UNIVERSITY PARTNER

UNIVERSITY OF WOLVERHAMPTON

HERALD COLLEGE KATHMANDU

4MM013 - Computational Mathematics

Mathematics Assignment-2

Full Marks: 20

University ID : 2330473

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1. Using Cramer's rule obtain the solutions to the following set of equations:

$$2\,x_1 + x_2 - x_3 = 0$$

$$x_1 + x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

Canvas

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Q.NO 1) Given system

$$2x_1 + x_2 - x_3 = 0$$
$$x_1 + x_3 = 4$$
$$x_1 + x_2 + x_3 = 0$$

Writing in matrix form

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

Swapping row 2 with row 3, we get

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

Now,

for x_1

$$\det \text{ of } x = 2\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= 2(1-0) - 1(1-1) - 1(0-1)$$

$$= 2+1$$

$$= 3$$

$$\Delta \text{ of } x_1 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 1 \\ 4 & 0 & 1 \end{bmatrix}$$

$$= 0\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} - 1\begin{bmatrix}0 & 1\\ 4 & 1\end{bmatrix} - 1\begin{bmatrix}0 & 1\\ 4 & 0\end{bmatrix}$$

$= 0 - 1(0-4) - 1\ (0-4)$

$= 0 + 4 + 4$

$= 8$

$\therefore x_1 = \frac{\Delta x_1}{det.x} = \frac{8}{3}$

for x_2

$$\Delta x_2 = \begin{bmatrix}3 & 0 & -1\\ 1 & 0 & 1\\ 1 & 4 & 1\end{bmatrix}$$

$$= 2\begin{bmatrix}0 & 1\\ 4 & 1\end{bmatrix} - 0\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} - 1\begin{bmatrix}1 & 0\\ 1 & 4\end{bmatrix}$$

$= 2(0-4) - 0 - 1(4)$

$= -8 - 4$

$= -12$

$\therefore x_3 = \frac{\Delta x_2}{det.x}$

$= \frac{-12}{3}$

$= -4$

for x_3

$$\Delta x_3 = \begin{bmatrix}2 & 1 & 0\\ 1 & 1 & 0\\ 1 & 0 & 4\end{bmatrix}$$

2.

a) Solve the following using Gauss elimination:

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

$$= 2\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 1 & 4 \end{bmatrix} + 0\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= 2(4-0) - 1(4-0) + 0$$

$$= 8-4$$

$$= 4$$

$$\therefore x_3 = \frac{\Delta x_3}{x} = \frac{4}{3}$$

$\therefore$ Req^d value of x_1, x_2 and x_3 are $\frac{8}{3}$, -4 and $\frac{4}{3}$ respectively.

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Q.NO 2. a) Sol^n Here.

$$x_1 + x_2 + x_3 = 2$$

$$2x_1 + 3x_2 + 4x_3 = 3$$

$$x_1 - 2x_2 - x_3 = 1$$

Writing in matrix from

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Working in augmented from

$$R_2 \to R_2 - 2R_1 \quad \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & -2 & -1 & 1 \end{array}\right]$$

b) Find the inverse of the matrix from (a) using elimination.

$R_3 \to R_3 - R_1$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right]$

$R_3 \to R_3 + 1R_2$ $\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right]$

Doing backward Substitution

$4x_3 = -4$ $\quad$ $x_2 + 2x_3 = -1$

so, $x_3 = \frac{-4}{4}$ $\quad$ so, $x_2 + 2(-1) = -1$

$\quad$ so, $x_2 - 2x - 1$

$\therefore x_3 = -1$ $\quad$ $\therefore x_2 = 1$

$\therefore$ Req^d value fo x_1, x_2 and x_3 are 2, 1 and -1 respectively

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Q. NO 2 (b)

Sol^n

Now,

$x_1 + x_2 + x_3 = 2$

$2x_1 + 3x_2 + 4x_3 = 3$

$x_1 - 2x_2 - x_3 = 1$

Writing in matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

To find inverse of the given matrix

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & -3 & -2 & -1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 - R_1 - 2R_1 \\ R_3 = R_1 - R_1 \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right] \quad R_2 = R_3 + 3R_1$$

$$\left[\begin{array}{ccc:ccc} 1 & 4 & 0 & 13 & -3 & -1 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right] \quad \begin{array}{l} \cancel{R_2 = 7R_2} - R_1 \\ R_1 = 4R_3 - R_3 \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & -3 & -2 & -3 \\ 0 & 0 & 4 & -7 & 3 & 1 \end{array}\right] \quad R_1 = R_2 - 4R_3$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 & -3 \\ -3 & -1 & -1 \\ -7/4 & 3/4 & 1/4 \end{bmatrix}$$

(4)

3. Determine whether the following sequence converges or diverges.

$$t_n = (-1)^{n+1}\,\frac{n+1}{n^2+3}$$

(4)

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Q. No. 3 Here,

$t_n = (-1)^{n+1}\ \frac{n+1}{n^2+3}$

To check the sequence converges or diverges

$\lim_{n\to\infty} t_n$

$\lim_{n\to\infty} (-1)^{n+1}\ \frac{n+1}{n^2+3}$

$\lim_{n\to\infty} (-1)^{n+1}\ \frac{1+1/n}{1+3/n^2}$

$(-1)^{\infty+1}\ \frac{1+1/\infty}{n(1+3/\infty)}$

$= (-1)^{\infty}\cdot\frac{1}{\infty}$

$= (-1)^{\infty}\cdot 0$

0

$\therefore$ Since, the Value of given function is specified value. The given sequence is convergent.

4. Find the Maclaurin series expansion of **Sinx,** also calculate the radius of convergence. (4).

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Q. no: 4

Soln

Here,

Let $f(x) = \sin x$

For maclaurin series we need

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) + \dots$$

So,

$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$

$f'''(0) = -\cos 0 = -1$

$f''''(0) = \sin 0 = 0$

$f'''''(0) = \cos 0 = 1$

So, the series will be

$$= 0 + x.1 + \frac{x^2}{2!} 0 + \frac{x^3}{3!} (-1) + \frac{x^4}{4!} . 0 + \frac{x^5}{5!} . (1)$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

The End