# Theory Of Automata (TOA) (Assignment 02)



Session (2022-2026)

Program

**BS-Computer Science** 

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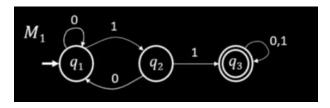
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## **Question No 1:**

Convert the following FA Machine into equivalent Regular Expression using Kleene's Method. Specify each step clearly.



#### **Answer:**

- States: q1, q2, q3
- Alphabet: {0, 1}
- Transitions:
  - $ullet q_1 \stackrel{0}{\longrightarrow} q_1$
  - $ullet q_1 \stackrel{1}{\longrightarrow} q_2$
  - $ullet q_2 \stackrel{0}{\longrightarrow} q_1$
  - $ullet q_2 \stackrel{1}{\longrightarrow} q_3$
  - $ullet q_3 \stackrel{0}{\longrightarrow} q_3$
  - $ullet q_3 \stackrel{1}{\longrightarrow} q_3$

# Steps to convert the FA to a regular expression:

# 1. Initial Regular Expressions:

- $\bullet \quad \text{For} \, q_1 \to q_1 \text{:} \, R_{11} = 0 \cup \epsilon$
- $\bullet \quad \text{For } q_1 \to q_2 \text{:} \, R_{12} = 1$
- ullet For  $q_2 
  ightarrow q_1$ :  $R_{21} = 0$
- ullet For  $q_2 o q_3$ :  $R_{23}=1$
- $\bullet \quad \text{For } q_3 \to q_3 \text{: } R_{33} = 0 \cup 1 \cup \epsilon$

## 2. Remove State q2:

- Update transitions involving q<sub>2</sub>:
  - ullet For  $q_1 o q_3$ :  $R_{13} = R_{12} \cdot R_{23} = 1 \cdot 1 = 11$
  - For  $q_1 o q_1$  (through  $q_2$ ):  $R_{11}=R_{11}\cup (R_{12}\cdot R_{21})=(0\cup\epsilon)\cup (1\cdot 0)=0\cup\epsilon\cup 10$
  - For  $q_3 o q_1$  (if there were a loop through  $q_2$ , but not in this case, hence unchanged)

## 3. Final expression (removing q1):

- q<sub>1</sub> is the start state and q<sub>3</sub> is the final state.
- We need to connect paths from  $q_1$  to  $q_3$  using the updated expressions.
- For  $q_1 o q_3$  directly and via loops:
  - $R_{13}=11$  (direct path through  $q_2$ )
- Consider loops at q<sub>1</sub>:
  - $R_{11} = 0 \cup \epsilon \cup 10$
- For loops at q<sub>3</sub>:
  - $R_{33} = (0 \cup 1)^*$
- · Combine the expressions:
  - The final regular expression is  $(0 \cup \epsilon \cup 10)^*11(0 \cup 1)^*$

# 4. Simplify the regular expression:

• Since  $\epsilon$  in  $R_{11}$  denotes zero or more occurrences of 0 or 10, the final regular expression can be simplified to  $(0 \cup 10)^*11(0 \cup 1)^*$ .

Thus the equivalent regular expression is:

$$(0 \cup 10)^*11(0 \cup 1)^*$$

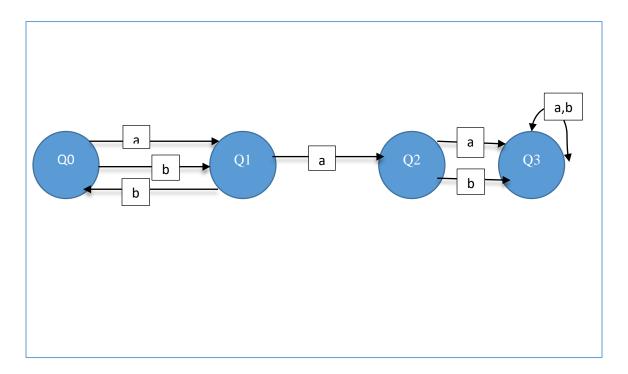
This regular expression represents the language accepted by the give FA using Kleene's method.

# Question No 2:

Using the technique discussed by Martin , Build an FA that accept the following language.

 $L=\{w \text{ belong to } \{a,b\}^*: Length(w) >= 2 \text{ and second letter of } w, \text{ from right is } a\}.$ 

## Answer:



#### **Question No 3:**

Build an FA for the following regular Language L defined over  $\Sigma = \{a,b\}$ 

```
L= {a, ab, bab, bb}
```

#### **Answer:**

For constructing an FA for this language, we need to create states and transitions for each of the strings:

#### 1. States:

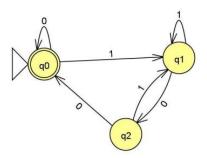
- o q0 (start state)
- q1 (accepting state for 'a')
- q2 (accepting state for 'ab')
- q3 (intermediate state)
- q4 (accepting state for 'bab')
- q5 (accepting state for 'bb')

#### 2. Transitions:

- From **q0**:
  - Read 'a': Go to q1 (final for 'a')
  - Read 'b': Go to q3
- From q1:
  - Read 'b': Go to q2 (final for 'ab')
- From q3:
  - Read 'a': Go to **q4** (intermediate state)
- From q4:
  - Read 'b': Go to q2 (final for 'bab')
- From q3:
  - Read 'b': Go to q5 (final for 'bb')

## **Question No 4:**

Give the 5-tuple representation and draw the transition table for the following FA.



#### **Answer:**

## 5-Tuple Representation

The 5-tuple representation of a finite automaton (FA) is given by  $(Q,\Sigma,\delta,q_0,F)$ , where:

- $\bullet \quad Q \text{ is the set of states} \\$
- ullet  $\Sigma$  is the alphabet
- ullet  $\delta$  is the transition function
- ullet  $q_0$  is the start state
- ullet F is the set of accept states

From the provided FA diagram:

- 1. Set of States (Q):  $Q=\{q_0,q_1,q_2\}$
- 2. Alphabet ( $\Sigma$ ):  $\Sigma = \{0,1\}$
- 3. Transition Function (δ):
  - $\delta(q_0,0) = q_0$
  - $\bullet \quad \delta(q_0,1)=q_1$
  - $\bullet \quad \delta(q_1,0)=q_2$
  - $\bullet \quad \delta(q_1,1)=q_1$
  - $\delta(q_2,0)=q_2$
  - $\bullet \quad \delta(q_2,1)=q_0$

- 4. Start State (q\_0):  $q_0$
- 5. Set of Accept States (F): The diagram does not indicate accept states explicitly. Assuming there are no special markers, we need this information to be provided. If we assume  $q_2$  is the accepting state (common in simple diagrams), then  $F=\{q_2\}$ .

#### **Transition Table**

The transition table represents the state transitions based on the input symbols. Here is the transition table for the given FA:

Current State	Input	Next State
$q_0$	0	$q_0$
$q_0$	1	$q_1$
$q_1$	0	$q_2$
$q_1$	1	$q_1$
$q_2$	0	$q_2$
$q_2$	1	$q_0$

# Summary of the 5-Tuple Representation:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{0,1\}$
- ullet  $\delta$  is as defined in the transition table above
- ullet  $q_0$  is the start state
- ullet  $F=\{q_2\}$  (assuming  $q_2$  is the accept state; please adjust based on actual accept state information if different)