4MM013 Computational mathematics Tutorial - 07

Instruction:

Complete all questions in 1 hr.

1. Explain the necessary condition for a series to be convergent.Determine which of the following series can not be convergent?

   a. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$

   b. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

Tutorial 7

a) Writing in summation form

$\sum_1^\infty \left(\frac{n}{n+1}\right)$

Now,

$\lim_{n \to \infty} \frac{n}{n+1}$

$\lim_{n \to \infty} = \frac{1}{1 + 1/n}$

$= \frac{1}{1 + 1/\infty}$

$= 1/1$

$= 1 \neq 0$, so this series can never be convergent

b) Writing in summation form

$\sum_1^\infty \left(1/n\right) (-1)^{n+1}$

$\lim_{n \to \infty} 1/\infty$

$= 1/\infty$

$= 0$, so this series may be convergent

2. Explain the alternating series test. Determine whether the following series are convergent or divergent by using alternating series test.

a. $$\sum_{p=1}^{\infty} (-1)^p \frac{2p-1}{2p+1}$$

b. $$\sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{p^2}$$

3. Determine whether the following series are convergent or divergent by using ratio test. a.

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + .$$

b. $$\sum_{p=1}^{\infty} \frac{1}{\sqrt{p}}$$

c. $$\frac{1}{\ln 3} + \frac{8}{(\ln 3)^2} + \frac{27}{(\ln 3)^3} + ..$$

2a) Here,

$$\sum_{n=0}^{\infty} (-1)^n \frac{2n-1}{2n+1}$$

$$\lim_{n \to \infty} \frac{2 - 1/n}{2 + 1/n}$$

$$= \frac{2-0}{2+0}$$

$$= \frac{2}{2}$$

$= 1 \neq 0$, so the series is divergent

b) Here

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$a_n = 1/n^2$$

$$\lim_{n \to \infty} 1/n^2$$

$$= 1/\infty$$

$$= 0$$

2^{nd} test

When $n = 1$

$a_{n+1} \leq a_n$

$a_2 \leq a_1$

$1/2^2 \leq 1/1$

$a_2 \leq 1$ (True

Hence the series is convergent

3 (a) Here,

$$a_n = 1/n$$

For ratio test

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$\lim_{n \to \infty} 1/1 + 1/n$$

$$= 1/1$$

$$= 1 = \lambda$$

Since, $\lambda = 1$, test is inconclusive

P-series test

$$\sum_{n=1}^{\infty} 1/n^p$$

We get $p = 1$, falls under $p \leq 1$ divergence

b) Here,

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n}}$$

From ratio test

$$\lim_{n \to \infty} \left| \frac{\sqrt{n}}{\sqrt{n+1}} \right|$$

$$\lim_{n \to \infty} \frac{n^{1/2}}{(n+1)^{1/2}}$$

$$\lim_{n \to \infty} \frac{1}{(1+n)^{1/2}}$$

$$= \frac{1}{1}$$

$= 1$

Since $\lambda = 1$ test is conclusion.

P-Series test

$\sum_{n=1} \frac{1}{n^p}$

Comparing with we get

$p = 1/2$, falls under $p \leq 1$ so divergent

c) Here,

$$\sum_{1}^{\infty} \left(\frac{n^3}{(\ln 3)^n} \right)$$

From ratio test

$$\lim_{n \to \infty} \left| \frac{(n+1)^3 \cdot \ln 3^n}{(\ln 3)^{n+1} \, n^3} \right|$$

$$= \left| \frac{(n+1)^3 \cdot \ln 3^n}{(\ln 3)^n \cdot \ln 3 \, n^3} \right|$$

$$= \left| \frac{(n+1)^3}{\ln 3 \cdot n^3} \right|$$

$$= \frac{1}{\ln 3}$$

$= 0.93 < 1$, Convergent

4. Write short note on radius of convergence. Find the radius of convergence of the following power series.

   a. $1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + .$

   b. $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n!}$

5. Find the Maclaurin series expansion of ln(x + 1).
6. Find the Maclaurin series expansion of the **sinx** and **cosx** and also calculate the radius of the convergence.

4)

→ The most important statement one can make about a Power Series is that there exist a number R, called the radius of Convergence, Such that if $|x| < R$ the power series is absolutely Convergent and if $|x| > R$ the power series is divergent.

a) Here,

$$\sum_{n}^{\infty} \frac{1}{n+1} x^n$$

Radius of Convergence $R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right|$

$\lim_{n \to \infty} \left| \frac{n+2}{n+1} \right| = 1 = R$

So, the given series is absolutely convergent if $|x| < 1$ and is divergent if $|x| > 1$

b) Here

$$\sum_{1}^{\infty} (x+3)^n \times \frac{1}{n!}$$

So, $a_n = \frac{1}{n!}$ and $a_{n+1} = \frac{1}{(n+1)!}$

Radius of Convergence $= \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n \to \infty} \frac{\frac{1}{n!}}{\frac{1}{(n+1)!}}$

$= \frac{(n+1)n!}{n!}$

$= n+1$

$= \infty$

Since Radius $= \infty$ $|x|$ is never > 1 the Series is Convergent

6) Find the Maclaurin series expansion of the sin x and cos x and also calculate the radius of the convergence

Sin x

Here, $f(x) = \sin x$

$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$

$f'''(0) = -\cos 0 = -1$

$f^{iv}(0) = \sin(0) = 0$

$f^{v}(0) = \cos(0) = 1$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

$$= 0 + x \times 1 + \frac{x^2}{2!} \times 0 + \frac{x^3}{3!} \times -1 + \frac{x^4}{4!} \times 0 + \frac{x^5}{5!} \times 1$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

$$= x - \frac{x^3}{6} + \frac{x^5}{120}$$

$\cos x$

$f(x) = \cos x$

$f(0) = \cos 0 = 1$

$f'(0) = -\sin 0 = 0$

$f''(0) = -\cos 0 = -1$

$f'''(0) = \sin 0 = 0$

$f^{iv}(0) = \cos 0 = 1$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \dots$$

$$= 1 + x \times 0 + \frac{x^2}{2!} \times (-1) + \frac{x^3}{3!} \times 0 + \frac{x^4}{4!} \times 1$$

$$= 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!}$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

9) Find the tylor series expansion of $\frac{1}{x^2}$ about $x = -1$

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \frac{(x - x_0)^3}{3!} f'''(x_0) + \frac{(x - x_0)^4}{4!} f^{iv}(x_0) + \dots$$

$$f(-1) = \frac{1}{x^2} = \frac{1}{(-1)^2} = 1$$

7. Find the Maclaurin series of $(1 - x)^{-3}$ and calculate the radius of convergence.

8. Find the Taylor series expansion of e^x about x=0.

9 Find the Taylor series expansion of $\frac{1}{x^2}$ about x=-1.

10. Find the Tylor series expansion of $x^3 - 10x^2 + 6$ about x = 3. "The End"