25/11/19

PRACTICAL - 01

TOPIC : LIMITS AND CONTINUITY

1) $\lim\limits_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim\limits_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3] $\lim\limits_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4] $\lim\limits_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5] Examine the Continuity of the following function.

i] $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, $\dfrac{\pi}{2} < x < \pi$

} at $x = \dfrac{\pi}{2}$

Scanned with CamScanner

ii] $f(x) = \frac{x^2-9}{x-3}$ $\quad 0<x<3$

$= \frac{x^2-9}{x+3}$ $\quad 6\leq x<9$ } at $x=3$ & $x=6$

$= x+3$ $\quad 3\leq x\leq 6$

⑥ Find value of k so that the function of x is continuous at the indicated point

i] $f(x) = \frac{1-\cos 4x}{x^2}$, $x<0$

$= k$, $x=0$ } at $x=0$

ii] $f(x) = (\sec^2 x)^{\cot^2 x}$, $x\neq 0$

$= k$, $x=0$ } at $x=0$

iii] $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}$, $x\neq \frac{\pi}{3}$

$= k$, $x=\frac{\pi}{3}$ } at $x=\frac{\pi}{3}$

7] Discuss the continuity of the following function which of these function have removable discontinuity? Redefine function so as to remove the discontinuity.

i) $f(x) = \frac{1-\cos 3x}{x\tan x}, \quad x \neq 0$

$\quad = 9, \quad x = 0$ } at $x=0$

ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}, \quad x \neq 0$

$\quad = \frac{\pi}{60}, \quad x = 0$ } at $x=0$

8] If $f(x) = \frac{e^{x^2}-\cos x}{x^2}$

for $x \neq 0$ is continuous at $x=0$ find $f(0)$

] If $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \frac{\pi}{2}$

is continuous at $x = \frac{\pi}{2}$

find $f\left(\frac{\pi}{2}\right)$

Scanned with CamScanner

1] $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \dfrac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \dfrac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right.$

$\lim_{x \to a} \dfrac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \dfrac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\dfrac{1}{3} \lim_{x \to a} \dfrac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\dfrac{1}{3} \lim_{x \to a} \dfrac{\sqrt{4a}}{\sqrt{3a}} + \dfrac{2\sqrt{a}}{\sqrt{3a}}$

$= \dfrac{1}{3} \dfrac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \dfrac{1}{3} \dfrac{4\sqrt{a}}{2\sqrt{3a}}$

$= \dfrac{1}{3} \dfrac{2\sqrt{a}}{\sqrt{3a}}$

$= \dfrac{1}{3} \dfrac{\sqrt{a}}{\sqrt{3a}}$

$= \dfrac{2}{2\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$$\lim_{y \to 0} \left(\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$$

$$\lim_{y \to 0} \frac{a + y - a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\,(\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3] $\lim_{x \to \pi/6} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

$x \to \pi/6 = h \quad x = h + \pi/6 \quad h \to 0$

$$\lim_{h \to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)} \quad \cancel{h \to 0}$$

$$\lim_{h \to 0} \frac{(\cos h . \cos\pi/6) - \sinh . \sin\pi/6) - \sqrt{3}(\sinh\cos\pi/6 + \cosh\sin\pi/6}{\pi - 6\left(\frac{6h+\pi}{c}\right)}$$

$$\lim_{h \to 0} \frac{\cos h - \frac{\sqrt{3}}{2} . \sinh \frac{1}{\sqrt{2}} - \sqrt{3}(\sin \sqrt{3}/2 + \cosh 1/\sqrt{2})}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h \to 0} \frac{\cos h \frac{\sqrt{3}h}{2} - \frac{\sin h}{2} - \frac{\sin 3h}{2} - \cos \frac{\sqrt{3}}{4} h}{-6h}$$

$$\lim_{h \to 0} \frac{\cancel{+\frac{\sin 4h}{2}}}{+6h}$$

$$\lim_{h \to 0} \frac{\sin 4h}{12h} = \frac{1}{3} \lim_{h \to 0} \frac{\sinh}{h}$$

$= \frac{1}{3} \times 1$

$= 1/3$

8

c) $\lim_{x\to\infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

$\lim_{x\to\infty} \left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}} \times \dfrac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \right]$

$\lim_{x\to\infty} \dfrac{(x^2+5-x^2+3)\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\times(\sqrt{x^2+5}+\sqrt{x^2-3})}$

$\lim_{x\to\infty} \dfrac{8}{2} \left(\dfrac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \right)$

$\lim_{x\to\infty} 4 \dfrac{\left(\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)} \right)}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$

$4 \lim_{x\to\infty} \dfrac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$

$= \underline{\underline{4}}$

5]

i] $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, $\pi/2 < x < \pi$ } at $x = \pi/2$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$$

$$f(\pi/2) = 0$$

f at $x = \pi/2$

$$\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$$

$$x - \pi/2 = h$$

$$x = h + \pi/2$$

where $h \to 0$.

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \pi/2\right)}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \pi/2\right)}{\cancel{\pi} - 2h - \cancel{\pi}}$$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h . \cos\frac{\pi}{2} - \sin h \; \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h \to 0^+} \cos h \cdot 0 - \frac{\sin h}{-2h}$$

$$\lim_{h \to 0^+} \frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$= \frac{1}{2}$$

$$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1 - \cos 2x}}$$

$$\lim_{x \to \pi/2} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x \to \pi/2} \frac{2\, \cancel{\sin x} \cos x}{\sqrt{2}\, \cancel{\sin x}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x$$

L.H.L. $\neq$ R.H.L.

f is not continuous at $x = \pi/2$

Q5]
ii]
$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \le x \le 6 \\ = \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$$ at $x = 3$ & $x = 6$.

at $x = 3$

$f(3) = \dfrac{x^2-9}{x-3} = 0$

f at $x = 3$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+}$

$f(3) = x+3 = 3+3 = 6$.

f is define at $x = 3$.

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (x+3) = 6$

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \dfrac{x^2-9}{x-3} = \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$=$ L.H.L. $=$ R.H.L.

$\therefore$ f is cont. at $x = 3$.

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}^{3}}{\cancel{9}}$$

$$\lim_{x\to 6^+} = \frac{x^2-9}{x+3}$$

$$\lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$$

$$\lim_{x\to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x\to 6^-} x+3 = 3+6 = 9$$

$\therefore$ L.H.L. $\neq$ R.H.L.

function is not continuous.

6]
i) $f(x) = \dfrac{1-\cos 4x}{x^2}$ $x<0$
$= k$ $x=0$ $\Big\}$ at $x=0$.

Sol^n :- f is continuous at $x=0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

$$2 \lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x\to 0} \left(\frac{\sin^2 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$\therefore \underline{\underline{k=8}}$$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$ \} at $x = 0$.

$= k$ $\qquad x = 0$

$= (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$

$= e$

$= k$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$

$= K$ $\quad x = \pi/3$ $\quad \}$ at $x = \pi/3$

$x - \dfrac{\pi}{3} = h$

$x = h + \pi/3$

$h \to 0$

$$f\left(\pi/3 + h\right) = \frac{\sqrt{3} - \tan\left(\pi/3 + h\right)}{\pi - 3\left(\pi/3 + h\right)}$$

$$= \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\pi/3 \cdot \tan h}}{\pi - \pi - 3h}$$

$$= \frac{\dfrac{\cancel{\sqrt{3}} - \sqrt{3} \times \sqrt{3}\tan h - \cancel{\sqrt{3}} - \tan h}{1 - \sqrt{3}\cdot\tan h}}{-3h}$$

$$= \frac{\dfrac{\cancel{\sqrt{3}} - 3\tan h - \cancel{\sqrt{3}}\cdot\tan h}{1 - \sqrt{3}\cdot\tan h}}{-3h}$$

$$= \frac{\dfrac{-4\tan h}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \frac{4\tan h}{3h\,(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\lim_{h\to 0}\frac{\tan h}{h}\;\lim_{h\to 0}\frac{1}{1-\sqrt{3}\tan h}$$

$$= \frac{4}{3}\;\frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}$$

7]

i] $f(x) = \dfrac{1-\cos 3x}{x\tan x}$ $\quad x \neq 0$

$= 9$ $\quad x = 0$ $\Bigg\}$ at $x=0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$= \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$= \frac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2}{\dfrac{x.\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= \cancel{2}^{1} \times \frac{9}{\cancel{4}2}$$

$$= \frac{9}{2}$$

$\neq 9$

$\neq f(0)$

$\therefore$ f is not continuous at $x=0$.

ii) $f(x) = \dfrac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$

$= \dfrac{\pi}{60}$, $x = 0$ } at $x = 0$

$\lim_{x \to 0} \dfrac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$3 \lim_{x \to 0} \dfrac{e^{3x}-1}{3x} \quad \lim_{x \to 0} \dfrac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$3 \log e \dfrac{\pi}{180} = \dfrac{\pi}{60} = f(0)$

$\therefore$ f is continuous at $x = 0$.

Scanned with CamScanner

8] $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$

$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$= \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{2 \sin^2 x/2}{x^2}$

$= \log e + 2 \left(\frac{\sin x/2}{x} \right)^2$

$= 1 + \cancel{2}^{1} \times \frac{1}{\cancel{4}\,2}$

$= 3/2$

$= f(0)$

9) $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(\circ)$ is continuous at $x = \pi/2$

$$\frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1 \cancel{+} \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2\times 2\sqrt{2}}$$

19 $$= \cancel{\frac{1}{4\sqrt{2}}}$$

$$f(\pi/2) = \frac{1}{4\sqrt{2}}$$

Scanned with CamScanner

PRACTICAL - 02

Topic: Derivative

Q1] Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x . \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(\cancel{a}+h-\cancel{a})\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan(\cancel{a} - \cancel{a} - h) - (1 + \tan a + \tan(a}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$.

ii) $\operatorname{Cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

Put $x - a = h$

$x = a + h$

as $x \to a, h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a . \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) . \sin\left(\frac{\cancel{a}-\cancel{a}-h}{2}\right)}{h \times \sin a . \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= \frac{-1}{2} \times 2\cos\left(\frac{2a+0}{2}\right)$$

$$= -\frac{\cos a}{\sin^2 a}$$

$$= -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$D f(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\frac{-h}{2} \times \frac{-1}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}}$$

$$= \frac{-\frac{1}{2} \times -2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= \frac{-1}{2} \times -2 \frac{\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

Q2] If $f(x) = 4x+1$, $x \leq 2$

$= x^2+5$, $x > 0$, at $x = 2$, then

find function is differentiable or not.

Sol^n :-

L.H.D. =

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \cancel{\lim_{x \to 2^-}} \frac{4x-8}{x+2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)}$$

$$= 4$$

R.H.D. =

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2$

$= 4$

R.H.D = L.H.D.

$\therefore f$ is differentiable at $x=2$.

Q3] If $f(x) = 4x+7, \quad x<3$
$\quad\quad = x^2+3x+1, \quad x\geq 3$ at $x=3$, then find f is differentiable or not?

Sol^n :-

R.H.D:

$$Df(3+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}} = 3+6 = \underline{9}$$

$$L.H.D = Df(3^-)$$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= \underline{4}$$

$\therefore$ R.H.D $\neq$ L.H.D.

$\therefore$ f is not differentiable at $x = 3$.

Q4] If $f(x) = 8x - 5 \qquad x \leq 2$

$\qquad\qquad = 3x^2 - 4x + 7 \qquad x > 2$ at $x = 2$.

Sol^n :-

$$f(2) = 8 \times 2 - 5 = 16 - 5 = \underline{\underline{11}}$$

$\therefore$ R.H.D. $= Df(2^+)$

$$= \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2$$

$$= \underline{\underline{8}}$$

$$L.H.D. = Df(2^-)$$

$$= \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \underline{\underline{8}}$$

$\therefore$ L.H.D. = R.H.D.

$\therefore$ It is differentiable at $x=3$.

PRACTICAL-03

Topic : Application of Derivative.

1) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 69 - 24x - 9x^2 + 2x^3$

v) $y = 2x^3 + x^2 - 20x + 4$

Sol^n :-

$f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f(x) > 0$

$\therefore$ f is increasing,

$3x^2 - 5 > 0$

$3(x^2-5/3)>0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})>0$

+ − +

$-\sqrt{5/3}$ $+\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x)<0$

$3x^2-5<0$

$\therefore 3(x^2-5/3)<0$

$(x-\sqrt{5/3})(x+\sqrt{5/3})<0$

+ − +

$-\sqrt{5/3}$ $+\sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

ii) $f(x) = x^2-4x$

$f'(x) = 2x-4$

$\because f(x)$ is increasing iff $f'(x)>0$

$\therefore 2x-4>0$

$\therefore 2(x-2)>0$

$\therefore x-2>0$

$x \in (2,\infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 2x-4<0$

$\therefore 2(x-2)<0$

$x-2<0$

$x \in \cancel{(2,\infty)}\ (-\infty, 2)$

iii) $f(x) = 2x^3 \cancel{+x^2} -20x+4$

$\therefore f'(x) = 6x^2+2x-20$

$\because f$ is increasing iff $f'(x)>0$

$\therefore 6x^2+2x-20>0$

$\therefore 3x^2+x-10>0$

$(x+2)(3x-5)>0$

+ − +

-2 $5/3$

f is decreasing iff $f'(x)$

$\therefore 6x^2+2x-20<0$

$3x^2+6x-5x-10<0$

$3x(x+2)-5(x+2)<0$

$\therefore (x+2)(3x-5)<0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-2, 5/3)$

iv) $f(x) = x^3-27x+5$

$f'(x) = 3x^2-27$

$\because$ f is increasing iff $f'(x)>0$

$\therefore 3(x^2-9)>0$

$\therefore (x-3)(x+3)>0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x)<0$

$\therefore 3x^2-27<0$

$\therefore 3(x^2-9)<0$

$\therefore (x-3)(x+3)<0$

$+ \quad \quad +$

$-3 \quad 3$

$x \in (-3, 3)$

v) $f(x) = 2x^3-9x^2-24x+69$

$f'(x) = 6x^2-18x-24$

$\because$ f is increasing iff $f'(x)>0$

$\therefore 6x^2-18x-24>0$

$\therefore x^2-4x+x-4>0$

$\therefore (x-4)(x+1)>0$

and f is decreasing iff $f'(x)<0$

$\therefore 6x^2-18x-24<0$

$x^2-4x+x-4<0$

$\therefore (x-4)(x+1)<0$

$+ \quad - \quad +$

$-1 \quad 4$

$\therefore x\in(-1,4)$

Q2]

i) $y=3x^2-2x^3$

$\therefore f(x)=3x^2-2x^3$

$\therefore f'(x)=6x-6x^2$

$\therefore f''(x)=6-12x$

f is concave upward if $f''(x)>0$

$\therefore (6-12x)>0$

$\therefore 12(6/12-x)>0$

$x-1/2>0$

$x>1/2$

$\therefore f''(x)>0$

$x\in(1/2,\infty)$

2) $y=x^4-6x^3-12x^2+5x+7$

$f'(x)=4x^3-18x^2+24x+5$

$f''(x)=12x^2-36x+24$

f is concave upward if $f''(x)>0$

$\therefore 12x^2-36x+24>0$

$\therefore x^2-2x-x+2>0$

$\therefore (x-2)(x-1)>0$

$+ \quad - \quad +$ $\qquad x\in(-\infty,1)\cup(2,\infty)$

3] $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$x \in (0, \infty)$

4] $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$x \in (3/2, \infty)$

5] $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

Scanned with CamScanner

$\therefore x < -1/6$

$\therefore f''(x) \not> 0$

$\therefore$ The exist no interval.

$x \in (-1/6, \infty)$

AK 16/12/19

112

PRACTICAL No. 4

TOPIC : Application of Derivative & Newton's Method

Q1] Find maximum & minimum values of following functions

(i) $f(x) = x^2 + \frac{16}{x^2}$

(ii) $f(x) = 3 - 5x^3 + 3x^5$

(iii) $f(x) = x^3 - 3x^2 + 1$ in $[-\frac{1}{2}, 4]$

(iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[-2, 3]$

Q2] Find the root of the following equation by Newton's Method.

(Take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Solⁿ :-

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider,

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has minimum value at x

$\therefore$ Function reaches minimum val

at $x = 2$ and $x = -2$.

ii) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider,

$f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$\therefore f(1) = -30 + 60$

$= +30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

Scanned with CamScanner

$f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore \underline{\underline{x = 0}}$ or $\underline{\underline{x = 2}}$

$f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$

$\therefore f(0) = 0^3 - 3(0)^2 + 1$

$= 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$.

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 of $x = 0$ and

f has ~~max~~ minimum value -3 a[illegible] $x = 2$.

iv) $f(x)=2x^3-3x^2-12x+1$

$f'(x)=6x^2-6x-12$

Consider,

$f'(x)=0$

$\therefore 6x^2-6x-12=0$

$6(x^2-x-2)=0$

$x^2-x-2=0$

$x^2+x-2x-2=0$

$x(x+1)-2(x+1)=0$

$(x-2)(x+1)=0$

$\therefore x=2$ or $x=-1$

$\therefore f''(x)=12x-6$

$\therefore f''(2)=12(2)-6$

$=24-6$

$=18>0$

$\therefore f$ has minimum value at $x=2$

$\therefore f(2)=2(2)^3-3(2)^2-12(2)+1$

$=2(8)-3(4)-24+1$

$=16-12-24+1$

$=-19$

$f''(-1)=12(-1)-6$

$=-12-6$

$=-18<0$

$\therefore f$ has maximum value at $x=-1$

$\therefore f(-1)=2(-1)^3-3(-1)^2-12(-1)+1$

$=-2-3+12+1$

$=8$

$\therefore f$ has maximum value 8 at $x=-1$ and

f has minimum value -19 at $x=2$

Q2)

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0 + \frac{9.5}{55}$$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 65$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0829}{55.9467}$$

$= \underline{\underline{0.1712}}$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$
$= 0.0050 - 0.0879 - 9.416 + 9.5$
$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$
$= 0.0879 - 1.0272 - 55$
$= -55.9393$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.1712 + \frac{0.0011}{55.9393}$$

$= 0.1712$

$\therefore$ The root of the equation is 0.1712.

ii) $f(x) = x^3 - 4x - 9$
$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6$

Let $x_0 = 3$ be the initial approximation,
$\therefore$ By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$

$$= 20.5528 - 10.9568 - 9$$

$$= 0.596$$

$$f'(x_1) = 3(2.7392)^2 - 4$$

$$= 22.5096 - 4$$

$$= 18.5096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.596}{18.5096}$$

$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$

$$= 19.8386 - 10.8284 - 9$$

$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$

$$= 21.9851 - 4$$

$$= 17.9851$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7071 - 0.0056$$

$$= 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$

$$= 19.7158 - 10.806 - 9$$

$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$

$$= 21.8943 - 4$$

$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7015 + 0.0050$$

$$= 2.7065$$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$

$$f(x) = 3x^2 - 3.6x - 10$$

$$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$$

$$= 1.8 - 10 + 17$$

$$= 6.2$$

$$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$$

$$= 8.7.2 - 20 + 17$$

$$= -2.2$$

let $x_0 = 2$ be initial approximation,

By Newton's Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_0 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2 - \frac{2.2}{5.2}$$

$$= 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 + 17$$
$$= 3.9219 - 4.4764 - 15.77 + 17$$
$$= 0.6755$$

$$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$$
$$= 7.46808 - 5.6772 - 10$$
$$= -8.2164$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6755}{8.2164}$$

$$= 1.577 + 0.0822$$

$$= 1.6592$$

$$f(x_1) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$$
$$= 4.5677 - 4.9553 - 16.592 + 17$$
$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= 8.2588 - 5.97312 - 10$$
$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.6592 + \frac{0.0204}{7.7143}$$
$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 4.5892 - 4.9708 - 16.618 + 17$$
$$= 0.0004$$

$$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$
$$= 8.2847 - 5.9824 - 10$$
$$= -7.6977$$

23/12/19

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$
$$= 1.618 + \frac{0.0004}{7.6977}$$
$$= \underline{1.6618}$$

The root of

PRACTICAL No. 5

TOPIC : Integration

Q1] Solve the following integration

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ (ii) $\int (4e^{3x}+1)\,dx$ (iii) $\int (2x^e - 3\sin x + 5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$ (v) $\int t^7 \sin(2t^4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ (ix) $\int e^{\cos^2 x}\sin 2x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

Soln :-

Q.1]

i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x(1)-3}}$

$= \int \frac{1}{\sqrt{x^2+2x(1)+(1)^2-(1)^2-3}}$

$= \int \frac{1}{\sqrt{(x+1)^2-(1)^2-3}}$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}$

$= \int \frac{1}{\sqrt{(x+1)^2-(2)^2}}$

$= \log\left|x+\sqrt{x^2\cancel{+a^2}}\right|+c$

$= \log\left|x+1\sqrt{(x+1)^2-(2)^2}\right|+c$

$= \log\left|x+1+\sqrt{x^2+2x+1-4}\right|+c$

$= \log\left|x+1+\sqrt{x^2+2x-3}\right|+c$

2] $\int (4e^{3x}+1)\,dx$

$= \int (4e^{3x} + x^0)\,dx$

$= 4\int e^{3x}\,dx + \int x^0\,dx$

$= \frac{4e^{3x}}{3} + \frac{x^{0+1}}{0+1} + c$

$= \frac{4e^{3x}}{3} + x + c$

3] $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= 2\int x^2\,dx - 3\int \sin x\,dx + 5\int x^{1/2}\,dx$

$= 2\frac{x^3}{3} - 3(\cos x) + \frac{5x^{1/2+1}}{1/2+1} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^{3/2}}{3/2} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt{x}^{\,3}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4] $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\, dx$

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2}\, dx + \int 3x^{1/2}\, dx + \int \frac{4}{x^{1/2}}\, dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5] $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2\times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{2\times 4}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du$

$= \int \frac{t^4 \times \sin(2t^4)}{8}\, du$

substitute t^4 with $\frac{u}{2}$

$= \int \frac{u/2 \times \sin(2t^4)}{8} du$

$= \int \frac{u}{2} \times \sin(u) / 8 \, du$

$= \int \frac{u \times \sin(u)}{16} du$

$= \frac{1}{16} \int u \times \sin(u) \, du$

$\int u \, dv = uv - \int v \, du$

$dv = \sin(u) \times du$

$du = 1 du \quad v = -\cos(u)$

$= \frac{1}{16} \times (u \times (-\cos(u)) - \int -\cos(u) du$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

c] $\int \sqrt{x}\,(x^2 - 1)\, dx$

$= \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2} dx$

$= \int x^{5/2} - x^{1/2} dx$

$= \int x^{5/2} - \int x^{1/2} dx$

$I_1 \ \frac{x^{5/2+1}}{5/2} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$$= \frac{2\sqrt{x}^3}{3}$$

$$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x}^3}{3}$$

$$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x}^3}{3} + c$$

viii)

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$t = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

Let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3} dx = dt$

$I = -\frac{1}{2}\int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2}\int \sin t$

$= -\frac{1}{2}(-\cos t) + c$

$= \frac{1}{2}\cos t + \cancel{c}$

Resubstituting $t = 1/x^2$

$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + c.$

8] $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)} dt$

$= \frac{1}{\sin x^{3/2}} dt$

$= \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3-1)t^{2/3-1}}$

$= \frac{-1}{-1/3 t^{2/3-1}} = \frac{1}{1/3 t^{-1/3}} = \frac{t^{1/3}}{1/3}$

$= 3t^{1/3}$

$= 3\sqrt[3]{t}$

Resubstituting $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + c$

9] $\int e^{\cos^2 x} \sin 2x \, dx$

put $\cos^2 x = t$

$2 \cdot \cos x - \sin x \, dx = dt$

$-\sin 2x \, dx = dt$

$\therefore \sin 2x \, dx = -dt$

$\therefore I = \int e^{\cos^2 x} \sin 2x \, dx$

$= -\int e^t \, dt$

$= -e^t + c$

$I = -e^{\cos^2 x} + c$

10] $\int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1} \right) dx$

put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3 \times 2x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x} dt$

$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)} dt$

$= \int \frac{1}{x^3 - 3x^2 + 1} \times 1/3 \, dt$

$= \int \frac{1}{3(x^3 - 3x^2 + 1)} dt$

$= \int \frac{1}{3t} dt$

$= 1/3 \int 1/t \, dt \quad = \int 1/x = \ln |x)$

$= 1/3 \times \ln |t| + c$

$= 1/3 \times \ln \left(|x^3 - 3x^2 + 1| \right) + c$

AK
08/01/2020

Topic: Application of Integration & Numerical Integration

Q1] Find the length of the following curve.

① $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$

② $y = \sqrt{4-x^2}$ $x \in (-2, 2)$

③ $y = x^{3/2}$ in $[0, 4]$

④ $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$

⑤ $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2] Using Simpson's Rule, solve the following.

① $\int_0^2 e^{x^2}\,dx$ with $n=4$

② $\int_0^4 x^2\,dx$ with $n=4$

③ $\int_0^{\pi/3} \sqrt{\sin x}\,dx$ with $n=6$

Sol :-

i) $x = t\sin t$, $y = 1 - \cos t$ $\in [0, 2\pi]$
for t belong to $[0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$

$\frac{dx}{dt} = t - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$$l = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2 \left|\sin\frac{t}{2}\right| dt \quad - \sin^2\frac{t}{2} - \frac{1 - \cos t}{2}$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$\left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$4 + 4 = 8$

ii) $y = \sqrt{4-x^2} \quad x \in [-2, 2]$

Sol^n:-

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi$$

iii) $y = x^{3/2}$ in $[0, 4]$

Sol^n:-

$$f'(x) = \frac{3}{2} x^{1/2}$$

$$[f'(x)]^2 = \frac{9}{4} x$$

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1 + \frac{9}{4}x}\, dx$$

put $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}\,dx$

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\,du = \left[\frac{4}{9}\cdot\frac{2}{2}\left(u^{3/2}\right)\right]_1^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$$

4) $x = 3\sin t$, $y = 3\cos t$

Sol:- $\frac{dx}{dt} = 3\cos t$

$\frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} \cancel{3\sqrt{x}\,dt}$$

$$= 3\int_0^{2\pi} x\,dt$$

$$= 3\left[x\right]_0^{2\pi}$$

$$= 3\left[2\pi - 0\right]$$

$L = 6\pi$ units

Scanned with CamScanner

5] $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1,2]$

Sol^n:- $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$$

$$= \frac{17}{12} \text{ units.}$$

Q2] i) $\int_0^2 e^{x^2}\,dx$ with $n=4$

$\int_0^2 e^{x^2}\,dx = 16.4526$

In each case the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

and so the sub intervals will be [0, 0.5] [0.5, 1] [1, 1.5] [1.5, 2]

∴ By Simpsons rule

$$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right)$$

$$\approx \underline{\underline{17.3536}}$$

$\int_0^4 x^2\,dx$ ~~$n=4$~~

$\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{64}{3}$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6$

Sol^n:-

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + 1/8) + 2(y_2 + y_4) + \right.$$

$$\approx \frac{\pi/18}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 0.930\right)$$

$$\approx 2(0.584 + 0.801) + 0.930$$

$$\approx \underline{\underline{0.681}}$$

Al
09/01/2020

PRACTICAL No. 7

9/1/20

TOPIC : Differential Equation

Q1] Solve the following differential Equation.

1) $x \frac{dy}{dx} + y = e^{x}$

2) $e^{x} \frac{dy}{dx} + 2e^{x} y = 1$

3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

4) $x \frac{dy}{dx} + 3y = \frac{\sin x}{x^{2}}$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

6) $\sec^{2} x \tan y \, dx + \sec^{2} y \tan x \, dy = 0$

7) $\frac{dy}{dx} = \sin^{2}(x - y + 1)$

8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

Sol^n:-

i) $x\frac{dy}{dx}+y=e^x$

$\therefore \frac{dy}{dx}+\frac{1}{x}y=\frac{e^x}{x}$

Comparing with $\frac{dy}{dx}+P(x)y=Q(x)$

$P(x)=\frac{1}{x}$; $Q(x)=\frac{e^x}{x}$

$I.F=e^{\int 1/x\,dx}$

$=e^{\log x}$

$I.F=x$

$y(I.F)=\int Q(x)(I.F)dx$

$y(x)=\int \frac{e^x}{x}\cdot x\,dx$

$=\int e^x dx$

$=e^x+c$

ii) $e^x\frac{dy}{dx}+2e^xy=1$

$e^x\left(\frac{dy}{dx}+2y\right)=1$

$\frac{dy}{dx}+2y=\frac{1}{e^x}$

Comparing with $\frac{dy}{dx}+P(x)y=Q(x)$

$\therefore P(x)=2$; $Q(x)=\frac{1}{e^x}$

Scanned with CamScanner

$$I.F. = e^{\int 2\,dx}$$
$$= e^{2x}$$

$$y(I.F.) = \int Q(x)\,(I.F.)\,dx$$

$$ye^{2x} = \int \frac{1}{e^{x}} e^{2x}\,dx$$

$$ye^{2x} = \int e^{-x} \cdot e^{2x}\,dx$$
$$ye^{2x} = \int e^{x}\,dx$$
$$ye^{2x} = e^{x} + c$$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$x\frac{dy}{dx} + 2y = \frac{\cos x}{x}$$

$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = \frac{2}{x}$, $Q(x) = \frac{\cos x}{x^2}$

$$I.F. = e^{\int \frac{2}{x}\,dx}$$
$$= e^{2\log x}$$
$$= e^{\log x^2}$$
$$I.F. = x^2$$
$$y(I.F.) = \int Q(x)(I.F.)\,dx$$
$$y(x^2) = \int \frac{\cos x}{\cancel{x^2}}(\cancel{x^2})\,dx$$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$

Comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$

$P(x) = 3x^{-1}$ $\quad Q(x) = \frac{\sin x}{x^3}$

$$\text{I.F.} = e^{\int \frac{3}{x}dx}$$
$$= e^{3\log x}$$
$$= e^{\log x^3}$$
$$= x^3$$

$$y\,(\text{I.F.}) = \int Q(x)(\text{I.F.})\,dx$$
$$y(x^3) = \int \frac{\sin x}{\cancel{x^3}}\,(\cancel{x^3})\,dx$$

$$x^3 y = -\cos x + c$$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$$\cancel{e^{2x}}\left(\frac{dy}{dx} + 2y\right) = 2x$$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

Comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$

$\therefore P(x) = 2$

$Q(x) = \ldots$

$\therefore \; I.F = e^{\int 2dx}$

$= e^{2x}$

$\therefore y(I.F) = \int Q(x)\,(I.F)\,dx$

$y(e^{2x}) = \int \frac{2x}{e^{2x}}(e^{2x})dx$

$ye^{2x} = \cancel{2}\frac{x^2}{\cancel{2}} + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$\frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$

$\therefore \int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x| + \log|\tan y| = c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$$1-\sin^2 v = \frac{dv}{dx}$$

$$dx = \frac{dv}{1-\sin^2 v}$$

$$\int dx = \int \sec^2 v \, dv$$

$$x = \tan v + c$$

But, $v = x+y-1$

$$\therefore \; x = \tan(x+y-1) + c$$

viii) $\frac{dy}{dx} = \frac{2\frac{x}{3}+3y-1}{6x+9y+6}$

$$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$$

Put $2x+3y\cancel{3} = v$

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

Scanned with CamScanner

$$\frac{v+2}{3(v+1)}\,dv = dx$$

$$\frac{1}{3}\int \frac{(v+1+1)}{(v+1)}\,dv = \int dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + c$$

But $v = 2x + 3y$

$$\therefore\ 2x + 3y + \log|2x+3y+1| = 3x + c$$

$$\therefore\ 3y = x - \log|2x+3y+1| + c$$

20/1/20

PRACTICAL - 08

Topic :- Euler's Method

1) $\frac{dy}{dx} = y + e^{x} - 2$ $\quad$ $y(0) = 2$, $h = 0.5$, find $y(2)$

2) $\frac{dy}{dx} = 1 + y^{2}$, $\quad$ $y(0) = 0$, $h = 0.2$ find $y(1)$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $\quad$ $y(0) = 1$, $h = 0.2$ find $y(1)$

4) $\frac{dy}{dx} = 3x^{2} + 1$, $\quad$ $y(1) = 2$, find $Y(2)$
for $h = 0.5$, $h = 0.25$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$, $\quad$ $y(1) = 1$
find $y(1.2)$ with $h = 0.2$

$\frac{dy}{dx} = y + e^x - 2$

$f(x,y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

By Eulers Formula,

$y(2) = 9.2831$

$\frac{dy}{dx} = 1 + y^2$

$f(x,y) = 1 + y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

using Euler's iteration formula,

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942
5	1	1.2942		

$\therefore$ By Euler's formula,

$y(1) = 1.2942$.

(3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $\quad y(0)=1, \quad x_0 = 0, \quad h = 0.2$

Using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

(4) $\frac{dy}{dx} = 3x^2 + 1$ $\quad y_0 = 2, \; x_0 = 1, \; h = 0$

For $h = 0.5$

Using Euler's Formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$
0	1	2	4
1	1.5	4	49
2	2	28.5	

By Euler's Formula,
$y(2) = 28.5$

For $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$
0	1	2	4
1	1.25	3	5.6875
2	1.5	4.4219	7.75
3	1.75	6.3594	10.1815
4	2	8.9048	

By Euler's Formula, $y(2) = 8.9048$.

$\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y_0 = 1, x_0 = 1, h = 0.2$.

Using Euler's iteration formula,
$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's formula,
$y(1.2) = 1.6$

Practical No-9

TOPIC: LIMITS & PARTIAL ORDER DERIVATIVES

① Evaluate the following limits.

(1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$

(2) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

(3) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

② Find f_x, f_y for each of the following f.

(i) $f(x,y) = xye^{x^2+y^2}$ (ii) $f(x,y) = e^x\cos y$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

③ Using definition find values of f_x, f_y at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

④ Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2-xy}{x^2}$ ii) $f(x,y) = x^3+3x^2y^2-\log(x^2+1)$

iii) $f(x,y) = \sin(xy)+e^{x+y}$

(5) Find the linearization of $f(x,y)$ at given point.

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$.

70

Sol^n :-

1)

i) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

$$= \lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

Applying limits

$$= \frac{(-4)^3-3(-4)+(-1)^2-1}{(-4)(-1)+5}$$

$$= \frac{-64+12+1-1}{4+5}$$

$$= \frac{-52}{9}$$

ii) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Apply Limit

$$= \frac{(0+1)[(2)^2+(0)^2-4(2)]}{2+3(0)}$$

$$= \frac{1(4+0-8)}{2}$$

$= \frac{4-8}{2}$

$= \frac{-4}{2}$

$= \underline{\underline{-2}}$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

Apply limit

$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)^2}$

$= \frac{1-1}{1-1}$

$= \frac{0}{0} = \underline{\underline{0}}$

Limit does not exist.

~~$f(x,y) = xe^{x^2+y^2}$~~

$f_{xy} = y(1.e^{x^2+y^2}) + xy(e^{x^2+y^2} \cdot 2x)$

$=$

Q2]

i) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore f_y = 2yxe^{x^2+y^2}$

ii) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6xy$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q3]

i) $f(x,y) = \frac{2x}{1+y^2}$

$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2 + 2y^2 - 0}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)}$

$= \frac{2}{1+y^2}$

At (0,0)

$= \frac{2}{1+0}$

$= 2$

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At (0,0)

$$= \frac{-4(0)(0)}{(1+0)^2}$$

$= 0$

Q4]

i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$= \frac{x^2\frac{\partial}{\partial x}(y^2-xy) - (y^2-xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$= x^2(-y) - (y^2-xy)(2x)$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y-x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(y^2-xy)\}}{x^4}\right)$$

$$= x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2x^2y)}{x^6}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y-x}{x^2}\right)$$

$$= \frac{2-0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y-x}{x^2}\right)$$

$$= x^2 \frac{\partial}{\partial x} \frac{(2y-x) - (2y-x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$\therefore \underline{\underline{f_{xy} = f_{yx}}}$$

(ii) $f(x,y)\ x^3+3x^2y^2-\log(x^2+1)$

$f_x = \frac{\partial}{\partial x}(x^3+3x^2y^2-\log(x^2+1))$

$= 3x^2+6xy^2-\frac{2x}{x^2+1}$

$f_y = \frac{\partial}{\partial y}(x^3+3x^2y^2-\log(x^2+1))$

$= 0+6x^2y-0$

$= 6x^2y$

$f_{xx} = 6x+6y^2-\left(\frac{x^2+1\frac{\partial(2x)}{\partial x} - 2x\frac{\partial(x)}{\partial x}}{(x^2+1)^2}\right)-①$

$f_{yy} = \frac{\partial}{\partial y}(6x^2y) = \underline{\underline{6x^2}}$

$f_{xy} = \frac{\partial}{\partial y}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$

$= 0+12xy-0 = \underline{\underline{12xy}}$

$f_{yx} = \frac{\partial}{\partial x}(6x^2y) = \underline{\underline{12xy}}$

$\therefore f_{xy} = f_{yx}$

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}$

$= y\cos(xy) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}$

$= x\cos(xy) + e^{x+y}$

$f_{xx} = \frac{\partial}{\partial x}(y\cos(xy) + e^{x+y})$

$= -y\sin(xy)\cdot(y) + e^{x+y}$

$= -y^2\sin(xy) + e^{x+y}$

$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$

$= -x\sin(xy)(x) + e^{x+y}$

$= -x^2\sin(xy) + e^{x+y}$

$f_{xy} = \frac{\partial}{\partial y}(y\cos(xyy + e^{x+y}))$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$

$\therefore f_{xy} \neq f_{yx}$

6

Scanned with CamScanner

Q5]

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad = \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$ $\quad f_y$ at $(1,1) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$$

$$= \cancel{\sqrt{2}} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\cancel{\sqrt{2}}}$$

$$= \frac{x+y}{\sqrt{2}}$$

Scanned with CamScanner

ii) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0) = -1 + 0$
$= \underline{\underline{-1}}$

$f_y = 0 - 0 + \sin x$
$= \sin\frac{\pi}{2} = \underline{\underline{1}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$
$= 1 - \frac{\pi}{2} + (-1)(x - \pi/2) + 1(y - 0)$
$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$
$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$

$f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$

f_y at $(1,1) = 1$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$
$\cancel{= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)}$
$= 0 + 1(x-1) + 1(y-1)$
$= x - 1 + y - 1$
$= \underline{\underline{x + y - 2}}$

25

PRACTICAL 10

TOPIC :- Directional derivative, gradien Vector & maxima, minima Tangent & normal values.

Q1] Find the directional derivative of the following function at given points & in the direction of given vector.

(i) $f(x,y) = x+2y-3$, $a=(1,-1)$ $u = 3i-j$

(ii) $f(x,y) = y^2-4x+1$, $a=(3,4)$, $u = i+5j$

(iii) $f(x,y) = 2x+3y$, $a=(1,2)$ $u = 3i+4j$

Q2] Find gradient vector for the following function at given point.

① $f(x,y) = x^y + y^x$, $a=(1,1)$

② $f(x,y) = (\tan^{-1}x) \cdot y^2$, $a=(1,-1)$

③ $f(x,y) = xyz - e^{x+y+z}$, $a=(1,-1,0)$

Q3] Find the equation of tangent & normal to each of the following using curves at given points.

① $x^2\cos y + e^{xy} = 2$ at $(1,0)$

② $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

Q4] Find the equation of tangent & normal line to each of the following surfaces.

① $x^2-2yz+3y+xz=7$ at $(2,1,0)$

② $3xyz-x-y+z=-4$ at $(1,-1,2)$

Q5] Find the local maxima & minima for the following functions.

① $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

② $f(x,y) = 2x^4 + 3x^2y - y^2$

③ $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

Solution:-

i) $f(x,y) = x+2y-3$ $\quad a=(1,-1) \quad u = 3i-j$

Here, $u = 3i-j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{v}{|v|} = \frac{1}{\sqrt{10}}(3,-1) = \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= \left(1+\frac{3}{\sqrt{10}}\right) - 2 - \frac{2h}{\sqrt{10}} - 3$

$= -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} + \cancel{4}}{\cancel{h}}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4) \quad u = i+5j$

Here, $u = i+5j$ is not a unit vector.

$|\bar{u}| = \sqrt{(1)^2+(5)^2} = \sqrt{26}$

Unit vector along u is $\frac{v}{|v|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \dfrac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} \cancel{+5} \cancel{-5}}{h}$

$= \dfrac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$

$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$

) $2x + 3y$ $\quad a = (1,2)$ $\quad u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8.$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$f(a+hu) = 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$Duf(a) = \lim_{h \to 0} \frac{\frac{18\cancel{h}}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}$$

$$= \frac{18}{5}$$

Q2]

i) $f(x,y) = x^4 + y^x \qquad a = (1,1)$

$fx = y \cdot x^4 + y^x \log y$

$fy = x^3 \log x + xy^{xy}$

$Df(x,y) = (fx, fy)$

$= (yx + y^2 \log y, x^3 \log x + xy^{xy})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \qquad a = (1,-1)$

$fx = \frac{1}{1+x^2} \cdot y^2$

$fy = 2y \tan^{-1} x$

$Df(x,y) = (fx, fy)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right)$

Scanned with CamScanner

$= \left(\frac{1}{2}, \frac{\pi}{4} \frac{(-2)}{2}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$f(1,-1,0) = \left((-1)(0) - e^{(1-1+0)}, (1)(0) - e^{(1+(-1)+0)}, ((1)(-1) - e^{(1-1+0)})\right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3) Find the equation.

i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy}.y$

$f_y = x^2(-\sin y) + e^{xy}.x$

$(x_0, y_0) = (1, 0) = x_0 = 1; y_0 = 0$

Equation of tangent

$f(x)(x - x_0) + f_y(y - y_0) = 0$

$f(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0.1$

$= 0 + 1.1$

$= \underline{\underline{1}}$

$$2(x-1)+1(y-0)=0$$
$$2x-2+y=0$$
$$2x+y-2=0$$

It is the required eq^n of tangent

Equation of Normal

$$= ax+by+c=0$$
$$= bx+ay+d=0$$
$$\therefore 1(1)+2(y)+d=0$$
$$1+2y+d=0$$
$$1+2(0)+d=0$$
$$d+1=0$$
$$\underline{\underline{d=-1}}$$

ii) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$$f_x = 2x+0-2+0+0$$
$$= 2x-2$$
$$f_y = 0+2y-0+3+0$$
$$= 2y+3$$
$$(x_0, y_0) = (2,-2) \quad \therefore x_0 = 2, \; y_0 = -2$$
$$f_x = (x_0, y_0) = 2(2)-2 = 2$$
$$f_y = (x_0, y_0) \; 2(-2)+3 = -1.$$

Equation of tangent

$f_x (x - x_0) + f_y (y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$

It is required eqn of tangent

Equation of normal.

$= ax + by + c = 0$

$= bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\underline{\underline{d = 6}}$

$x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + 2$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$f_y = +2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3$

$= 0$

$4x + 3y - 11 = 0 \rightarrow$ This is required eq^n of tangent

$(4, 3, -11)$

Equation of Normal at

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $\therefore x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Scanned with CamScanner

Equation of tangent

$-7(x-1)+5(y+1)-2(z-2)=0$

$= -7x+7+5y+5 \cancel{z} -2z+4=0$

$= -7x+5y-2z+16=0$

This is required eqn of tangent.

Equation of Normal at $(-7, 5, -2)$.

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

ot tangent.

$f(x,y) = 3x^2+y^2-3xy+6x-4y$

$f_x = 6x+0-3y+6-0$

$= 6x-3y+6$

$f_y = 0+2y-3x+0-4$

$= 2y-3x-4$

$f_x = 0$

$6x-3y+6=0$

$3(2x-y+2)=0$

$2x-y+2=0$

$2x-y=-2. \quad -(1)$

$f_y=0$

$2y-3x-4=0$

$\cancel{2y-y=-2}.$

$2y-3x=4. \quad -(2)$

$c^2.$

16

multiply eq^n ① with ②

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substituting value of x in eq^n ①

$2(0) - y = -2$

$+y = +2$

$\therefore y = 2$

Critical points at $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here, $r > 0$

$= rt - 3^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ is has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$fx = 8x^3 + 6xy$

$fy = 3x^2 - 2y$

$fx = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \quad -①$

$fy = 0$

$\therefore 3x^2 - 2y = 0 \quad -②$

Multiply eqⁿ ① with ③ & ② with ④

$12x^2 + 9y = 0$

$- 12x^2 - 8y = 0$

$+$

$y = 0$

Substitute value of y in eqⁿ ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is (0,0)

$r = fxx = 24x^2 + 6x = 0$

$t = fyy = 0 - 2 = -2$

$s = fxy = 6x - 0 = 6x = 6(0) = 0$

r at (0,0)

$= 24(0) + 6(0)$

$= 0$

8

$rt - s^2 = 0(-2) - (5)^2$

$= 0 - 0$

$= 0$

$r = 0 \quad rt - s^2 = 0$

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= \underline{\underline{0}}$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$fx = 2x + 2$

$fy = -2y + 8$

$fx = 0 \quad \therefore 2x + 2 = 0 \quad x = -2/2 \quad \underline{x = -1}$

$fy = 0 \quad \therefore -2y + 8 = 0 \quad y = -8/-2 \quad \therefore \underline{\underline{y = 4}}$

$\therefore$ Critical point is $(-1, 4)$

$r = fxx = 2$

$t = fyy = -2$

$s = fxy = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= -33$

★★ INDEX ★★