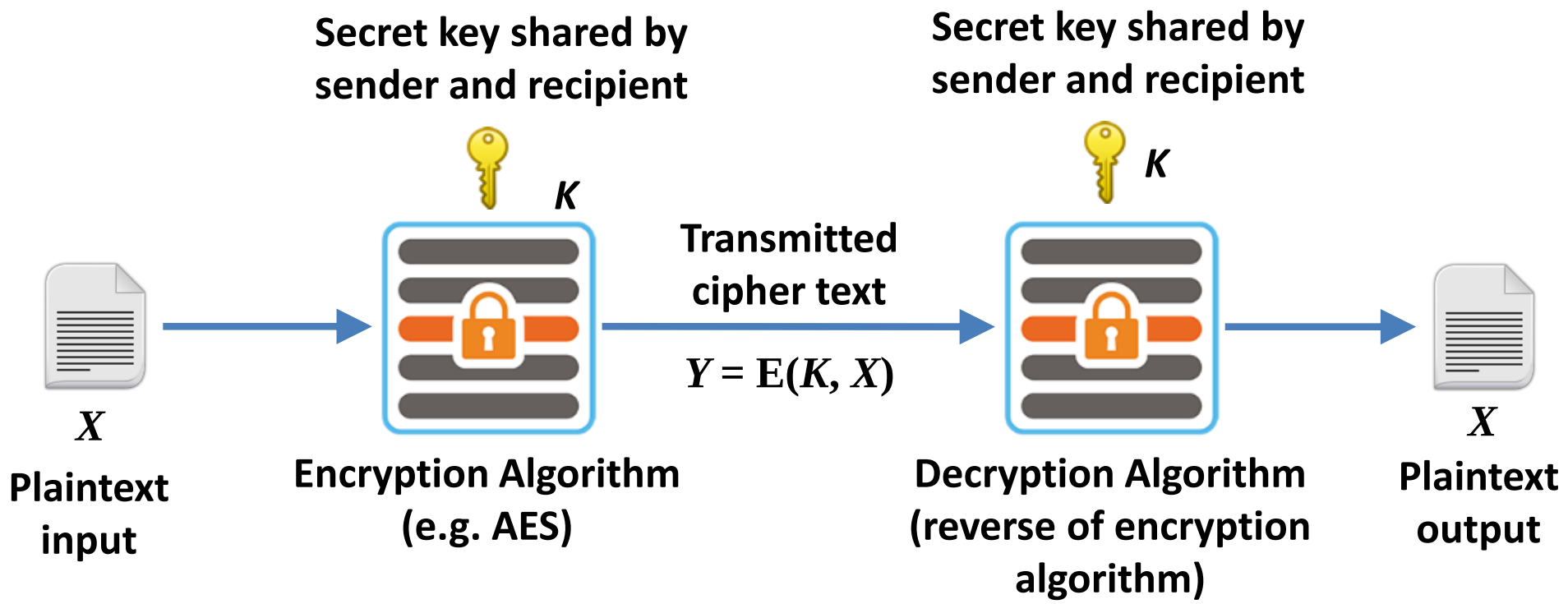
Asymmetric Ciphers

Outline

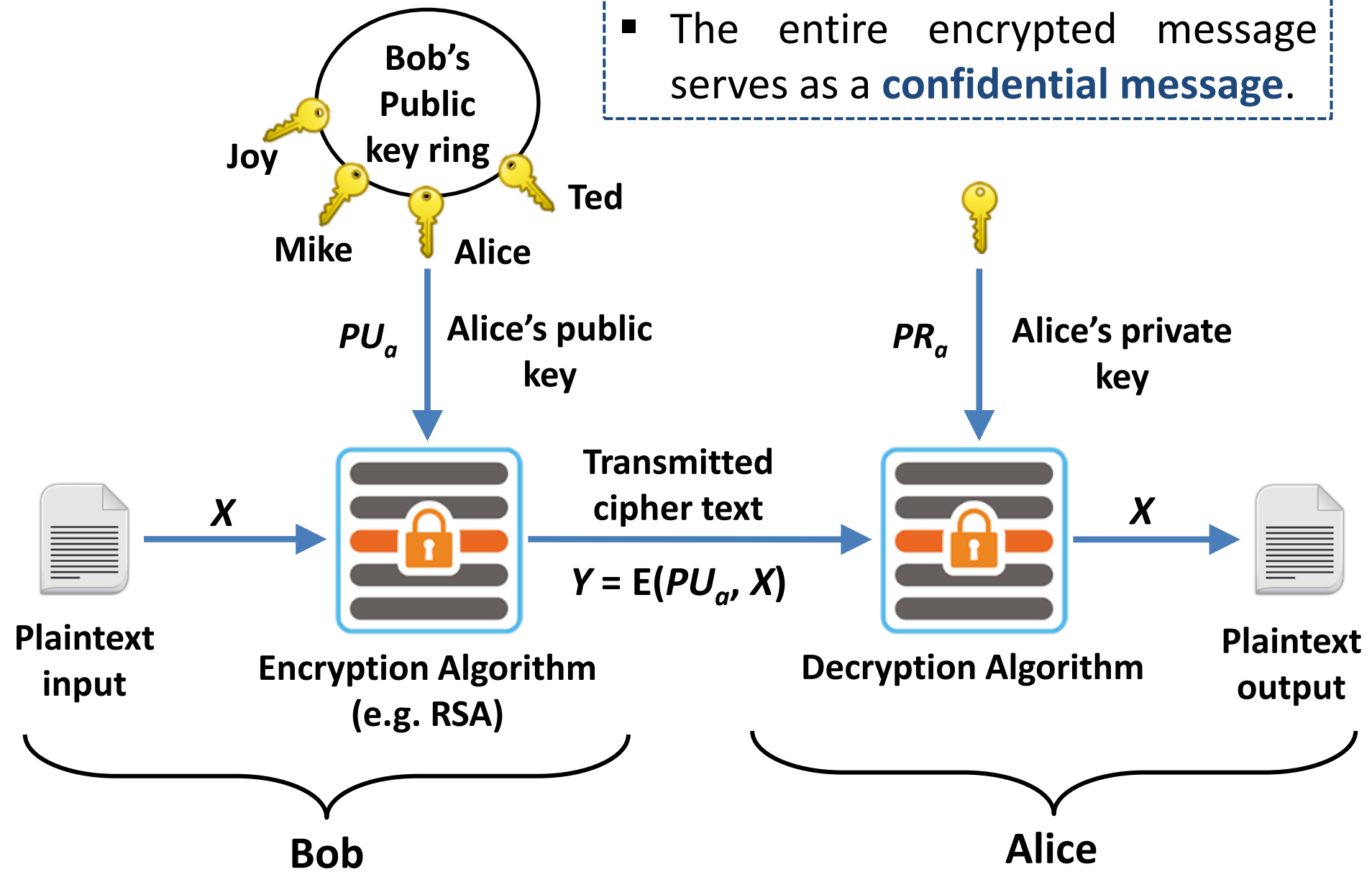
- Public Key Cryptosystems with Applications
- Requirements and Cryptanalysis
- RSA algorithm
- RSA computational aspects and security
- Diffie-Hillman Key Exchange algorithm
- Man-in-Middle attack

Symmetric Key Encryption



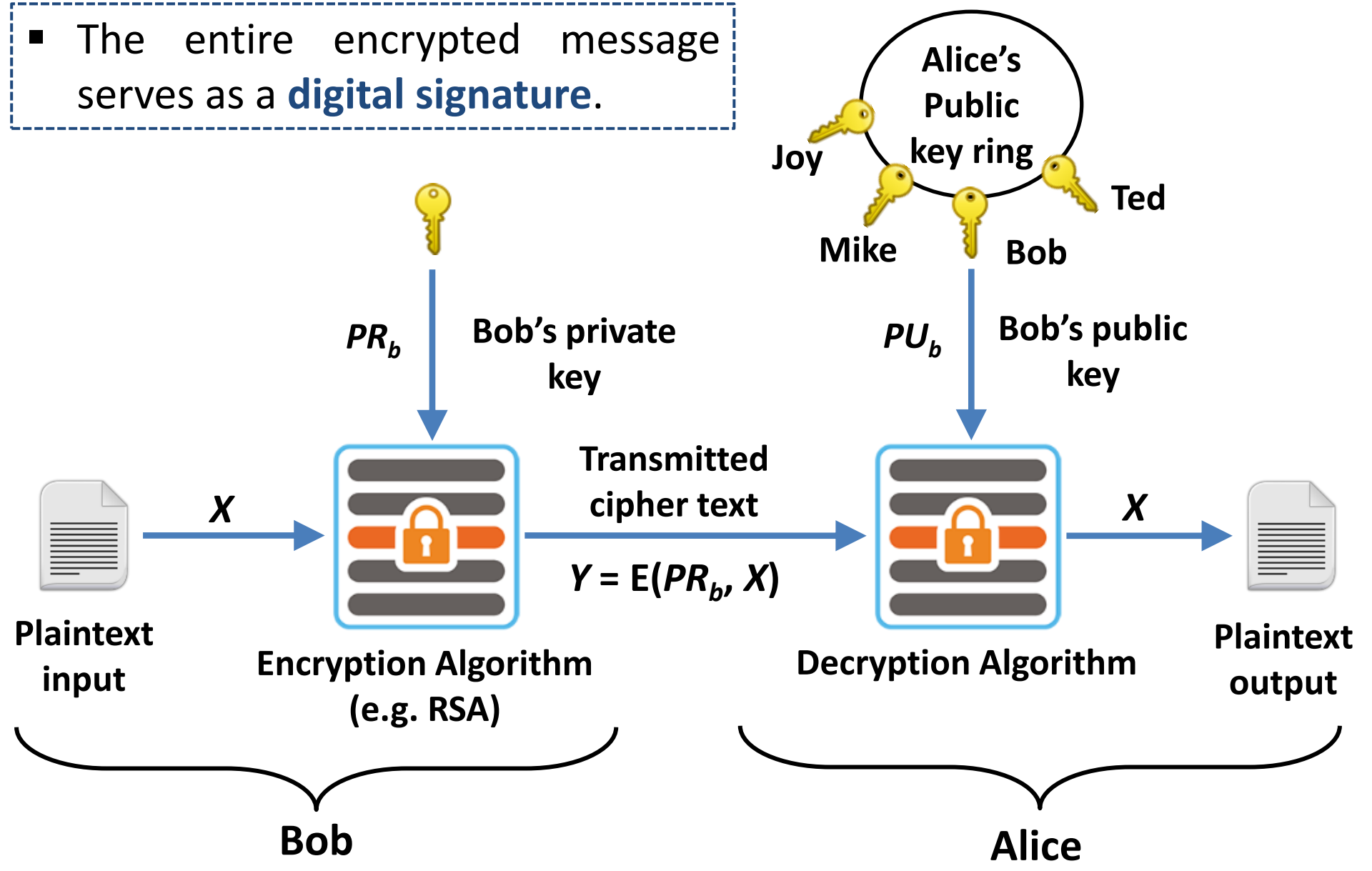
Asymmetric Key Encryption with Public Key

- The entire encrypted message serves as a **confidential message**.

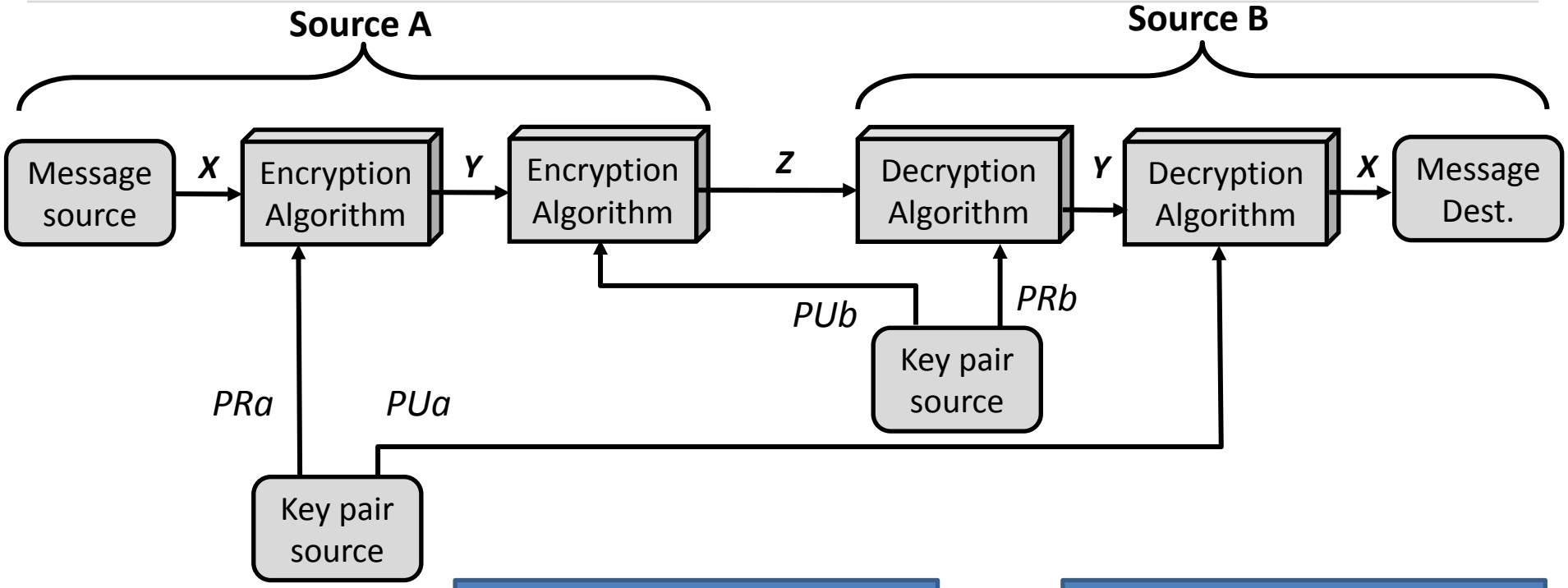


Asymmetric key Encryption with Private Key

- The entire encrypted message serves as a **digital signature**.



Authentication and Confidentiality



$Z = \mathrm{E}(PU_b, \mathrm{E}(PR_a, X))$

$X = \mathrm{D}(PU_a, \mathrm{D}(PR_B, Z))$

Applications for Public-Key Cryptosystems

- **Encryption/decryption:** The sender encrypts a message with the recipient's public key.
- **Digital signature:** The sender "signs" a message with its private key. Signing is achieved by a cryptographic algorithm applied to the message or to a small block of data that is a function of the message.
- **Key exchange:** Two sides cooperate to exchange a session key. Several different approaches are possible, involving the private key(s) of one or both parties. E.g. Diffie–Hellman key exchange scheme

RSA Algorithm

- **RSA** is a block cipher in which the Plaintext and Ciphertext are represented as integers between **0** and **n-1** for some n.
- Large messages can be broken up into a number of blocks.
- Each block would then be represented by an integer.

Step-1: Generate Public key and Private key

Step-2: Encrypt message using Public key

Step-3: Decrypt message using Private key

Step-1: Generate Public key and Private key

- Select two large prime numbers: **p** and **q**
- Calculate modulus : **n = p * q**
- Calculate Euler's totient function : **ϕ(n) = (p-1) * (q-1)**
- Select **e** such that **e** is **relatively prime** to **ϕ(n)** and **1 < e < ϕ(n)**

Two numbers are relatively prime if they have no common factors other than 1.

- Determine **d** such that **d * e ≡ 1 (mod ϕ(n))**
- Publickey : **PU = { e, n }**
- Privatekey : **PR = { d, n }**

Step-1: Generate Public key and Private key

- Select two large prime numbers: **p = 3** and **q = 11**
- Calculate modulus : **n = p * q, n = 33**
- Calculate Euler's totient function : **ϕ(n) = (p-1) * (q-1)**

  ϕ(n) = (3 – 1) * (11 – 1) = 20
- Select **e** such that **e** is **relatively prime** to **ϕ(n)** and **1 < e < ϕ(n)**
- We have several choices for **e : 7, 11, 13, 17, 19** Let's take **e = 7**
- Determine **d** such that **d * e ≡ 1 (mod ϕ(n))**
- **? * 7 ≡ 1 (mod 20), 3 * 7 ≡ 1 (mod 20)**

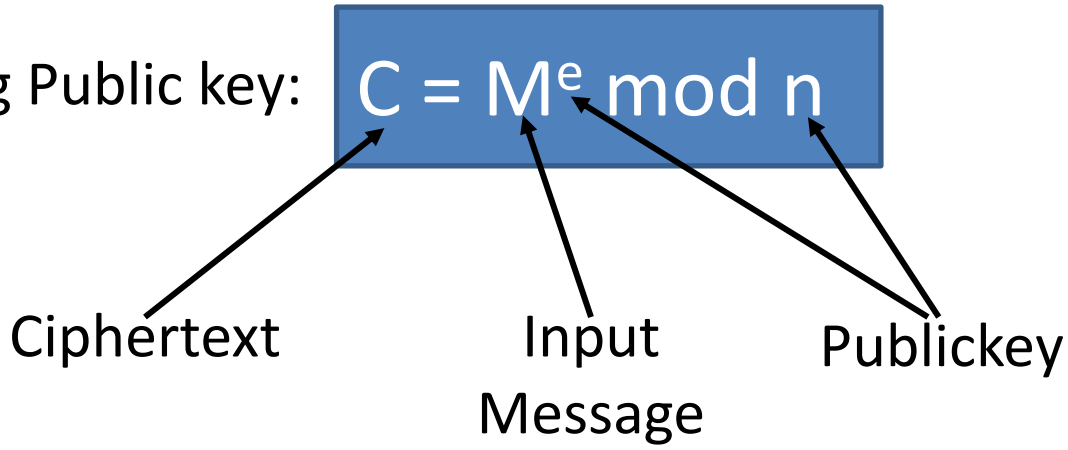
- Public key : **PU = { e, n } , PU = { 7, 33 }**
- Private key : **PR = { d, n }, PR = { 3, 33 }**

- This is equivalent to finding d which satisfies de = 1 + j.ϕ(n) where j is any integer.
- We can rewrite this as d = (1 + j. ϕ(n)) / e

*Find Modular Multiplicative Inverse using Extended Euclidean algorithm

Step-2 : Encrypt Message

- Encryption Using Public key: $C = M^e \bmod n$



PU = { e, n } , PU = { 7, 33 }

For message M = 14

$C = 14^7 \bmod 33$

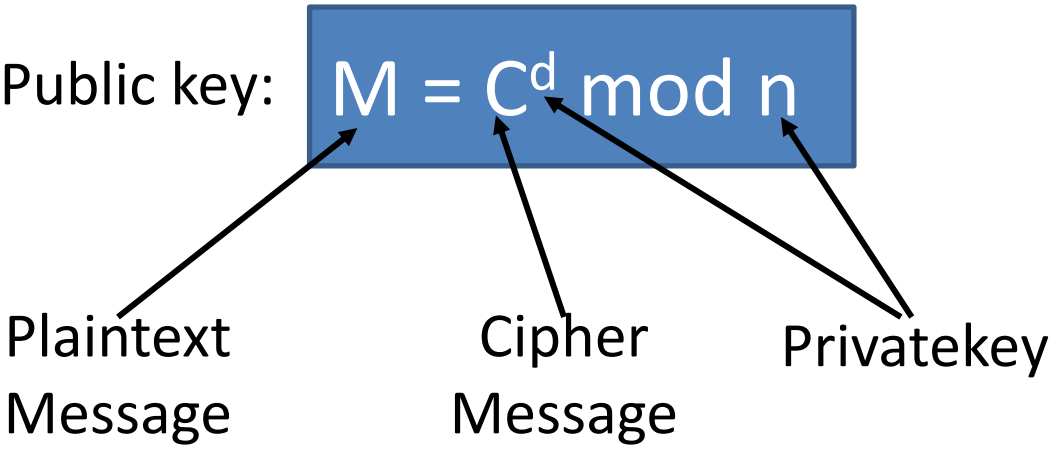
$C = [(14^1 \bmod 33) \times (14^2 \bmod 33) \times (14^4 \bmod 33)] \bmod 33$

$C = (14 \times 31 \times 4) \bmod 33 = 1736 \bmod 33$

$C = 20$

Step-3 : Decrypt Message

- Encryption Using Public key: $M = C^d \bmod n$



PR = { d, n } , PR = { 3, 33 }

For Ciphertext C = 20

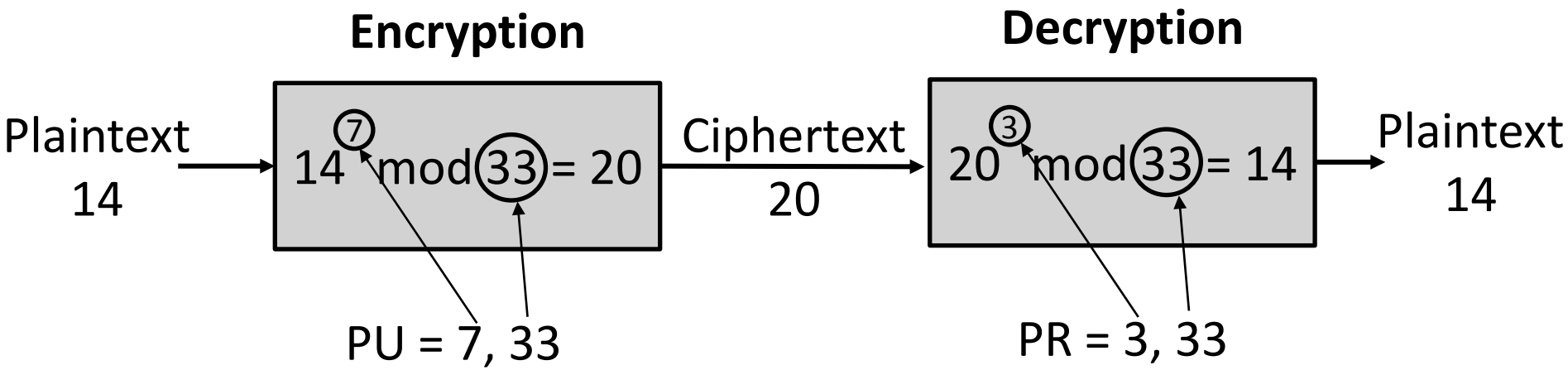
$M = 20^3 \bmod 33$

$M = [(20^1 \bmod 33) \times (20^2 \bmod 33)] \bmod 33$

$M = (20 \times 4) \bmod 33 = 80 \bmod 33$

$M = 14$

Example RSA Algorithm



RSA Example

- Find n, ϕ(n), e, d for p=7 and q= 19 then demonstrate encryption and decryption for M = 6

n = p * q = 7 * 19 = 133

ϕ(n) = (p – 1) * (q – 1) = 108

Finding e relatively prime to 108
e = 2 => GCD(2, 108) = 2 (no)
e = 3 => GCD(3, 108) = 3 (no)
e = 5 => GCD(5, 108) = 1 (Yes)

- Finding d such that (d * e) mod ϕ(n) = 1
- We can rewrite this as d = (1 + j . ϕ(n)) / e

j = 0 => d = 1 / 5 = 0.2 ← integer ? (no)
j = 1 => d = 109 / 5 = 21.8 ← integer ? (no)
j = 2 => d = 217 / 5 = 43.4 ← integer ? (no)
j = 3 => d = 325 / 5 = 65 integer ? (yes)
*OR Find Modular Multiplicative Inverse using Extended Euclidean algorithm

Public key :
PU = { e, n } = {5, 133}
Private key :
PR = { d, n } = {65, 133}

RSA Example – cont…

- Encryption:

$C = M^e \bmod n$

$PU = \{ e, n \}, PU = \{ 5, 133 \}$

For message $M = 6$

$C = 6^5 \bmod 133$
$C = 7776 \bmod 33$
$C = 62$

- Decryption:

$M = C^d \bmod n$

$PR = \{ d, n \}, PU = \{ 65, 133 \}$

For $C = 62$

$M = 62^{65} \bmod 133$
$M = 2666 \bmod 33$
$M = 6$

RSA Example

- P and Q are two prime numbers. P=7, and Q=17. Take public key E=5. If plain text value is 10, then what will be cipher text value according to RSA algorithm?
- n = 119
- $\phi(n) = 96$
- e = 5
- d = 77
- PU = { 5, 119 }
- PR = {77, 119}
- $C = 10^5 \bmod 119 \Rightarrow C = 40$

RSA Security

- possible approaches to attacking RSA are:
  - brute force key search - infeasible given size of numbers
  - mathematical attacks - based on difficulty of computing ø(n), by factoring modulus n
  - timing attacks - on running of decryption
  - chosen ciphertext attacks - given properties of RSA

Mathematical Attack

- mathematical approach takes 3 forms:
  - factor $n=p.q$, hence compute $\varnothing(n)$ and then d
  - determine $\varnothing(n)$ directly and compute d
  - find d directly
- currently assume 1024-2048 bit RSA is secure

Timing Attacks

- Developed by Paul Kocher in mid-1990's
- Exploit timing variations in operations
  - E.g. multiplying by small vs large number
- Infer operand size based on time taken
- Infer time taken in exponentiation
- Countermeasures
  - use constant exponentiation time
  - add random delays
  - blind values used in calculations

Chosen Ciphertext Attacks

- RSA is vulnerable to a Chosen Ciphertext Attack (CCA)
- Attackers can choose ciphertexts & get decrypted plaintext back
- Countermeasure with random pad of plaintext

Primitive root

- Let p be a prime number
- Then a is a primitive root for p, if the powers of a modulo p generates all integers from **1 to $p-1$** in some permutation.

$$a \bmod p, a^2 \bmod p, \ldots, a^{p-1} \bmod p$$

- Example: p = 7 then primitive root is 3 because powers of 3 mod 7 generates all the integers from 1 to 6

$$3^1 = 3 \equiv 3 \pmod 7$$
$$3^2 = 9 \equiv 2 \pmod 7$$
$$3^3 = 27 \equiv 6 \pmod 7$$
$$3^4 = 81 \equiv 4 \pmod 7$$
$$3^5 = 243 \equiv 5 \pmod 7$$
$$3^6 = 729 \equiv 1 \pmod 7$$

Discrete Logarithm

- For any integer b and a primitive root a of prime number p, we can find a unique exponent i such that

$$b = a^i (mod\ p)\ \ where\ 0 \leq i \leq (p-1)$$

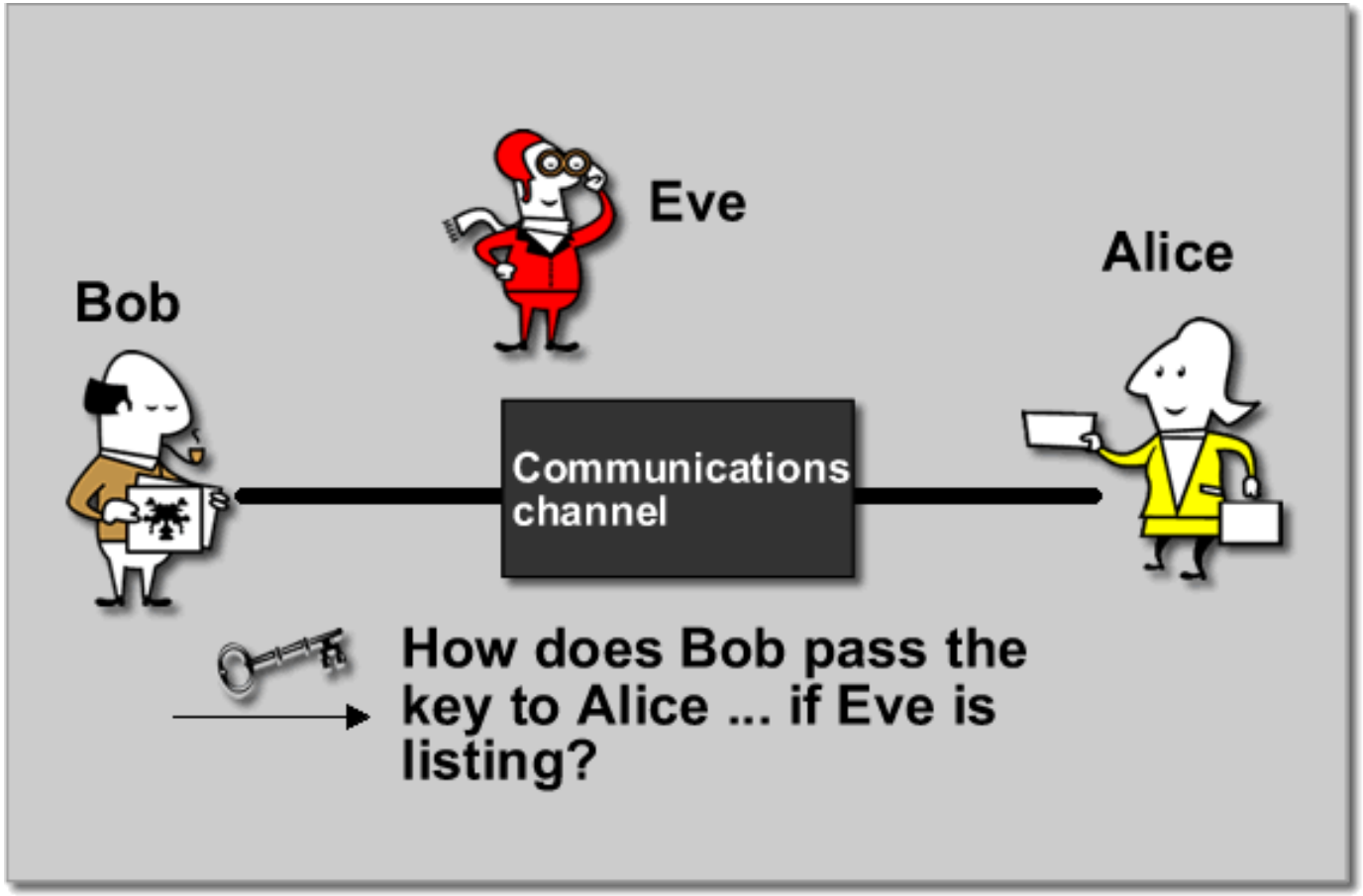
- The exponent i is referred as the discrete logarithm of b for the base a, mod p. It expressed as below.

$$\mathrm{dlog}_{a,p}(b)$$

Key Establishment Problem

- Securing communication requires that the data is encrypted before being transmitted
- Associated with encryption and decryption are keys that must be shared by the participants.
- The problem of securing the data then becomes the problem of securing the key establishment
- Task: If the participants do not physically meet, then how do the participants establish a shared key?

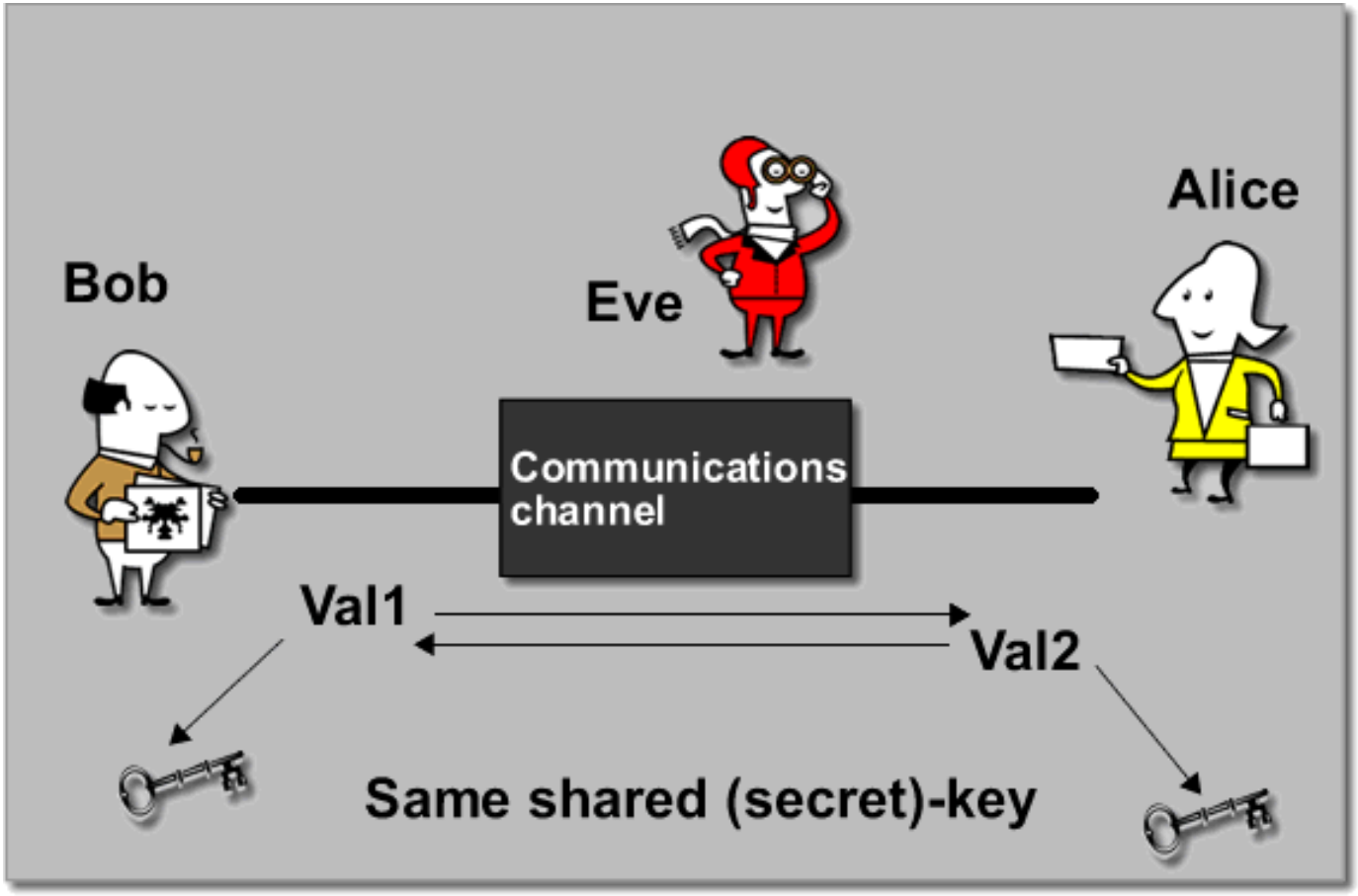
Key Establishment Problem (cont.)



Diffie-Hellman Key Agreement

- Discovered by Whitfield Diffie and Martin Hellman
- Diffie-Hellman key agreement protocol
  - Exponential key agreement
  - Allows two users to exchange a secret key
  - Requires no prior secrets
  - Real-time over an **untrusted** network

Diffie-Hellman Key Agreement (cont.)



Diffie-Hellman Algorithm

- Requires two large numbers, one prime p, and generator g (2 <= g <= p-2), a primitive root of p, (p and g are both publicly available numbers).
- Users pick random private values x (x < p) and y (y < p)
- Compute public values (keys)
  - $R1 = g^x \bmod p$
  - $R2 = g^y \bmod p$
- Keys R1 and R2 are exchanged
- Compute shared, private key
  - $k_{alice} = (R2)^x \bmod p$
  - $k_{bob} = (R1)^y \bmod p$
- Algebraically it can be shown that $k_{alice} = k_{bob}$
  - Users now have a symmetric secret key to encrypt

Proof

- We know

  $R1 = g^x \bmod p$

  $R2 = g^y \bmod p$

- $k_{alice} = (R2)^x \bmod p$

  $= (g^y \bmod p)^x \bmod p$

  $= (g^y)^x \bmod p$

  $= (g)^{yx} \bmod p$

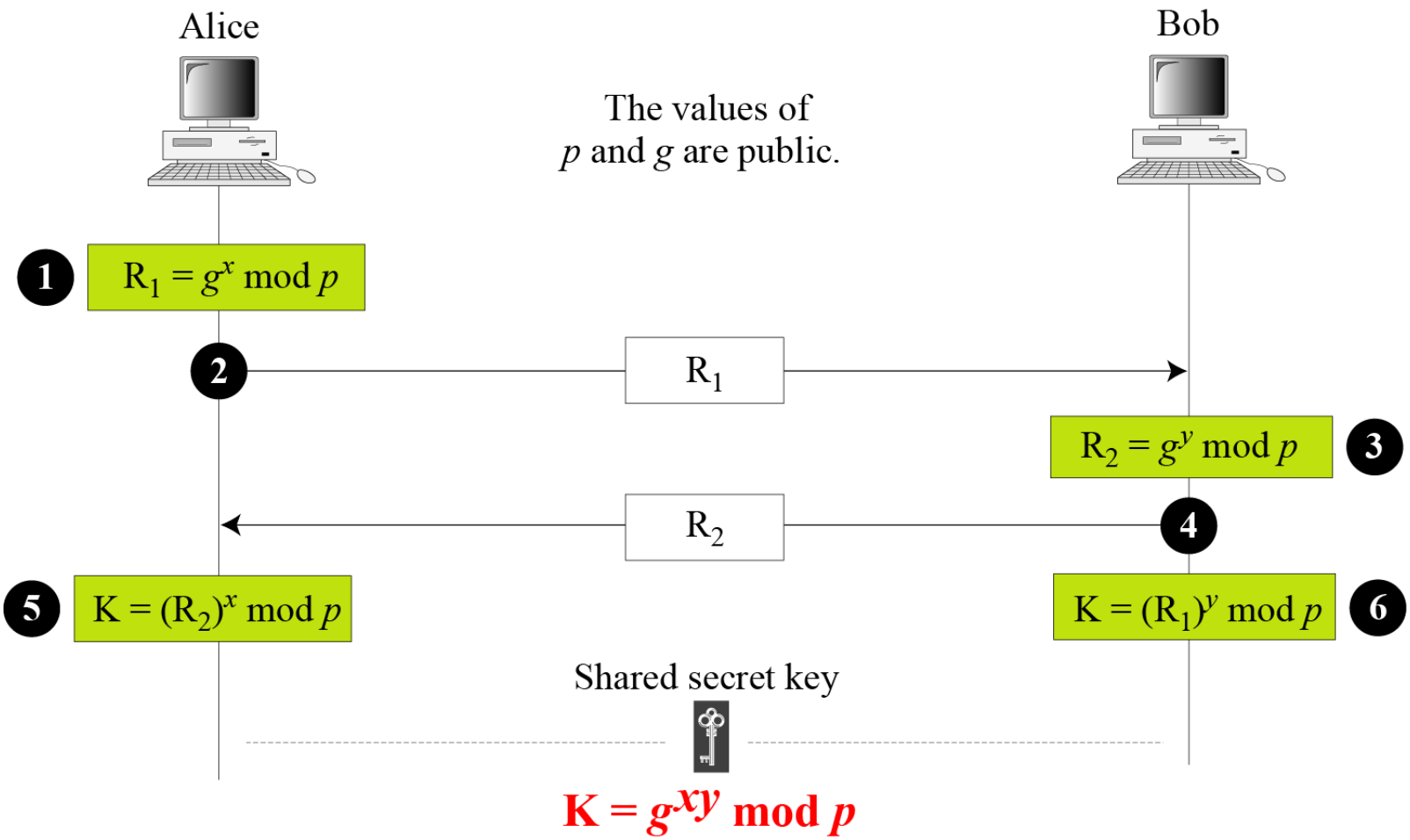
  $= (g^x)^y \bmod p$

  $= (g^x \bmod p)^y \bmod p$

  $= (R1)^y \bmod p$

  $= k_{bob}$

Key Exchange



Example

- Alice and Bob get public numbers
  - P = 19, G = 3 [Primitive roots of modulus 19 are 2,3,10,13,14,15]
- Alice and Bob pick private values x=15 & y=10 respectively
- Alice and Bob compute public values
  - R1 = 3^{15} mod 19 = 12
  - R2 = 3^{10} mod 19 = 16
  - Alice and Bob exchange public numbers
- Alice and Bob compute symmetric keys
  - kalice = $(R2)^x$ mod p = $(16)^{15}$ mod 19 = 7
  - kbob = $(R1)^y$ mod p = $(12)^{10}$ mod 19 = 7
- Alice and Bob now can talk securely!

Example: P=23 and G=5 ?

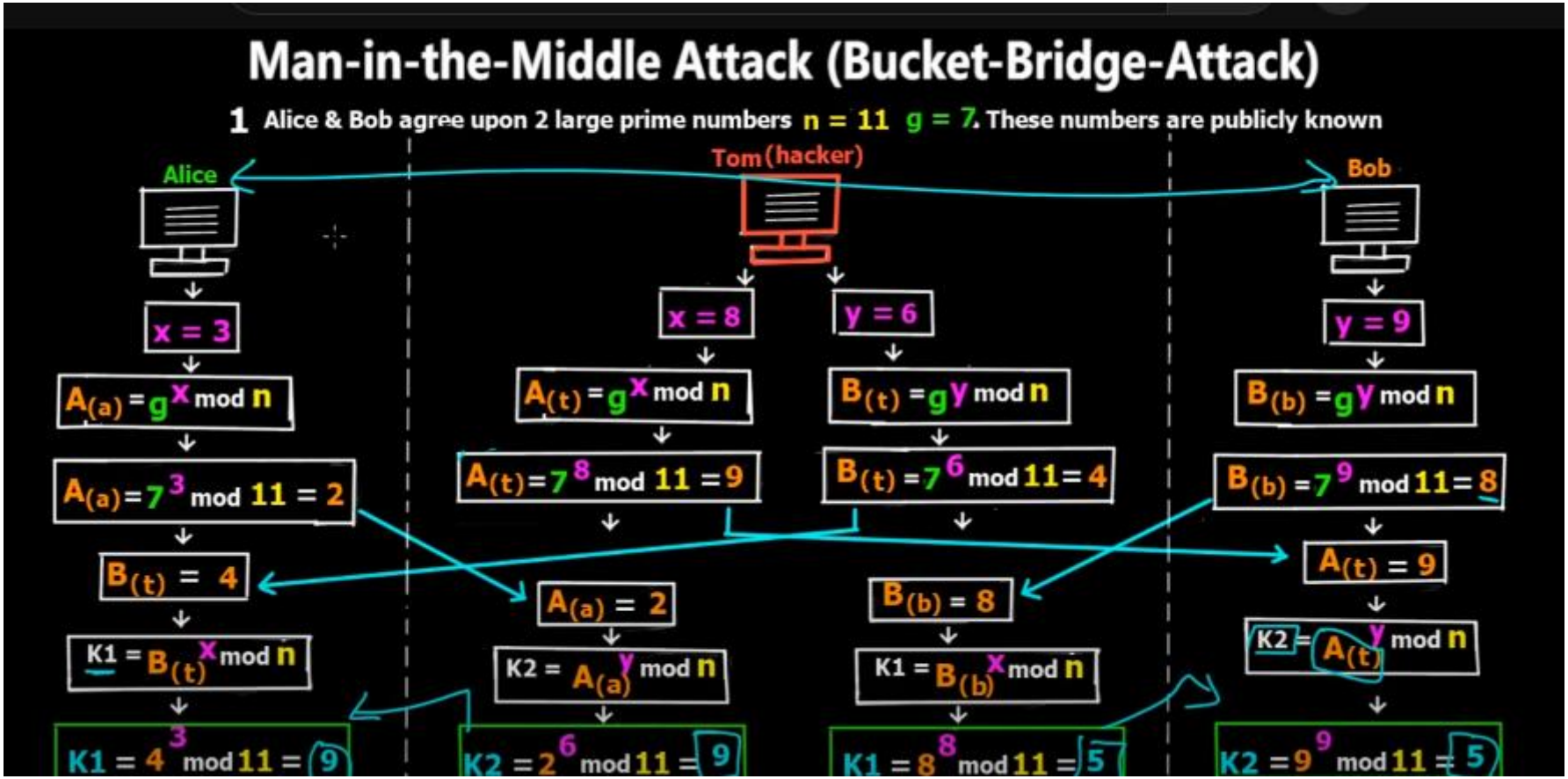
Security of Diffie-Hellamn

- This protocol susceptible to two attacks:
  - The Man-in-the-middle attack
  - The Discrete logarithmic attack

Man in the middle attack

- Suppose Alice and Bob wish to exchange keys, and Darth is the adversary.

1. Darth prepares for the attack by generating two random private keys X_{D1} and X_{D2} and then computes corresponding public keys Y_{D1} and Y_{D2}.
2. Alice transmits Y_A to Bob.
3. Darth intercepts Y_A and transmits Y_{D1} to Bob. Darth also calculates $K_2=(Y_A)^{XD2} \bmod q$.
4. Bob receives Y_{D1} and calculates $K_1 = (Y_{D1})^{XB} \bmod q$.
5. Bob transmits Y_B to Alice.
6. Darth intercepts Y_B and transmits Y_{D2} to Alice. Darth calculates $K_1=(Y_B)^{XD1} \bmod q$.
7. Alice receives Y_{D2} and calculates $K_2 = (Y_{D2})^{XA} \bmod q$.


Man-in-the-Middle Attack (Bucket-Bridge-Attack)
1 Alice & Bob agree upon 2 large prime numbers n = 11 g = 7. These numbers are publicly known
Alice
Tom (hacker)
Bob
x = 3
x = 8
y = 6
y = 9
A(a) = g^x mod n
A(t) = g^x mod n
B(t) = g^y mod n
B(b) = g^y mod n
A(a) = 7^3 mod 11 = 2
A(t) = 7^8 mod 11 = 9
B(t) = 7^6 mod 11 = 4
B(b) = 7^9 mod 11 = 8
B(t) = 4
A(a) = 2
B(b) = 8
A(t) = 9
K1 = B(t)^x mod n
K2 = A(a)^y mod n
K1 = B(b)^x mod n
K2 = A(t)^y mod n
K1 = 4^3 mod 11 = 9
K2 = 2^6 mod 11 = 9
K1 = 8^8 mod 11 = 5
K2 = 9^9 mod 11 = 5

ElGamal Encryption Algorithm

- **ElGamal encryption** uses asymmetric key encryption for communicating between two parties and encrypting the message. It is based on the Diffie–Hellman key exchange.
- This cryptosystem is based on the difficulty of finding **discrete logarithm** in a cyclic group that is even if we know g^a and g^k, it is extremely difficult to compute g^{ak}.
- ElGamal is generally used to encrypt only the symmetric key (Not the plaintext). This is because asymmetric cryptosystems like ElGamal are usually slower than symmetric ones

ElGamal Encryption

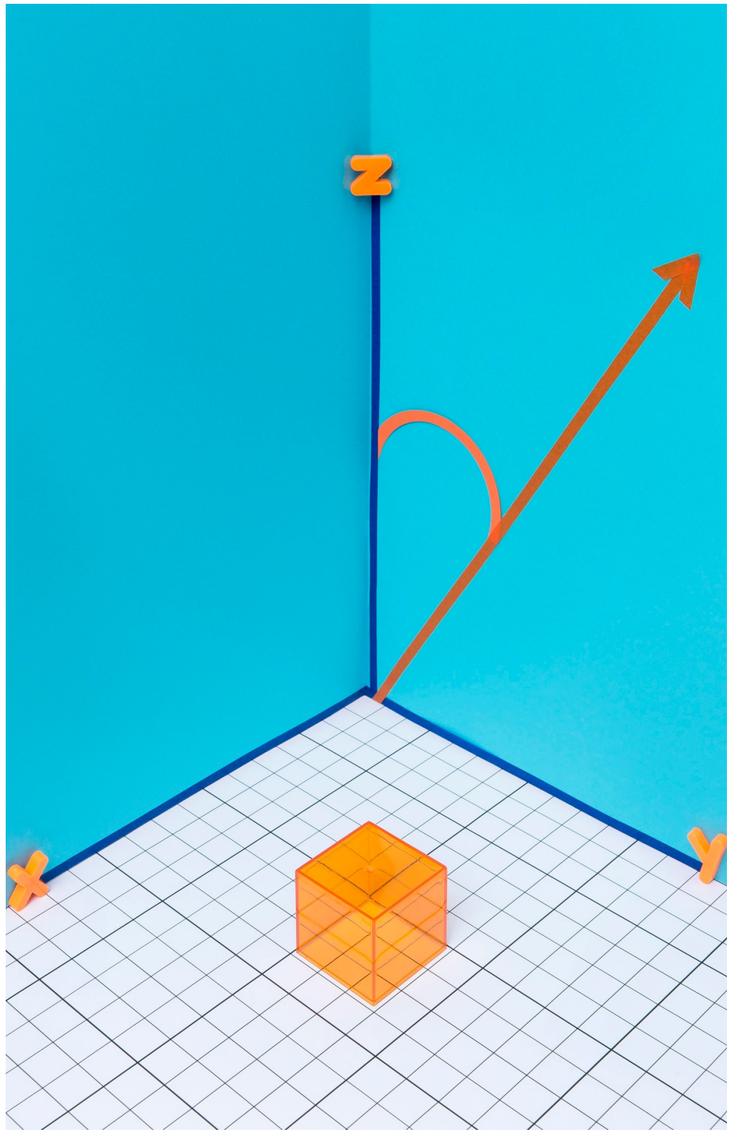
- **Keys & parameters**
  - Domain parameter = {p, g}
  - Choose x $\in$ [1, p-1] and compute $y = g^x \bmod p$
  - Public key (p, g, y)
  - Private key x

- **Encryption**: $m \rightarrow (C_1, C_2)$
  - Pick a random integer k $\in$ [1, p-1]
  - Compute $C_1 = g^k \bmod p$
  - Compute $C_2 = m \times y^k \bmod p$

- **Decryption**
  - $m = C_2 \times C_1^{-x} \bmod p$
  - $C_2 \times C_1^{-x} = (m \times y^k) \times (g^k)^{-x} = m \times (g^x)^k \times (g^k)^{-x} = m \bmod p$

ElGamal Example

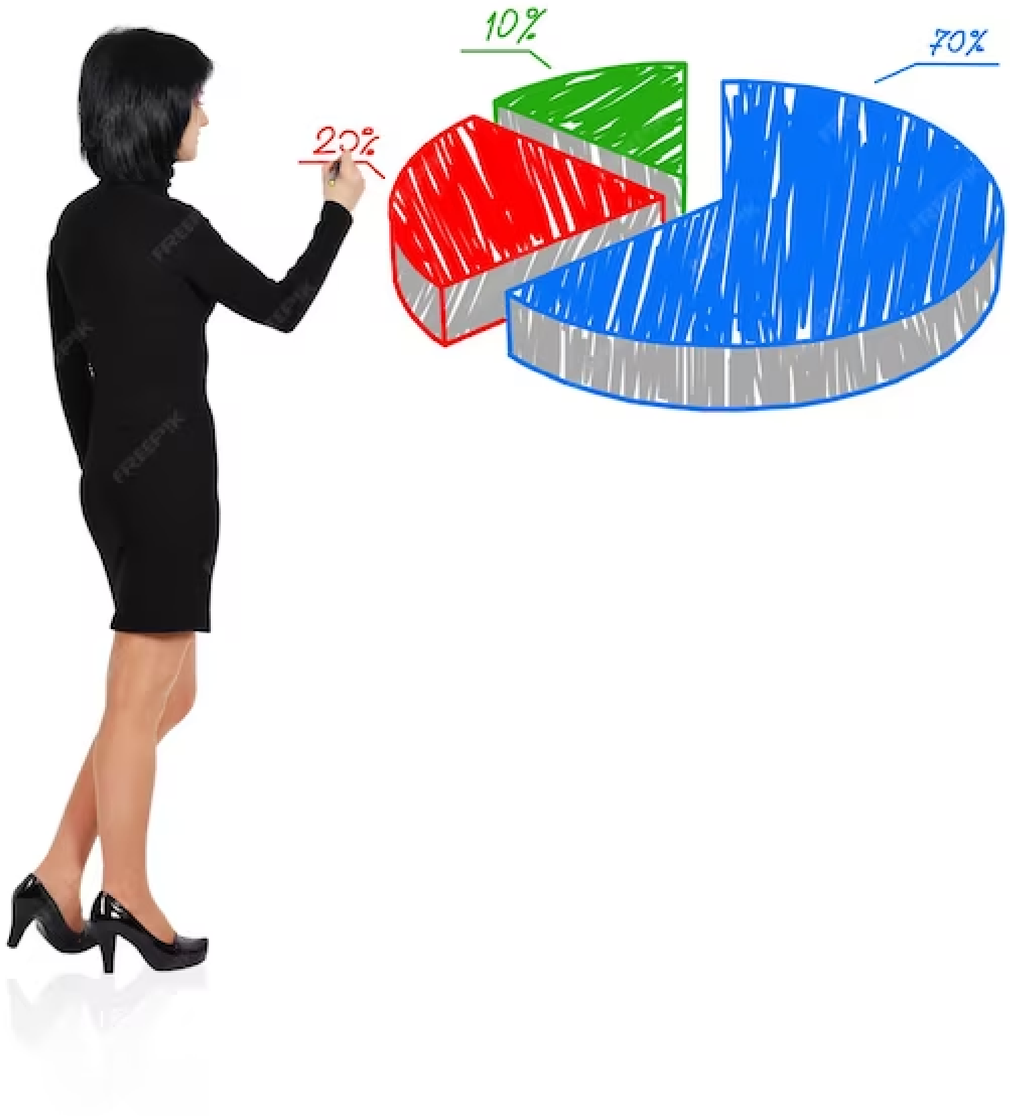
- Keys & parameters
  - Let prime number p=23 and generator g=7
  - Choose x=9 and y= g^x mod p= 7^9 mod 23=15
  - Publick key: {23, 7, 15}
  - Private key=9
- Encryption for m=20
  - Choose random k=3
  - C1= 7^3 mod 23=21
  - C2=20 x 15^3 mod 23=20x17 mod 23=18
  - Send (C1,C2)=(21,18) as a ciphertext
- Decryption
  - M=18 x 21^{-9} mod 23 = 20



Unraveling the ECC Encryption and Decryption Process: A Comprehensive Overview

Introduction

Welcome to the presentation on **Unraveling the ECC Encryption and Decryption Process**. This comprehensive overview will delve into the intricacies of the **Elliptic Curve Cryptography** algorithm, highlighting its strengths and applications in modern cryptography.



What is ECC?

Elliptic Curve Cryptography (ECC) is a public-key cryptographic algorithm based on the mathematics of elliptic curves over finite fields. It offers a high level of security with smaller key sizes compared to other encryption methods. ECC is widely used in various applications, including secure communication, digital signatures, and secure key exchange.

Key Generation

The ECC encryption process begins with key generation. A private key is randomly generated, and a corresponding public key is derived using mathematical operations on the elliptic curve. The security of ECC relies on the difficulty of solving the **elliptic curve discrete logarithm problem**.

Encryption

To encrypt a message using ECC, the sender converts the plaintext into a point on the elliptic curve. The sender then generates a random number, performs mathematical operations on the curve, and combines the result with the plaintext point to produce the ciphertext. Only the intended recipient with the private key can decrypt the ciphertext.

Decryption

Decryption in ECC involves the recipient using their private key to perform mathematical operations on the ciphertext point, resulting in the recovery of the original plaintext point. The private key operation exploits the properties of the elliptic curve to reverse the encryption process and retrieve the original message.

Strengths of ECC

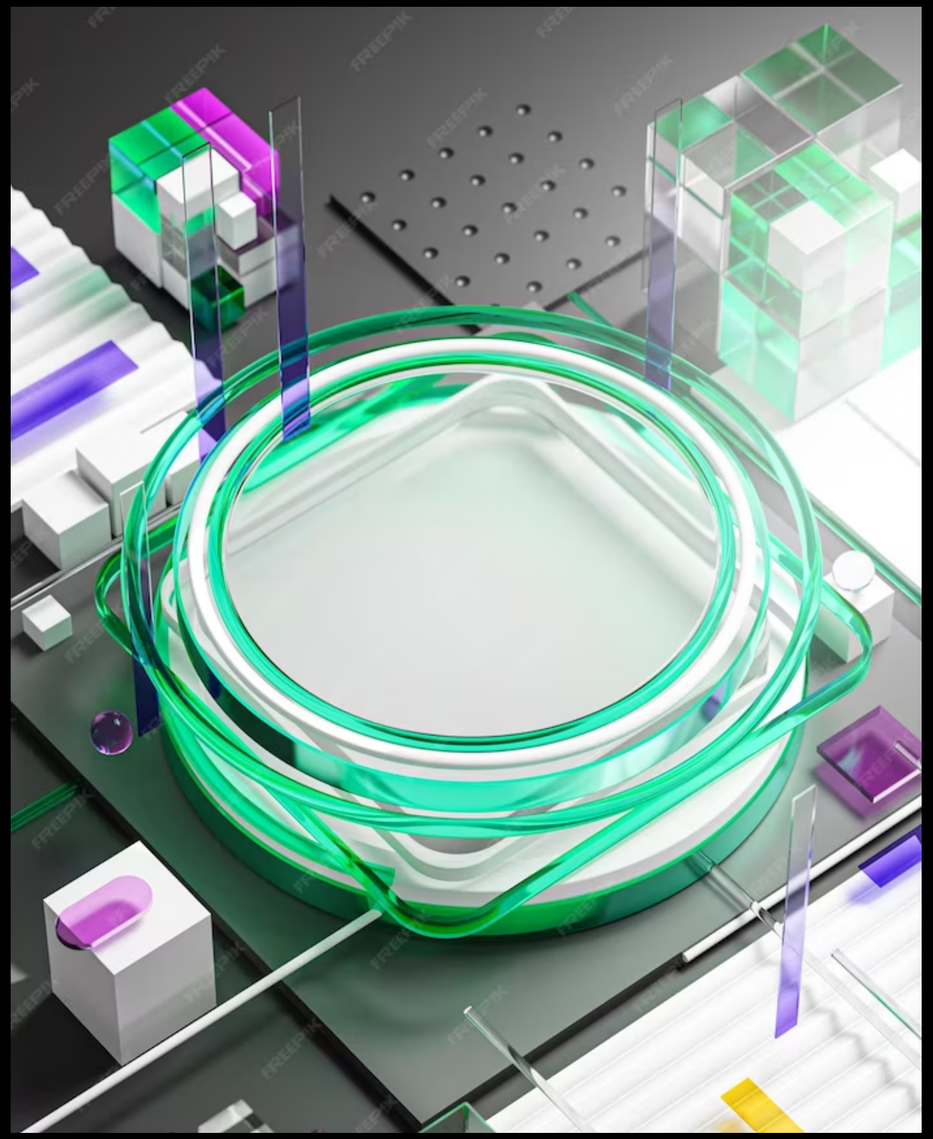
ECC offers several advantages over other encryption methods. It provides a high level of security with smaller key sizes, making it more efficient in terms of computational resources and storage. ECC is resistant to attacks such as brute force and integer factorization, making it suitable for resource-constrained devices and applications.

Applications of ECC

ECC finds applications in various domains. It is widely used in secure communication protocols like **TLS/SSL** to ensure encrypted data transmission. ECC is also utilized in **digital signatures** to verify the authenticity and integrity of digital documents. Additionally, ECC plays a crucial role in **secure key exchange** algorithms such as **Diffie-Hellman**.

Challenges and Considerations

While ECC offers numerous benefits, it also poses challenges and considerations. Implementation errors, side-channel attacks, and the selection of appropriate elliptic curves are critical factors to address. Additionally, the choice of key size and the need for standardized ECC parameters require careful consideration to ensure optimal security.



Future Developments

Ongoing research and development in ECC aim to further enhance its security and efficiency. Advancements include the exploration of post-quantum ECC, which focuses on developing ECC variants resistant to attacks by quantum computers. Additionally, efforts are being made to optimize ECC implementations for different platforms and improve interoperability.

Conclusion

In conclusion, **Elliptic Curve Cryptography** is a powerful encryption and decryption algorithm that provides robust security with smaller key sizes. Its widespread adoption in various applications underscores its effectiveness in protecting sensitive data. As technology advances, ECC continues to evolve, ensuring secure communication and data integrity in an increasingly interconnected world.

Thanks!

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