D E C L A R A T I O N

by the B.Tech. Student

I/We hereby *declare* that the Project Work Report entitled MODELLING OF A LIQUID LEVEL CONTROL TANK SYSTEM ....

....

which is being submitted to the **National Institute of Technology Karnataka, Surathkal** for the award of the Degree of Bachelor of Technology in Electrical and Electronics Engineering ....

....

is a *bonafide report of the work carried out by me/us*. The material contained in this Project Work Report has not been submitted to any University or Institution for the award of any degree.

Register Number, Name & Signature of the Student(s):

(1) 2110268, S V Sowndarya

(2) 2110286, Shah Charu Samir

(3) 2110510, Gaurangee Parashar

(4)

Department of Electrical and Electronics Engineering ....

Place: NITK, SURATHKAL

Date: 17/11/23

[declaration to be signed by the student(s) and incorporated as part of the Project Work Report]

C E R T I F I C A T E

This is to *certify* that the B.Tech. Project Work Report entitled

MODELLING OF A LIQUID LEVEL CONTROL TANK SYSTEM

submitted by :

Sl.No. Register Number & Name of Student(s)

(1) 2110268, S V Sowndarya

(2) 2110286, Shah Charu Samir

(3) 2110510, Gaurangee Parashar

(4)

as the record of the work carried out by him/her/them, is *accepted as the B.Tech. Project Work Report submission* in partial fulfillment of the requirements for the award of degree of **Bachelor of Technology** in Electrical and Electronics Engineering

Guide(s)
(Name and
Signature with Date)

1. Dr. Dharavath Kishan
2. Dr. Ravi Raushan

Chairman - DUGC
(Signature with Date and Sea

MODELLING OF A LIQUID LEVEL CONTROL TANK SYSTEM

1 Abstract

In certain applications such as chemical and industrial processes, it is essential to maintain the level of liquid in a tank at a certain desired level. In this work, we present a PID (Proportional-Integral-Derivative) based controller system where the liquid level is controlled by adjusting the rate of incoming liquid flow into the container. This adjustment is achieved by varying the speed of a DC motor-driven pump. The accuracy of the PID-based control system is demonstrated using MATLAB simulation software.

2 Introduction

PID controllers are widely used in practical control systems, from consumer electronics to industrial processes, due to their ability to achieve desired set-points in minimal time with minimal overshoot and steady-state error. Basically, it is used to adjust the output, in this case the liquid level at the desired set point so that in the ideal case there is no error between the sensed output and the desired reference level.The performance of PID controllers is evaluated based on parameters such as overshoots, rising time, settling time, and steady-state error.

In this study, we consider a system comprising a DC motor-driven pump that fills a tank. The motor's speed determines the rate of liquid flow into the tank. DC motors are commonly used in industries due to their variable speed, making them suitable for various applications. They offer precise speed-torque characteristics, reliability, and ease of control. The application of a PID control theory and feedback system modelling is used to design the overall system. By maintaining the required liquid level, this system not only increases process productivity but also conserves liquid by preventing overflow.

3 Modelling of the system

We set a reference input height level, $h_o(t)$, which shows the desired level the tank has to be filled. In forward path, we have PID controller which controls the speed of the DC motor. The motor's speed is directly related to the flow rate q_{in} supplying the tank. At the systems's overall output, we have the liquid level, h(t) and this information is feedback at the input and compared with the reference desired level. The error signal between the actual output and the reference, e(t) will be an input signal to the PID controller and the speed of the motor [ω(t) in rad/s] will be adjusted (either increased or decreased) to control the flow rate q_{in} until the required target level is achieved. We assume that the speed information of the motor is obtained from speed measurement device such as tachometer. The speed is transformed by the speed to height transformation (STH) block which relates the speed to the flow rate and later on to the level h(t).

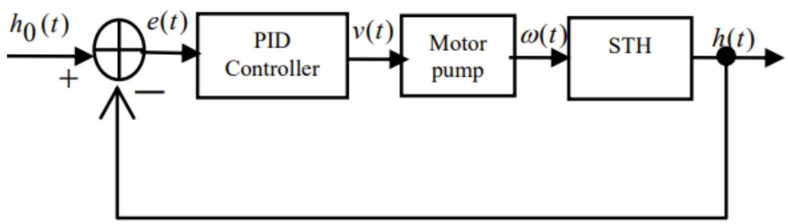

Figure 1: Block diagram of the liquid level controlling system

Motor pumps liquid to the container at a rate of q_{in} [m3/s] through an inlet pipe, the container with cross-sectional area A[in m2], is filled to liquid level, h and liquid is leaving the container at a rate of q_o [m3/s] through an outlet pipe. Using the flow balance into and out of the tank, the height, h is correlated to q_{in} and q_o as:

$$q_{in} - q_o = A\frac{dh}{dt} \tag{1}$$

The tank's out flow and height can also be related by assuming there's a linear resistance to flow for simplifying the analysis and it is given as:

$$q_o = \frac{h}{R_f} \tag{2}$$

where R_f [in s/m2] is the flow resistance.

We assume a simple linear relationship between the speed and the incoming flow rate to the tank:

$$\omega(t) = K_f q_{in}(t) \tag{3}$$

3.1 Modelling of the DC motor

The pump is a DC motor with the electric equivalent circuit of the armature and the free-body diagram of the rotor as shown in the given figure.

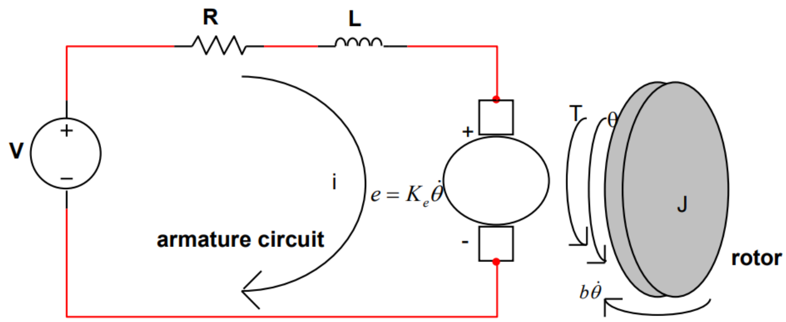

Figure 2: Equivalent circuit of the DC motor

The input to the motor is a voltage source (V) applied to the motor's armature, the output is the rotational speed of the shaft ω(t) =dθ/dt. Assuming armature controlled motor, the torque is proportional to the armature current (T=K_ti) and the back emf is proportional to the shaft's angular velocity, e=K_e ω. Using the above facts and based on Newton's second law and Kirchhoff's voltage law, the DC motor equations relating the motor speed to the physical parameters of the motor can be written as:

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = T = K_t i \tag{4}$$

$$L\frac{di}{d} + Ri = V - e = V - K_e\omega \tag{5}$$

Here, J: moment of inertia of the rotor [kg.m2]
b: motor viscous friction constant [in N.m.s]
K_e: electromotive force constant [in V/rad/sec]
K_t: motor torque constant [in N.m/Amp]
R: electric resistance [in Ω]
L: electric inductance [in H].

Using the properties of Laplace transform, we get:

$$\omega(s) = s\theta(s) \tag{6}$$

$$Js^2\theta(s) + bs\theta(s) = K_t I(s) \tag{7}$$

$$sLI(s) + RI(s) = V(s) - K_e\omega(s) \tag{8}$$

Using equations (6),(7) and (8), we get:

$$P_m(s) = \frac{\omega(s)}{V(s)} = \frac{K_t}{s^2 JL + s(JR + bL) + Rb + K_t K_i} \tag{9}$$

3.2 Modelling of the PID controller

The relationship between the error e(t) and the output v(t) is given as:

$$v(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{10}$$

where K_p, K_i and K_d are the proportional gain, the integral gain and the derivative gain respectively.

Using the Laplace domain, the input-output relation for the controller is given as:

$$C(s) = \frac{V(s)}{e(s)} = K_p + \frac{K_i}{s} + sK_d \tag{11}$$

3.3 Modelling of the speed to height (STH) transformation block

Let the impulse response of the STH system be, g(t) that represents the relation between h(t) and ω(t). We can find an expression for G(s) the Laplace transform of g(t) by combining (1), (2) and (3).

$$R_f\omega(t) - K_f h = K_f R_f A \frac{dh}{dt} \tag{12}$$

The relation in (12), in the Laplace domain is given as:

$$R_f\omega(s) - K_f h(s) = K_f R_f A s h(s) \tag{13}$$

Therefore the transfer function G(s) becomes:

$$G(s) = \frac{h(s)}{\omega(s)} = \frac{R_f}{K_f + sK_f R_f A} \tag{14}$$

3.4 Transfer function of the system

The overall transfer function of the forward path, F(s), can be obtained by combining (9), (11) and (14),

$$F(s) = \frac{h(s)}{e(s)} = C(s)P_m(s)G(s) \tag{15}$$

The overall transfer function, G_{sys}(s), of the overall system including the unity feedback loop is given as:

$$G_{sys}(s) = \frac{h(s)}{h_o(s)} = \frac{F(s)}{1 + F(s)} \tag{16}$$

4 Results and discussions

A unit step signal has been used as the reference liquid level for the overall system. We assumed the following parameter values for the motor, the tank, and the STH system.

- Motor: J = 0.01 Kg.m2, b = 0.1 N.m.s, Kt = 0.1 N.m/A, K_e=0.01 V/rad/s, R = 1 Ω, L = 0.5 H.
- Tank: A=0.5 m2, R_f =0.5 s/m2
- STH: K_f =1

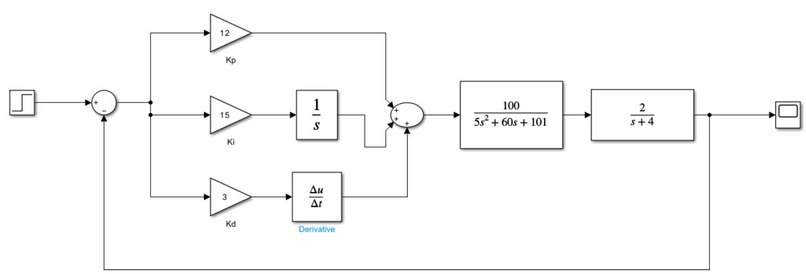

Figure 3: MATLAB simulation of the liquid level controlling system

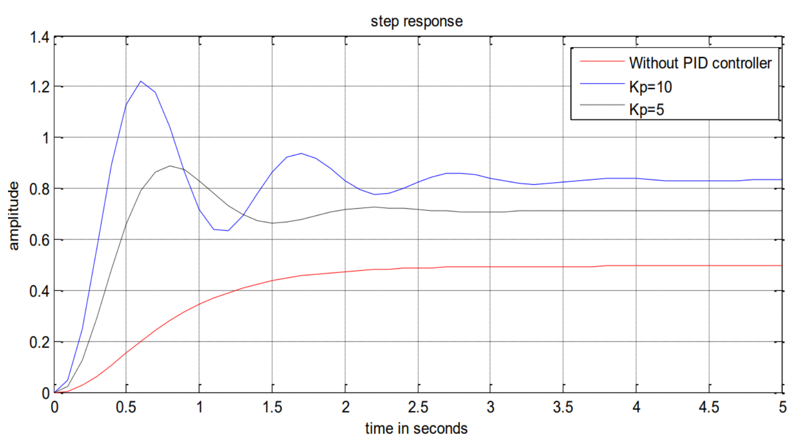

Figure 4: Step response with and without proportional controller

In the figure above, the results without the PID controller (red color) is the step response of the DC motor combined with STH block. The performance without controller is bad with respect to settling time with very high steady state error and a final value of 50 % of the required reference target.

The proportional controller reduces the rise time, improves the steady state error to some extent but a ringing and an over shoot starts to appear. A proportional controller with K_p=10 is better for reducing the steady state error compared with K_p=5.

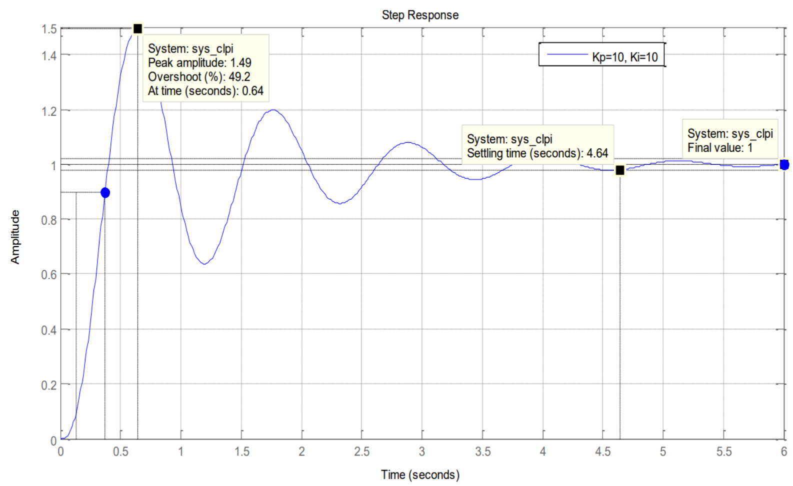

Figure 5: Step response with proportional-integral (PI) controller

The figure given above shows the system response with the proportional-integral (PI) controller with K_p=K_i=10. The addition of the integral part to the proportional part eliminates the steady state error. However, there is still a high overshoot and the settling time also gets larger.

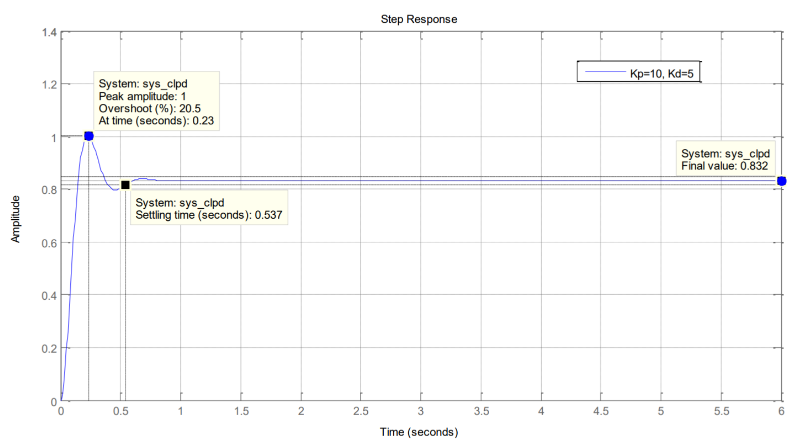

Figure 6: . Step response with proportional-derivative (PD) controller

The figure given above shows the system response with the proportional-derivative (PD) controller with K_p=10 and K_d=5. It reduces the over shoot, improves the rise time and settling time but with a significant amount of steady state error.

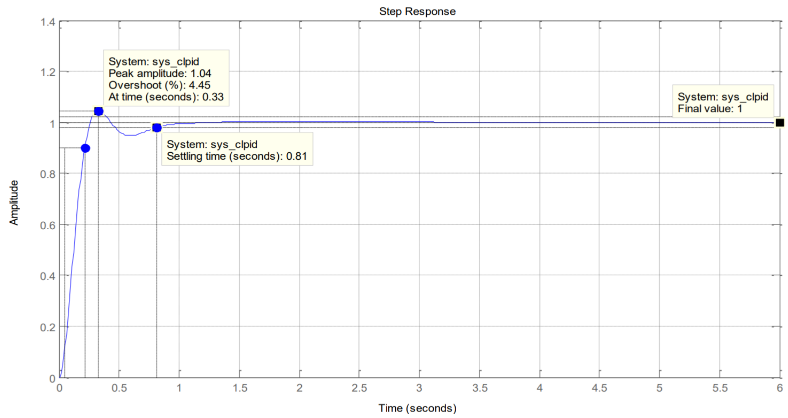

Figure 7: Step response with proportional-integral derivative (PID) controller

The figure given above shows the proportional-integral and derivative (PID) controller with K_p=12, K_i=15 and K_d=3. In this case the overshoot is less than 5%, settling time less than 2 seconds,steady state value is one and the rise time is also reduced.

Table 1: COMPARISON OF THE DIFFERENT CONTROLLERS

Type of controller	Rise time(s)	Settling time(s)	Overshoot (%)	Steady state value
Proportional(P), K_p=10	0.228	2.95	47.2	0.832
PI, K_p=10, K_i=10	0.237	4.64	49.2	1.0
PD, K_p=10, K_d=5	0.1	0.537	20.5	0.832
PID, K_p=12, K_i=15, K_d=3	0.17	0.81	4.45	1.0

5 Conclusion

The speed of the motor and hence the rate of liquid flow into the tank is controlled by adjusting the parameters of the PID controller. Such controllers are powerful in controlling any similar processes that essentially require close monitoring (tight control) of the process variables or parameters that have significant impact on quality and amount of production.