Another example

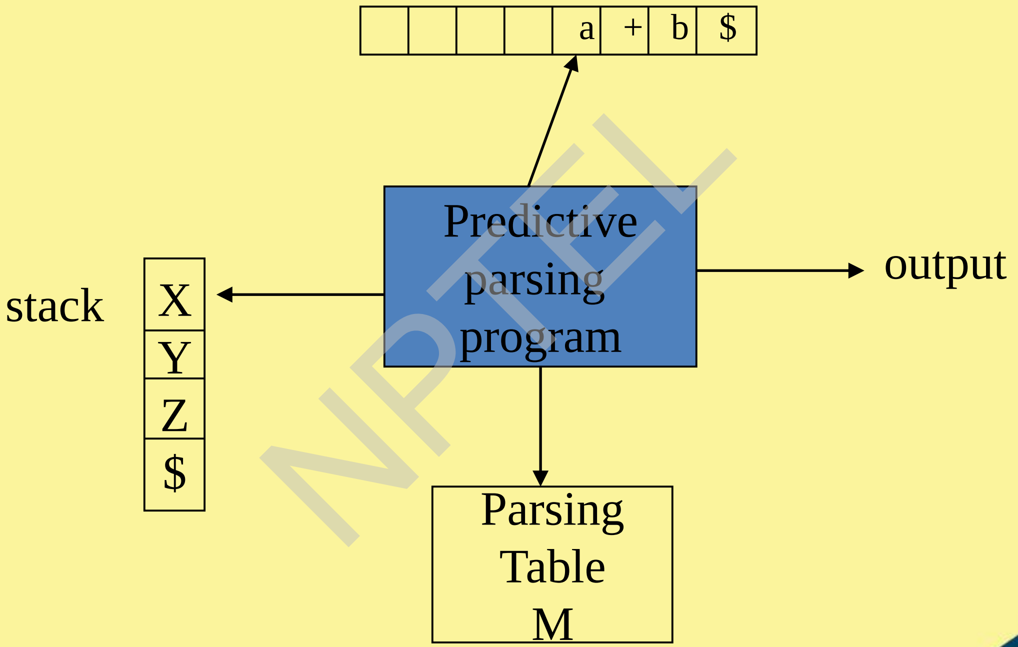
S -> iEtSS' | a
S' -> eS | ε
E -> b

First(S) = {i,a}
First(S') = {e, ε}
First(E) = {b}

Follow(S) = {$,e}
Follow(S') = {$,e}
Follow(E) = {t}

Non - terminal	Input Symbol					
	a	b	e	i	t	$
S	S -> a			S -> iEtSS'		
S'			S' -> ε S' -> eS			S' -> ε
E		E -> b				

Non-recursive predicting parsing



Predictive parsing algorithm

```
While (stack is not empty) do
    Let X be the top symbol in the stack;
    Let a be the next input symbol;
    if (X is a) pop the stack and advance input pointer;
    else if (X is a terminal) error();
    else if (M[X,a] is an error entry) error();
    else if (M[X,a] = X->Y1Y2..Yk) {
            output the production X->Y1Y2..Yk;
            pop the stack;
            push Yk,…,Y2,Y1 on to the stack with Y1 on top;
    }
```

Example

id+id*id$

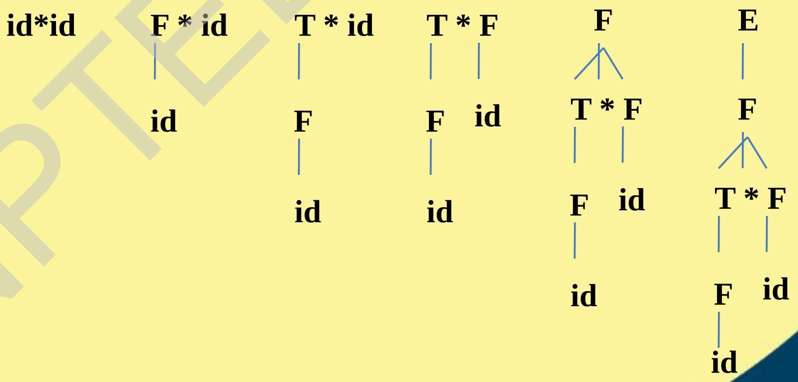
Non - terminal	id	+	*	(	)	$
E	E -> TE'			E -> TE'		
E'		E' -> +TE'			E' -> ε	E' -> ε
T	T -> FT'			T -> FT'		
T'		T' -> ε	T' -> *FT'		T' -> ε	T' -> ε
F	F -> id			F -> (E)		

Stack	Input	Action
E	id+id*id$	Parse E -> TE’
E’T	id+id*id$	Parse E’ ->FT’
E’T’F	id+id*id$	Parse F -> id
E’T’id	id+id*id$	Advance input
E’T’	+id*id$	Parse T’ -> Ɛ
E’	+id*id$	Parse E’ -> +TE’
E’T+	+id*id$	Advance input
E’T	id*id$	Parse T ->FT’
E’T’F	id*id$	Parse F -> id
E’T’id	id*id$	Advance input
E’T’	*id$	Parse T’->*FT’
E’T’F*	*id$	Advance input
E’T’F	id$	Parse F -> id
E’T’id	id$	Advance input
E’T’	$	Parse T’ -> Ɛ
E’	$	Parse E’ -> Ɛ
	$	Done

Bottom-up Parsing

Introduction

- Constructs parse tree for an input string beginning at the leaves (the bottom) and working towards the root (the top)
- Example: id*id



E -> E + T | T
T -> T * F | F
F -> (E) | **id**

Shift-reduce parser

- The general idea is to shift some symbols of input to the stack until a reduction can be applied
- At each reduction step, a specific substring matching the body of a production is replaced by the nonterminal at the head of the production
- The key decisions during bottom-up parsing are about when to reduce and about what production to apply
- A reduction is a reverse of a step in a derivation
- The goal of a bottom-up parser is to construct a derivation in reverse:
  - E=>T=>T*F=>T*id=>F*id=>id*id

Handle pruning

- A Handle is a substring that matches the body of a production and whose reduction represents one step along the reverse of a rightmost derivation

Right sentential form	Handle	Reducing production
id*id	id	F->id
F*id	F	T->F
T*id	id	F->id
T*F	T*F	E->T*F

Shift reduce parsing

- A stack is used to hold grammar symbols
- Handle always appear on top of the stack
- Initial configuration:

Stack	Input
$	w$

- Acceptance configuration

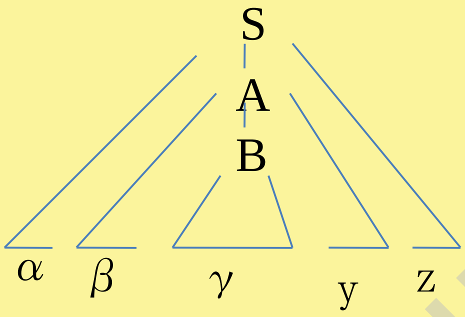
Stack	Input
$S	$

Shift reduce parsing (cont.)

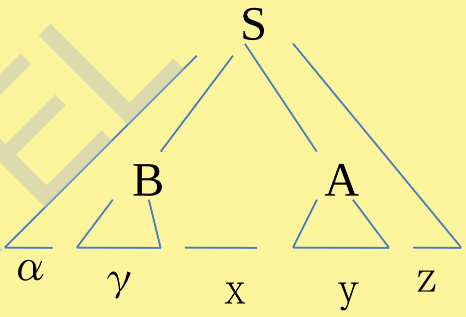
- Basic operations:
  - Shift
  - Reduce
  - Accept
  - Error
- Example: id*id

Stack	Input	Action
$	id*id$	shift
$id	*id$	reduce by F->id
$F	*id$	reduce by T->F
$T	*id$	shift
$T*	id$	shift
$T*id	$	reduce by F->id
$T*F	$	reduce by T->T*F
$T	$	reduce by E->T
$E	$	accept

Handle will appear on top of the stack



Stack	Input
$\$\alpha\ \beta\ \gamma$	yz$
$\$\alpha\ \beta B$	yz$
$\$\alpha\ \beta By$	z$



Stack	Input
$\$\alpha\ \gamma$	xyz$
$\$\alpha Bxy$	z$

Conflicts during shift reduce parsing

- Two kind of conflicts
  - Shift/reduce conflict
  - Reduce/reduce conflict
- Example:

stmt → If expr **then** stmt
| If expr **then** stmt **else** stmt
| other

Stack	Input
… if expr then stmt	else …$

Reduce/reduce conflict

stmt -> id(parameter_list)
stmt -> expr:=expr
parameter_list->parameter_list, parameter
parameter_list->parameter
parameter->id
expr->id(expr_list)
expr->id
expr_list->expr_list, expr
expr_list->expr

Stack	Input
… id(id	,id) …$

Bottom-Up Parsing

- Operator Precedence Parsing
- LR Parsing

Operator Precedence Parsing

Operator Grammar

- No Ɛ-transition
- No two adjacent non-terminals

Eg.

$E \rightarrow E \text{ op } E \mid id$

$op \rightarrow + \mid *$

The above grammar is not an operator grammar but:

$E \rightarrow E + E \mid E * E \mid id$

Operator Precedence

- If a has higher precedence over b; $a \cdot> b$
- If a has lower precedence over b; $a <\cdot b$
- If a and b have equal precedence; $a \doteq b$

Note:

- id has higher precedence than any other symbol
- $ has lowest precedence.
- if two operators have equal precedence, then we check the **Associativity** of that particular operator.

Precedence Table

	id	+	*	$
id		·>	·>	·>
+	<·	·>	<·	·>
*	<·	·>	·>	·>
$	<·	<·	<·	·>

Example: w = $id + id * id$

$ <· id ·> + <· id ·> * <· id ·> $

Basic Principle

- Scan input string left to right, try to detect ·> and put a pointer on its location.
- Now scan backwards till reaching <·
- String between <· and ·> is our handle.
- Replace handle by the head of the respective production.
- REPEAT until reaching start symbol.

Algorithm

```
Initialize stack to $
while true do
    let U be the topmost terminal in the stack
    let V be the next input symbol
    if U = $ and V = $ then return
    if U <· V or U ≐ V then
        shift V onto stack
        advance input pointer
    else if U ·> V then
        do
            pop the topmost symbol, call it V, from the stack
        until the top of the stack is <· V
    else
        error
end
```

Example

STACK	INPUT	ACTION/REMARK
$	id + id * id$	$ <· id
$ id	+ id * id$	id ·> +
$	+ id * id$	$ <· +
$ +	id * id$	+ <· id
$ + id	* id$	id ·> *
$ +	* id$	+ <· *
$ + *	id$	* <· id
$ + * id	$	id ·> $
$ + *	$	* ·> $
$ +	$	+ ·> $
$	$	accept

	id	+	*	$
id		·>	·>	·>
+	<·	·>	<·	·>
*	<·	·>	·>	·>
$	<·	<·	<·	·>

Establishing Precedence Relationships

- Construct two lists: Firstop+, Lastop+
- Firstop+: List of all terminals which can appear first on any right hand side of a production
- Lastop+: List of terminals that can appear last on any right hand side of a production

Firstop and Lastop

- For X -> a...|Bc, put a, B, c in Firstop(X)
- For Y -> ...u|...vW, put u, v, W in Lastop(Y)
- Compute Firstop+ and Lastop+ using Closure algorithm
  - Take each nonterminal in turn, in any order and look for it in all the Firstop lists. Add its own first symbol list to any other in which it occurs
  - Similarly process Lastop list
  - Drop all nonterminals from the lists

Constructing Precedence Matrix

- Whenever terminal a immediately precedes nonterminal B in any production, put a <·α where α is any terminal in the Firstop+ list of B
- Whenever terminal b immediately follows nonterminal C in any production, put β ·>b where β is any terminal in the Lastop+ list of C
- Whenever a sequence aBc or ac occurs in any production, put a ≐ c
- Add relations $<· a and a ·> $ for all terminals in the Firstop+ and Lastop+ lists, respectively of S

Example

E -> E + T | T
T -> T * F | F
F -> (E) | id

Firstop(E) = {E, +, T}
Firstop(T) = {T, *, F}
Firstop(F) = {(, id}

Lastop(E) = {+, T}
Lastop(T) = {*, F}
Lastop(F) = {), id}

Firstop+(E) = {E, +, T, *, F, (, id}
Firstop+(T) = {T, *, F, (, id}
Firstop+(F) = {(, id}

Lastop+(E) = {+, T, *, F,), id}
Lastop+(T) = {*, F,), id}
Lastop+(F) = {), id}

Firstop+(E) = {+, *, (, id}
Firstop+(T) = {*, (, id}
Firstop+(F) = {(, id}

Lastop+(E) = {+,*,), id}
Lastop+(T) = {*,), id}
Lastop+(F) = {), id}

Example (Contd.)

E -> E + T | T
T -> T * F | F
F -> (E) | id

Firstop+(E) = {+, *, (, id}
Firstop+(T) = {*, (, id}
Firstop+(F) = {(, id}

Lastop+(E) = {+,*,), id}
Lastop+(T) = {*,), id}
Lastop+(F) = {), id}

	$	(	)	id	+	*
$		<·		<·	<·	<·
(		<·	≐	<·	<·	<·
)	·>		·>		·>	·>
id	·>		·>		·>	·>
+	·>	<·	·>	<·	·>	<·
*	·>	<·	·>	<·	·>	·>