

* Arrays, Records, Pointers.

Operations on Linear structures

- ① Traversal
- ② Searching
- ③ Insertion
- ④ Deletion
- ⑤ Sorting
- ⑥ Merging

⇒ Linear Arrays

- ↳ List of finite no. 'n' of homogeneous data elements (elements of same type).
- ↳ Elements are referenced by 'index set' $[1 \text{ to } n]$
- ↳ Array elements are stored in successive memory locations.

$$\text{Length or size of array} = \text{UB} - \text{LB} + 1$$

Example :- DATA = 247, 56, 429, 135, 89, 156.

AUTO [k] = no. of automobiles sold in year 'k'.

Representation in memory -

1000	
1001	
1002	
1003	⋮

Computer Memory

Let - $\text{LOC}[A[k]] = \text{address of element } A[k]$
of array A .

As elements are stored in continuous memory locations, no need to keep track of addresses of all elements. We can only track address of first element.

$\text{Base}(A) \rightarrow \text{base address of } A$.

then -

$$\text{LOC}(A[k]) = \text{Base}(A) + w(k - \text{LB})$$

where -

w - no. of words per memory cell.

Ex: $\text{LOC}(\text{AUTO}[1932]) = 200$

$\text{LOC}(\text{AUTO}[1933]) = 204$

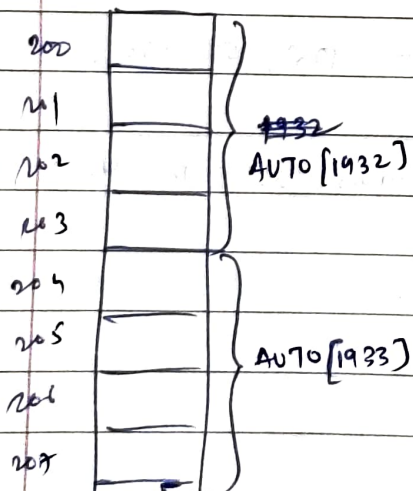
Address for $k=1965$ -

$$\text{LOC}(\text{AUTO}[1965]) = \text{Base}(\text{AUTO}) + w(1965 - \text{LB})$$

$$= 200 + 4(1965 - 1932)$$

$$= 200 + 4 \times 33$$

$$= \underline{232}$$



* Traversing the array

Algo:-

1. Repeat for $k = LB$ to UB .

Apply process to $A[k]$.

[End of loop]

2. ~~Exit~~

or

1. Set $k := LB$.

2. Repeat steps ③ & ④ while $k \leq UB$.

3. Apply process to $A[k]$.

4. $k = k + 1$

[End of while loop]

5. Exit

Example :- Find no. of years when sales of automobiles is greater than 300.

If $AUTO[k] > 300$ then set $COUNT = COUNT + 1$

or

Print Year and no. of automobiles sold

Write $k, AUTO[k]$.

* Insertion and Deletion

Insert :- Add element to array

Delete :- Remove element from array

INSERT (A, N, K, ITEM). \rightarrow Insert item at K^{th} position

1. Set $J = N$
2. Repeat 3 and 4 while $J \geq K$.
3. Set $A[J+1] = A[J]$
4. Set $J = J - 1$;

[End of loop]

5. Set $A[K] = \text{ITEM}$
6. Set $N = N + 1$
7. Exit.

DELETE (A, N, K, ITEM)

1. Set $\text{ITEM} = A[K]$
2. Repeat for $J = K$ to $N - 1$
 • Set $A[J] = A[J + 1]$.

[End of loop]

3. Set $N = N - 1$
4. Exit.

Note :- Array is not an efficient way of storing data when we have to frequently insert and delete items.

Assignment 1.

- 1) Write an algorithm for Bubble sort.
 Do its complexity analysis: Best, Average, Worst.
 Write a working example for 5 elements.
- 2) Write an algorithm for Linear search.
 Do its complexity analysis: Best, Average, Worst.

* Binary Search

BINARY (DATA, LB, UB, ITEM, LOC)

- ① Set $BEG = LB$ $END = UB$ $MID = \text{INT}((BEG + END) / 2)$
- ② Repeat steps ③ & ④ while $BEG \leq END$ and $DATA[MID] \neq ITEM$
 - ③ If $ITEM < DATA[MID]$ then:

Set $END = MID - 1$

Else

Set $BEG = MID + 1$

[End of 'if']
 - ④ Set $MID = \text{INT}((BEG + END) / 2)$
 - ⑤ If $DATA[MID] = ITEM$ then:

Set $LOC = MID$.

Else

Set $LOC = \text{NULL}$

[End of IF structure].
- ⑥ Exit.

Example:-

DATA = 11, 22, 30, 33, 40, 44, 55, 60, 66, 77, 80, 88, 99
ITEM = 40.

- ① $BEG = 1$ $END = 13$ $MID = (1+13)/2 = 7$ $DATA(7) = 55$
- ② $40 < 55$ so $END = MID - 1 = 6$
 $MID = \text{INT}((1+6)/2) = 3$ $DATA(3) = 30$
- ③ $40 > 30$ so $BEG = MID + 1 = 4$
 $MID = (4+6)/2 = 5$ $DATA(5) = 40$
We have found ITEM then $LOC = MID = 5$

Suppose ITEM = 85

	BEG	END	MID	DATA [MID]	Condition
①	1	13	7	55	$55 < 85$
②	8	13	10	77	$77 < 85$
③	11	13	12	88	$88 > 85$
④	11	12	11	80	$80 < 85$
5	⑫	⑪	$\rightarrow \text{BEG} > \text{END} \rightarrow \text{out of loop.}$		

Complexity for Binary search

won't work when value is not present or present at extreme ends

Iterations	Size of Array
0	n
1	$n/2$
2	$n/4 = n/2^2$
\vdots	
k	$n/2^k = 1$

$$n = 2^k$$

$$k = \log_2 n$$

or from the recurrence equation.

$$T(n) = T(n/2) + 1$$

$$T(n) = O(\log n)$$

Example: DATA contains 1000000 elements

the $2^{10} = 1024 > 1000$

$2^{20} \geq (1000)^2 = 1000000$ } only 20 comparisons

Multidimensional Array

Columns -

	A(1,1)	A(1,2)	A(1,3)	A(1,4)
Rows	A(2,1)	A(2,2)	A(2,3)	A(2,4)
	A(3,1)	A(3,2)	A(3,3)	A(3,4)

Two dimensional 3x4 Array

$$\text{Length} = \text{UB} - \text{LB} + 1$$

Ex : In FORTRAN INTEGER NUMB (4:5, -3:1)

Length of 1st dimension: $5 - 4 + 1 = 2$

Length of 2nd dimension: $1 - (-3) + 1 = 5$

Representation of 2-D Array in Memory

Recall $\text{LOC}(A(K)) = \text{Base}(A) + w(K - \text{LB})$

A		A	
	(1,1)		(1,1)
	(2,1)		(1,2)
	(3,1)		(1,3)
	(1,2)		(1,4)
	(2,2)		(2,1)
	(3,2)		(2,2)
	(1,3)		(2,3)
	(2,3)		(2,4)
	(3,3)		(3,1)
	(1,4)		(3,2)
	(2,4)		(3,3)
	(3,4)		(3,4)

(a) Column-Major order

(b) Row Major order

Column Major order.

$$\text{Loc}(A[J, K]) = \text{Base}(A) + w[M(K-1) + (J-1)]$$

Row Major order

$$\text{Loc}(A[J, K]) = \text{Base}(A) + w[N(J-1) + (K-1)]$$

Example :-

Consider 25×4 Matrix SCORE.

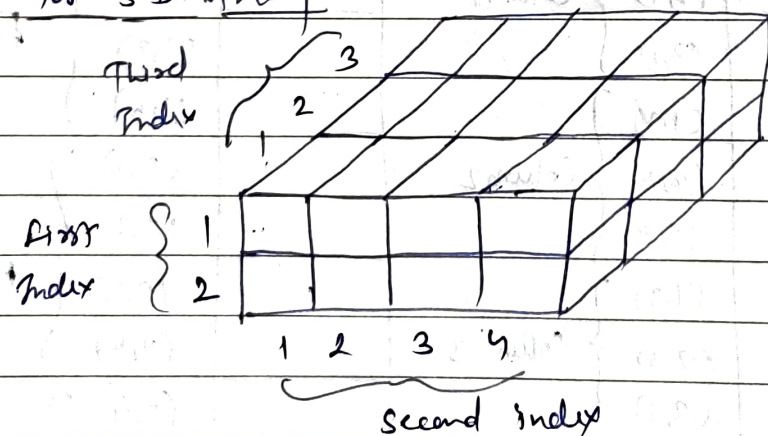
$\text{Base}(\text{SCORE}) = 200$ and $w = 4$.

It is stored using row major order.

$$\text{SCORE}(12, 3) = ?$$

$$\begin{aligned} \text{Loc}(\text{SCORE}[12, 3]) &= 200 + 4[4(12-1) + (3-1)] \\ &= 200 + 4[44 + 2] \\ &= \underline{\underline{384}} \end{aligned}$$

For 3D Array



$2 \times 4 \times 3$

3D Array

$L_1, L_2, L_3 \rightarrow \text{dimensions}$

$$\begin{aligned} E_1 &= k_1 - LB & E_2 &= k_2 - LB & E_3 &= k_3 - LB \\ &= k_1 - 1 & &= k_2 - 1 & &= k_3 - 1 \end{aligned}$$