DLD
The Karnaugh Map

Department of Computer science & IT

Introduction

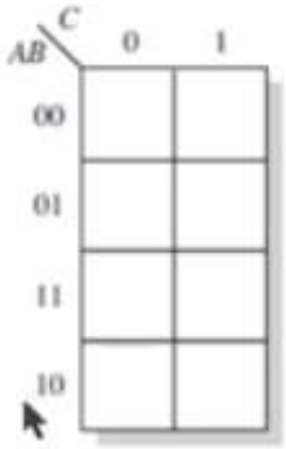
- A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression.
- A **Karnaugh map** is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value.
- Instead of being organized into columns and rows like a truth table, the Karnaugh map is an array of **cells** in which each cell represents a binary value of the input variables.

Introduction

- The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells.
- The number of cells in a Karnaugh map, as well as the number of rows in a truth table, is equal to the total number of possible input variable combinations.
- For three variables, the number of cells is $2^3 = 8$.
- For four variables, the number of cells is $2^4 = 16$.

The 3-Variable Karnaugh Map

- The 3-variable Karnaugh map is an array of eight cells (2^3).



Point to be Noted…

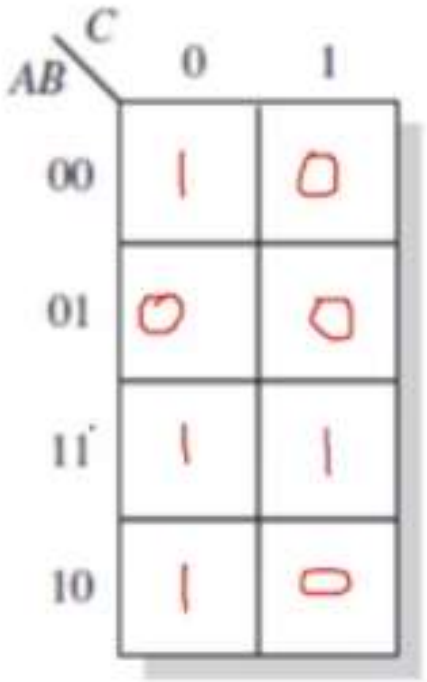
- In case of K-maps the row and column differ by a single bit i.e. to write numerically the index it is (00 01 11 10). as we move along, there is a change of only one bit and this is the functionality of Gray code.
- Cells that differ by only one variable are adjacent.
- Cells with values that differ by morethan one variable are not adjacent.

Wrap-around Adjacency

- Physically, each cell is adjacent to the cells that are immediately next to it on any of its four sides.
- A cell is not adjacent to the cells that diagonally touch any of its corners.
- Also, the cells in the top row are adjacent to the corresponding cells in the bottom row and the cells in the outer left column are adjacent to the corresponding cells in the outer right column.
- This is called "wrap-around" adjacency because you can think of the map as wrapping around from top to bottom to form a cylinder or from left to right to form a cylinder.

Mapping Directly from a Truth Table

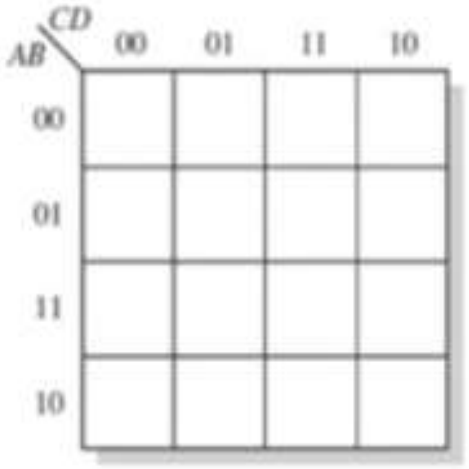
Inputs			Output
A	*B*	*C*	*X*
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Four Variable Karnaugh Map

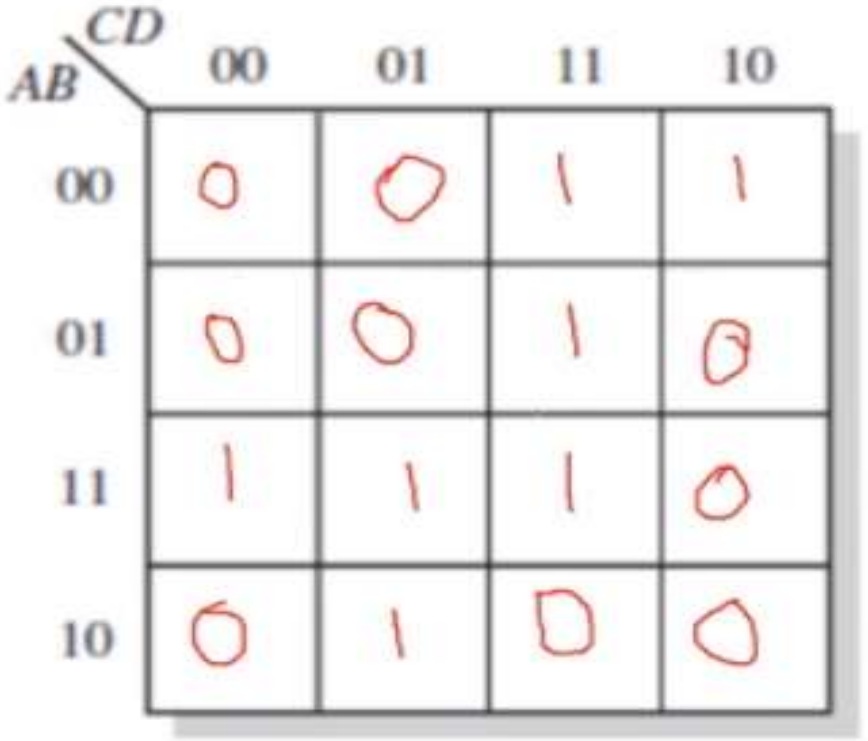
The 4-Variable Karnaugh Map

- The 4-variable Karnaugh map is an array of sixteen cells.



Mapping from Truth Table

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

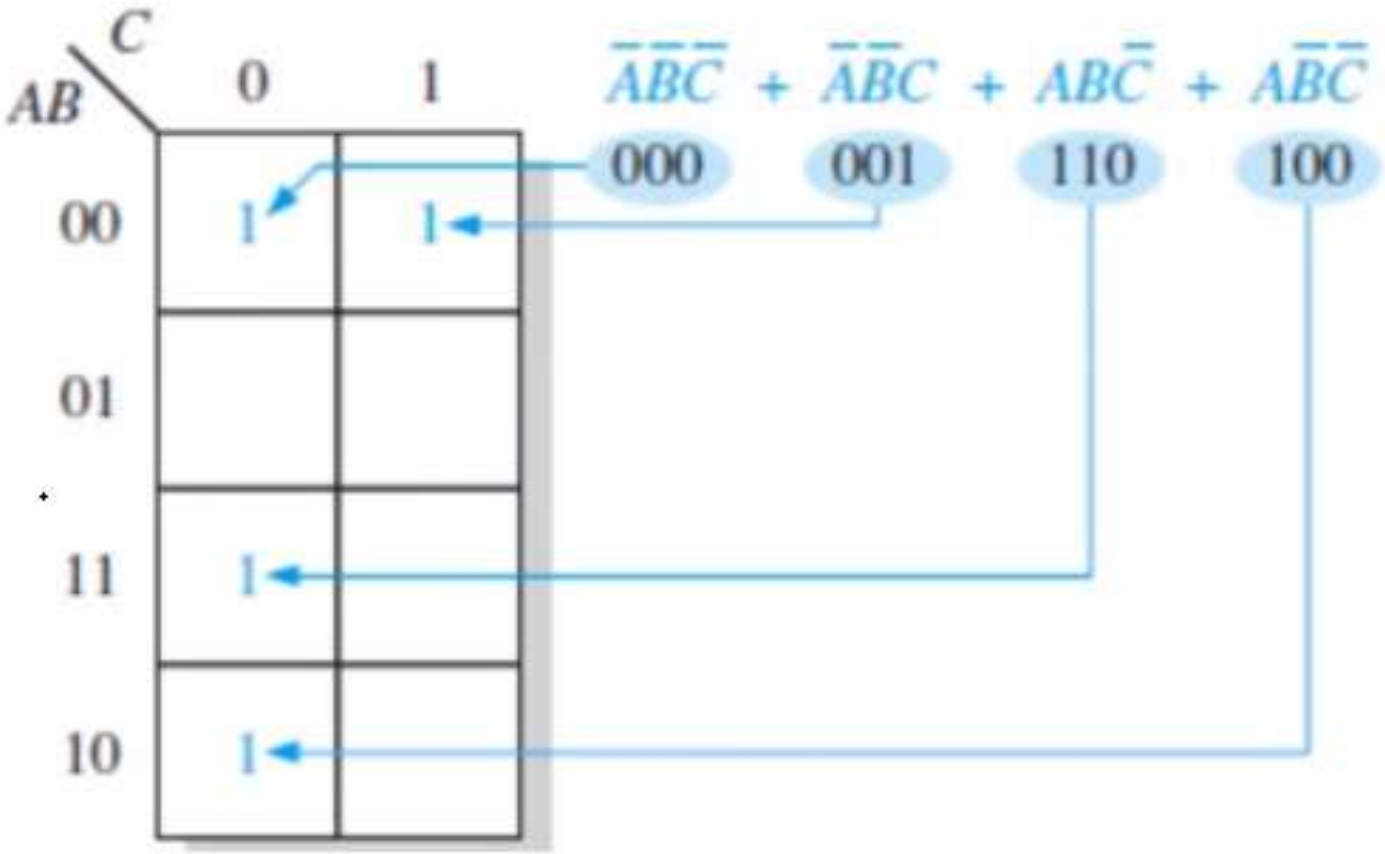


Mapping a standard SOP Expression on K-Map

Mapping a Standard SOP Expression

- Following steps to be followed when mapping a standard SOP expression:
- **Step 1:** Determine the binary value of each product term in the standard SOP expression.
- After some practice, you can usually do the evaluation of terms mentally.
- **Step 2:** As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

Mapping a Standard SOP Expression

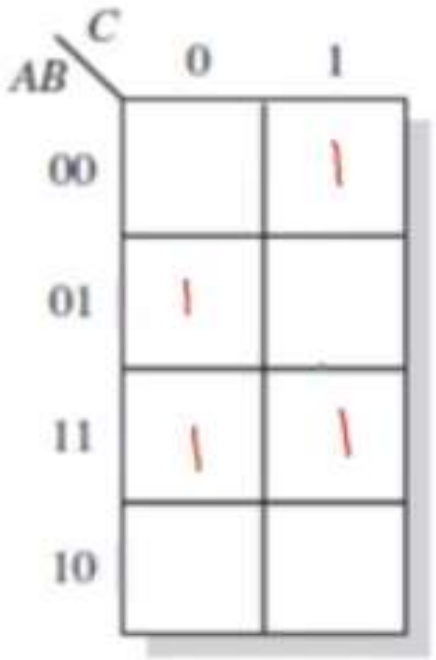


EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

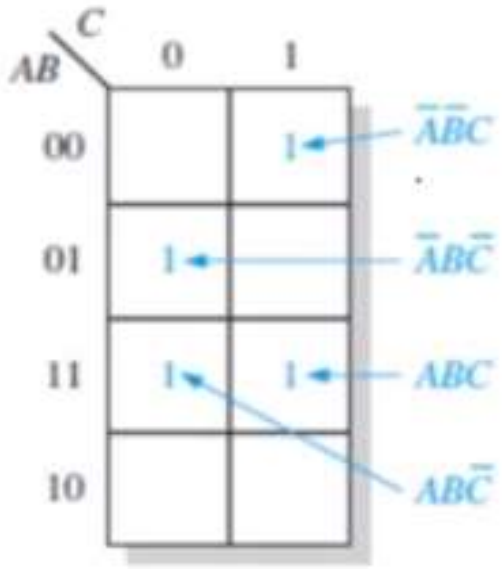
001 010 110 111



EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

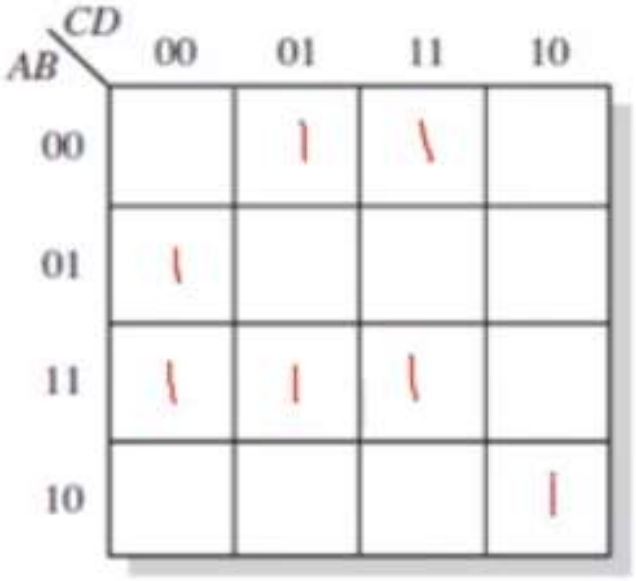


EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D}$$

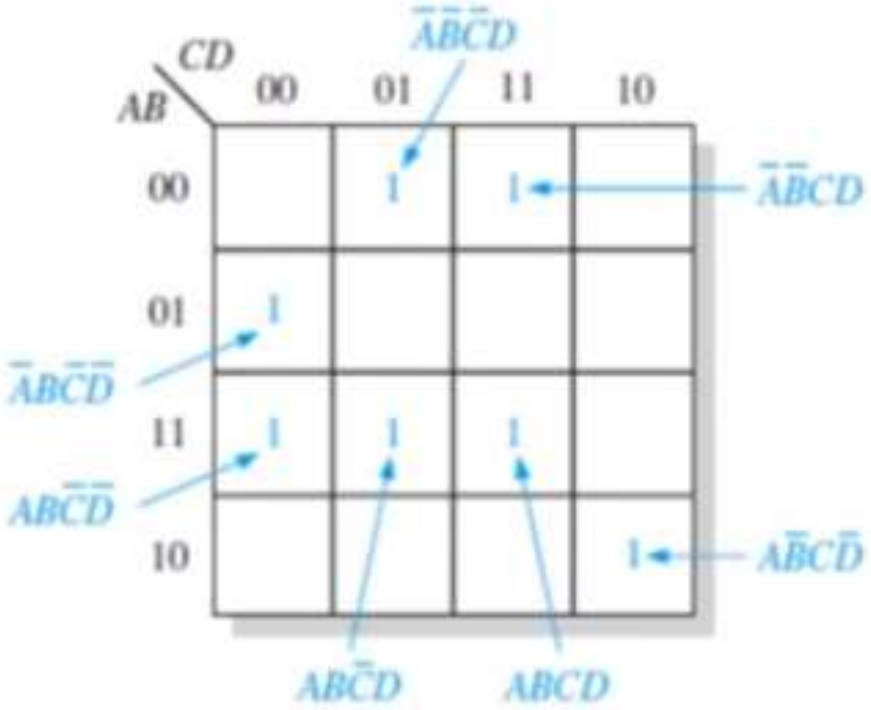
0011 0100 1101 1111 1100 0001 1010



EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}\overline{C}\,\overline{D}$$



Mapping Non Standard SOP on K-Map

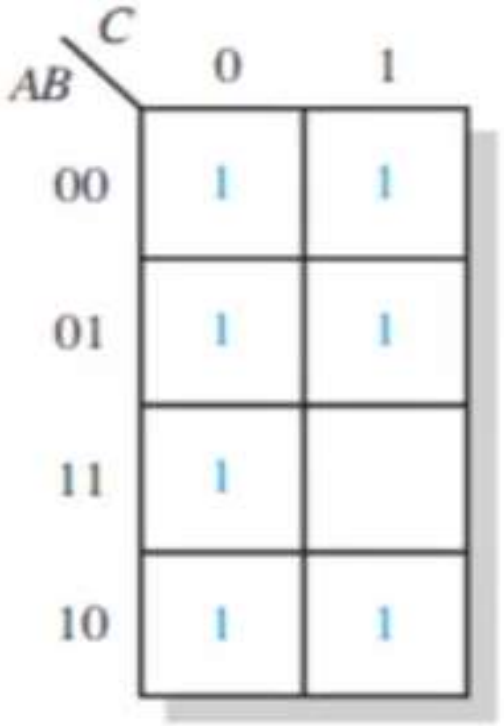
Mapping a SOP Expression

- Usually, when working with SOP expressions, the 0s are left off the map.
- Following steps to be followed when mapping a standard SOP expression:
- **Step 1:** Determine the binary value of each product term in the standard SOP expression.
- After some practice, you can usually do the evaluation of terms mentally.
- **Step 2:** As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		



EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}B\overline{C}D + A\overline{B}CD$$

$\overline{B}\overline{C}$	$A\overline{B}$	$AB\overline{C}$	$A\overline{B}C\overline{D}$	$\overline{A}B\overline{C}D$	$A\overline{B}CD$
0000	1000	1100	1010	0001	1011
0001	1001	1101			
1000	1010				
1001	1011