DLD
Boolean Algebra, Operation and Expression

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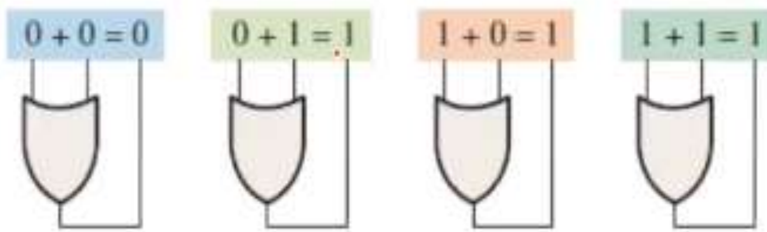
Boolean Operations and Expressions

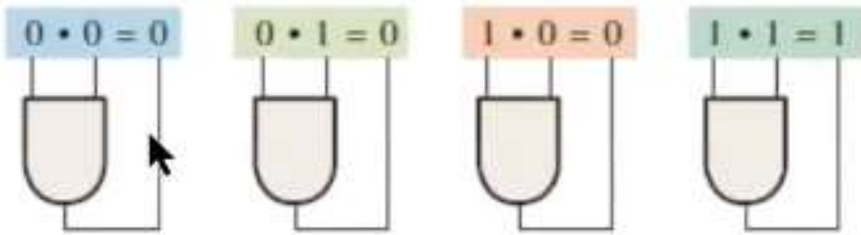
- Boolean algebra is the mathematics of digital logic.
- **Boolean Addition:** Boolean addition is equivalent to the OR operation.
- The basic rules are illustrated below:



- In logic circuits, a sum term is produced by an OR operation.
- Some examples of sum terms are $A + B$, $A + B + C$, and $A + B + C + D$.

Boolean Operations and Expressions

- **Boolean multiplication** is equivalent to the AND operation.
- The basic rules are illustrated with their relation to the AND gate:



- Examples are: AB, $A\overline{B}$, ABC, and $A\overline{B}C\overline{D}$.
- A product term is equal to 1 only if each of the literals in the term is 1.
- A product term is equal to 0 when one or more of the literals are 0.

Boolean Operations and Expressions

- A sum term is equal to 1 when one or more of the literals in the term are 1.
- A sum term is equal to 0 only if each of the literals is 0.

EXAMPLE 4–1

Determine the values of A, B, C, and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

Solution

For the sum term to be 0, each of the literals in the term must be 0. Therefore, $A = \mathbf{0}$, $B = \mathbf{1}$ so that $\overline{B} = 0$, $C = \mathbf{0}$, and $D = \mathbf{1}$ so that $\overline{D} = 0$.

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

EXAMPLE 4–2

Determine the values of A, B, C, and D that make the product term $A\overline{B}C\overline{D}$ equal to 1.

Solution

For the product term to be 1, each of the literals in the term must be 1. Therefore, $A = \mathbf{1}$, $B = \mathbf{0}$ so that $\overline{B} = 1$, $C = \mathbf{1}$, and $D = \mathbf{0}$ so that $\overline{D} = 1$.

$$A\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

Related Problem

Determine the values of A and B that make the product term $\overline{A}\,\overline{B}$ equal to 1.

DIY

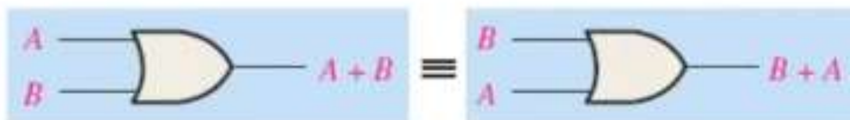
1. If $A = 0$, what does $\overline{A}$ equal?
2. Determine the values of A, B, and C that make the sum term $\overline{A} + \overline{B} + C$ equal to 0.
3. Determine the values of A, B, and C that make the product term $A\overline{B}C$ equal to 1.

Laws of Boolean Algebra

Commutative Laws

The *commutative law of addition* for two variables is written as

$$A + B = B + A$$



The *commutative law of multiplication* for two variables is

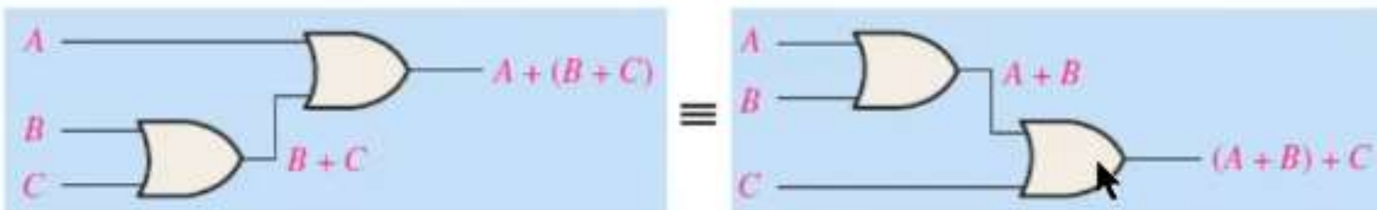
$$AB = BA$$

Laws of Boolean Algebra

Associative Laws

The *associative law of addition* is written as follows for three variables:

$$A + (B + C) = (A + B) + C$$

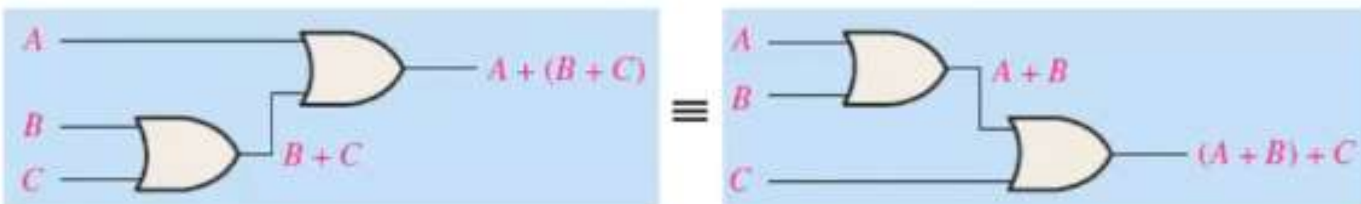


Laws of Boolean Algebra

Associative Laws

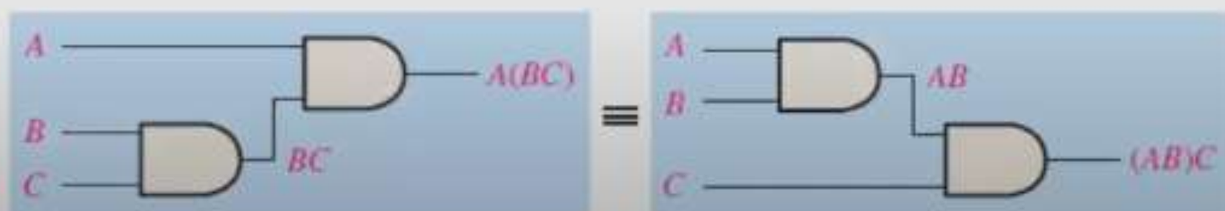
The *associative law of addition* is written as follows for three variables:

$$A + (B + C) = (A + B) + C$$



The *associative law of multiplication* is written as follows for three variables:

$$A(BC) = (AB)C$$

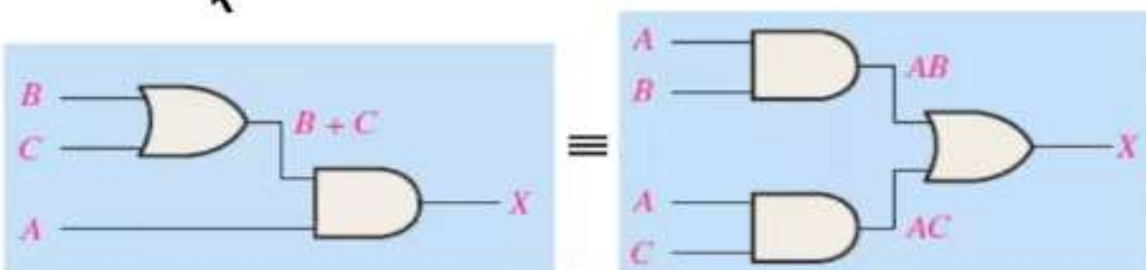


Laws of Boolean Algebra

Distributive Law

The distributive law is written for three variables as follows:

$$A(B + C) = AB + AC$$



$X = A(B + C)$ $\quad$ $X = AB + AC$

Rules of Boolean Algebra

Basic rules of Boolean algebra.

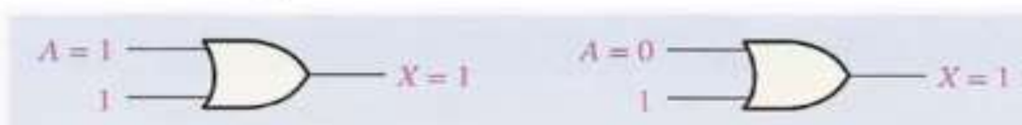
1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

Rules of Boolean Algebra

Rule 1. $A + 0 = A$

$X = A + 0 = A$

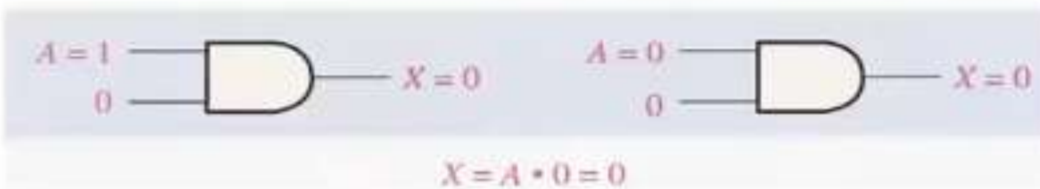
Rule 2. $A + 1 = 1$



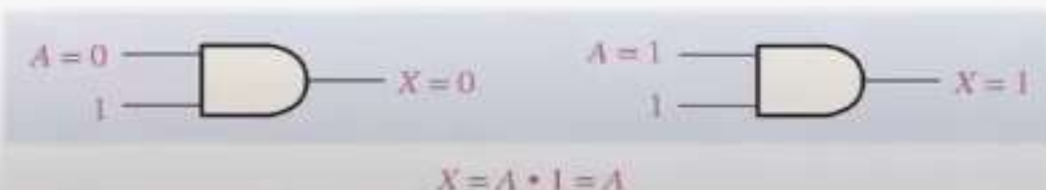
$X = A + 1 = 1$

Rules of Boolean Algebra

Rule 3. $A \cdot 0 = 0$

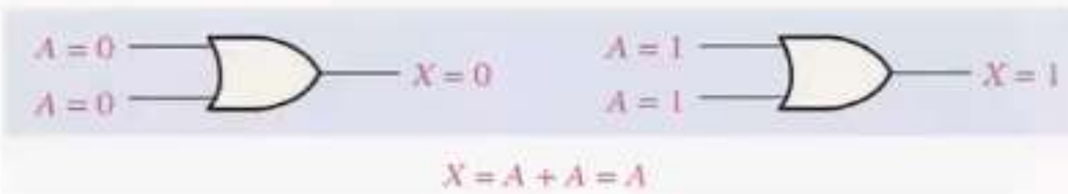


Rule 4. $A \cdot 1 = A$

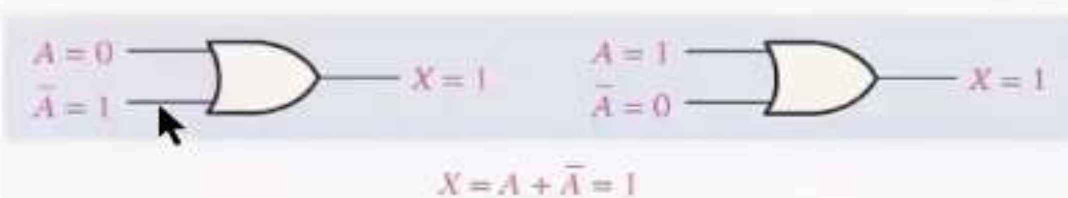


Rules of Boolean Algebra

Rule 5. $A + A = A$

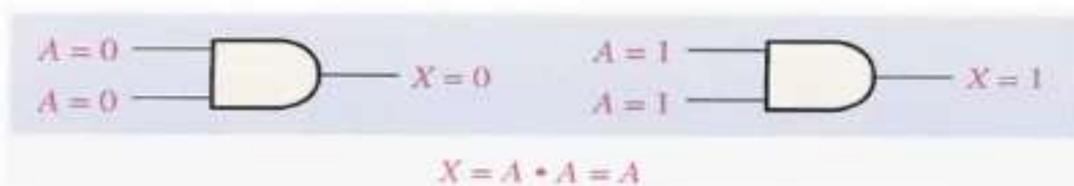


Rule 6. $A + \bar{A} = 1$

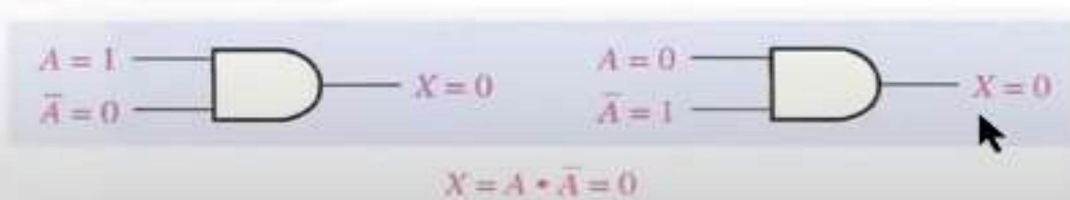


Rules of Boolean Algebra

Rule 7. $A \cdot A = A$

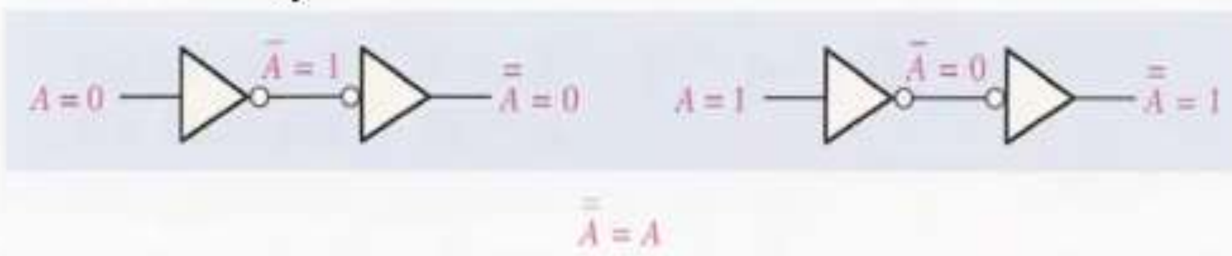


Rule 8. $A \cdot \bar{A} = 0$



Rules of Boolean Algebra

Rule 9. $\overline{\overline{A}} = A$



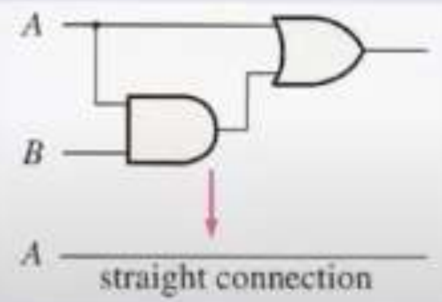
Rules of Boolean Algebra

Rule 10. $A + AB = A$ This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows:

$$A + AB = A(1 + B) \quad \text{Factoring (distributive law)}$$
$$= A \cdot 1 \quad \text{Rule 2: } (1 + B) = 1$$
$$= A \quad \text{Rule 4: } A \cdot 1 = A$$

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



Rules of Boolean Algebra

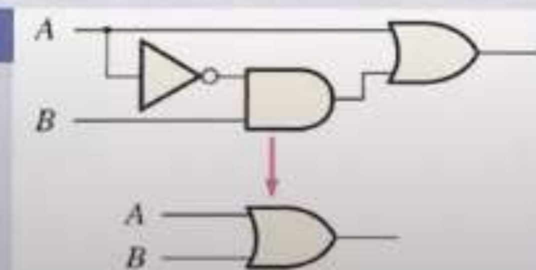
Rule 11. $A + \bar{A}B = A + B$ This rule can be proved as follows:

$$A + \bar{A}B = (A + AB) + \bar{A}B \qquad \text{Rule 10: } A = A + AB$$
$$= (AA + AB) + \bar{A}B \qquad \text{Rule 7: } A = AA$$
$$= AA + AB + A\bar{A} + \bar{A}B \qquad \text{Rule 8: adding } A\bar{A} = 0$$
$$= (A + \bar{A})(A + B) \qquad \text{Factoring}$$
$$= 1 \cdot (A + B) \qquad \text{Rule 6: } A + \bar{A} = 1$$
$$= A + B \qquad \text{Rule 4: drop the 1}$$

$A(A+B)+\bar{A}(A+B)$

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rules of Boolean Algebra

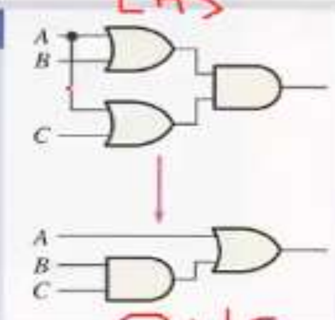
Rule 12. $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= \underline{A + AC} + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= \underline{A \cdot 1} + AB + BC && \text{Rule 2: } \underline{1 + C = 1} \\
&= \underline{A(1 + B)} + BC && \text{Factoring (distributive law)} \\
&= \underline{A \cdot 1 + BC} && \text{Rule 2: } 1 + B = 1 \\
&= \underline{A + BC} && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal

LHS



RHS

Expression Simplification

EXAMPLE 4–9

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

$$AB + AC + B + BC \rightarrow B(1+C)$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B.$$

EXAMPLE 4-9

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

$AB + AB + AC + BB + BC$

$AB + AC + B + BC$

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6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

$B + AB$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$

At this point the expression is simplified as much as possible. Once you gain experience in applying Boolean algebra, you can often combine many individual steps.

EXAMPLE 4–9

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or *C* can represent a single variable or a combination of variables.

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

B+AB

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$

At this point the expression is simplified as much as possible. Once you gain experience in applying Boolean algebra, you can often combine many individual steps.