

①

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}$$

a) 2×3

b) not square $\rightarrow \underline{D}, E, G$

c) not symmetric \rightarrow

d) $X \rightarrow B$

e) $B \rightarrow A$

f) F, E

g) G, E

h)

$$A^T = \begin{bmatrix} -1 & 0 \\ 23 & -2 \\ 10 & -11 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -6 & 3 & 5 & 1 \\ 2 & -3 & -11 & -1 \\ 10 & 4 & 9 & 9 \end{bmatrix}$$

$1+0+2$

②

$$A \cdot B = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 4 & -5 \end{bmatrix}$$

$$B \cdot C = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ 1 & -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -3 & 29 & -5 & 7 \end{bmatrix}$$

not possible

③

$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix}$$

~~$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix}$$~~

$$|M| = 15 \times 2 - 30 = 0$$

$$M = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{pmatrix}$$

$$2 \begin{pmatrix} -2 & -6 \\ -8 \end{pmatrix} - 3 \begin{pmatrix} 1 & -9 \\ -8 \end{pmatrix} + 1 \begin{pmatrix} -2 & -6 \\ -8 \end{pmatrix} = -16 + 24 - 8 = 0$$

④

$$A = \begin{pmatrix} -3 & -2 \\ 3 & 3 \end{pmatrix}$$

$$|A| = (-9 + 6) = -3$$

$$A^{-1} = \frac{1}{-3} \begin{pmatrix} 3 & 2 \\ -3 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{2}{3} \\ 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} + & - & + \\ 1 & 0 & 1 \\ 0 & + & - \\ 1 & - & 1 \end{pmatrix}$$

$$|A| = 1(-1) - 0 + 1(0-1) = -1$$

$$-1 \times \begin{pmatrix} 0 & +1 & -1 \\ 1 & 0 & -1 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

⑤ b

⑥

$$F(x,y) = X^T A y + X^T B x + Cy + D$$

$$X \in \mathbb{R}^M$$

m Component

$$y \in \mathbb{R}^N$$

N Component

$$X^T \rightarrow 1 \times 1$$

$$X^T A y$$

$$1 \times M \times M \times N$$

$$X^T B x$$

$$1 \times M \times M \times 1 \times M$$

$$B \rightarrow 1 \times 1$$

$$C y$$

$$1 \times 1 \times N$$

$$A \rightarrow M \times N$$

$$B \rightarrow M \times M$$

$$x \in \mathbb{R}^M \rightarrow \begin{pmatrix} \end{pmatrix}_{M \times 1}$$

$$y \in \mathbb{R}^N \rightarrow \begin{pmatrix} \end{pmatrix}_{N \times 1}$$

$$x^T A y$$

Diagram showing the dimensions of the terms in the expression $x^T A y$:

- x^T is $1 \times M$
- A is $M \times N$
- y is $N \times 1$

$$A \rightarrow M \times N$$

$$x^T B x$$

Diagram showing the dimensions of the terms in the expression $x^T B x$:

- x^T is $1 \times M$
- B is $M \times M$
- x is $M \times 1$

$$B \rightarrow M \times M$$

$$C y$$

Diagram showing the dimensions of the terms in the expression $C y$:

- C is $1 \times N$
- y is $N \times 1$

$$C \rightarrow 1 \times N$$

$$D \rightarrow 1 \times 1$$