# Conditions for a given grammar to be for LL(1)

A grammar $G$ is LL(1) if and only if whenever $A \to \alpha \mid \beta$ are two distinct productions of $G$, the following conditions hold:

1. For no terminal $a$ do both $\alpha$ and $\beta$ derive strings beginning with $a$.
2. At most one of $\alpha$ and $\beta$ can derive the empty string.
3. If $\beta \overset{*}{\Rightarrow} \epsilon$, then $\alpha$ does not derive any string beginning with a terminal in FOLLOW($A$). Likewise, if $\alpha \overset{*}{\Rightarrow} \epsilon$, then $\beta$ does not derive any string beginning with a terminal in FOLLOW($A$).

The first two conditions are equivalent to the statement that FIRST($\alpha$) and FIRST($\beta$) are disjoint sets. The third condition is equivalent to stating that if $\epsilon$ is in FIRST($\beta$), then FIRST($\alpha$) and FOLLOW($A$) are disjoint sets, and likewise if $\epsilon$ is in FIRST($\alpha$).

Example:

Q. Check if the grammar is LL(1) or not.

(i) $S \to aSa \mid bS \mid c$

let us take $aSa \to \alpha$, $bS \to \beta$, $c \to \gamma$

first $(\alpha) = \{a\}$
first $(\beta) = \{b\}$
first $(\gamma) = \{c\}$

→ According to the rule the first $(\alpha) \cap$ first $(\beta) \cap$ first $(\gamma)$ should be a disjoint set i.e. $\emptyset$.

first $(\alpha) \cap$ first $(\beta) \cap$ first $(\gamma)$
$= a \cap b \cap c$
$= \emptyset$.

→ This states that the given grammar is LL(1).

(ii) $S \rightarrow iCtSS_1 | (a)$

$S_1 \rightarrow eS | \epsilon$

$C \rightarrow b$

→ Here, there are 3 production rules so we'll check each of the rules individualy. If any of rule does satisfies the condition then whole grammar is not LL(1).

→ $S \rightarrow (iCtSS_1 | (a)$

$iCtSS_1 \rightarrow \alpha$ , $(a) \rightarrow \beta$

first $(\alpha) = i$

first $(\beta) = c$

first $(\alpha) \cap$ first $(\beta)$

$= i \cap c$

$= \emptyset$

→ $S_1 \rightarrow eS_1 | \epsilon$

$eS \rightarrow \alpha$ , $\epsilon \rightarrow \beta$

first $(\alpha) = e$

and as $\beta = \epsilon$ so, we'll apply rule (3) i.e

= first $(\alpha) \cap$ follow $(S_1)$

= first $(\alpha) \cap$ follow $(S)$

$= e \cap e\$$

$= e \neq \emptyset$

→ Thus, here the condition does not satisfies hence the given grammar is LL(1).

Video link for reference:

https://youtu.be/GimDicPMQ68?si=pd48pB81ou8X6FyS