80513 TOPICS IN GRAPH THEORY - Exercise 3

Deadline: March 28th, 2017

1) We described in class a collection of $|E| - |V| + 1$ cycles in a connected graph $G = (V, E)$ that is based on selecting a fixed spanning tree and associating a cycle with each non-tree edge. Show that this collection of cycles is basis for G's cycle space.

2) Let $M = (S, \mathcal{I})$ be a matroid with ground set S and a family $\mathcal{I}$ of independent sets, and let $T \subseteq S$. Show that all independent sets that are subsets of T and are inclusion-maximal have the same cardinality. This cardinality is denoted $r_{\mathcal{I}}(T)$.

3) We showed in class two ways to define matorids. Here we want to prove that the two definitions coincide by constructing a bijection between **families of independent sets** and **rank functions**. Let S be a fixed ground set.

   (a) Show that $r_{\mathcal{I}}$ from the previous item is a rank function.

   (b) Given a rank function r, show that $\mathcal{I}_r := \{T \subseteq S \ : \ |T| = r(T)\}$ is a family of independent sets.

   (c) Prove that the mappings $\mathcal{I} \mapsto r_{\mathcal{I}}$ and $r \mapsto \mathcal{I}_r$ are inverses of each other.