Tutorial - 9 (solutions)

(1) The input to the circuit is a 4-bit binary and the output of the circuit is a 4-bit Gray code. The 4-bit binary and the corresponding Gray code are shown in the conversion table below.

4-bit binary					4-bit gray			
B_4	B_3	B_2	B_1		G_4	G_3	G_2	G_1
0	0	0	0	$\rightarrow$	0	0	0	0
0	0	0	1	$\rightarrow$	0	0	0	1
0	0	1	0	$\rightarrow$	0	0	1	1
0	0	1	1	$\rightarrow$	0	0	1	0
0	1	0	0	$\rightarrow$	0	1	1	0
0	1	0	1	$\rightarrow$	0	1	1	1
0	1	1	0	$\rightarrow$	0	1	0	1
0	1	1	1	$\rightarrow$	0	1	0	0
1	0	0	0	$\rightarrow$	1	1	0	0
1	0	0	1	$\rightarrow$	1	1	0	1
1	0	1	0	$\rightarrow$	1	1	1	1
1	0	1	1	$\rightarrow$	1	1	1	0
1	1	0	0	$\rightarrow$	1	0	1	0
1	1	0	1	$\rightarrow$	1	0	1	1
1	1	1	0	$\rightarrow$	1	0	0	1
1	1	1	1	$\rightarrow$	1	0	0	0

From the truth table, we observe that

1. The entries for G_4 are exactly the same as those for B_4. Therefore, $G_4 = B_4$.
2. The entries for G_3 are:

$G_3 = 1$, only when either $B_4 = 1$ or $B_3 = 1$.

$G_3 = 0$ for $B_4 = B_3 = 0$, and

$B_4 = B_3 = 1$

This is an X-OR operation of B_4 and B_3.

Therefore, $G_3 = B_4 \oplus B_3$

3. The entries for G_2 are:

$G_2 = 1$, only when either $B_3 = 1$ or $B_2 = 1$.

$G_3 = 0$ for both $B_3 = B_2 = 1$, and

$B_3 = B_2 = 0$.

This is an X-OR operation of B_3 and B_2.

Therefore, $G_2 = B_3 \oplus B_2$.

4. The entries for G_1 are:

$G_1 = 1$, only when $B_2 = 1$ or $B_1 = 1$.

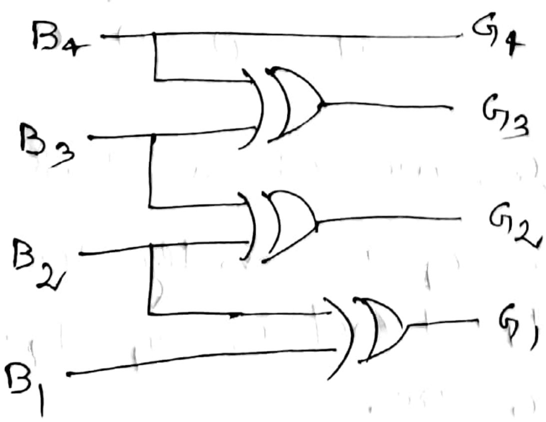
$G_1 = 0$, for both $B_2 = B_1 = 1$ and

$B_2 = B_1 = 0$.

This is an X-OR operation of B_2 and B_1.

Therefore, $G_1 = B_2 \oplus B_1$

So, the conversion can be achieved by using three X-OR gates as shown in Figure below.



NOTE: The same circuit can be obtained by implementing the minimal expressions for G_4, G_3, G_2, and G_1 in terms of B_4, B_3, B_2 and B_1. obtained by minimizing the K-maps. Thus, the minimal expressions for G_4, G_3, G_2 and G_1 are:

$$G_4 = B_4$$

$$G_3 = \overline{B_4} B_3 + B_4 \overline{B_3} = B_4 \oplus B_3$$

$$G_2 = \overline{B_3} B_2 + B_2 \overline{B_3} = B_3 \oplus B_2$$

$$G_1 = \overline{B_2} B_1 + B_1 \overline{B_2} = B_2 \oplus B_1$$

The K-map for G_4 and its minimization is shown below.

$B_4 B_3$ \ $B_2 B_1$				
	0	1	3	2
	4	5	7	6
	12: 1	13: 1	15: 1	14: 1
	8: 1	9: 1	11: 1	10: 1

$$G_4 = B_4$$

(2) The 4-bit input Gray code and the corresponding output binary numbers are shown in the table next page.

4-bit gray				4-bit binary			
G_4	G_3	G_2	G_1	B_4	B_3	B_2	B_1
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	1
0	1	0	1	0	1	1	0
0	1	0	0	0	1	1	1
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	1
1	0	1	ϕ	1	0	1	0
1	0	1	0	1	0	1	1
1	1	1	0	1	1	0	0
1	1	1	1	1	1	0	1
1	1	0	1	1	1	1	0
1	1	0	0	1	1	1	1

From the truth table, we observe that

1. The entries for B_4 are exactly the same as those for G_4. Therefore, $B_4 = G_4$.

2. The entries for B_3 are:
$B_3 = 1$, only when the number of 1s in G_4 and G_3 is an odd number. Otherwise $B_3 = 0$.
So, B_3 is the modulo sum of G_4 and G_3.
Therefore, $B_3 = G_4 \oplus G_3$

3. The entries for B_2 are

$B_2 = 1$, only when the number of 1s in G_4, G_3 and G_2 is an odd number. Otherwise $B_2 = 0$.

So, B_2 is the modulo sum of G_4, G_3, and G_2, i.e. modulo sum of B_3 and G_2. Therefore,

$B_2 = B_3 \oplus G_2$.

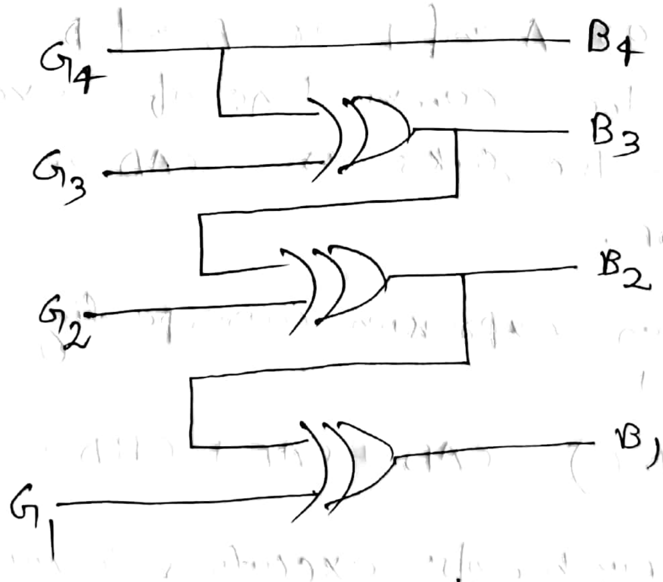
4. The entries for B_1 are:

$B_1 = 1$, only when the number of 1s in G_4, G_3, G_2 and G_1 is an odd number. Otherwise $B_1 = 0$.

So, B_1 is the modulo sum of G_4, G_3, and G_2 and G_1, i.e. modulo sum of B_2 and G_1.

Therefore, $B_1 = B_2 \oplus G_1$.

So, the conversion can be achieved by using three X-OR gates as shown in Figure below.



NOTE: The same circuit can be obtained by implementing the minimal expression for G_4, G_3, G_2, and G_1 in terms of B_4, B_3, B_2 and B_1 obtained by minimizing the K-maps.

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3. The following table indicates the executives and the locks they can open.

Executive	Keys for locks: v	w	x	y	z
Mr. A	✓		✓		
Mr. B	✓			✓	
Mr. C		✓		✓	
Mr. D			✓		✓
Mr. E	✓				✓

We see that the key for lock w is only with Mr C. So, Mr. C is the essential executive, without whom the safe cannot be opened. Once C is present, he can open lock y too. As seen from the table, the remaining locks v, x, and z can be opened by A and D or A and E or B and D or D and E. So the combinations of executives who can open the locks are CAD or CAE or CBD or CDE.

The boolean expression corresponding to the above statement is

$$f(A,B,C,D,E) = CAD + CAE + CBD + CDE$$

The minimal number of executives required is 3.

(4) From the statement, the boolean expression must be in the POS form given by

$$f = (A_3 + \overline{A_2} + \overline{A_1} + \overline{A_0})(\overline{A_3} + A_2 + \overline{A_1} + \overline{A_0})(\overline{A_3} + \overline{A_2} + \overline{A_1} + \overline{A_0})$$

where each non-complemented variable represents a 1 and the complemented variable a 0.

since $X \cdot X = X$, the minimal expression is given by

$$f_{min} = (A_3 + \overline{A_2} + \overline{A_1} + \overline{A_0})(\overline{A_3} + A_2 + \overline{A_1} + \overline{A_0})(\overline{A_3} + \overline{A_2} + \overline{A_1} + \overline{A_0})$$

$$= (\overline{A_2} + \overline{A_1} + \overline{A_0})(\overline{A_3} + \overline{A_1} + \overline{A_0})$$

$$= (\overline{A_2}\,\overline{A_3} + \overline{A_1} + \overline{A_0})$$

(5) Let the variables $w, x, y,$ and z assume the truth value in the following cases.

$w = 1$, if the applicant has been involved in a car accident.

$x = 1$, if the applicant is married

$y = 1$, if the applicant is a male

$z = 1$, if the applicant is under 25

The policy can be issued when any one of the conditions 1, 2, 3, 4 or 5 is met.

The conditions 1, 2, 3, 4, and 5 are represented algebraically by $x\bar{y}\bar{z}$, $\bar{y}z$, $xyz\bar{w}$, xyw, $xy\bar{z}\bar{w}$.

Therefore,

$$f(w, x, y, z) = x\bar{y}\bar{z} + \bar{y}z + xyz\bar{w} + xyw + xy\bar{z}\bar{w}$$

$$= xy\bar{w}(z+\bar{z}) + xyw + \bar{y}(z+x\bar{z})$$

$$= xy\bar{w} + xyw + \bar{y}(z+\bar{z})(z+x)$$

$$= xy(w+\bar{w}) + \bar{y}(z+x)$$

$$= xy + x\bar{y} + \bar{y}z$$

$$= x(y+\bar{y}) + \bar{y}z$$

$$= x + \bar{y}z$$

So the policy can be issued if the applicant is either married or is a female under 25.