80513 TOPICS IN GRAPH THEORY - Exercise 1

Deadline: March 14th, 2017

1. Let T be a tree and let $\mathcal{T}$ be a family of subtrees of T. Suppose that every two trees in $\mathcal{T}$ have a nonempty intersection. Show that all trees in $\mathcal{T}$ have a nonempty intersection.

2. It is given that the graph G has a cycle C and that there are two vertices in C that are connected by a simple path of length k. Show that G contains a cycle of length $\geq \sqrt{2k}$. Bonus: How tight is this bound?

3. Consider the following bipartite graph G with parts A and B. Both A and B are related to the complete graph K_6. The vertices in A are identified with the 15 edges of K_6. The vertices in B are identified with the 15 perfect matchings in K_6. Vertex $e \in A$ and $m \in B$ are neighbors iff e is one of the edges of m. Find the girth and the diameter of G.