HW1, CSE 569 Fall 2019

Due 11:59pm Sep 21, 2019

Maximum score: 70

Q.1) (**5 pt**) Given that $\min[a, b] \leq \sqrt{ab}$, show that the error rate for a two-category Bayes classifier must satisfy,

$$P(error) \leq \sqrt{P(\omega_1)P(\omega_2)}\ \rho \leq \rho/2,$$

where, ρ is the *Bhattacharyya coefficient* $\rho = \int \sqrt{p(\boldsymbol{x}|\omega_1)p(\boldsymbol{x}|\omega_2)}d\boldsymbol{x}$

Q.2) (**5 pt**) Consider a two-category two dimensional classification problem with,

$$p(\boldsymbol{x}|\omega_1) \sim \mathcal{N}([0, 0]^\top, I),\ p(\boldsymbol{x}|\omega_2) \sim \mathcal{N}([1, -1]^\top, I),\ \text{and}\ P(\omega_1) = P(\omega_2) = \frac{1}{2},$$

where I is the identity matrix.

(a) Calculate the Bayes decision boundary.

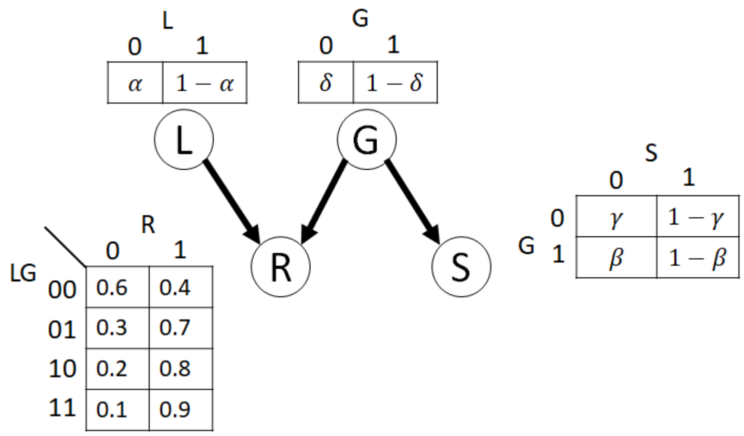
(b) Calculate the Bhattacharyya error bound.

Q.3) (**10 pt**) Consider a two-category one dimensional classification problem with $p(x|\omega_i) \sim \mathcal{N}(\mu_i, \sigma^2)$ with prior probabilities $P(\omega_1) = P(\omega_2) = \frac{1}{2}$. Show that the minimum probability of error is given by,

$$P(error) = \frac{1}{\sqrt{2\pi}} \int_a^\infty \exp(-u^2/2) du,$$

where, $a = |\mu_2 - \mu_1|/2\sigma$.

Q.4) (**10 pt**) Consider the problem with 4 random variables: G = 'Gray', L = 'London', R = 'Rain' and S = 'Sad'. They are modeled by the Bayes Net below. Write down the expression for $P(S = 1|L = 1)$ and $P(S = 1|L = 0)$ in terms of α, β, γ and δ.

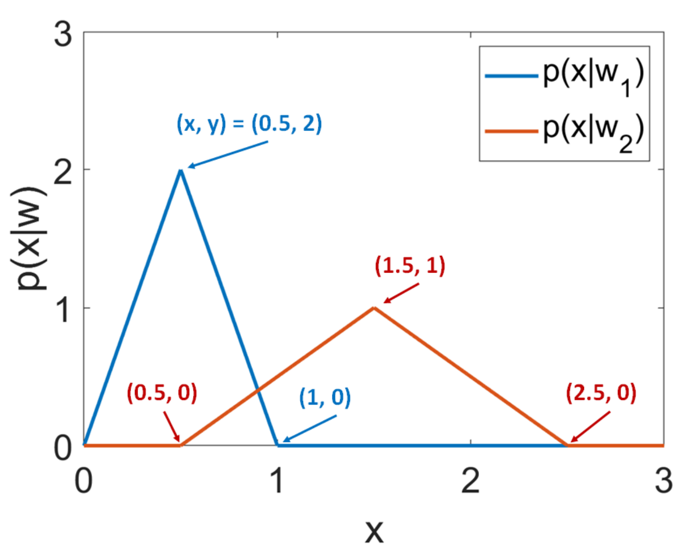


Q.5) (**30 pt**) Consider a two-category one dimensional classification problem with $P(\omega_1) = P(\omega_2) = \frac{1}{2}$ and triangular conditional densities $p(x|\omega_1)$ and $p(x|\omega_2)$ depicted in the below figure.

(a) Estimate the Bayes decision boundary and the Bayes error

(b) Estimate the Minimax decision boundary and the Minimax error

(c) If the maximum acceptable error rate for classifying a pattern that is ω_2 as ω_1 is 0.03, what is the optimal decision boundary under this constraint. If the maximum acceptable error rate is instead 0.05, what is the optimal decision boundary.



Q.6) (**10 pt**) Illustrate that the aggregation of multiple random variables yields a normal distribution by sampling from a dice which takes random values in $x \in \{1, 2, 3, 4, 5, 6\}$. You can assume uniform probabilities or random probabilities for the dice. Follow the below steps to demonstrate the Central Limit Theorem. Let $N = 1000$ and do the experiment for sample size $n = \{2, 5, 10\}$.

Algorithm 1 Central Limit Theorem Illustration

for $s := 1$ to N **do**
    Sample n random values from the dice distribution
    Estimate the mean μ_s of the sample of n values
end for
Estimate the mean μ_x and the variance σ_x^2 of all the $\{\mu_s\}_{s=1}^N$

Estimate the expected value $\mu := \mathbb{E}(x)$ and the variance $\sigma^2 := \mathbb{E}(x^2)$ of the population (random variable x). Empirically demonstrate that $\mu_x \approx \mu$ and $\sigma_x^2 \approx \sigma^2/n$ with increasing values of n. For each n, plot the histogram for $\{\mu_s\}_{s=1}^N$ and the normal distribution $\mathcal{N}(\mu_x, \sigma_x^2/n)$ in the same figure (overlay them in order to compare) - 3 figures. Your answer should contain the figures, the empirical analysis and the code in the same pdf file.