Digital Logic Design

Boolean Expression Standard Forms:
Product of Sum

The Product-of-Sums (POS) Form

- When two or more sum terms are multiplied, the resulting expression is a **Product-of-Sums (POS)**.
- Some examples are:

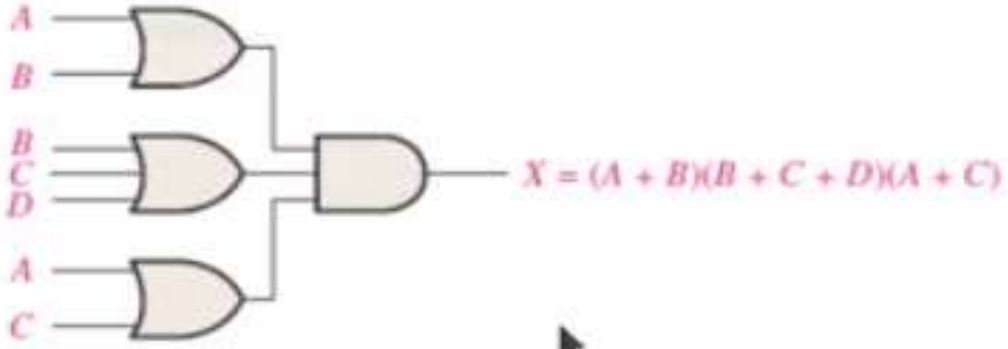
$$(\overline{A} + B)(A + \overline{B} + C)$$
$$(\overline{A} + \overline{B} + \overline{C})(C + \overline{D} + E)(\overline{B} + C + D)$$
$$(A + B)(A + \overline{B} + C)(\overline{A} + C)$$

- In a POS expression, a single overbar cannot extend over more than one variable; however, more than one variable in a term can have an overbar.
- For example,

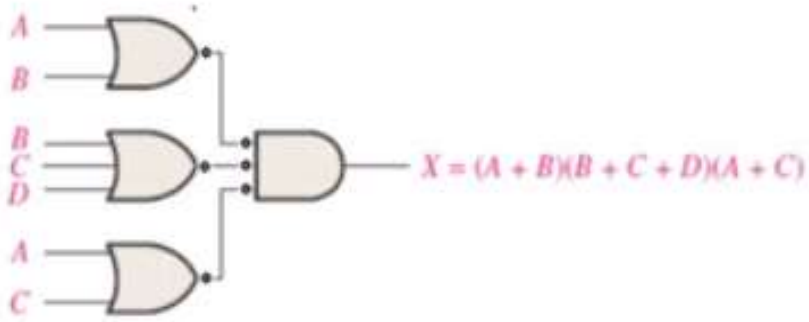
POS expression can have the term $\overline{A} + \overline{B} + \overline{C}$ but not $\overline{A + B + C}$.

Implementation of a POS Expression

- A POS expression can be implemented by logic in which the outputs of a number (equal to the number of sum terms in the expression) of OR gates connect to the inputs of an AND gate.



NOR/NOR Implementation of an POS Expression



The Standard Product of Sum (POS) Form

The Standard POS Form

- A standard POS expression is one in which all the variables in the domain appear in each sum term in the expression.
- For example,

$$(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + \overline{B} + C + D)(A + B + \overline{C} + D)$$

- Each sum term in a POS expression that does not contain all the variables in the domain can be expanded to standard form to include all variables in the domain and their complements.
- A nonstandard POS expression is converted into standard form using Boolean algebra rule 8 $(A \cdot \overline{A} = 0)$ and 12 $(A + B)(A + C) = A + BC$

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

$$= (A + \overline{B} + C + D\overline{D})(\overline{A}\,\overline{A} + \overline{B} + C + \overline{D})$$

$$(A + \overline{B} + \overline{C} + D)$$

$$= (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})$$

$$(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

$$= (A + \overline{B} + C + D\overline{D})(\overline{A}\,\overline{A} + \overline{B} + C + \overline{D})$$

$$(A + \overline{B} + \overline{C} + D)$$

$$= (A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})$$

$$(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

POS Expression to Truth Table Format

Binary Representation of a Standard Sum Term

- A POS expression is equal to 0 only if one or more of the sum terms in the expression is equal to 0.
- Remember, a sum term is implemented with an OR gate whose output is 0 only if each of its inputs is 0.
- Inverters are used to produce the complements of the variables as required.
- For Example;

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

Converting POS Expressions to Truth Table Format

- A POS expression is equal to 0 only if at least one of the sum terms is equal to 0.
- To construct a truth table from a POS expression, list all the possible combinations of binary values of the variables just as was done for the SOP expression.
- Finally, place a 0 in the output column (X) for each binary value that makes the expression a 0 and place a 1 for all the remaining binary values.

Converting POS Expressions to Truth Table Format

EXAMPLE 4–21

Determine the truth table for the following standard POS expression:

$$(A + B + C)(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$$

A	B	C	Output X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$

A	B	C	D	Output X
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	0
1	1	1	0	
1	1	1	1	

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$

A	B	C	D	Output X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

Determining Standard POS Expressions from a Truth Table

- **To determine the standard POS expression represented by a truth table,**

1. list the binary values for which the output is 0.
2. Convert each binary value to the corresponding sum term by replacing each 1 with the corresponding variable complement and each 0 with the corresponding variable.

EXAMPLE 4–22

From the truth table in Table 4–8, determine the standard POS expression.

TABLE 4–8

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

ST

$A+B+C$

$A+B+\bar{C}$

$A+\bar{B}+C$

$\bar{A}+B+\bar{C}$

$$X=(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+\bar{C})$$

Converting SOP to POS

Converting Standard SOP to Standard POS

Step 1: Evaluate each product term in the SOP expression. That is, determine the binary numbers that represent the product terms.

Step 2: Determine all of the binary numbers not included in the evaluation in Step 1.

Step 3: Write the equivalent sum term for each binary number from Step 2 and express in POS form.

- Using a similar procedure, you can go from POS to SOP.

EXAMPLE 4–19

Convert the following SOP expression to an equivalent POS expression:

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

000 010 011 101 111

$X = (A+B+\bar{C})(\bar{A}+B+C)$

$(\bar{A}+\bar{B}+C)$

A	B	C	Sum Term
~~0~~	~~0~~	~~0~~	
0	0	1	$A+B+\bar{C}$
~~0~~	~~1~~	~~0~~	
~~0~~	~~1~~	~~1~~	
1	0	0	$\bar{A}+B+C$
~~1~~	~~0~~	~~1~~	
1	1	0	$\bar{A}+\bar{B}+C$
~~1~~	~~1~~	~~1~~	

Converting POS to SOP Form

Converting Standard POS to Standard SOP

Step 1: Evaluate each sum term in the POS expression. That is, determine the binary numbers that represent the sum terms.

Step 2: Determine all of the binary numbers not included in the evaluation in Step 1.

Step 3: Write the equivalent product term for each binary number from Step 2 and express in SOP form.

- Using a similar procedure, you can go from SOP to POS.

- Consider the following POS expression, convert it into an equivalent SOP expression.

$$(A + B + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

$$X = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C}$$

A	B	C	Product Term
~~0~~	~~0~~	~~0~~	
0	0	1	$\overline{A}\,\overline{B}C$
0	1	0	$\overline{A}B\overline{C}$
~~0~~	~~1~~	~~1~~	
~~1~~	~~0~~	~~0~~	
~~1~~	~~0~~	~~1~~	
1	1	0	$AB\overline{C}$
~~1~~	~~1~~	~~1~~	