" Shahd Hesham "

Sheet(1)   / /

Neural Network

A- Forward pass :-

* the total net input of h1 :-

* $net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$

$= (0.15 \times 0.05) + (0.2 \times 0.1) + 0.35 = 0.3775$

* using Activation sigmoid function to get the output of h1:-

* $out_{h1} = \frac{1}{1 + e^{-net_{h1}}} = \frac{1}{1 + e^{-0.3775}} = 0.593269992$

* the total net input of h2:-

$net_{h2} = w_3 * i_1 + w_4 * i_2 + b_1 * 1$

$net_{h2} = (0.25 \times 0.05) + (0.3 \times 0.1) + 0.35 = 0.3925$

* using Activation sigmoid function to get the output of h2

* $out_{h2} = \frac{1}{1 + e^{-0.3925}} = 0.596884378$

* the output of O1

* $net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$

$= (0.4 \times 0.593269992) + (0.45 \times 0.596884378) + 0.6$

$= 1.105905967$

* $out_{o1} = \dfrac{1}{1 + e^{-net_{o1}}} = 0.75136507$

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* the output of O2

* $net_{o2} = w_7 * out_{h1} + w_8 * out_{h2} + b_2 * 1$

$= (0.5 \times 0.593269992) + (0.55 \times 0.5968843783) + 0.6$

$= 1.224921404$

* $out_{o2} = \dfrac{1}{1 + e^{-net_{o2}}} = 0.772928465$

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* Calculating the error in each O_1 & O_2

* $E_{o1} = \frac{1}{2}(Target - out_{o1})^2$

$= \frac{1}{2}[0.01 - 0.75136507]^2 = 0.274811083$

* $E_{o2} = \frac{1}{2}\left[0.99 - 0.772928465\right] = 0.023560026$

* the Total Error is the sum of $E_{o1} + E_{o2}$

$$E_{total} = 0.298371109$$

B. Backword pass:

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* In output layer:

By Applying chain Rule:

$$\Rightarrow \frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$* \frac{\partial E_{total}}{\partial w_5} = -(target_{o1} - out_{o1})^* \, out_{o1}(1 - out_{o1})^* \, out_{h1}$$

$$= -(0.01 - 0.75136507)^* \, 0.75136507(1 - 0.75136507) * 0.593269992$$

$$= 0.082167041$$

$$\Rightarrow \frac{\partial E_{total}}{\partial w_6} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial w_6}$$

$$= -(0.01 - 0.75136507)^* \, 0.75136507(1 - 0.75136507) * 0.596884378$$

$$= 0.0826676278$$

$$\Rightarrow \frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_7}$$

$$= -(Target - out_{o2}) * out_{o2}(1 - out_{o2}) * out_{h1}$$

$$= -(0.99 - 0.772928465) * 0.772928465(1 - 0.772928465) * 0.593269992$$

$$=$$

$$\Rightarrow \frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial w_8}$$

$$= -(target - out_{o2}) * out_{o2}(1 - out_{o2}) * out_{h2}$$

$$= -(0.217071535) * 0.175510053 * 0.596884378$$

$$= -0.02274024227$$

Assumming that the learning Rate is $\eta = 0.5$

→ then by updating weights w_5, w_6, w_7, w_8

$$* \; w_5' = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5}$$

$$= 0.4 - [0.5 * 0.082167041] = 0.35891648$$

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By Applying the same Rule on each weight

So, $w_6' = 0.408666186$

$w_7' = 0.511301270$

$w_8' = 0.561370121$

In Hidden layer :-

By Applying chain Rule.

$$* \frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}} * \frac{\partial net_{h1}}{\partial w_1}$$

where :

$$* \frac{\partial net_{o1}}{\partial out_{h1}} = \frac{\partial}{\partial out_{h1}} \left[out_{h1} * w_5 + out_{h2} * w_6 + b_2 \right]$$

$= w_5 = 0.4$ ~~$= w_5 + w_6 = 0.4 + 0.45 = 0.85$~~

$$* \frac{\partial out_{h1}}{\partial net_{h1}} = out_{h1}(1 - out_{h1})$$

$= 0.593269992\,(1 - 0.593269992)$

$= 0.2413007086$

$$* \frac{\partial net_{h1}}{\partial w_1} = input_1 = 0.05$$

Then by Multiplying :-

$$* \frac{\partial E_{total}}{\partial w_1} = 0.00066839602$$

then, $w_1' = w_1 - \eta \frac{\partial E_{total}}{\partial w_1} = 0.15 - [0.5 \times 0.00066839602] = 0.149665802$

$$\boxed{\therefore w_1' = 0.149665802}$$

$$* \frac{\partial E_{total}}{\partial w_3} = \underbrace{\frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial out_{h1}} * \frac{\partial out_{h1}}{\partial net_{h1}}}_{\text{"Given"}} * \frac{\partial net_{h1}}{\partial w_3}$$

Since, $\frac{\partial net_{h1}}{\partial w_3} = i_2 = 0.1$

then, $\frac{\partial E_{total}}{\partial w_3} = 0.1384985615 * 0.4 * 0.2413007086 * 0.1$

$= 0.0013367920 4$

then $w_3' = w_3 - \eta \frac{\partial E_{total}}{\partial w_3}$

$= 0.25 - [0.5 \times 0.0013367920 4]$

$$\boxed{\therefore w_3' = 0.249331604}$$

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$$* \ \frac{\partial E_{total}}{\partial w_2} = \frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_{h2}} * \frac{\partial out_{h2}}{\partial net_{h2}} * \frac{\partial net_{h2}}{\partial w_2}$$

where,

$$* \ \frac{\partial net_{o2}}{\partial out_{h2}} = \frac{\partial}{\partial out_{h2}}\left[h_1 * w_7 + h_2 * w_8 + b \right] = w_8$$

$$= 0.55$$

$$* \ \frac{\partial out_{h2}}{\partial net_{h2}} = out_{h2}(1 - out_{h2}) = 0.2406134172$$

$$= 0.5968843783(1 - 0.5968843783)$$

$$* \ \frac{\partial net_{h2}}{\partial w_2} = input_1 = 0.05$$

then, $$\frac{\partial E_{total}}{\partial w_2} = 0.0380982366 * 0.55 * 0.2406134172 * 0.05$$

$$= 0.00025209103$$

then, $$w_2' = w_2 - \eta \frac{\partial E_{total}}{\partial w_2}$$

$$= 0.2 - \left[0.5 \times 0.00025209103 \right]$$

$$\boxed{\therefore \ w_2' = 0.1998739545}$$

$$* \quad \frac{\partial E_{total}}{\partial w_4} = \underbrace{\frac{\partial E_{total}}{\partial out_{o2}} * \frac{\partial out_{o2}}{\partial net_{o2}} * \frac{\partial net_{o2}}{\partial out_{h2}} * \frac{\partial out_{h2}}{\partial net_{h2}}}_{\text{Givens}} * \frac{\partial net_{h2}}{\partial w_4}$$

$$\text{Since}, \quad \frac{\partial net_{h2}}{\partial w_4} = input_2 = 0.1$$

$$\text{then}, \quad \frac{\partial E_{total}}{\partial w_4} = 0.0380982366 * 0.55 * 0.2406134172 * 0.1$$

$$= 0.00050418206$$

$$\text{then}, \quad w_4' = w_4^* - \eta \frac{\partial E_{total}}{\partial w_4}$$

$$= 0.3 - [0.5 * 0.00050418206]$$

$$\rightarrow \boxed{w_4' = 0.29950229}$$