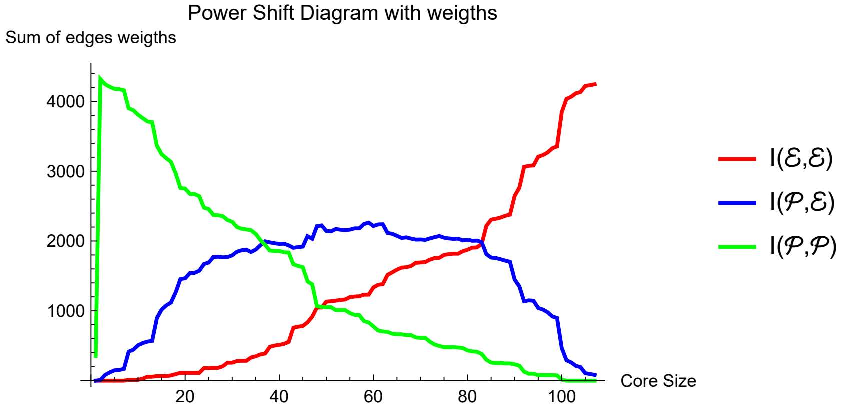
```
In[◦]:= Rich[G_, k_, deg_] := VertexList[G]⟦Ordering[deg, k, Greater]⟧
        SymmetryPointSize[G_, deg_]  := First@
          First@Position[Accumulate[deg], Select[Accumulate[deg], # ≥ Total[deg] / 2 &, 1]⟦1⟧]
        RenameGraphN[g_, sl_] :=
         Module[{G = g, VrDg = sl, VrLs, VrLst, SVrLst, NewName, edges, NewEdges, ed},
          VrLs = VertexList[G];
          VrLst = Table[{VrDg⟦i⟧, VrLs⟦i⟧}, {i, Length[VrDg]}];
          SVrLst = Sort[VrLst, #1⟦1⟧ > #2⟦1⟧ &];
          NewName = Table[SVrLst⟦i⟧⟦2⟧, {i, Length[VrDg]}];
          edges = EdgeList[G];
          If[DirectedGraphQ[G],
           NewEdges = Table[Position[NewName, edges⟦i⟧⟦1⟧]⟦1⟧⟦1⟧ →
               Position[NewName, edges⟦i⟧⟦2⟧]⟦1⟧⟦1⟧, {i, Length[edges]}],
           NewEdges = Table[Position[NewName, edges⟦i⟧⟦1⟧]⟦1⟧⟦1⟧ —
               Position[NewName, edges⟦i⟧⟦2⟧]⟦1⟧⟦1⟧, {i, Length[edges]}]];
          { NewName, Table[SVrLst⟦i⟧⟦1⟧, {i, Length[VrDg]}],
           Graph[Range[Length[VrLst]], NewEdges]}]

        SymmetryPoint[G_, M_] := Module[
          {s, EE = {}, EP = {}, PE = {}, PP = {}, ee = 0, ep = 0, pe = 0, pp = 0, tc, bc, tr, br},
          s = Length[VertexList[G]];
          ee = M⟦1⟧⟦1⟧;
          pe = Total[Total[Take[M, {2, s}, {1, 1}]]];
          ep = Total[Total[Take[M, {1, 1}, {2, s}]]];
          pp = Total[Total[Take[M, {2, s}, {2, s}]]];
          EE = {0, ee / 2};
          EP = {0, ep / 2};
          PE = {0, pe / 2};
          PP = {EdgeCount[G], pp / 2};
          For[i = 1, i < s - 1, i++;
           tc = Total[Total[Take[M, {1, i - 1}, {i, i}]]];
           bc = Total[Total[Take[M, {i + 1, s}, {i, i}]]];
           tr = Total[Total[Take[M, {i, i}, {1, i - 1}]]];
           br = Total[Total[Take[M, {i, i}, {i + 1, s}]]];
           ee = ee + tc + tr + M⟦i⟧⟦i⟧;
           ep = ep + br - tc;
           pe = pe + bc - tr;
           pp = pp - bc - br - M⟦i⟧⟦i⟧;;
           EE = Join[EE, {ee / 2}];
           EP = Join[EP, {ep / 2}];
           PE = Join[PE, {pe / 2}];
           PP = Join[PP, {pp / 2}];];
          {EE, EP + PE, PP}]
```

```
In[•]:= (*calculate for weigth graph the "degree" as the sum of weigths*)
        (*Compute the weighted adjacency matrix*)
        UnWight = AdjacencyMatrix[GraphT];
        WightT = Normal[WeightedAdjacencyMatrix[GraphTWight]];
        (*Compute the degree of each vertex using the weighted adjacency matrix*)
        degreesW  = Total /@ WightT;
        degrees = Total /@ UnWight;
```

```
In[•]:= namelistcoreW  = VertexList[GraphTWight];
        namelistcore  = VertexList[GraphT];
        resW = RenameGraphN[GraphTWight, degreesW];
        res = RenameGraphN[GraphT, VertexDegree[GraphT]];
        sortDegW = resW⟦2⟧;
        namelistSortW = resW⟦1⟧;
        DegreeOrderGraphW  = resW⟦3⟧;
        sortDeg = res⟦2⟧;
        namelistSort = res⟦1⟧;
        DegreeOrderGraph  = res⟦3⟧;
```

plot with weigths

```
In[•]:=
        ListLinePlot[SymmetryPoint[RenameGraphN[GraphTWight, degreesW]⟦3⟧, WightT],
         PlotLegends → {"I(ℰ,ℰ)", "I(𝒫,ℰ)", "I(𝒫,𝒫)"}, PlotRange → All,
         AxesLabel → {"Core Size", "Sum of edges weigths"},
         PlotLabel → "Power Shift Diagram with weigths",
         PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}}]
Out[•]=
```

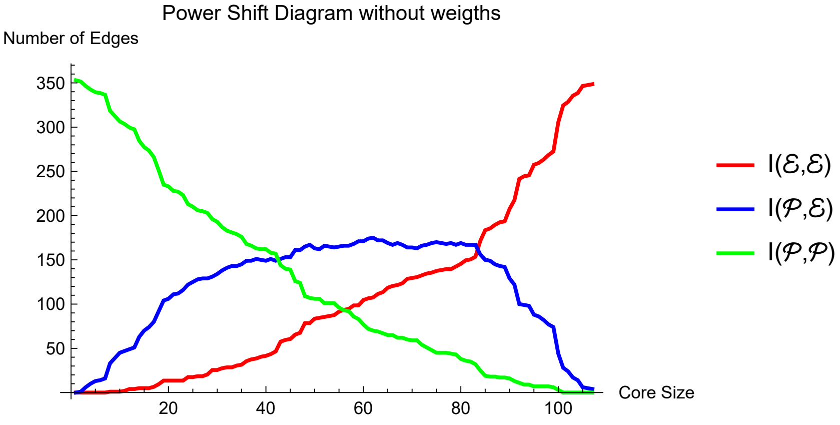


plot without weigths

In[∘]:=

```
ListLinePlot[SymmetryPoint[RenameGraphN[GraphT, VertexDegree[GraphT]][[3]], UnWight],
 PlotLegends → {"I(ℰ,ℰ)", "I(𝒫,ℰ)", "I(𝒫,𝒫)"},
 PlotRange → All, AxesLabel → {"Core Size", "Number of Edges"},
 PlotLabel → "Power Shift Diagram without weigths",
 PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}}]
```

Out[∘]=



Core graph

- Core graph whit Wight

In[∘]:=

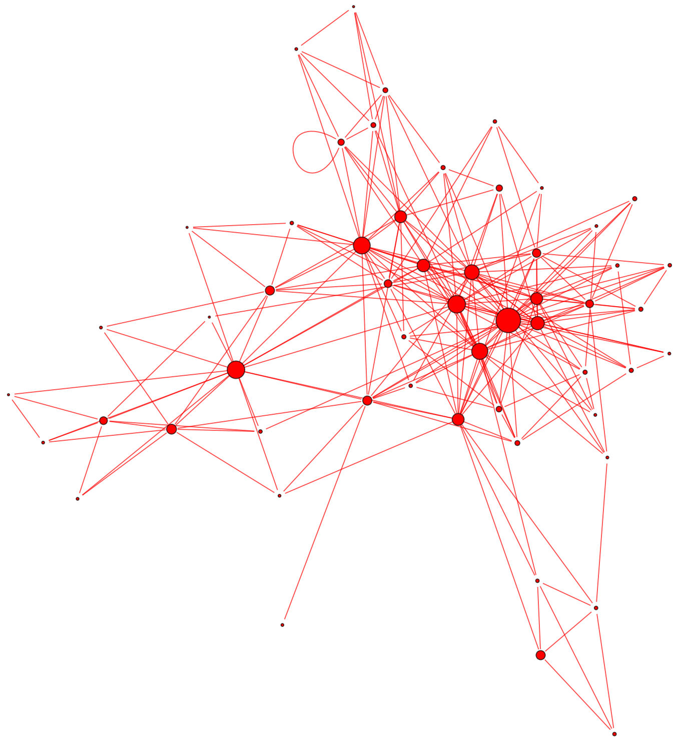
```
Rich[GraphTWight, SymmetryPointSize[DegreeOrderGraphW, Flatten[sortDegW]], degreesW]
```

Out[∘]=

```
{Tyrion, Jon, Sansa, Jaime, Bran, Robb, Samwell,
 Arya, Joffrey, Daenerys, Cersei, Tywin, Catelyn, Hodor, Mance}
```

before

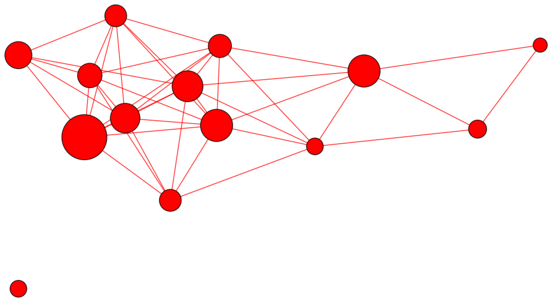
```
In[◦]:= CoreWbefore = Subgraph[GraphTWight,
          Rich[GraphTWight, SymmetryPointSize[DegreeOrderGraph, VertexDegree[GraphTWight]],
           VertexDegree[GraphTWight]], VertexLabels → Table[VertexList[GraphTWight][[i]] →
              Placed[VertexList[GraphTWight][[i]], Tooltip], {i, 1, VertexCount[GraphTWight]}],
          VertexSize → Thread[VertexList[GraphTWight] → Rescale[VertexDegree[GraphTWight]]],
          VertexStyle → Red, EdgeStyle → Red]
        EdgeCount[CoreWbefore]
        VertexCount[CoreWbefore]
Out[◦]=
```



```
Out[◦]=
        234
Out[◦]=
        55
```

After

```
In[•]:= CoreW = Subgraph[CoreWbefore, Rich[GraphTWight,
            SymmetryPointSize[DegreeOrderGraphW, Flatten[sortDegW]], degreesW]]
        Rich[GraphTWight, SymmetryPointSize[DegreeOrderGraphW, Flatten[sortDegW]], degreesW]
        EdgeCount[CoreW]
        VertexCount[CoreW]
Out[•]=
```



```
Out[•]=
        {Tyrion, Jon, Sansa, Jaime, Bran, Robb, Samwell,
         Arya, Joffrey, Daenerys, Cersei, Tywin, Catelyn, Hodor, Mance}
Out[•]=
        42
Out[•]=
        14
```

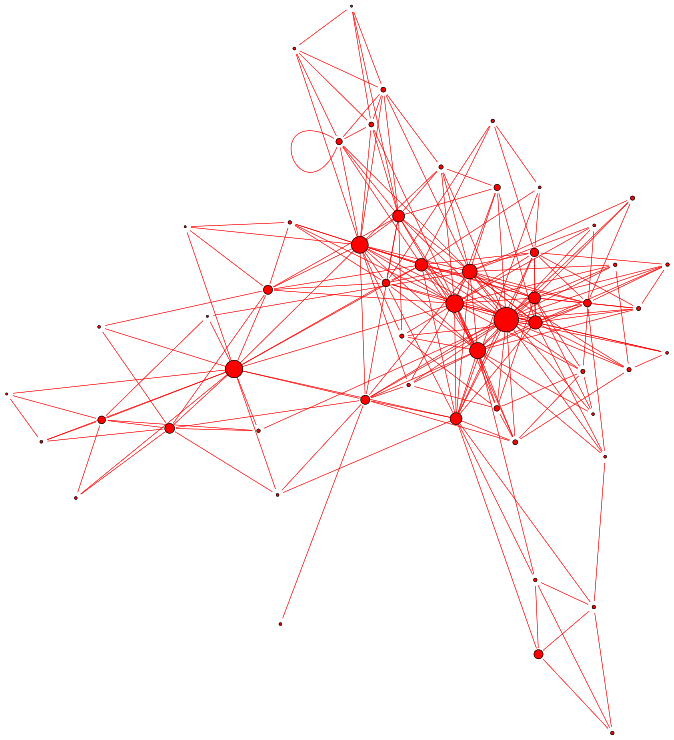
Core graph

- Core graph without Wight

```
In[•]:= Rich[GraphT, SymmetryPointSize[DegreeOrderGraph, Flatten[sortDeg]], degrees]
Out[•]=
        {Tyrion, Sansa, Jon, Robb, Jaime, Tywin, Cersei, Arya, Robert, Joffrey,
         Catelyn, Samwell, Stannis, Daenerys, Bran, Sandor, Mance, Gregor, Eddard}
```

before

```
In[∘]:= Corebefore = Subgraph[GraphT,
          Rich[GraphT, SymmetryPointSize[DegreeOrderGraph, VertexDegree[GraphT]],
           VertexDegree[GraphT]], VertexLabels → Table[VertexList[GraphTWight]⟦i⟧ →
              Placed[VertexList[GraphT]⟦i⟧, Tooltip], {i, 1, VertexCount[GraphT]}],
          VertexSize → Thread[VertexList[GraphTWight] → Rescale[VertexDegree[GraphT]]],
          VertexStyle → Red, EdgeStyle → Red]
        EdgeCount[Corebefore]
        VertexCount[Corebefore]
Out[∘]=
```

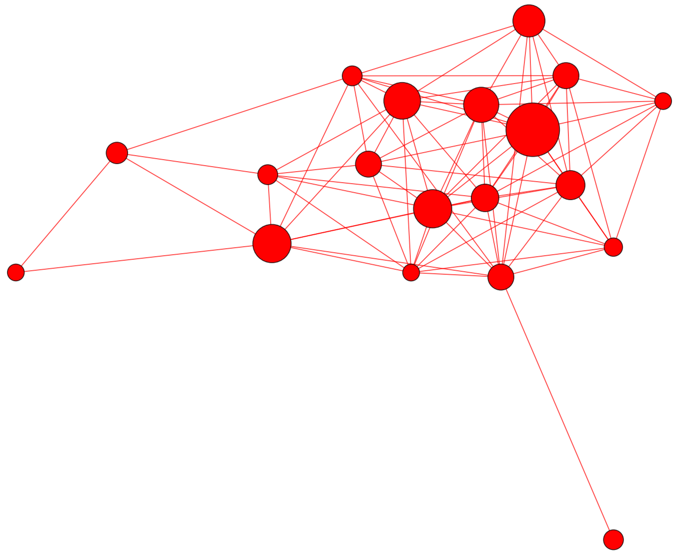


```
Out[∘]=
        234
Out[∘]=
        55
```

After

```
In[∘]:= Core = Subgraph[Corebefore,
          Rich[GraphT, SymmetryPointSize[DegreeOrderGraph, Flatten[sortDeg]], degrees]]
        Rich[GraphT, SymmetryPointSize[DegreeOrderGraph, Flatten[sortDeg]], degrees]
        EdgeCount[Core]
        VertexCount[Core]
```

Out[∘]=



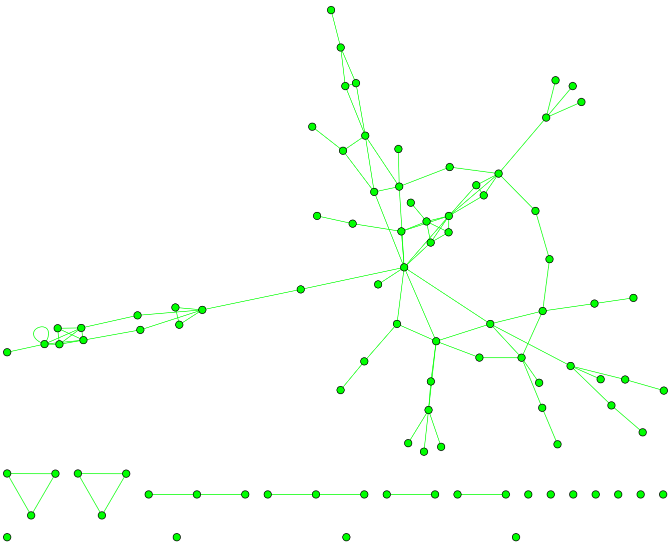
```
Out[∘]=
        {Tyrion, Sansa, Jon, Robb, Jaime, Tywin, Cersei, Arya, Robert, Joffrey,
         Catelyn, Samwell, Stannis, Daenerys, Bran, Sandor, Mance, Gregor, Eddard}
Out[∘]=
        82
Out[∘]=
        19
```

Periphery graph

- graph whit Wight

In[∘]:=
```
PeripW = Subgraph[GraphTWight, Complement[VertexList[GraphTWight], VertexList[CoreW]],
  VertexStyle → Green, EdgeStyle → Green]
VertexCount[PeripW]
EdgeCount[PeripW]
```

Out[∘]=



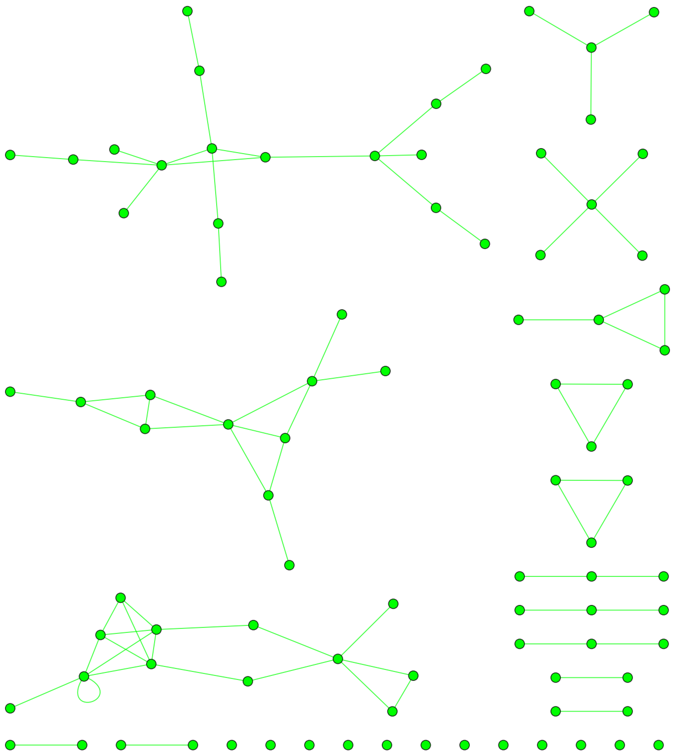
Out[∘]=
```
93
```

Out[∘]=
```
106
```

- Without weigths

In[∘]:=
```
Perip = Subgraph[GraphT, Complement[VertexList[GraphT], VertexList[Core]],
  VertexStyle → Green, EdgeStyle → Green]
VertexCount[Perip]
EdgeCount[Perip]
```

Out[∘]=



Out[∘]=

```
88
```

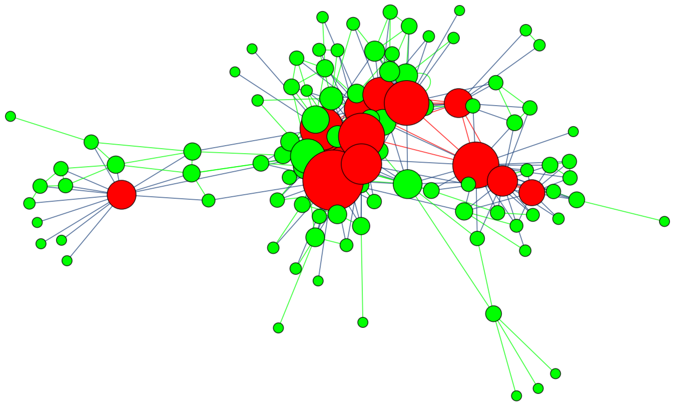
Out[∘]=

```
77
```

Putting it Together

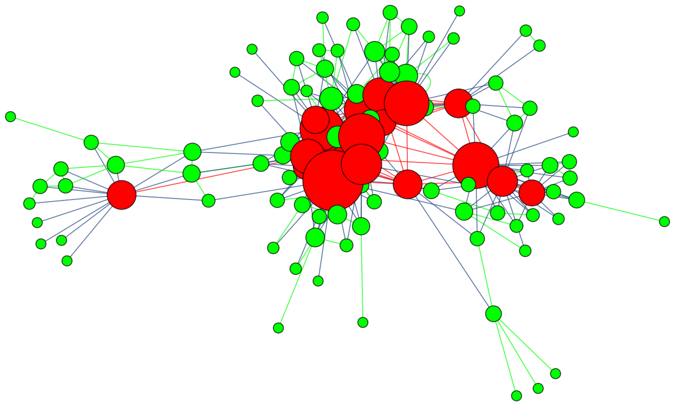
- With weigths

```
In[•]:= HighlightGraph[GraphTWight,
         {Style[VertexList[CoreW], Red], Style[VertexList[PeripW], Green],
          Style[EdgeList[CoreW], Red], Style[EdgeList[PeripW], Green]}, VertexSize →
          Thread[VertexList[GraphTWight] → 1 + 5 * Rescale[VertexDegree[GraphTWight]]]]
Out[•]=
```



- Without weigths

```
In[•]:= HighlightGraph[GraphT, {Style[VertexList[Core], Red], Style[VertexList[Perip], Green],
          Style[EdgeList[Core], Red], Style[EdgeList[Perip], Green]},
        VertexSize → Thread[VertexList[GraphT] → 1 + 5 * Rescale[VertexDegree[GraphT]]]]
Out[•]=
```

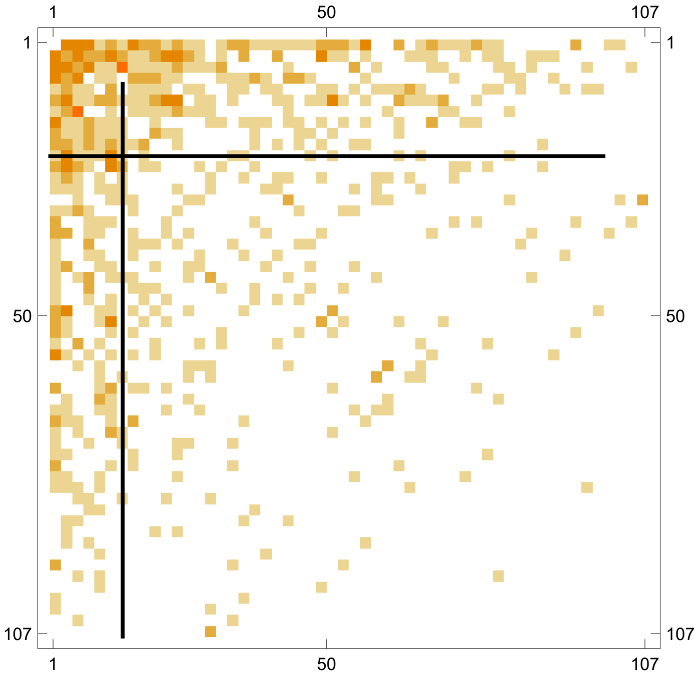


Matrix with symmetry point

- With weigths

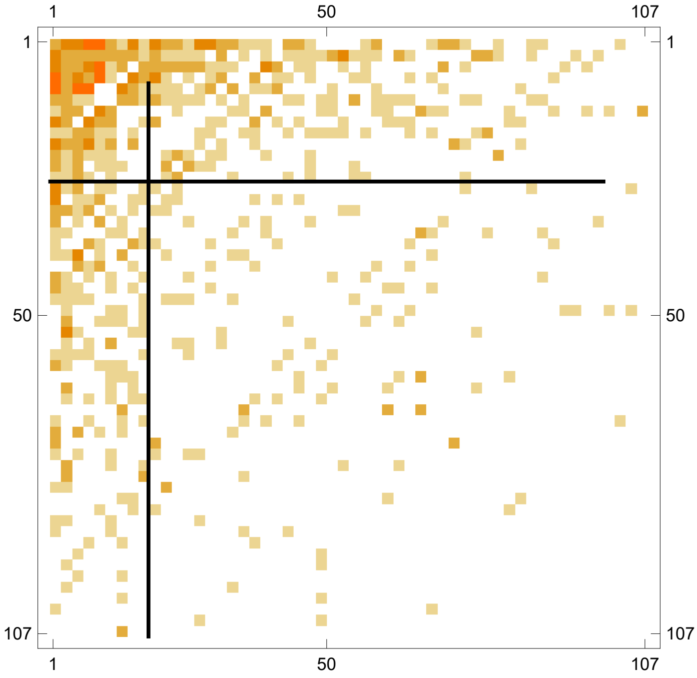
In[∘]:=
```
CoreRatioW = VertexCount[CoreW] / VertexCount[GraphTWight];
Show[MatrixPlot[AdjacencyMatrix[DegreeOrderGraphW]],
 Graphics[{{Thick, Line[{{CoreRatioW * 50, 0}, {CoreRatioW * 50, 50}}]},
   {Thick, Line[{{0, 50 - CoreRatioW * 50}, {50, 50 - CoreRatioW * 50}}]}}]]
```

Out[∘]=



In[∘]:=
```
CoreRatio = VertexCount[Core] / VertexCount[GraphT];
Show[MatrixPlot[AdjacencyMatrix[DegreeOrderGraph]],
 Graphics[{{Thick, Line[{{CoreRatio * 50, 0}, {CoreRatio * 50, 50}}]},
   {Thick, Line[{{0, 50 - CoreRatio * 50}, {50, 50 - CoreRatio * 50}}]}}]]
```

Out[∘]=

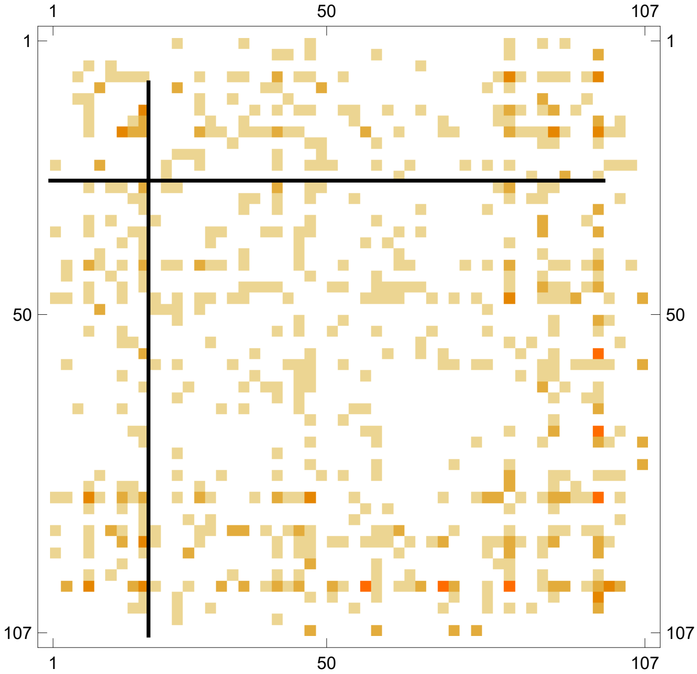


compare result to shift diagram of random order of nodes and to random configuration model

- Random order

```
In[∘]:= DegreeOutput = Map[VertexDegree[GraphT, #] &, RandomOrderGraph];
        resRand = RenameGraphN[GraphT, RandomOrderGraph, DegreeOutput];
        RandomOrderGraph = resRand⟦1⟧;
        Show[MatrixPlot[AdjacencyMatrix[RandomOrderGraph]],
         Graphics[{{Thick, Line[{{CoreRatio * 50, 0}, {CoreRatio * 50, 50}}]},
           {Thick, Line[{{0, 50 - CoreRatio * 50}, {50, 50 - CoreRatio * 50}}]}}]]
```

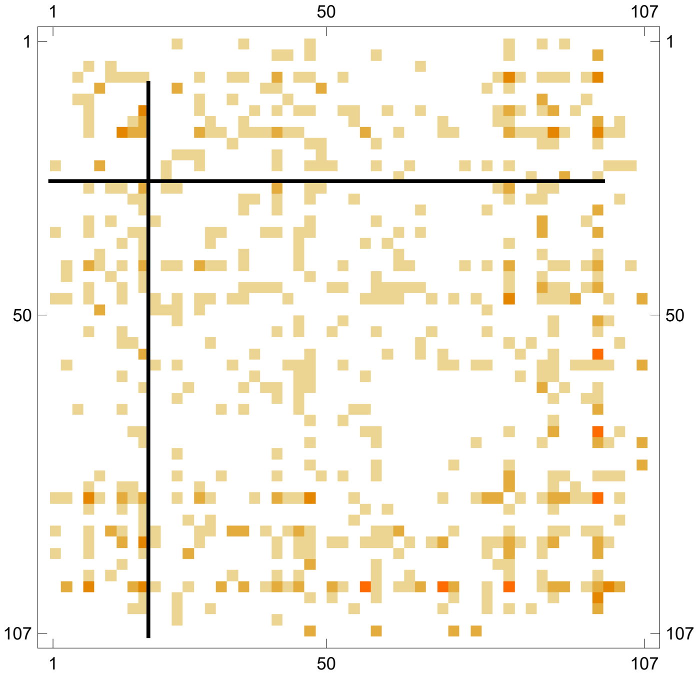
Out[∘]=



- configuration model

In[∘]:=
```
DegreeOutput = Map[VertexDegree[GraphT, #] &, RandomOrderGraph];
resRand = RenameGraphN[GraphT, RandomOrderGraph, DegreeOutput];
RandomOrderGraph = resRand[[1]];
Show[MatrixPlot[AdjacencyMatrix[RandomOrderGraph]],
 Graphics[{{Thick, Line[{{CoreRatio * 50, 0}, {CoreRatio * 50, 50}}]},
   {Thick, Line[{{0, 50 - CoreRatio * 50}, {50, 50 - CoreRatio * 50}}]}}]]
```

Out[∘]=


1
50
107
1
50
107
1
50
107
1
50
107