OBJECT: DATE / /

- $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

- $y(n) = \sum_{k=-\infty}^{\infty} x(k)\,\underset{\uparrow \text{ impulse response}}{h(n-k)} = \sum_{k=-\infty}^{\infty} x(n-k)\,h(k)$

  step response

  ↳ $h(n) = \delta(n) - \delta(n-1)$

  $h_1(n) \quad h_2(n)$

  Cascaded ↳ $h_1(n) * h_2(n)$ parallel ↳ $h_1(n) + h_2(n)$

- $y(n) = x(n) * h(n) = h(n) * x(n)$

  ↳ both finite : Table

  ↳ one finite other infinite

  change to → deltas $\delta(n), \delta(n-1)$

  $x(n) * \delta(n) = x(n)$

  $x(n) * \delta(n-k) = x(n-k)$

- impulse response $h(n)$

  Step response $h(n) * \underset{\uparrow \text{input}}{u(n)} \rightsquigarrow y(n) = \sum_{k=-\infty}^{n} h(k)$

- shift invariant $x(n-n_0) \to y(n-n_0)$

for LSI

- Stability $\sum_{-\infty}^{\infty} |h(k)| < \infty$

  Causality $h(n) = 0 \quad n < 0$

  invertability $h(n) * h^{-1}(n) = \delta(n)$

  Memory except $h(n) = c\,\delta(n)$

  ↑ only existing

for LSI system input $x(n) = e^{jwn}$

- $y(n) = e^{jwn} \sum_{k=-\infty}^{\infty} \underset{\downarrow}{h(k)\, e^{-jwk}}$

$$= e^{jwn} \quad H(e^{jw})$$

Freq response • $H(e^{jw}) = \sum_{n=-\infty}^{\infty} \underset{\uparrow \text{impulse response}}{h(n)} e^{-jwn}$

($\hat{}$ h(n) from) • $h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw}) e^{jwn} dw$

D.T.F.T

- $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$

- $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(w)|^2 dw = \sum_{-\infty}^{\infty} |x(n)|^2$

- $x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} dw$

conv. • $x(n) * h(n) \longleftrightarrow X(e^{jw})\ H(e^{jw})$

stability $\sum_{-\infty}^{\infty} |x(n)| < \infty$ stable system $\longleftrightarrow$ $H(e^{jw})$ converges

---

Z - Transform

- $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$ $\quad z = re^{jw}$

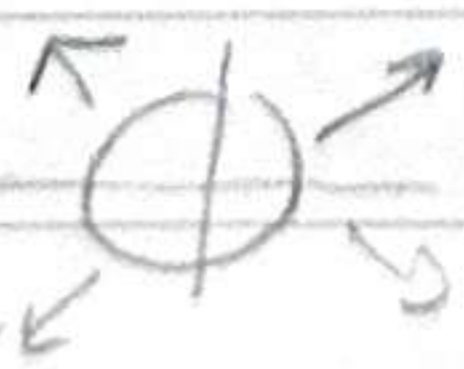
$\hookrightarrow X(e^{jw}) = X(z)\big|_{z=e^{jw}}$ mag unity

stability $\sum_{n=-\infty}^{\infty} |x(n)\, r^{-n}| < \infty$ or $X(e^{jw})$ exists

R.O.C includes unity

R.O.C

finite all

right sided  $\longleftrightarrow$ Causal

outer most pole

left sided  inner most pole

two sided $|a| < |z| < |b|$

$n\,x(n) \longleftrightarrow -z \frac{\partial X(z)}{\partial z}$

$a^n x(n) \longleftrightarrow X(a^{-1} z)$

$x(-n) \longleftrightarrow X\left(\frac{1}{z}\right)$

inverse $H(z)_{inverse} = \frac{1}{H(z)}$ $\quad h(n)_{inverse} = Z^{-}\{H(z)_{inverse}\}$

OBJECT: DATE / /

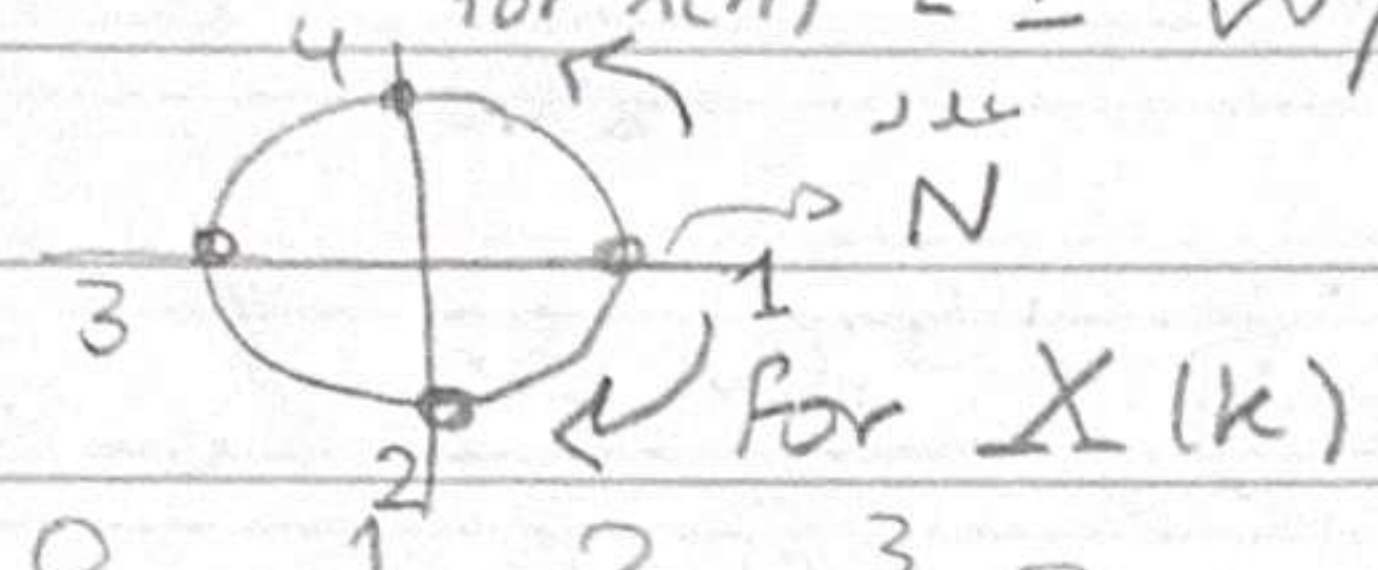
DFT

DFT • $X(K) = \sum_{n=0}^{N-1} x(n) \, W_N^{nK} \quad [K: 0 \rightarrow N-1]$, $W_N^{nK} \rightarrow e^{-j\frac{2\pi}{N}nk}$

• $\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{0}^{N-1} |X(k)|^2$

IDFT • $x(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(K) \, W_N^{-nK} \quad [K: 0 \rightarrow N-1]$

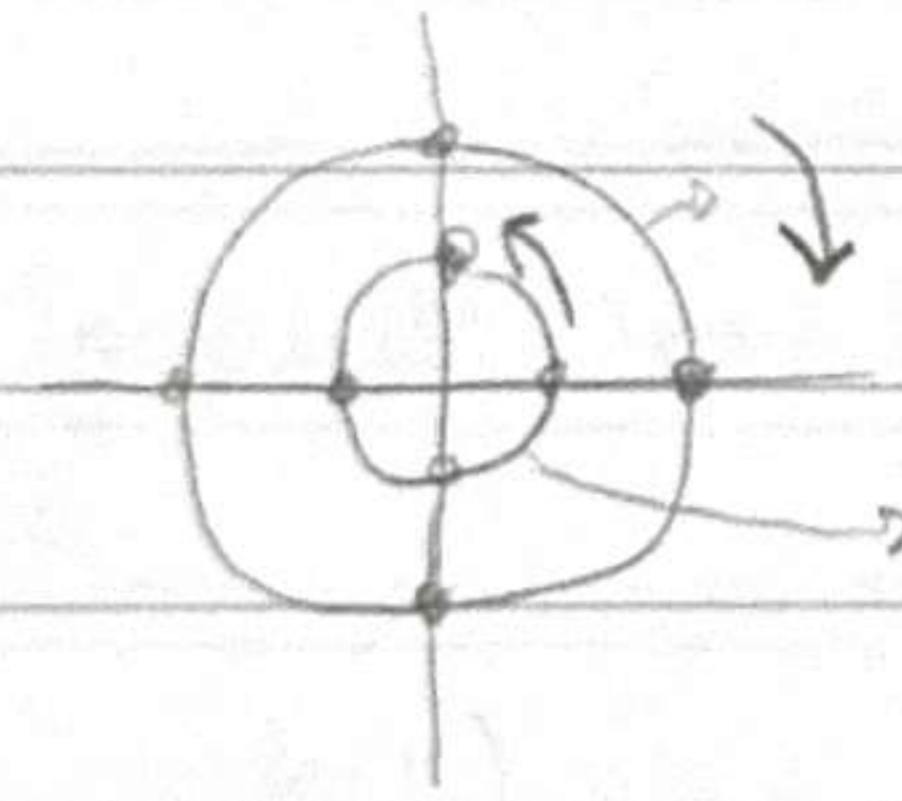
$$X(K) = X(Z) \Big|_{Z = W_N^{-K}} \quad [K: 0 \rightarrow N-1]$$



DFT
$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(4) \end{bmatrix} = \begin{array}{c|cccc} & n\;0 & 1 & 2 & 3 \\ \hline K\;0 & 1 & 1 & 1 & 1 \\ 1 & 1 & W_N^1 & W_N^2 & W_N^3 \\ 2 & 1 & W_N^2 & W_N^4 & W_N^6 \\ 3 & 1 & W_N^3 & W_N^6 & W_N^9 \end{array} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = \begin{bmatrix} \text{mag} \\ \vdots \\ \vdots \\ \; \end{bmatrix} \overset{\theta}{\text{phase}}$$

Circular Convolution

rotating circles



divide circle to N

rotating clockwise ↓

convolution • make convolution with table then overlap every N

as if linear • Choose $N = N_{x_1} + N_{x_2} - 1$ pad x_1 & x_2 with zeros then make convolution with table

$$x(0) = \frac{1}{N} \sum_{0}^{N-1} \underbrace{X(k)}_{\uparrow \text{ wanted}}$$

$$X(0) = \sum_{0}^{N-1} x(n)$$

OBJECT : DATE / /

$e^{jw} = \cos w + j \sin w$ $\cos w = \frac{1}{2}(e^{jw} + e^{-jw})$

$e^{-jw} = \cos w - j \sin w$ $\sin w = \frac{1}{2}(e^{jw} - e^{-jw})$

Billinear Transformation

Step ① Get N $\quad |H(w)|^2 = \frac{H_0^2}{1+(\frac{w}{w_c})^{2N}}$

Step ② $H(S)_{\text{normalized}}$ $\quad$ (N=1) $\frac{1}{S+1}$ $\quad$ (N=2) $\frac{1}{S^2+\sqrt{2}S+1}$ $\quad$ (N=3) $\frac{1}{S^3+2S^2+2S+1}$

Step ③ $W_{cd} = 2\pi f_c$ $\quad W_{ca} = \frac{2}{T_s} \tan\left(\frac{W_{cd} T_s}{2}\right)$

Step ④ $H(S)_{\text{denormalized}}$ $\quad$ (LPF) $S \rightarrow \frac{S}{W_{ca}}$ $\quad$ (HPF) $S \rightarrow \frac{W_{ca}}{S}$

(BPF) $S \rightarrow \frac{S^2+W_o^2}{S.B}$ $\quad$ (BSF) $S \rightarrow \frac{S.B}{S^2+W_o^2}$

$B = \underset{\text{High}}{W_{ca}} - \underset{\text{low}}{W_{ca}}$ $\quad W_o^2 = \sqrt{W_{ca_H} + W_{ca_L}}$

Step ⑤ Apply Transf. $S = \frac{2}{T_s} \times \frac{Z-1}{Z+1}$

Step ⑥ implement

---

impulse invariance $\qquad\qquad$ Step invariance

① H(S) ✓

② $h(t) \xleftarrow[L^{-1}]{} \;\big|\; \xrightarrow[L^{-1}]{\frac{H(S)}{S}} S(t)$

③ $h(n) \longleftarrow$ Sampling $t = nT_s \longrightarrow S(n)$

④ $H(Z) \longleftarrow$ Z-Transform $\longrightarrow H(z) = S(z) \times \frac{Z-1}{Z}$