

\* Finding First() and Follow() from given grammar

First(A) contains all terminals present in first place of every string derived by A.

First(Terminal) = Terminal

First( $\epsilon$ ) =  $\epsilon$

E.g.  $S \rightarrow abc \mid defghi$

First(S) = {a, d, g}

E.g.  $S \rightarrow ABC \mid ghi \mid jkl$  ( $A, g, j$ )  $\Rightarrow (a, g, j) (a, b, c, g, j)$

$A \rightarrow alb \mid c$  ( $a, b, c$ )

$B \rightarrow b$  ( $b$ )

$D \rightarrow \delta$  ( $\delta$ )

E.g.  $S \rightarrow ABC$

$A \rightarrow alb \mid c$

$B \rightarrow c \mid d \mid e$

$C \rightarrow e \mid f \mid e$

$F(C) = \{e, f, \epsilon\}$ ,  $F(B) = \{c, d, \epsilon\}$ ,  $F(A) = \{a, b, e\}$

$F(S) = F(A) = \{a, b, \epsilon\} \cup F(B) = \{c, d\} \cup F(C) = \{e, f, \epsilon\}$   
 $= \{a, b, c, d, e, f, \epsilon\}$

E.g.  $E \rightarrow TE'$

$E' \rightarrow *TE' | \epsilon$

$T \rightarrow FT'$

$T' \rightarrow \epsilon | +FT'$

$F \rightarrow id | ( E )$

$\text{first}(F) = \{id, ()\}$ ,  $\text{first}(T) = \text{first}(F) = \{id, ()\}$ ,

$\text{first}(T') = \{\epsilon, +\}$ ,  $\text{first}(E') = \{*, \epsilon\}$ ,

$\text{first}(E) = \text{first}(T) = \{id, ()\}$

$\text{Follow}(A)$  contains set of all terminals present immediate in right of A. It never contains  $\epsilon$ .  
Follow of start symbol is  $\$$ .

E.g.  $S \rightarrow ACD$

$C \rightarrow a1b$

$F_0(A) = \text{first}(C) = \{a, b\}$

$F_0(D) = F_0(S) = \{\$\}$

E.g.  $S \rightarrow aSbs | bsas | \epsilon$

$F_0(S) = \{\$, b, a\}$

E.g.  $S \rightarrow AaAb | BbBa$

$A \rightarrow E$

$B \rightarrow E$

$F_0(A) = \{a, b\}$ ,  $F_0(B) = \{b, a\}$

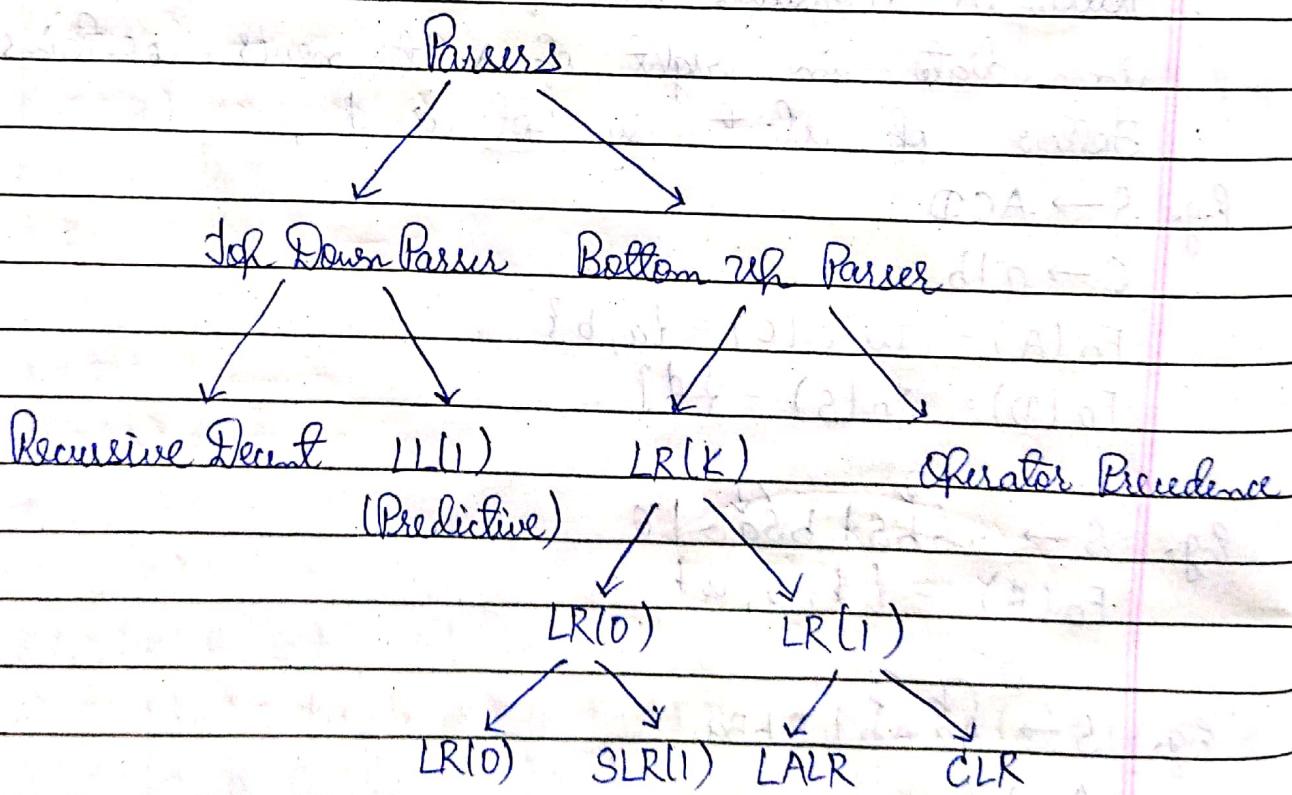
E.g.  $S \rightarrow ABC$   
 $A \rightarrow DEF$

$B, C \rightarrow \epsilon, D \rightarrow \epsilon, E \rightarrow \epsilon, F \rightarrow \epsilon$

$F_0(A) = \text{First}(B) = \text{First}(C) = \text{Follow}(S) = \{\$\}$

### \* Parsing

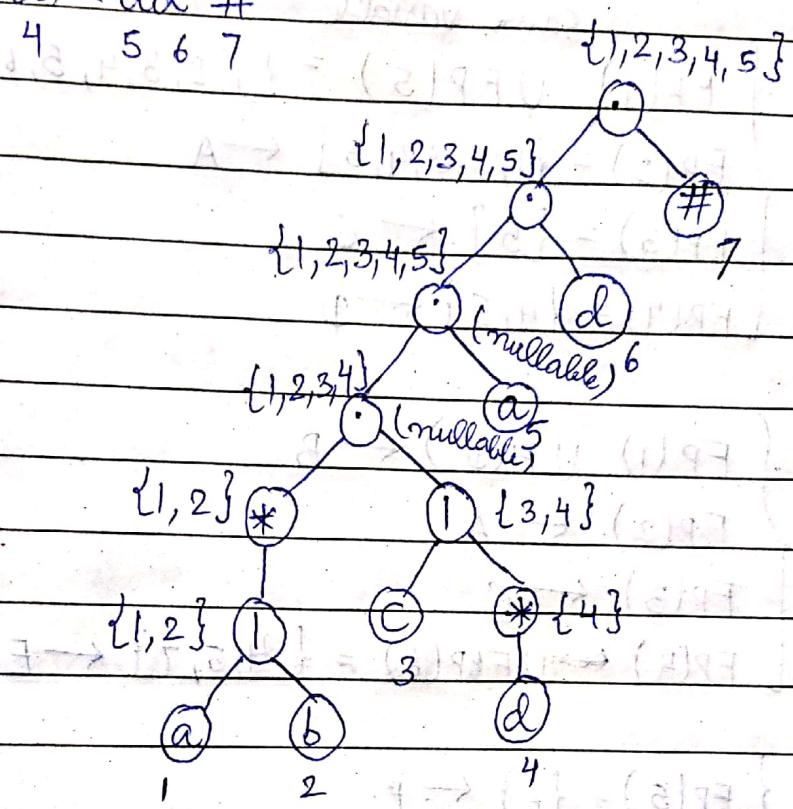
It is a process of deriving string from a given grammar.



\* RE to DFA using Syntax tree method

E.g.  $(ab)^* \cdot (cd)^* \cdot ad \#$

1 2 3 4 5 6 7  $\{1, 2, 3, 4, 5\}$



Union of

FIRSTPOS(1) = FIRSTPOS(child nodes)

FIRSTPOS(\* · \*) = FIRSTPOS(child node)

FIRSTPOS(.) = FIRSTPOS(left node)

(OR)

Union of FIRSTPOS(left node) and

FIRSTPOS(right node) if left node  
contains \* (that is nullable)

FOLLOWPOS - which symbols can follow the given symbol

FIRSTPOS(root) = {1, 2, 3, 4, 5}  $\leftarrow A$

FOLLOWPOS(1) = {1, 2, 3, 4, 5}

FOLLOWPOS(2) = {1, 2, 3, 4, 5}

FOLLOWPOS(3) = {5}

FOLLOWPOS(4) = {4, 5}

FOLLOWPOS(5) = {6}

$\text{FOLLOWPOS}(6) = \{7\}$

$\text{FOLLOWPOS}(7) = \#\emptyset$

same variable

$$A \left\{ \begin{array}{l} FP(1) \cup FP(5) = \{1, 2, 3, 4, 5, 6\} \leftarrow B \\ FP(2) = \{1, 2, 3, 4, 5\} \leftarrow A \\ FP(3) = \{5\} \leftarrow C \\ FP(4) = \{4, 5\} \leftarrow D \end{array} \right.$$

$$B \left\{ \begin{array}{l} FP(1) \cup FP(5) \leftarrow B \\ FP(2) \leftarrow A \\ FP(3) \leftarrow C \\ FP(4) \leftrightarrow FP(6) = \{4, 5, 7\} \leftarrow E \end{array} \right.$$

$$C \{ FP(5) = \{6\} \leftarrow F \}$$

$$D \{ FP(4) \leftarrow D \}$$

$$\{ FP(5) \leftarrow F \}$$

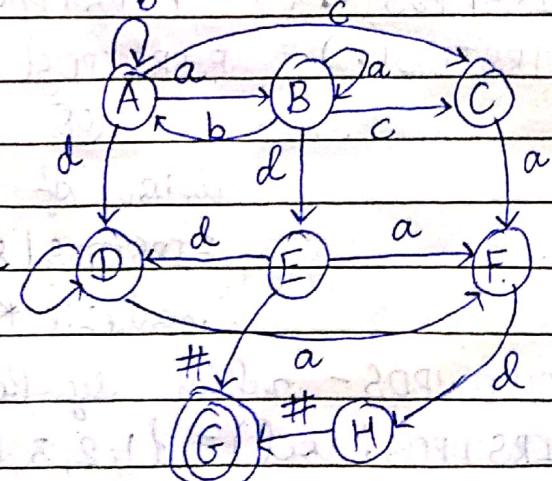
$$E \{ FP(4) \leftarrow D \}$$

$$\{ FP(5) \leftarrow F \}$$

$$\{ FP(7) \leftarrow \# \leftarrow G \}$$

$$F \{ FP(6) = \{7\} \leftarrow H \}$$

$$G \{ FP(7) \leftarrow \# \}$$



## \* LL(1) Parser (Top-Down)

(\*) How to check whether the string is LL(1) or not  
 e.g.  $S \rightarrow (L) | a$

$$L \rightarrow SL'$$

$$L' \rightarrow \epsilon, SL'$$

$$F(S) = \{ (, a \} \quad F_0(S) = \{ \$, \geq, ), \}$$

$$F(L) = F(s) = \{ (, a \} \quad F_0(L) = \{ ) \}$$

$$F(L') = \{ \epsilon, \geq, ' \} \quad F_0(L') = \{ ) \}$$

Make Parsing Table where rows are non-terminals and columns are terminals and  $\$$ .

	(	)	a	,	\$
S	$S \rightarrow (L)$	e	$S \rightarrow a$	e	e
L	$L \rightarrow SL'$	e	$L \rightarrow SL'$	e	e
L'	$L' \rightarrow \epsilon$	e		$L' \rightarrow SL'$	e
<u>FOLLOW</u>					

Make entry for every row of the table where terminal's first is non-terminal (column). The entry would be equation whose first is non-terminal. If first contains  $\epsilon$  then use follow of that non-terminal. All other entries will be  $e$ . If one cell maximum one entry then the grammar will be accepted by LL(1) Parser.

The above grammar is LL(1).

E.g.

$$S \rightarrow aSbS \mid bSaS \mid \epsilon$$

$$F(S) = \{a, b, \epsilon\}$$

$$FD(S) = \{b, a, \$\}$$

S

a

b

\$

$$S \rightarrow aSbS$$

$$S \rightarrow bSaS$$

$$\text{or } S \rightarrow bSaS \epsilon \text{ or } S \rightarrow aSbS \epsilon$$

$$S \rightarrow \epsilon$$

Here, there are two entries twice for one cell so the above grammar is not LL(1).



How LL(1) parser works?

$$S \rightarrow AA$$

$$A \rightarrow aA$$

$$A \rightarrow b$$

$$a$$

$$b$$

$$\$$$

$$S$$

$$S \rightarrow AA$$

$$S \rightarrow AA$$

$$\epsilon$$

$$A$$

$$A \rightarrow aA$$

$$A \rightarrow b$$

$$\epsilon$$

String: ababb

Initially, there is first non-terminal on the top of stack followed by \$ and string is also followed by \$. So at every point; non-terminal at top of stack is replaced with the function present in row of non-terminal of stack and column of terminal of string where pointer is. If top of stack and string pointer has same terminal

Then stack is popped and string pointer is incremented. If there are no rules present for a given non-terminal in stack according to table then grammar is not LL(1). If top of stack becomes \$ and string pointer also becomes \$ then string is LL(1).

Note: While replacing non-terminal with rule, all the contents are added in reverse order.

Stack	ab ab \$	Rule
① \$ S	$\uparrow \uparrow \uparrow \uparrow$	$S \rightarrow AA - A$
\$ AA		$A \rightarrow aA$
\$ AAA (pop & increment)	$\uparrow \uparrow \uparrow$	$A \rightarrow aA$
② \$ AA	$\uparrow \uparrow \uparrow$	$A \rightarrow b - A$
\$ Ab (pop & increment)	$\uparrow \uparrow \uparrow$	$A \rightarrow aA$
\$ A	$\uparrow \uparrow \leftarrow T$	$A \rightarrow aA$
\$ Aa (pop & increment)	$\uparrow \uparrow \uparrow$	$A \rightarrow b$
\$ A	$\uparrow \uparrow \leftarrow T$	$A \rightarrow aA$
\$ b (pop & increment)	$\uparrow \uparrow \uparrow$	$A \rightarrow b$
\$ (Accepted)	$\uparrow \uparrow \uparrow$	$A \rightarrow aA$

- Check table for rules in row S and column a.
- Check table for rule in row A and column b.

## \* Conditions for Top-Down Parser

The grammar should not contain:

- Left Recursion
- Ambiguity (Left Factoring) Non-Determinism

### 1. Eliminating Left Recursion

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

$$F \rightarrow (E) \mid id$$

$$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_m \mid B_1 \mid B_2 \mid \dots \mid B_n$$

$$A \rightarrow B_1 A' \mid B_2 A' \mid B_3 A' \mid \dots \mid B_n A'$$
$$A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_m A' \mid e$$

$$E \rightarrow TAE'$$

$$T \rightarrow FT'$$

$$E' \rightarrow +TA'E' \mid -TA'E' \mid \epsilon$$

$$S \rightarrow abA \mid abc \mid Bba \mid Sdd \mid bs$$

$$A \rightarrow a$$

$$B \rightarrow b$$

$$S \rightarrow abAS' \mid Bbas' \mid bss'$$

$$S' \rightarrow aes' \mid dds' \mid e$$

$$A \rightarrow a$$

$$B \rightarrow b$$

2. Eliminating left - factoring  
 2.  $A \rightarrow \alpha B_1 | \alpha B_2 | \dots | \alpha B_n$

$$A \rightarrow \alpha A'$$

$$A' \rightarrow B_1 | B_2 | \dots | B_n$$

e.g.  $S \rightarrow Aa | Ab | AS$

$$S \rightarrow AS'$$

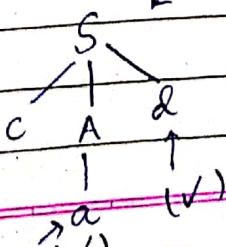
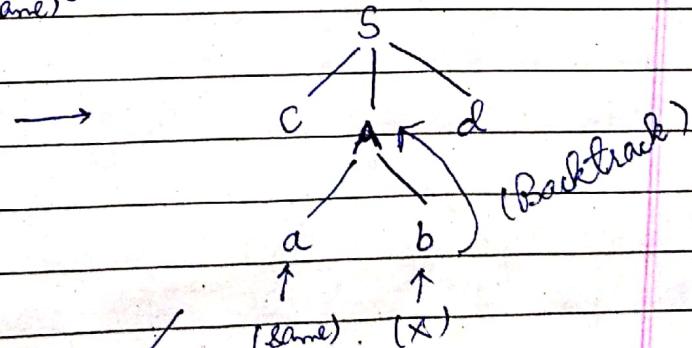
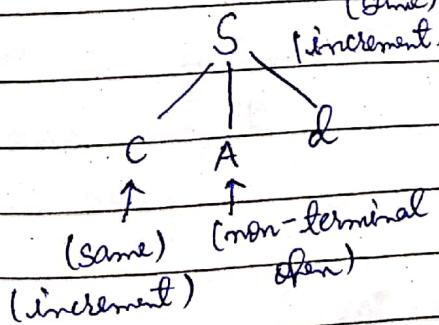
$$S' \rightarrow ab | S$$

\* Recursive Descent Parser (Top - Down)

Recursive Descent Parser is a top-down parser built from a set of mutually recursive procedures where each such procedure implements one of the non-terminals of the grammar.

e.g.  $S \rightarrow cAd$   
 $A \rightarrow ab | a$

Input string: cad



E.g.  $S \rightarrow aSbS \mid bSas \mid \epsilon$

String: abab

$S \rightarrow aSbS$

$\Rightarrow a \cdot aSbS \ bS \quad \times \text{(Backtracking)}$

$\Rightarrow a \ bSas \ bS$

$\Rightarrow ab \ aSbS \ aS \ bS \quad \times \text{(Backtracking)}$

$\Rightarrow ab \ bSas \ aS \ bS \quad \times \text{(Backtracking)}$

$\Rightarrow ab \ \epsilon \ aSbS$

$\Rightarrow aba \ aSbS \ bS \quad \times \text{(Backtracking)}$

$\Rightarrow aba \ bSas \ bS \quad \times \text{(Backtracking)}$

$\Rightarrow aba \ \epsilon \ bS$

$\Rightarrow abab \ S$

$\Rightarrow abab \ aSbS \quad \times \text{(Backtracking)}$

$\Rightarrow abab \ bSas \quad \times \text{(Backtracking)}$

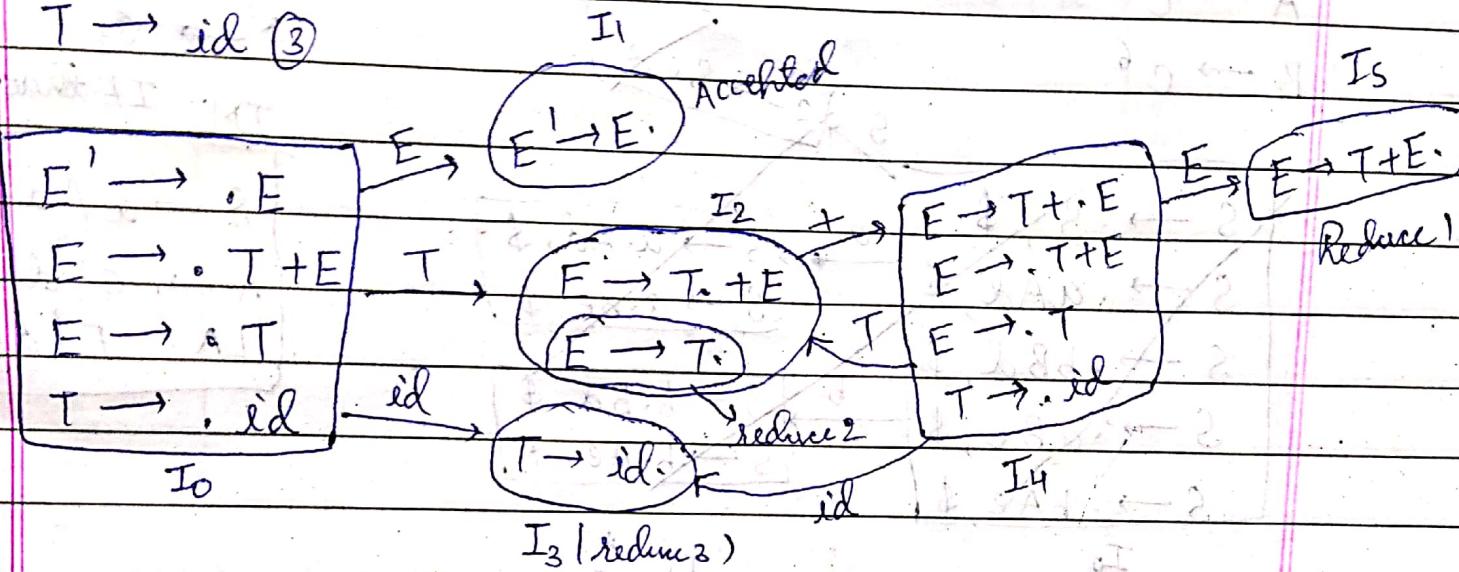
$\Rightarrow abab \ \epsilon$

$\Rightarrow abab$

\* SLR(1) (Bottom-up)

E.g.  $E \rightarrow T + E \mid T$

$T \rightarrow id \quad (3)$



state	id + \$	$E \rightarrow T$
0	S <sub>3</sub>	
1		Accepted
2	S <sub>4</sub>	$\text{FO}(E)$
3	S <sub>4</sub>	S <sub>3</sub>
4	S <sub>3</sub>	5 $\leftarrow$ 2
5		21

Find follow of LHS of reduce equation

$E \rightarrow T$ .

$\text{FO}(E) = \{ \$ \}$

$\text{FO}(T) = \{ \$, + \}$

Write reduce in row having same state number and shift in the row from where equation is going.

SLR(1) ✓ (Not more than one entries in any cell)

\* CLR (Bottom - Up)

IMP

$S \rightarrow a^1 Ad | b^2 Bd | a^3 Be | b^4 Ae$

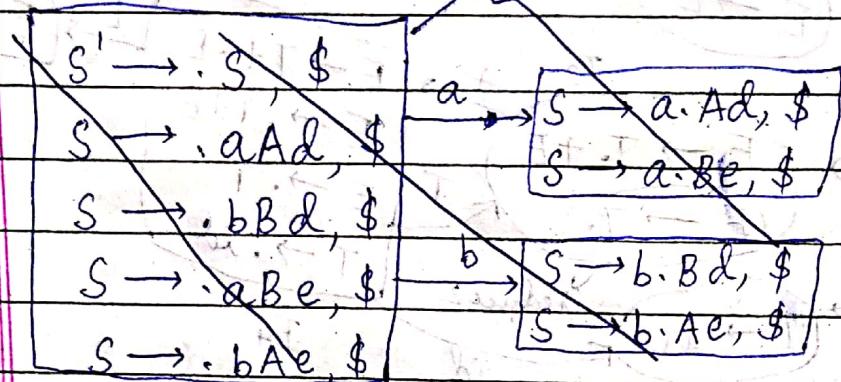
$A \rightarrow c^5$

$B \rightarrow c^6$

[By default,  
lookahead is \$]

~~\$~~  $\rightarrow$   $\$$

IMP IF there NT after \$



$S \rightarrow a.Ad, \$$

$A \rightarrow .c, d$

$$LA = F(d, \$) = d$$

$I_0$

$I_1 *$

$S' \rightarrow .S, \$$

$S \rightarrow .aAd, \$$

$S \rightarrow .bBd, \$$

$S \rightarrow .aBe, \$$

$S \rightarrow .bAe, \$$

$S \rightarrow .S, \$$

$S \rightarrow a.Ad, \$$

$A \rightarrow .c, d$

$S \rightarrow a.Be, \$$

$B \rightarrow .c, e$

$I_4$

$I_6 *$

$A \rightarrow S \rightarrow aAd, \$$

$B \rightarrow S \rightarrow ab.e, \$$

$A \rightarrow C., d$

$I_{11}$

$B \rightarrow C., e$

$I_0$

$b$

$I_3$

$I_9$

$S \rightarrow b.Bd, \$$

$B \rightarrow .c, d$

$S \rightarrow b.Ae, \$$

$A \rightarrow .c, e$

$S \rightarrow bB.d, \$$

$A \rightarrow S \rightarrow bA.e, \$$

$C \rightarrow B \rightarrow C., d$

$A \rightarrow C., e$

If state is accepted then write reduce in column of its lookahead.

State	a	b	c	d	e	\$	A	B
0	$s_2$	$s_3$						
1							$s_1$	
2								
3				$s_6$	$s_7$	$s_8$		
4				$s_9$				
5						$s_{10}$		
6							$s_{11}$	
7							$s_5$	$s_6$
8							$s_{12}$	
9							$s_{13}$	
10								$s_{10}$
11								$s_{11}$
12								$s_{12}$
13								$s_{10}, s_4$

### \* LALR (Bottom-up)

LALR is similar to CLR but in LALR similar states are merged.

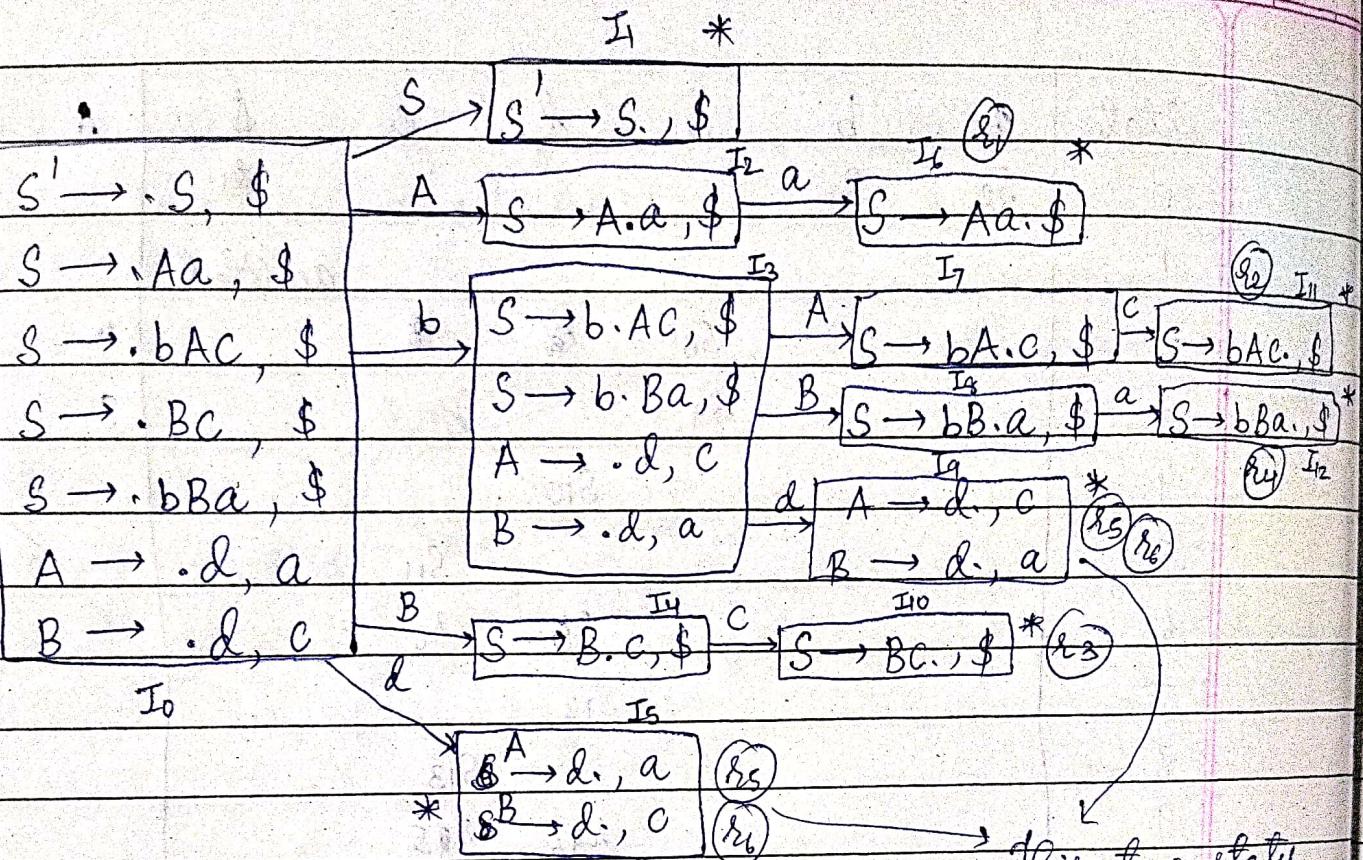
$$S \rightarrow Aa \mid bAc \mid Bc \mid bBa$$

$$A \rightarrow d^5$$

$$B \rightarrow d^6$$

In LALR, two reduce with different look ahead can be merged.

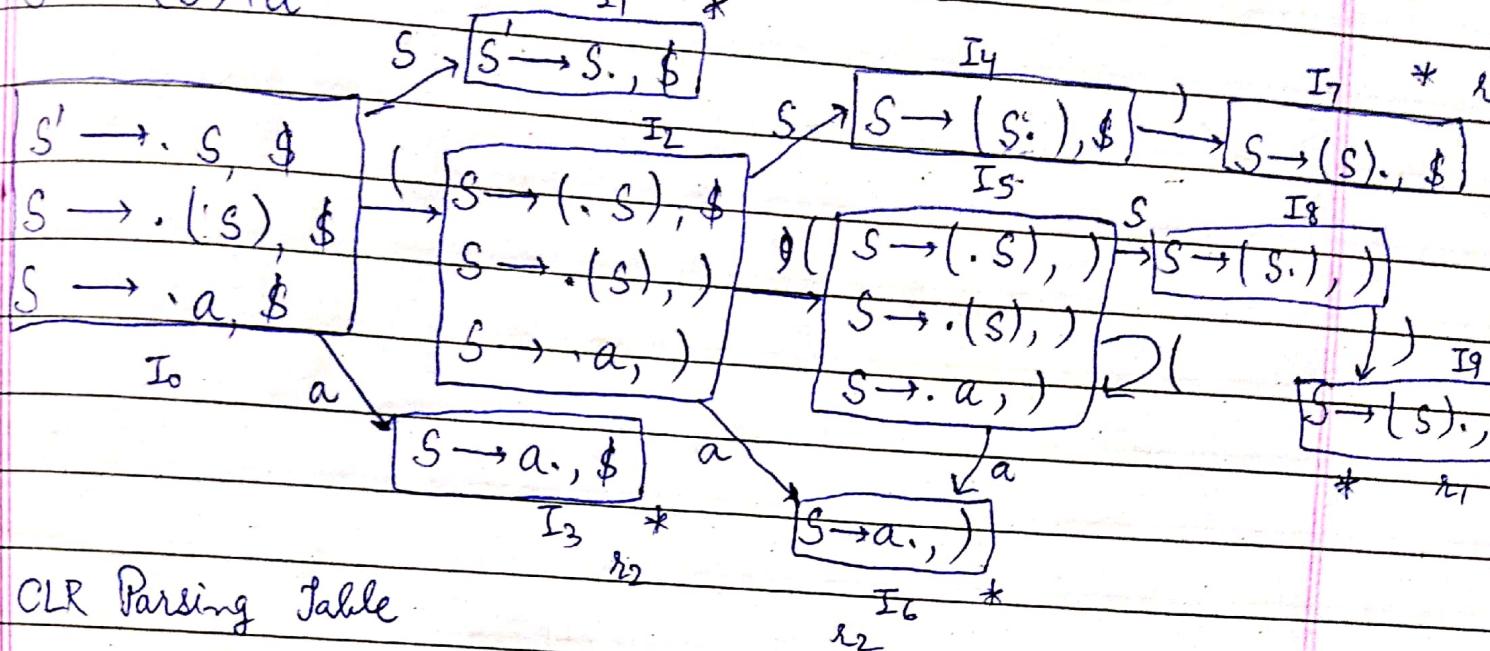
IMP Sometimes, a grammar can be CLR but is not LALR.



This two states  
cannot be  
merged as there  
would be multiple  
entries in the same  
column for LALR.  
So, the above  
grammar is not  
LALR but is  
CLR.

\* Difference between CLR and LALR.

$S \rightarrow (S) | a^2$



CLR Parsing Table

	(	)	a	\$	S
0	$S_2$				1
1				accepted	
2	$S_5$	$\textcircled{S}_5$		$S_6$	4
3					$r_2$
4		$S_7$			
5	$S_5$		$S_6$		8
6		$r_2$			
7				$r_1$	
8		$S_9$			
9		$r_1$			

LALR Parsing Table

	(	)	a	\$	S
0	$S_2$				1
1				accepted	
2	$S_5$		$S_6$		48
3		$r_2$		$r_2$	
4		$S_7$			
5		$r_1$		$r_1$	