80513 TOPICS IN GRAPH THEORY - Exercise 2

Deadline: March 21st, 2017

1) Here is a construction of a family of graphs G:

   Let q be an odd prime power, and $\mathbb{F}_q$ the finite field of order q. The vertices of G are all nonzero triples (x, y, z) of elements from $\mathbb{F}_q$ modulo a multiplication by a nonzero element of $\mathbb{F}_q$. Two triples are adjacent in G if their inner product is zero.

   You are requested to:

   a. Explain why G is irregular and how to turn it into a regular graph H by adding some properly chosen edges.

   b. Prove that H's diameter is 2

   c. Find the relationship between H's degree and its number of vertices.

2) How tight is the bound in Mader's Theorem?