Topic: Elementry Row & Columns Operation on Matrix.

e.g $A = \begin{bmatrix} 2 & 3 \\ -6 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

1. $R_i \Longleftrightarrow R_j$ (Interchanging ith row by jth row)

$R_1 \longleftrightarrow R_2$ $A = \begin{bmatrix} -6 & 8 \\ 2 & 3 \end{bmatrix}$

$C_i \Longleftrightarrow C_j$

$C_1 \longleftrightarrow C_2$ $A \begin{bmatrix} 3 & 2 \\ 8 & -6 \end{bmatrix}$

2. Scalar Multiplication of row

$R_i \longrightarrow kR_1$

$R_1 \longrightarrow 2R_1$ $A = \begin{bmatrix} 4 & 6 \\ -6 & 8 \end{bmatrix}$

$C_2 \to -3C_2$

$A = \begin{bmatrix} 2 & -9 \\ -6 & -24 \end{bmatrix}$

3. Addition of two rows.

$$\left.\begin{aligned} &\text{i} \quad R_1 \longrightarrow R_1 + R_2 \\ &\text{ii} \quad R_2 \longrightarrow R_2 - 2R_1 \\ &\text{iii} \quad R_1 \longrightarrow R_1 + 3R_2 \end{aligned}\right\} (A)$$

$A = \begin{bmatrix} 2 & 3 \\ -6 & 8 \end{bmatrix}$

(i) $A = \begin{bmatrix} -4 & 11 \\ -6 & 8 \end{bmatrix}$ $R_1 \longrightarrow R + R_2$

(ii) $A = \begin{bmatrix} 4 & 6 \\ -6 & 8 \end{bmatrix}$ $2R_1$

$A = \begin{bmatrix} 2 & 3 \\ -8 & 2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$

$A = \begin{bmatrix} -4 & 11 \\ -6 & 8 \end{bmatrix}$

$A = \begin{bmatrix} 8 & 22 \\ -6 & 8 \end{bmatrix}$ $2R_1$

$A = \begin{bmatrix} -4 & 11 \\ 2 & 30 \end{bmatrix}$ $R_2 \longrightarrow R_2 - 2R_1$

iii) $A = \begin{bmatrix} -4 & 11 \\ 2 & 30 \end{bmatrix}$ $A = \begin{bmatrix} -4 & 11 \\ 6 & 90 \end{bmatrix}$ $3R_2$

$A = \begin{bmatrix} 2 & 101 \\ 2 & 30 \end{bmatrix}$

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$\to C_1 + 2C_2$

$B = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ $\quad \left.\begin{matrix} 0 & 2 \\ 2 & -2 \end{matrix}\right| \begin{matrix} 2 \\ 0 \end{matrix}$

$B = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ $C_1 \to C_1 + 2C_2$.

Rank of Matrix.

→ The rank of a Matrix is the order of the highest ordered non-zero minor.

→ The maximum no. of linearly independent columns or rows of a matrix. is called rank of a Matrix

The rank of 'A' is denoted by $\rho(A) = r$.

1) Minor method (determinant method)
2) Echolon form of matrix.
3) Canonical form or Normal form
4) Using row & colum operation method

Rank of the identity Matrix is its order.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\rho(A) = 2$ $\quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\rho(B) = 3$

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Example 1 Find The rank of Matrix.

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}_{3\times3}$$

Sol
$$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{vmatrix}_{3\times3}$$

$$|A| = 2(-9+8) - 1(0+4) - 1(0-6)$$
$$= -2(-1) - 1(4) - 1(-6)$$
$$= -2 - 4 + 6$$
$$|A| = 0$$

One of the property of determinant is That if the value of determinant is zero Then it means that atleast one of its row or column must be zero.
Means ~~on~~ we can convert the above matrix with the help of row & column operation so That one of its row or column should be zero.

Means $\rho(A) \neq 3$.
So it must be < 3

If rank=2 we can take any minor whose determinat value is not zero

Taking a order two minor

$$m_{11} = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$$
$$= 6 - 0 \neq 0$$

Rank of A or $\rho(A) = 2$.

Exp 2 Find the rank of matrix $A = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{bmatrix}_{3\times3}$

Sol

$$|A| = \begin{vmatrix} -1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{vmatrix}_{3\times3}$$

$$= -1(28+2) + 2(-14-1) + 3(-4+4)$$

$$= -30 - 30$$

$$|A| = -60 \neq 0$$

$\therefore$ The rank of $A = 3$

The determinant method is usefull in 3x3, 3x4 4x3 or less Than That; but if The order is 4x4 or 5x5 or more Then we use use elementry row or Columns operation.

Q// Find the Rank of Matrix $A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}_{4\times4}$

Sol As we know that

$R_1 = R_3$ & $R_2 = R_4$

$R_3 \to R_3 - R_1$ & $R_4 \to R_4 - R_2$

$$A \sim \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Rank of matrix } A = 2$$

nd the rank of Matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}_{3\times4}$

Sol Taking minors

$\cancel{|A| = 1(12-12) - 2(12-6) + 1(8-4)}$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{vmatrix} \quad \begin{vmatrix} 2 & 3 & 1 \\ 4 & 6 & 2 \\ 2 & 3 & 2 \end{vmatrix} \quad \begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 2 \end{vmatrix} \quad \begin{vmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \\ 1 & 3 & 2 \end{vmatrix}$$

Taking minor.

$\cancel{|A|} = 1(12-12), 2(12-6), 1(8-4), 1(12-6)$

$|A| = 0 = 0,\ 12 \neq 0,\ 4 \neq 0,\ 6 \neq 0$

The rank of Matrix $A = 3$

Q Find the rank of Matrix using Echelon form

There should be a Specific form.

i.e $\begin{bmatrix} 1 & - - - - - - \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ in which there is unity or 1 & below 1 there should be zero

In Echelon form there is only row operation no column operation is allow.

Find the rank of Matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 2 & 2 \end{bmatrix}_{3\times 4}$

Sol: Finding the rank by E.F

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$A \approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} \to R_2 \to R_2 - 2R_1 \\ \to R_3 \to R_3 - R_1 \end{matrix}$

$$\begin{array}{cccc} 2 & 4 & 6 & 2 \\ 2 & 4 & 6 & 2 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

$A \approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to$ making this +ve xing by (−)

$A \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \to -R_2$

Now Matrix A is Converted to Echelon form.

& There are two non-zero Rows

$\Rightarrow$ Rank of Matrix $A = 2$.

Find Rank of Matrix by E.F.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

a_{11} should be equal to zero.

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_4$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_2 \to R_2 / -4$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/4 & -5/4 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/4 & -5/4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/4 & -5/4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 / 3$

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/4 & -5/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-zero rows are 3

$\therefore$ Rank of $A = 3$

R_2	2	4	3	2
$2R_1$	2	4	6	0
	0	0	-3	2
R_3	3	2	1	3
$3R_1$	3	6	9	0
	0	-4	-8	3
	6	8	7	5
	6	12	18	0
	0	-4	-11	5

Nullity:

$=$ order of square matrix $- \rho(A)$

$= 4 - 3$

$= 1$

Nullity means how many rows are "0"

Rank by determinant Method of order 3x3

Q. Find the rank of the Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

Sol $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix}$,

$= 1(20-12) - 2(5-4) + 3(6-8)$

$= 8 - 2 - 6 = 0$

$\Rightarrow \rho(A) \neq 0$ but $\rho(A) < 3$

Now choosing a minor of A a order 2.

say $\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 4 - 2 = 2 \neq 0$

It means we find a minor of ~~2~~ order 2 ~~whose~~ which is not zero. So the order of 2 which is not 0 & its higher order which is 3 & it is 0, So Then we say that the rank of The given Matrix is 2.

If all the minors are equal to zero then the rank of the given matrix is 1

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Rank by row ~~& column~~ operations

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

Note: Column operation is not allowed in Echelon form

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - 2R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - 2R_1 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \cancel{R_2/2} \\ R_3 \to R_3 - R_2 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2/2 \\ \\ \end{matrix}$$

$\therefore$ $\rho(A)$ = no. of non-zero rows

$\rho(A) = 2$

Nullity of $A = N(A)$

= order of matrix $A - \rho(A)$

$= 3 - 2$

$= 1$

Find the rank of Matrix A

where $A = \begin{bmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_3$

Ans

$$A \sim \begin{bmatrix} 1 & 4 & 9 & 16 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 11 \\ 7 & 9 & 11 & 13 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 9 & 16 \\ 3 & 5 & 7 & 9 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_3 \end{array}$$

$R_4 \to R_4 - R_3$, $R_3 \to R_3/2$

$$A \sim \begin{bmatrix} 1 & 4 & 9 & 16 \\ 3 & 5 & 7 & 9 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 5 & 7 & 9 \\ 1 & 4 & 9 & 16 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \longleftrightarrow R_1$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 8 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 8 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2/2 \\ R_3 \to R_3 - 3R_2 \end{array}$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 3R_2$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3/2$$

$\therefore$ Rank = no; of non-zero rows

$\Rightarrow \rho(A) = 3$

& Nullity $= 4 - 3$
$= 1$

Means 1 row must be completely 0