Multi-path Propagation
Theoretical Laboratory Session

WIRELESS COMMUNICATIONS 371-1-1903

Part 1 – General Theoretical Information

Path Loss

Path loss, or path attenuation, is the reduction in power density (attenuation) of an electromagnetic wave as it propagates through space. Path loss is a major component in the analysis and design of the link budget of a telecommunication system.

Path loss may be due to many effects, such as **free-space loss**, refraction, diffraction, reflection, aperture-medium coupling loss, and absorption. Path loss is also influenced by terrain contours, environment (urban or rural, vegetation and foliage), propagation medium (dry or moist air), the distance between the transmitter and the receiver, and the height and location of antennas.

Free Space Path Loss

Free-space path loss (FSPL) is the attenuation of radio energy between the feedpoints of two isotropic antennas that results from the combination of the receiving antenna's capture area and the obstacle-free, line-of-sight (LOS) path through free space (usually air).

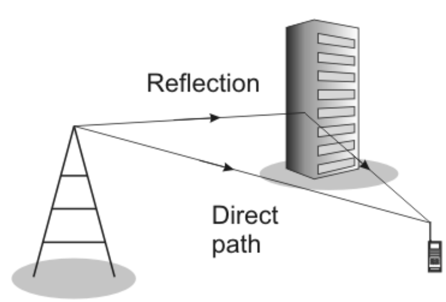
The free-space path loss (FSPL) formula derives from the Friis transmission formula and states:

$$\frac{P_r}{P_t} = D_t D_r \left(\frac{\lambda}{4\pi d}\right)^2$$

- λ is the signal wavelength. Substitute $\lambda = \frac{c}{f}$ for to calculate from frequency.
- d is the distance between the antennas.
- D_t is the isotropic directivity of the transmitting antenna in the direction of the receiving antenna.
- D_r is the isotropic directivity of the receiving antenna in the direction of the transmitting antenna.

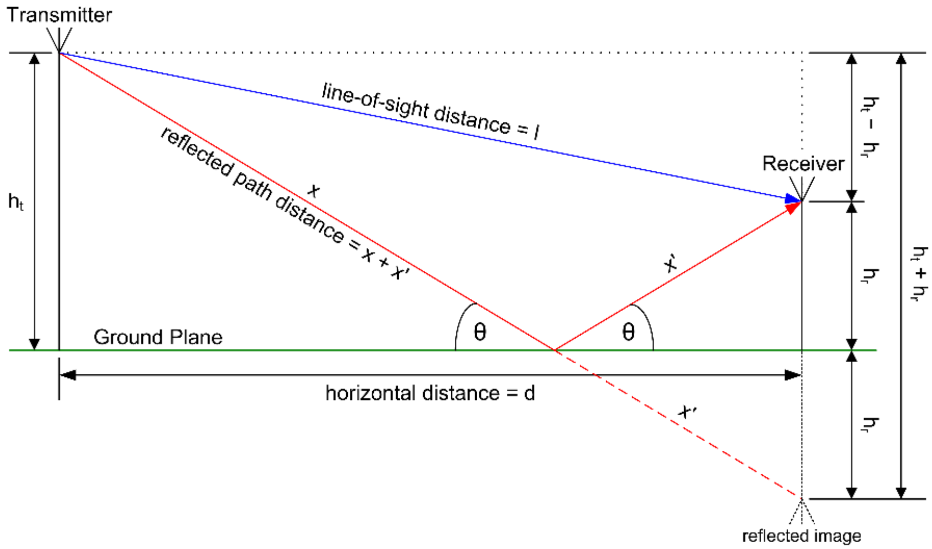
Multipath

Multipath results from the fact that we don't have a single path for the signal to travel from the transmitter to the receiver in most communication environments. Any time there is an object that is reflective to the signal, a new path can be established between the two nodes. Surfaces like buildings, signs, trees, people, cats, etc., can all produce signal reflections. Each of these reflective paths will show up at the receiver at different times based on the length of the path. Summing these together at the receiver causes distortions, both constructively and destructively.



Two Ray Model

The Two-Rays Ground Reflected Model is a radio propagation model that predicts path losses between transmitting and receiving antennas when in LOS. Generally, the two antennae each have different heights. The received signal has two components, the LOS and multipath components, formed predominantly by a single ground-reflected wave.



From the geometry of the figure above, the LOS distance is:

$$l = \sqrt{(h_t - h_r)^2 + d^2}$$

and the ground reflected ray distance:

$$x + x' = \sqrt{(h_t + h_r)^2 + d^2}$$

From these distances, we can obtain the LOS propagation time delay:

$$t_{LOS} = \frac{l}{C} = \frac{\sqrt{(h_t - h_r)^2 + d^2}}{C}$$

and the ground reflected ray propagation time delay:

$$t_{GR} = \frac{x + x'}{C} = \frac{\sqrt{(h_t + h_r)^2 + d^2}}{C}$$

Resources

https://en.wikipedia.org/wiki/Radio_propagation_model
https://en.wikipedia.org/wiki/Free-space_path_loss
https://en.wikipedia.org/wiki/Two-ray_ground-reflection_model
Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

Theoretical Questions

- Under a free space path loss model, find the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas ($G_l = 1$) and a carrier frequency $f = 5\ GHz$, assuming a distance $d = 10m$. Repeat for $d = 100m$.
- Consider an indoor wireless LAN with $f_c = 900\ MHz$, cells of radius $100\ m$, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of $10\ \mu W$. How does this change if the system frequency is $5\ GHz$?

Part 2 – Two-Ray Simulation

We will construct an example of a communication system using GNU Radio, adding more reflections in each part to create a multipath environment.

1. Change the samp_rate to 512e9.
2. Add the "Import" component and enter "import numpy" in the Import field.
3. Add another "Import" component and enter "import scipy.constants" in the Import field.

   (If not installed, write in terminal: sudo apt-get install python-scipy)

4. Add "QT GUI Range" for the frequency – from 1Hz to 6GHz in 100 Hz steps, with the default value of 900MHz.
5. Add another "QT GUI Range" for the distance – from 10m to 15km in 10m steps. The default value is 10m.
6. Add 4 "Variable" components:
   - ant_gain_r – 1
   - ant_gain_t – 1
   - ant_hight_t – 10m
   - ant_hight_r – 10m
7. Add "Variable" wavelength and enter the value scipy.constants.c/frequency
8. Add "Variable" LOS_distance and enter the value

   numpy.sqrt(numpy.square(ant_hight_t-ant_hight_r)+numpy.square(distance))

9. Add "Variable" GR_distance and enter the value

   numpy.sqrt(numpy.square(ant_hight_t+ant_hight_r)+numpy.square(distance))

10. Add "Variable" LOS_delay and enter the value LOS_distance/scipy.constants.c
11. Add "Variable" GR_delay and enter the value GR_distance/scipy.constants.c

12. Add a float Sine "Signal source" with an amplitude of 1V and a Frequency of frequency
13. Add 2 float "Delay" components:
    - Delay of int(LOS_delay*samp_rate)
    - Delay of int(GR_delay*samp_rate)

    Connect both input ports to the signal source's output port.

14. Add 2 float “Multiply Const” components:
    - Multiply by a constant of ant_gain_t*ant_gain_r*numpy.square(wavelength/(4*(numpy.pi)*LOS_distance))
    - Multiply by a constant of ant_gain_t*ant_gain_r*numpy.square(wavelength/(4*(numpy.pi)*GR_distance))

    Connect both input ports to the Delay output ports, respectively.

15. Add a float “Add” component, and connect both of the “Multiply Const” components’ output ports to its input ports.
16. Add a float “QT GUI Time Sink” and enter the following preferences:

    General tab:
    - Type: float
    - Y-Axis Label: Power (W)
    - Grid = Yes
    - Autoscale = Yes
    - Number of inputs: 3

    Trigger tab:
    - Trigger mode = Auto

    Config tab:
    - Control Panel = Yes
    - Line 1 Label: LOS
    - Line 2 Label: Ground Reflected
    - Line 3 Label: Combined

    Connect the In0 input port to the LOS Multiply Const output port.

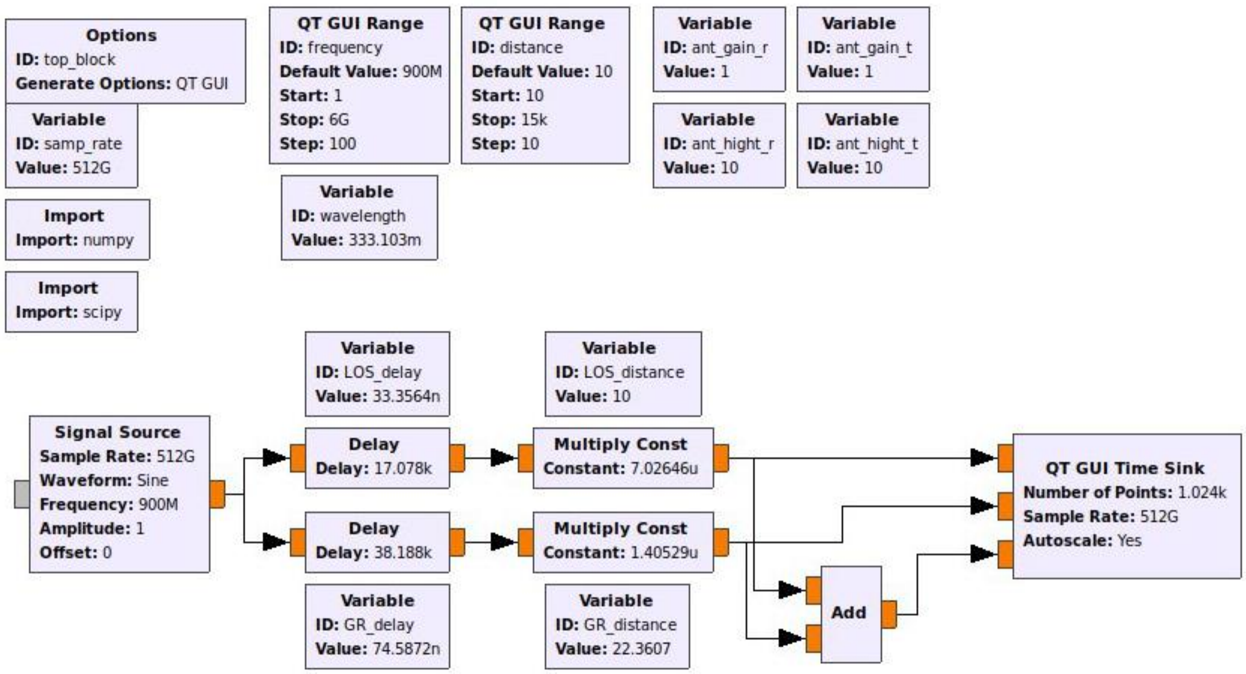
    Connect the In1 input port to the GR Multiply Const output port.

    Connect the In2 input port to the Add output port.

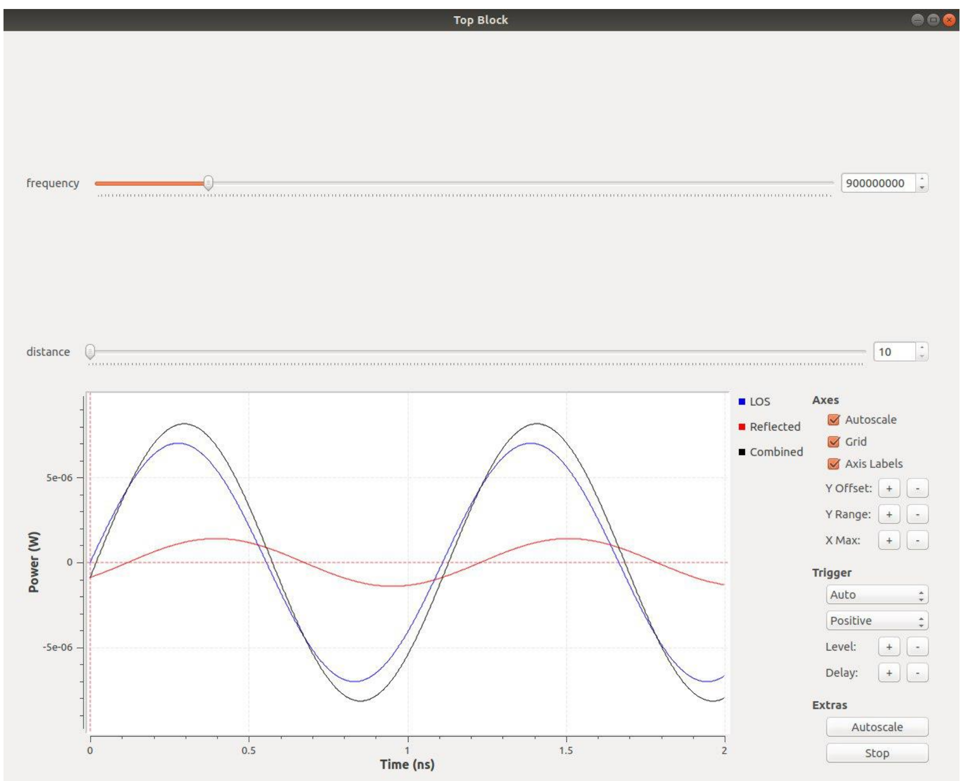
17. Save and run your code.

Save your code and add it to your submission!

If you performed all the stages correctly, your system's code should look like this:



These are the signals that you should see in your oscilloscopes:

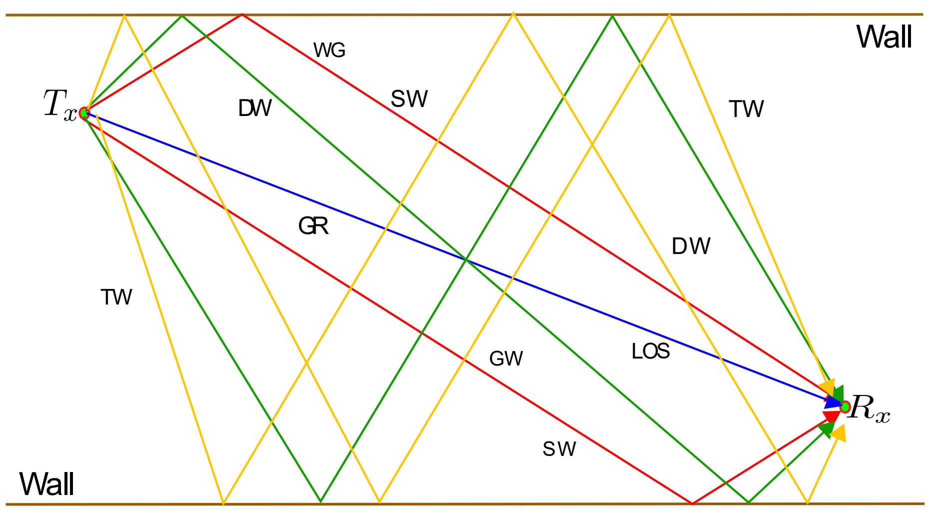


Theoretical Questions

3. Answer the following:
    a. Change the frequency while the distance is 10m. What is the power difference you witness for the combined signal? (min and max)
    b. Set the frequency (as you wish) while changing the distance. At which distance does the path loss affect the signals most? (both high and low) Why is that? Include snapshots of your results.
4. Find a frequency and a distance at which there is a complete constructive interference and a frequency and distance at which there is a complete destructive interference. Show simulation proof of the phenomena and explain why it accrues.
5. For $\Gamma = -1$ (complete reflection), derive a mathematical expression for the locations in which the signal nulls at the receiver at distance d.
6. For a two-path propagation model with transmitter-receiver separation $d = 80m$, $h_t = 10m$, and $h_r = 1m$, $f = 900MHz$ find the delay spread between the two signals (Show both mathematically and in simulation).
   * The delay spread is the time difference between the arrival of the fastest and slowed signals.

Part 3 – Ten-Ray Simulation

The ten-ray model is applied to transmissions in urban areas, where we typically add four more rays to the two-ray model. This incorporates paths from one to three reflections: specifically, there is the LOS, GR (ground reflected), SW (single-wall reflected), DW (double-wall reflected), TW (triple-wall reflected), WG (wall-ground reflected) and GW (ground-wall reflected paths) where each one of the paths bounces on both sides of the wall.



Experimentally, it has been demonstrated that the ten-ray model simulates or can represent the propagation of signals through a dielectric canyon. The rays traveling from a transmitter to a receiver bounce many times.

Let's expand our simulation of the two-ray model by adding more rays.

***Save your code as a new project to not overwrite the two-ray simulation.**

1. Expand the number of inputs in your "Add" component and the "QT GUI Time Sink" to 10.
2. In the ten-ray model, we have 2 walls, so set 4 "Variable" blocks: distance from wall 1 to the transmitting antenna and the receiving and distance from wall 2 to the transmitting antenna and the receiving. Set them as you wish.
3. We need to calculate each ray's distance. WARNING: The calculations are heavily detailed for your convenience, and the "final product" is given ***for each ray***; Make sure you understand the calculations before putting them into the simulation. Putting them blindly is unrecommended. Set a "Variable" for the distance for each of the following rays:
   - LOS

$$\sqrt{d^2 + (h_t - h_r)^2}$$

$$\mathrm{numpy.sqrt(numpy.square(ant\_hight\_r - ant\_hight\_t) + numpy.square(distance))}$$

   - GR

$$\sqrt{d^2 + (h_t + h_r)^2}$$

$$\mathrm{numpy.sqrt(numpy.square(ant\_hight\_r + ant\_hight\_t) + numpy.square(distance))}$$

   - SW (x2)
     it is necessary to find the angle between the direct distance d and the distance of LOS, $\sqrt{d^2 + (h_t - h_r)^2}$.

$$\cos\theta = \frac{d}{\sqrt{d^2 + (h_t - h_r)^2}}$$

Viewing the model with a side view, it is necessary to find a flat distance between the transmitter and receiver called d'.

$$d' = \sqrt{\mathrm{d}^2 - (\mathrm{w}_{r2} - \mathrm{w}_{t2})^2}$$

Now we deduce the remaining height of the wall from the height of the receiver called z by the similarity of triangles:

$$\frac{z}{a} = \frac{h_t}{d'} = \frac{h_t}{\sqrt{\mathrm{d}^2 - (\mathrm{w}_{r2} - \mathrm{w}_{t2})^2}}$$

$$z = \frac{h_t a}{d'}$$

By likeness of triangles, we can deduce the distance from where the ray collides with the wall until the perpendicular of the receiver called a, getting:

$$\frac{a'}{d} = \frac{w_{r2}}{w_{t2} + w_{r2}}$$

$$a = \frac{d' w_{r2}}{w_{t2} + w_{r2}}$$

The third ray is defined like in the two-ray model:

$$\frac{\sqrt{(h_t - h_r - z)^2 + (d' - a)^2} + \sqrt{z^2 + a^2}}{\cos\theta}$$

$$\mathrm{(numpy.sqrt(numpy.square(ant\_hight\_r - ant\_hight\_t - z) + numpy.square(d - a))}$$
$$\mathrm{+ numpy.sqrt(numpy.square(z) + numpy.square(a)))/cos\_theta}$$

Two rays collide once on the wall, so finding the fifth ray is the same as the third.

- WG (x2)

  It is possible to calculate this ray similarly to the SW by taking a side-view:

$$\frac{\sqrt{(h_t - h_r - z)^2 + (d' - a)^2} + \sqrt{(2h_r + z)^2 \ + a^2}}{\cos\theta}$$

$$((\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{ant\_hight\_r} - \mathrm{ant\_hight\_t} - \mathrm{z}) + \mathrm{numpy.square}(\mathrm{d} - \mathrm{a}))) + \mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{z} + 2 * \mathrm{ant\_hight\_r}) + \mathrm{numpy.square}(\mathrm{a})))/\mathrm{cos\_theta}$$

  Two rays collide once on the wall and then once on the ground, so finding the sixth ray is the same as the fourth.

- DW (x2)

  To model the rays that collide with the wall twice, we use the Pythagoras theorem on the direct distance d and the sum of the distances between the receiver to each wall with double the distance of the transmitter to the wall w_{t2}. We divide the result by the angle formed between the direct distance and the reflected ray.

$$\frac{\sqrt{d^2 + (w_{r2} + 2w_{t2} + w_{r1})^2}}{\cos\theta}$$

$$(\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{distance}) + \mathrm{numpy.square}(\mathrm{wall\_2\_distance\_r} + 2 * \mathrm{wall\_2\_distance\_t} + \mathrm{wall\_1\_distance\_r})))/\mathrm{cos\_theta}$$

  Two rays collide twice on the wall, so finding the ninth ray is the same as the seventh.

- TW (x2)

  The eighth ray is calculated using a series of auxiliary variables; each is connected to the distances and heights found by the likeness of different triangles.

  First, we take the flat distance between the wall (of the second "shock") and the receiver:

$$x = \frac{d' w_{r1}}{w_{r2} + w_{r1}}$$

  Similarly, we find the flat distance between the transmitter and the wall (the first "shock"):

$$y = \frac{d' w_{t2}}{w_{r2} + w_{r1}}$$

  Finding the distance between the height of the wall of the second shock with respect to the first shock, is obtain:

$$z_1 = \frac{h_t\big(d' - (x + y)\big)}{d'}$$

  Deducing also the distance between the height of the wall of the second shock with respect to the receiver:

$$z_2 = \frac{h_t x}{d'}$$

  Calculating the height of the wall where occurs the first hit:

$$h_{p1} = h_r + z_1 + z_2$$

  Calculating the height of the wall where occurs the second shock:

$$h_{p2} = h_r + z_2$$

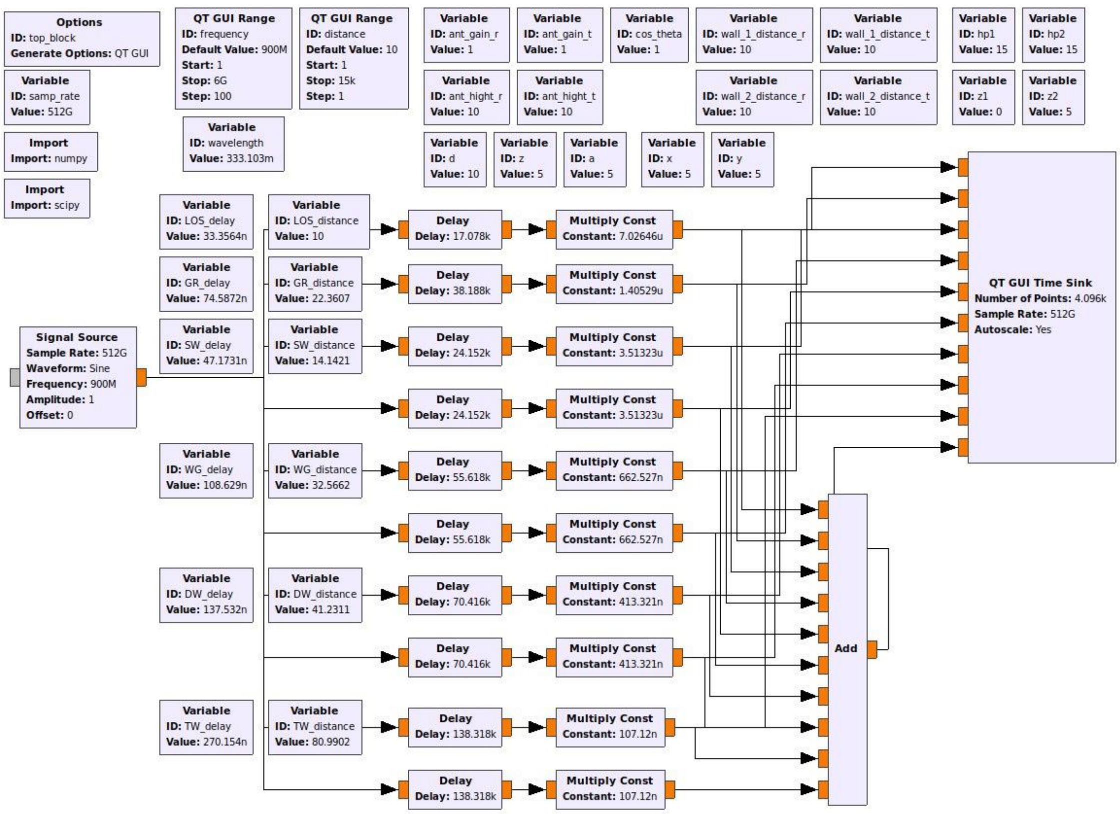
With these parameters is calculate the equation for the eighth ray:

$$\frac{\sqrt{y^2 + \left(h_t + h_{p1}\right)^2} + \sqrt{\left(d' - (x + y)\right)^2 + \left(h_{p1} + h_{p2}\right)^2} + \sqrt{x^2 + \left(h_r + h_{p2}\right)^2}}{\cos\theta}$$

$$\begin{aligned}(\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{y}) + \mathrm{numpy.square}(\mathrm{ant\_hight\_t} + \mathrm{hp1}))) \\ + (\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{d} - (\mathrm{x} + \mathrm{y})) + \mathrm{numpy.square}(\mathrm{hp2} + \mathrm{hp1}))) \\ + (\mathrm{numpy.sqrt}(\mathrm{numpy.square}(\mathrm{x}) + \mathrm{numpy.square}(\mathrm{ant\_hight\_r} \\ + \mathrm{hp2})))/\mathrm{cos\_theta}\end{aligned}$$

Due to its reflected shape, the tenth ray is the same as the eighth ray.

If you performed all the stages correctly, your system's code should look like this:



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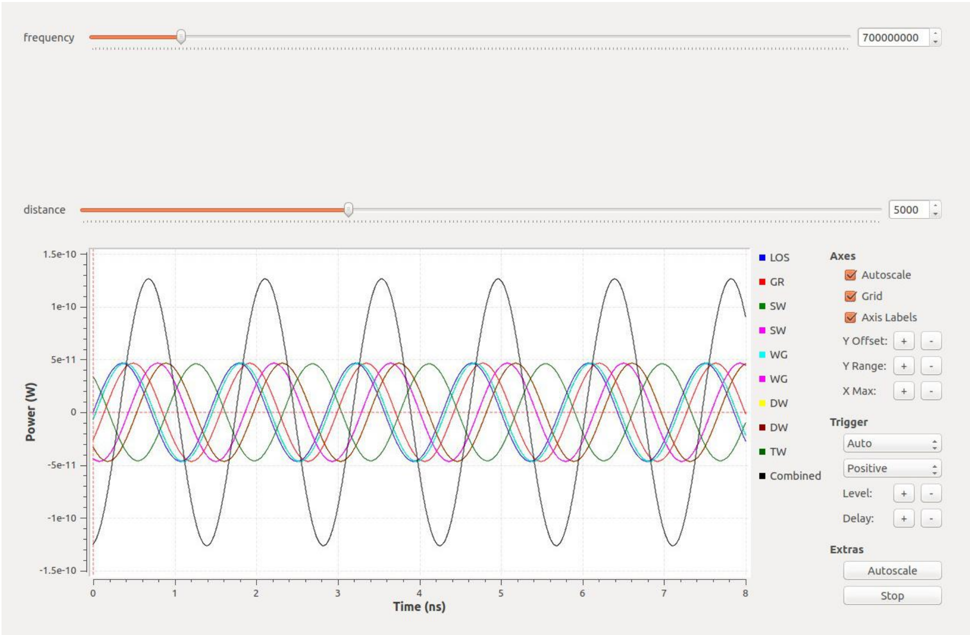


Save your code and add it to your submission!

Resources

https://en.wikipedia.org/wiki/Ten_rays_model

Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

Theoretical Questions

7. Change the frequency and the distance. Can you locate a place of complete destructive interference (signal null)? Why is it more difficult to find than with the two-ray model simulation? Explain and show proof from your simulation.
8. For the 10-ray model, assume the transmitter and receiver are in the middle of a street of width $50m$ and are at the same height. The transmitter-receiver separation is $500m$. Find the delay spread for this model. Find the power of the strongest and the weakest ray to arrive at the receiver relative to the transmitter power P.

Part 4 – Power Delay Profile

The received signal $r(t)$ is the sum of the LOS path and all resolvable multipath components. Writing them explicitly:

$$r(t) = \Re\left\{\sum_{n=0}^{N(t)} \alpha_n(t)u\big(t-\tau_n(t)\big)e^{j(2\pi f_c(t-\tau_n(t))+\phi D_n}\right\}$$

$n = 0$ corresponds to the LOS. The unknowns in this expression are the number of resolvable multipath components ($N(t)$), their path lengths ($r_n(t)$, which are incorporated in the delays $\tau_n(t) = r_n(t)/c$), Doppler phase shifts ($\phi D_n(t)$, details in the future), and amplitudes ($\alpha_n(t)$).

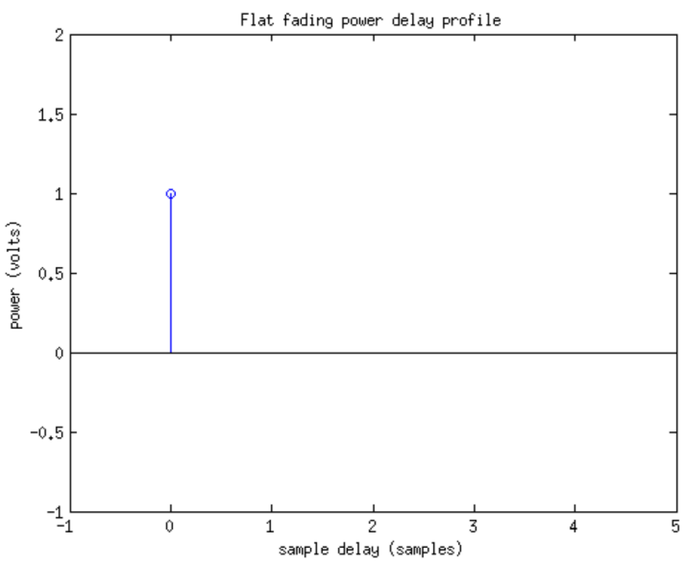
Since the parameters $\alpha_n(t)$, $r_n(t)$, and $\phi D_n(t)$ associated with each resolvable multipath component change over time, they are characterized as both stationary and ergodic *random processes*. Thus, the received signal is also a stationary and ergodic random process.

$r(t)$ is obtained by *convolving* the baseband input signal $u(t)$ with the equivalent lowpass time-varying channel impulse response $c(\tau, t)$ of the channel. If the channel is time-invariant then the time-varying parameters in $c(\tau, t)$ become constant, and $c(\tau, t) = c(\tau)$ is just a function of τ:

$$c(\tau) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n}\delta(\tau-\tau_n)$$

For stationary channels, the response to an impulse at time t_1 is just a shifted version of its response to an impulse at time t_2, $t_1 \neq t_2$. The ***Power-Delay Profile*** (PDP) is a discrete representation of the channel, $c(\tau, t)$.

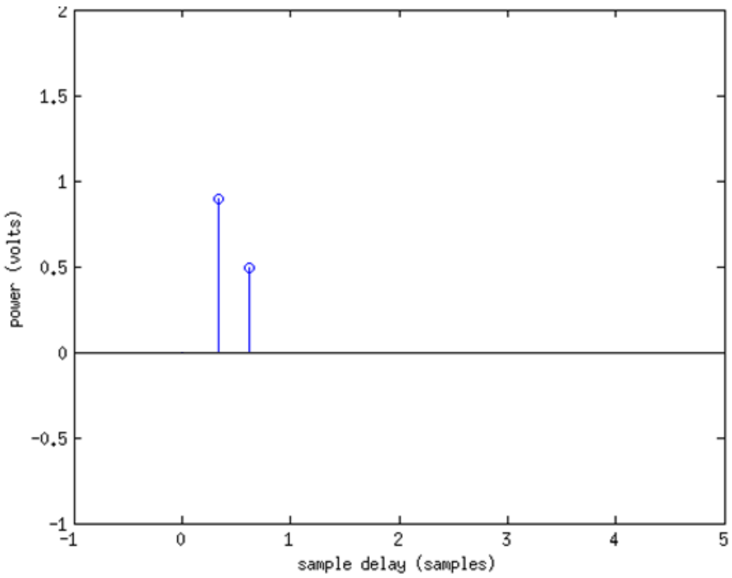
Observe 2 examples: A fadeless LOS (i.e., the signal is not attenuated or delayed) channel. In this case, $c(\tau) = \delta(\tau)$. I.e., $N = 1$, $\alpha_0 = 1$, $\phi_0 = 0$, $\tau_0 = 0$.:



For a two-ray model, we get the following:

$$c(\tau) = \alpha_0 e^{-j\phi_0}\delta(\tau - \tau_0) + \alpha_1 e^{-j\phi_1}\delta(\tau - \tau_1)$$

Where $N = 2$, $\alpha_0 = \left(\frac{\lambda\sqrt{G_l}}{4\pi l}\right)^2$, $\phi_0 = \frac{2\pi l}{\lambda}$, $\tau_0 = \frac{l}{c}$, $\alpha_1 = \left(\frac{\lambda\sqrt{RG_l}}{4\pi(x+x')}\right)^2$, $\phi_1 = \frac{2\pi(x+x')}{\lambda}$, $\tau_1 = \frac{x+x'}{c}$



Remark for future lectures: In statistical multi-path propagation, the statistical characterization of $c(\tau, t)$ is determined by its *autocorrelation function*[1]: $A_c(\tau, \Delta t)$. When stationary, the autocorrelation function is not a function of Δt, and it is acceptable to replace $\Delta t = 0$ and write $A_c(\tau) \triangleq A_c(\tau, 0)$.

[1] assuming wide-sense stationarity and uncorrelated scattering

Theoretical Questions

9. Consider a two-path channel with impulse response $h(t) = \alpha_1\delta(\tau) + \alpha_2\delta(\tau - 0.05\mu sec)$. Find the distance separating the transmitter and receiver, α_1 and α_2, assuming free space path loss on each path with a reflection coefficient of -1. Assume the transmitter and receiver are located 10 meters above the ground, and the carrier frequency is 900MHz.
10. Consider a wireless LAN operating near two conveyor belts in a factory. The transmitter and receiver have a LOS path with gain α_0, phase ϕ_0 and delay τ_0. Every T_0 seconds a metal item comes down the first conveyor belt, creating an additional reflected signal path with gain α_1, phase ϕ_1 and delay τ_1. Every $2T_0$ seconds another metal item comes down the second conveyor belt, creating an additional reflected signal with gain α_2, phase ϕ_2 and delay τ_2. Each object leaves the system after $2T_0$ seconds. Notice the periodical nature of the system. Find the time-varying impulse response $c(\tau, t)$ of this channel.
11. Calculate and sketch a PDP for the Ten-Ray model, N=10 (all rays amplitudes, arrival times and phases should be relative to the original signal). Explain your work. (You can use your simulation to confirm your sketching)
12. **Bonus (3pt):** Assume $N \to \infty$. How would the PDP look like? What is the distribution? Explain.

Resources

Wireless communications, Andrea Goldsmith, Stanford University, California, 2005, 9780511841224

https://www.site.uottawa.ca/~sloyka/elg4179/Lec_5_ELG4179.pdf

Rappaport Wireless Communications Principles And Practice 2Nd Edition, Prentice Hall, 2002 Theodore S. Rappaport, 978-0130422323

Good luck!