Lab-5 and Lab-6: Modeling Epidemics Spread by SIR

Dhruv Shah (202103017),[*] Pranav Patel (202103040),[†] and Vatsal Shah (202103022)[‡]
Dhirubhai Ambani Institute of Information & Communication Technology, Gandhinagar, Gujarat 382007, India
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In this report we will attempt to model the SIR model of epidemic spread and draw meaningful observations using numerical methods to plot the dynamics of the model.

I. INTRODUCTION

The SIR Model is a simple yet very crucial mathematical model used to study the spread of infectious diseases in a population. Developed by Kermack and McKendrick, this model is widely applied in epidemiology and is used by healthcare officials and policymakers to estimate the impact of various intervention strategies on the epidemic's spread. The acronym SIR stands for Susceptible, Infected, and Recovered, which are the three main compartments used to categorize individuals in the population based on their disease status. In this lab, we will employ the SIR model to the problem of influenza spread at a boys' boarding school. Subsequently, we will observe the impact of vaccination on the course of an epidemic. Later, we will also study the effect of lockdown on the epidemic's spread.

II. MODEL

The population has been divided into 3 compartments namely:

- **Susceptible[S]:** They have no immunity against the disease and hence can get it when they come in contact with an infected individual.
- **Infected[I]:** They have the disease and can potentially spread it to others.
- **Recovered[R]:** They are recovered from the disease and are immune to further infections

We take a closed population sample where there are no births, deaths, immigration, or immigration, basically, the total population(N) remains constant. Initially, everyone is susceptible and there is one infected individual. Let c denote the number of contacts per unit of time and let p denote the transmission probability, the product cp can be known as the transmission rate β_f. The number of individuals contacted per unit of time by the infected population is $\beta_f I(S/N)$. Let α be the recovery rate. Hence the differential equations of our SIR model can be written as:

$$\frac{dS}{dt} = -\beta_f I \frac{S}{N} \tag{1}$$

$$\frac{dI}{dt} = \beta_f I \frac{S}{N} - \alpha I \tag{2}$$

$$\frac{dR}{dt} = \alpha I \tag{3}$$

Eq.2 can be re-written as :

$$\frac{dI}{dt} = \alpha I(\frac{\beta_f S}{\alpha N} - 1) \tag{4}$$

Let $R_0 = \frac{\beta_f S}{\alpha N}$ is known as the reproduction number which basically signifies the number of secondary infections per unit time if an infected person was put in an entirely susceptible population. It determines the rate of change of I, when $R_0 < 1$, the epidemic grows, and when $R_0 < 1$ the epidemic decays. Simulating the differential equations of our model numerically, we get the following curves for the below-mentioned parametric values.

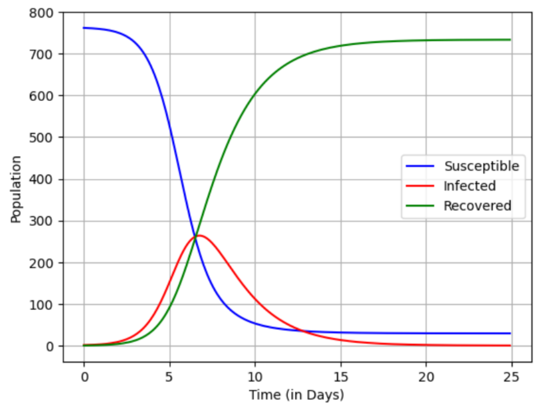

FIG. 1: SIR Model: Rate of infection= $\beta_f/N = 0.00218$, Rate of recovery=0.5, Initially Susceptibles=762 and Infected = 1

As we can infer from the graph, after a long time, the number of infected becomes zero and hence the epidemic dies out, note that the susceptible never reaches zero, as there is always someone who never got infected in large populations. We will now modify the SIR model to incorporate the effect of vaccination and lockdown.

[*]Electronic address: 202103017@daiict.ac.in
[†]Electronic address: 202103040@daiict.ac.in
[‡]Electronic address: 202103022@daiict.ac.in

III. RESULTS

A. Effect of Vaccination in the SIR Model

Vaccinating a population has a direct effect on the number of susceptible. Let ρ be the rate of vaccination. Initially, we assume that immunization begins immediately after vaccination.

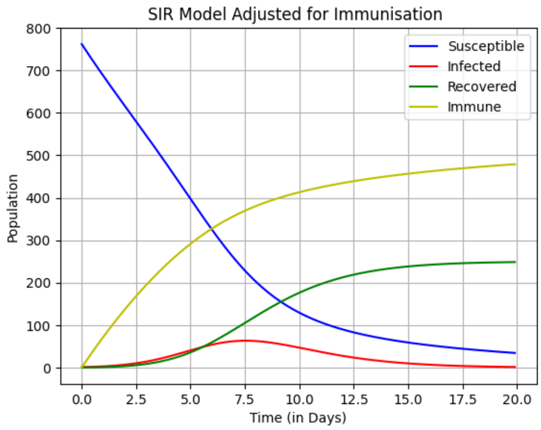

FIG. 2: $\rho = 0.10$, Rate of Infection = 0.00218, $\alpha = 0.5$

Fig.2 clearly shows how the peak of infected flattens in the case of vaccination of the susceptible population. We will now examine the peak of the Infected Population as the rate of vaccination is changed.

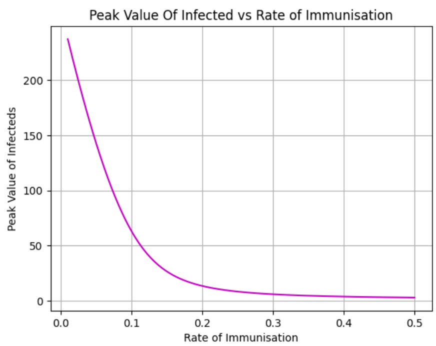

FIG. 3: Rate of Infection = 0.00218, Rate of Recovery = 0.5

We observe from Fig.3 that when the immunization rate increases, the peak value of the infected population decreases, hence decreasing the severity of the epidemic. Here, our assumption that immunization begins immediately after vaccination is far from the real-world scenario. Since most vaccines are built such that they take time to build antibodies to make an individual immune to the disease, we will examine some cases where the immunization does not occur immediately.

1. *Immunisation begins three days after vaccination*

In Fig.4 and Fig.5 we have simulated the case when the immunization begins three days after vaccination. We observe that here as people get immune three days after they are vaccinated the peak of the infected population comes early. We have taken the Rate of Infection = 0.00218 day^{-1}, Rate of Recovery = 0.5 day^{-1} and Rate of Immunisation = 0.15 day^{-1}. Initial number of Susceptibles = 762 with simulations done for 20 days.

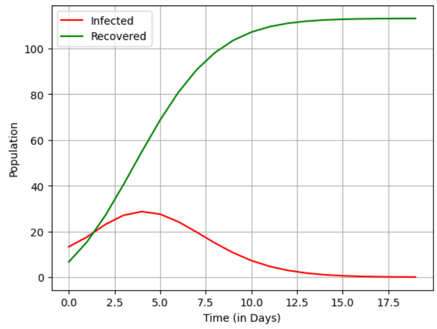

FIG. 4: Number of Infecteds and Recoverds vs Time

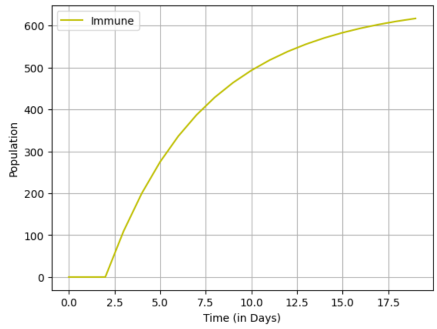

FIG. 5: Rate of Infection = 0.00218, Rate of Recovery = 0.5

2. *All susceptibles are vaccinated two days before a person gets infected and immunization begins after four days:*

Fig.6 and Fig.7 shows the evolution in the number of infected, recovered, and immune with time. We have assumed the Rate of Infection = 0.00218 day^{-1} and the Rate of Recovery = 0.5 day^{-1}

B. Effect of lockdown in SIR model

Lockdown affects the rate of contact c, basically affecting the transmission rate β_f. We assume that the basic reproduction number $R_0 > 1$ initially and assume

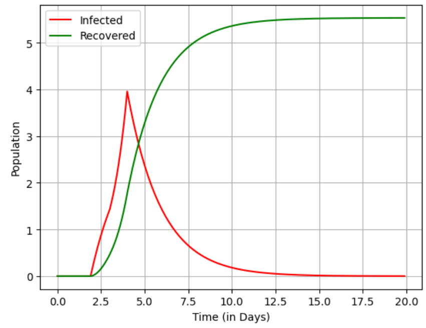

FIG. 6: Population Infected and Population Recovered vs Time

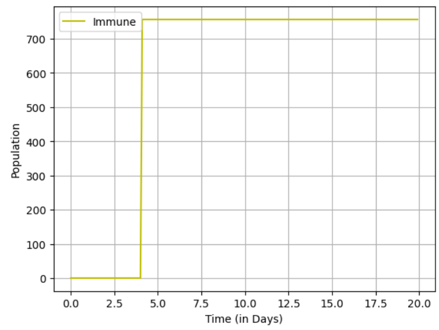

FIG. 7: Population Immune vs Time

$\beta_f(t) = (1-\theta_t)\beta_f$, where θ_t is the severity of the lockdown and $\beta_f(t)$ is resultant transmission coefficient due to lockdown.

$$\theta_t = \begin{cases} A & \text{if } t_1 \leq t \leq t_2 \\ 0 & otherwise \end{cases} \tag{5}$$

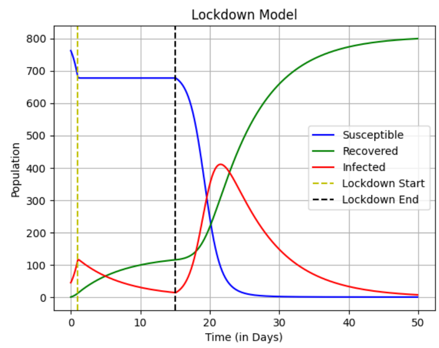

FIG. 8: Lockdown is implemented from the 2nd Day till the 15th Day. Assume, Rate of Recovery = $0.15day^{-1}$, Initial Number of Susceptible = 762, Initial Number of Infected = 45, A = 0.75

The value of β_f decreases as the lockdown period progresses, and after opening the lockdown, the β_f value will be the same as before the lockdown. During the lockdown period, the transmission rate decreases, and hence the Infected population decreases, then again increases after the lockdown ends as we can see in the Fig. 8. This shifts the peak toward the right, that is delays the peak, so the government can utilize the extra time to improve the medical facilities and be prepared for the epidemic.
Now, we implemented lockdown from day 7 to 20, that is after the peak of infection has already occurred, we see no significant effect in terms of peak shifting, the peak is the same as if there were no lockdown. So, the ideal duration for the lockdown should be before the peak.

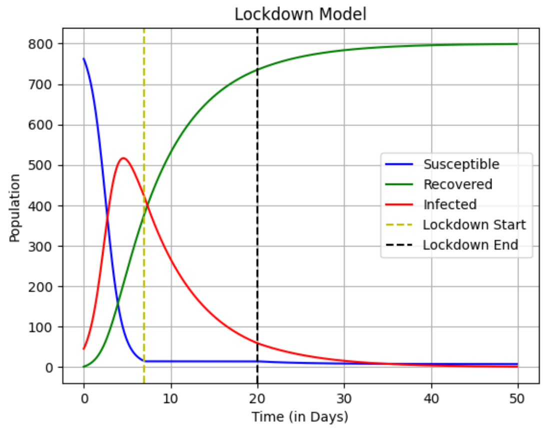

FIG. 9: Lockdown is implemented from the 7th Day till the 20th Day. Assume, Rate of Recovery = $0.15day^{-1}$, Initial Number of Susceptible = 762, Initial Number of Infected = 45, A = 0.75

Mathematically from Eq.4, when $R_0 > 1$, we know that a peak in infection will occur. Now before the peak time, $\frac{\beta_f S}{\alpha N} > 1$, hence there is a positive rate of change of infections, that is I constantly increases. But now if we implement lockdown β_f will decrease such that the rate of infections becomes negative, so the number of infections will decrease during the lockdown period and then again increase after the lockdown ends. Eventually, this causes the peak to shift towards the right.
In Fig.9, we observe that as the severity of the lockdown is increased, the time at which the Maximum number of Infected Occurs increases.

We will now attempt to include Human behaviour in the above lockdown model. Therefore we assume the following function for the effectiveness of lockdown.

$$\theta_t = \begin{cases} 2(t-t_1)/(t_2-t_1) & \text{if } t_1 \leq t \leq (t_1+t_2)/2 \\ 2-2(t-t_1)/(t_2-t_1) & \text{if } (t_1+t_2)/2 < t \leq t_2 \\ 0 & otherwise \end{cases} \tag{6}$$

We clearly observe from Fig.11 that initially when the lockdown is not severe we observe a rise in the infections, but as the severity of the lockdown increase according to the function defined above the number of infections decrease. But as soon as the lockdown is relaxed, the infections again rise.

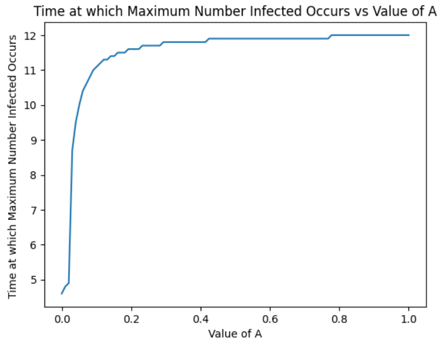

FIG. 10: Here the lockdown is implemented from 1st to 5th Day

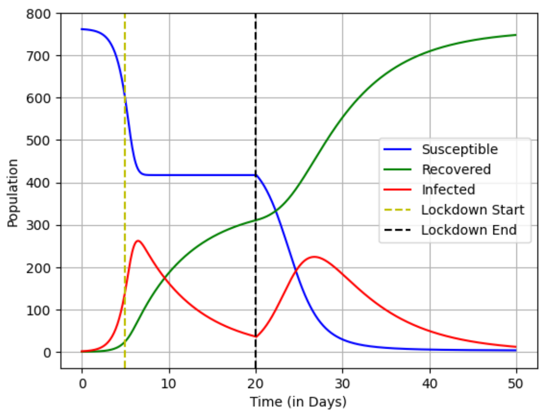

FIG. 11: Here the lockdown is implemented from 5th to 20th day, and β_f varies using

IV. CONCLUSIONS

In conclusion, the SIR model provides a valuable framework for understanding the dynamics of infectious disease spread in a population. Using the SIR model, we explored the impact of vaccination and lockdown measures on the spread of infectious diseases. The effect of vaccination was analyzed in terms of the reduction in the peak of infected individuals, with variations in the rate of vaccination and the timing of immunization after vaccination considered. It was evident that early vaccination and rapid immunization could significantly mitigate the severity of epidemics.

Additionally, the report investigated the role of lockdown measures in controlling disease outbreaks. It was found that implementing a lockdown before the peak of infections could effectively delay the peak and provide crucial time for healthcare systems to prepare. The severity of the lockdown (represented by parameter A) was shown to influence the timing of the peak, with stricter measures leading to more delay in the peak of infections.

[1] A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).