* Mathematical Notations and Functions

→ Floor and Ceiling functions

x = real number.

x lies between two integers called the floor and the ceiling of x

$\lfloor x \rfloor$ = floor of x = greatest integer that does not exceed x

$\lceil x \rceil$ = ceiling of x = least integer that ~~does not~~ is not less than x.

If x = integer then $\lfloor x \rfloor = \lceil x \rceil$

otherwise $\lfloor x \rfloor + 1 = \lceil x \rceil$.

Examples :-

$\lfloor 3.14 \rfloor = 3$ $\quad \lfloor \sqrt{5} \rfloor = 2$

$\lfloor -8.5 \rfloor = -9$ $\quad \lfloor 7 \rfloor = 7 = \lceil 7 \rceil$.

$\lceil 3.14 \rceil = 4$ $\quad \lceil \sqrt{5} \rceil = 3$ $\quad \lceil -8.5 \rceil = -8$

→ Remainder function (Modular Arithmatic)

Let 'k' be any integer and M be the integer then -

$$k \pmod{M}$$

denotes integer remainder when 'k' is divided by M.

$k \pmod{M}$ is unique integer 'r' such that

$k = Mq + r$ where $0 \le r \le M$.

Examples :-

when k is positive. -

25 (mod 7) = 4   25 (mod 5) = 0   35 (mod 11) = 2

3 (mod 8) = 3.

when 'k' is negative - divide |k| by modulus to obtain r'. Then $k \equiv -r' \pmod{M}$.

Hence, $k \pmod{M} = M - r'$ when $r' \neq 0$

$= M - (k \bmod M)$.

-26 = -5 mod 7

$-26 \pmod 7 = 7 - 5 = 2$ / $-371 \pmod 8 = 8 - 3 = 5$

-26, closest no less than -26 and divisible by 7 is -28 so Ans = $-26 - (-28) = \underline{2}$

Remainder theorem

$-6 \bmod 5 = 2$

$-6 = 5 \times q + r$ Range of r = 0 to 4.

Let q = -1

$-6 = 5 \times (-1) + r$

$\boxed{r = -1}$ This is wrong as r = 0 to 4

let q = -2

$-6 = 5 \times (-2) + r$.

$\boxed{r = 4}$, This is correct.

$-2345 \pmod 6 = 6 - 5 = 1$ $-39 \pmod 3 = 0$.

'mod' also used for mathematical congruence.

$a \equiv b \pmod{M}$ iff M divides b - a.

$0 \equiv M \pmod{M}$ and $a \pm M \equiv a \pmod{M}$.

Arithmatic modulo 'M' refers to arithmatic operations of Add, Subs, Multi. where arithmatic value is replaced by its equivalent value in set.

$$\{0, 1, 2, \ldots, M-1\}.$$

or

$$\{1, 2, 3, \ldots, M\}.$$

Example:

Arithmatic Modulo 12 –

$6+9 \equiv 3$, $7 \times 5 \equiv 11$, $1-5 \equiv 8$, $2+10 \equiv 0 \equiv 12$

⇒ **Integer and Absolute value function**

Let x = Real no., Integer value of x $INT(x)$ convert to integer by truncating fractional part.

$INT(3.14) = 3$ $INT(\sqrt{5}) = 2$ $INT(-8.5) = -8$.

$INT(7) = 7$.

$INT(x) = \lfloor x \rfloor$ or $\lceil x \rceil$ according to whether x is +ve or –ve.

Absolute value of 'x' written as $ABS(x)$ or $|x|$ is defined as greater of x or $-x$.

$x=0$ $ABS(0) = 0$

$x \neq 0$ $ABS(x) = x$ or $ABS(x) = -x$. depending on whether x is +ve or –ve.

$|-15| = 15$ $|-3.33| = 3.33$ $|4.4| = 4.4$ $|-0.75| = 0.75$.

→ **Summation symbol ; Sums.**

$a_1 + a_2 + \cdots + a_n$ and $a_m + a_{m+1} + \cdots + a_n$.

denoted by

$\sum_{j \in} \quad \sum_{j=1}^{n} a_j$ and $\sum_{j=m}^{n} a_j$

Σ = Greek letter sigma.

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 2 + 3 + \cdots + 50 = \frac{50(51)}{2} = 1275$$

→ **Factorial:**

$n! = 1 \cdot 2 \cdot 3 \cdots (n-2)(n-1)n$

$0! = 1$

Proof: $n! = (n)(n-1)!$

for $n = 1$

$1! = 1 \cdot (1-1)!$

$1 = 1 \cdot 0!$

$\boxed{0! = 1}$

→ **Permutations:**

for n elements $n!$ permutations are possible

$n = 3 \quad 3! = 6$

a, b, c

abc, acb

bac, bca

cab, cba

⇒ Exponent and Logarithms

$a^m = a \cdot a \cdot a \cdots$ (m times) $\quad a^0 = 1 \quad a^{-m} = \frac{1}{a^m}$

$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$\log_b x = y$ then $b^y = x$.

$\log_2 8 = 3$ since $2^3 = 8$

$\log_{10} 100 = 2$ since $10^2 = 100$.

for any base -

$\log_b 1 = 0$ since $b^0 = 1$

$\log_b b = 1$ since $b^1 = b$.

⇒ Algorithmic Notations

Algorithm Finite step-by-step list of well-defined instructions for solving a particular problem.

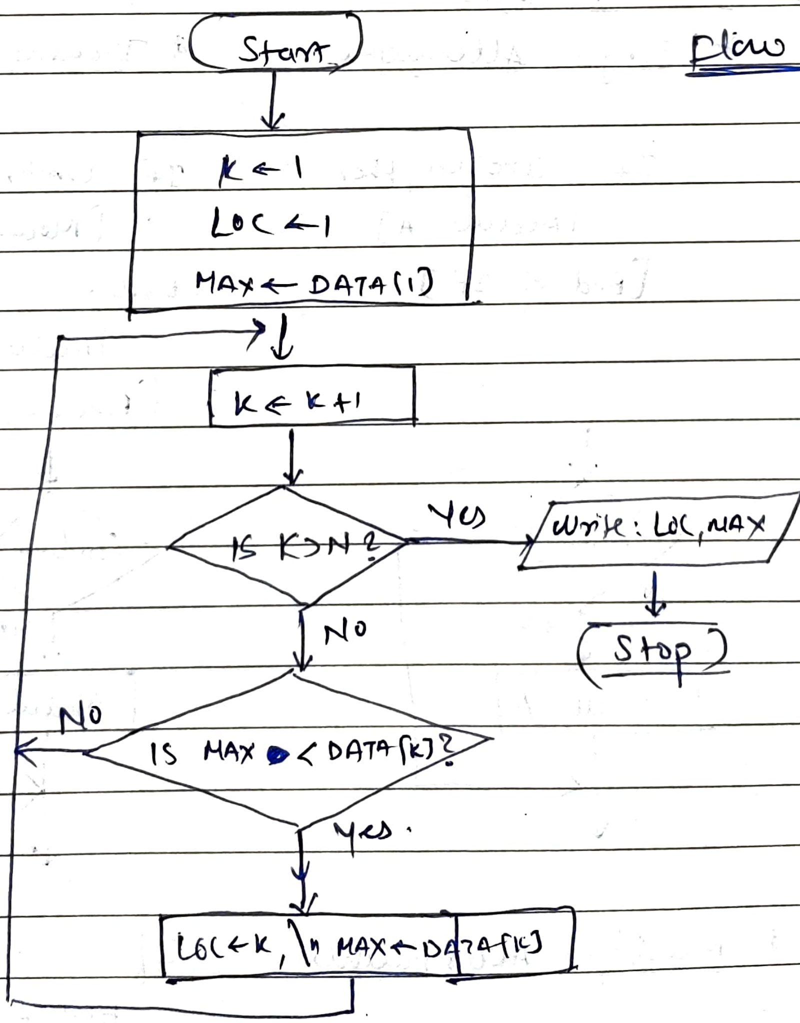
Algo to find max element in array and its location

Array - DATA []
N ⇒ size of array
LOC ⇒ store location of max element
MAX ⇒ largest element
K ⇒ counter variable

Step 1. [Initialize] Set K:=1, LOC:=1 and MAX:=DATA[1].
Step 2. [Increment counter] Set K:=K+1.
Step 3. [Test counter] If K>N then:
Write : LOC, MAX and Exit.
Step 4. [Compare and update] If MAX < DATA[K] then:
Set LOC:=K and MAX:=DATA[K].
Step 5. [Repeat loop] Go to step 2.

* Comments :- In square Brackets.
* Variable Names :- In capital letters.
* Assignment statement :- := Notation used in Pascal.
* I/p and O/p :- ~~READ~~ ~~and~~
~~Red~~ Read : Variable Name
Write : Variable Name / Message.

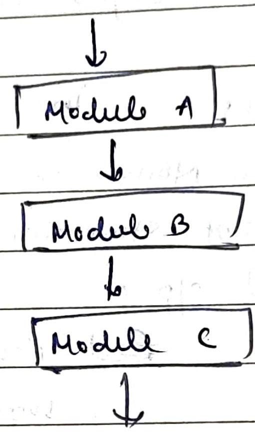
⇒ Flow chart



→ **Control Structures**

1. Sequence logic / sequential flow
2. Selection logic / Conditional flow
3. Iteration logic / Repetitive flow

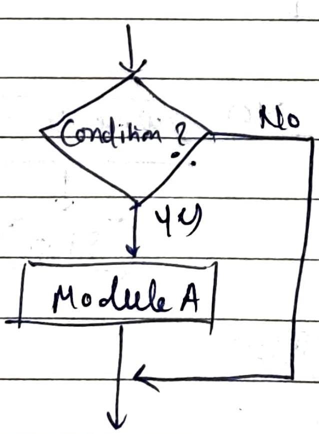
1. Sequential logic



2. Selection logic

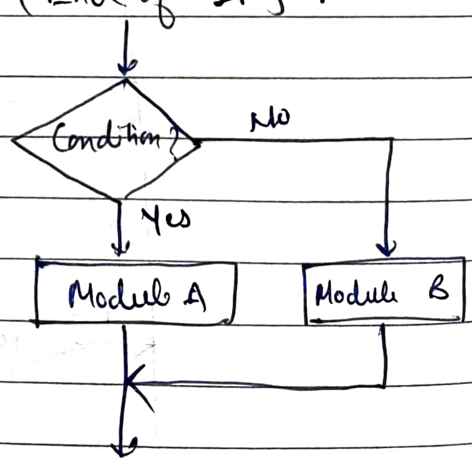
* Single Alternative

If condition, then
[Module A]
[End of If]



* Double Alternative

If condition, then
[Module A]
Else :
[Module B]
[End of If] .



* Multiple Alternatives

If

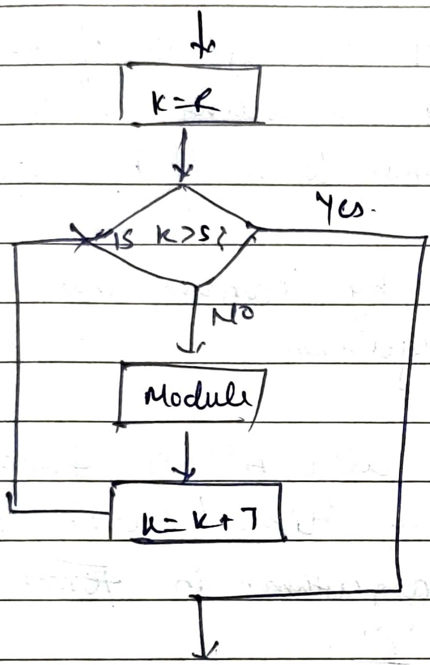
Else if .

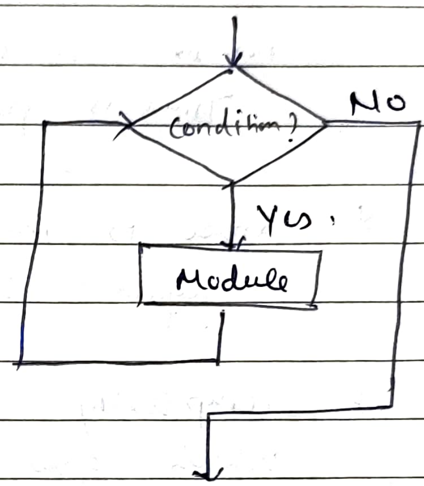
(3) Iteration logic

Begins with Repeat statement.

Repeat for K = R to S by T:
[Module].
[End of loop].

Repeat while cond^n
[Module]
[End of loop].





Repeat for structure

Repeat while structure

Algo: to find max element using loops -

1. Set K = 1 LOC = 1 and MAX = DATA(1).
2. Repeat Steps 3 and 4 while K ≤ N:
3. If MAX < DATA[K] then
 Set LOC = K and MAX := DATA[K].
 [End of If].
4. ~~Set~~ Set K = K+1.
 [End of Step 2 loop]
5. Write: LOC, MAX.
6. Exit.