Digital Logic Design

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Simplification of SOP Expression Using K-Map (Three Variable)

Karnaugh Map Simplification of SOP Expressions

- The process that results in an expression containing the fewest possible terms with the fewest possible variables is called **minimization**.
- After an SOP expression has been mapped, a minimum SOP expression is obtained by grouping the 1s and determining the minimum SOP expression from the map.

Grouping the 1s

- The goal is to maximize the size of the groups and to minimize the number of groups.
- You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing 1s.
- **Rule 1:** A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two.
- In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
- **Rule 2:** Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other. (Continue…)

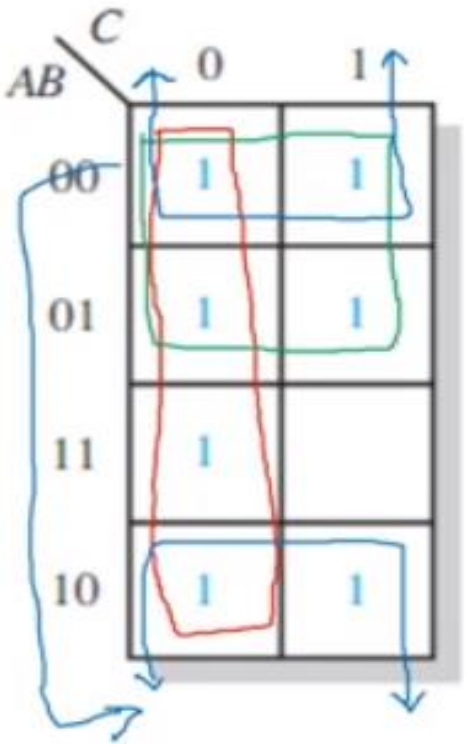
Grouping the 1s

- **Rule 3:** Always include the largest possible number of 1s in a group in accordance with rule 1.
- **Rule 4:** Each 1 on the map must be included in at least one group.
- The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		



Determining the Minimum SOP Expression from the Map

- When all the 1s representing the standard product terms in an expression are properly mapped and grouped, the process of determining the resulting minimum SOP expression begins.
- The following rules are applied to find the minimum product terms and the minimum SOP expression:
- **Rule 1:** Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group.
- Variables that occur both uncomplemented and complemented within the group are eliminated.
- These are called contradictory variables. (Continued…)

- **Rule 2:** Determine the minimum product term for each group.
- **(a) For a 3-variable map:**
  - **(i)** A 1-cell group yields a 3-variable product term
  - **(ii)** A 2-cell group yields a 2-variable product term
  - **(iii)** A 4-cell group yields a 1-variable term
  - **(iv)** An 8-cell group yields a value of 1 for the expression

- **Rule 2:** Determine the minimum product term for each group.
- **(a) For a 3-variable map:**
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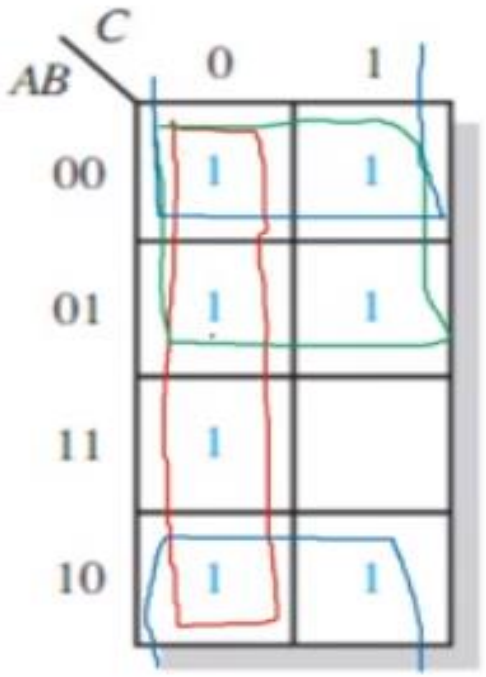
  - **(ii)** A 2-cell group yields a 2-variable product term
  - **(iii)** A 4-cell group yields a 1-variable term
  - **(iv)** An 8-cell group yields a value of 1 for the expression
- **(b) For a 4-variable map:**
  - **(i)** A 1-cell group yields a 4-variable product term
  - **(ii)** A 2-cell group yields a 3-variable product term
  - **(iii)** A 4-cell group yields a 2-variable product term
  - **(iv)** An 8-cell group yields a 1-variable term
  - **(v)** A 16-cell group yields a value of 1 for the expression

- **Rule 3:** When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression

EXAMPLE 4–25

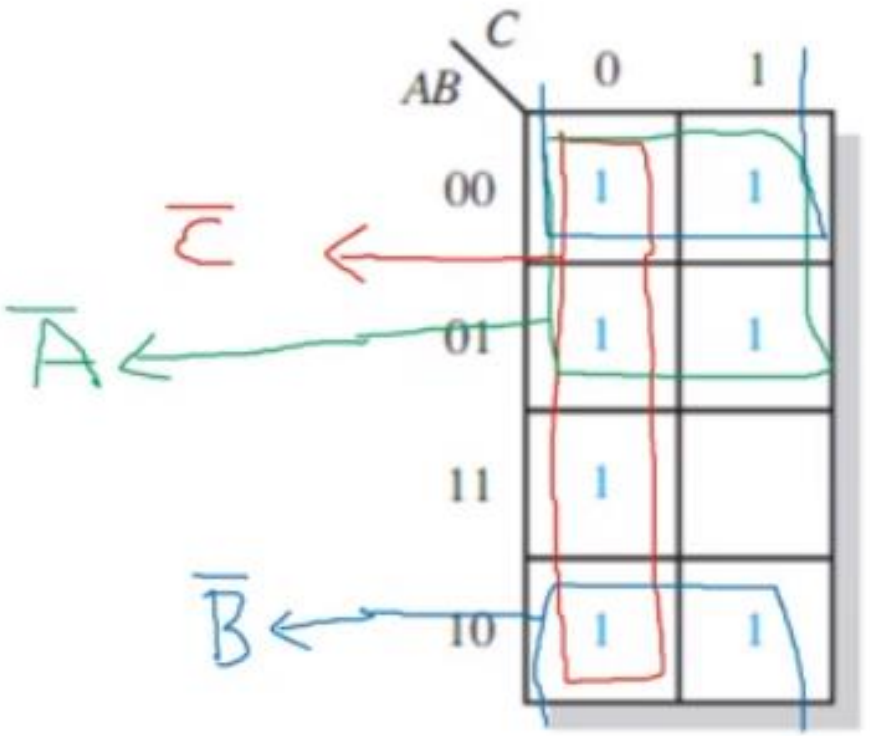
Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.



EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

$= \overline{A} + \overline{B} + \overline{C}$



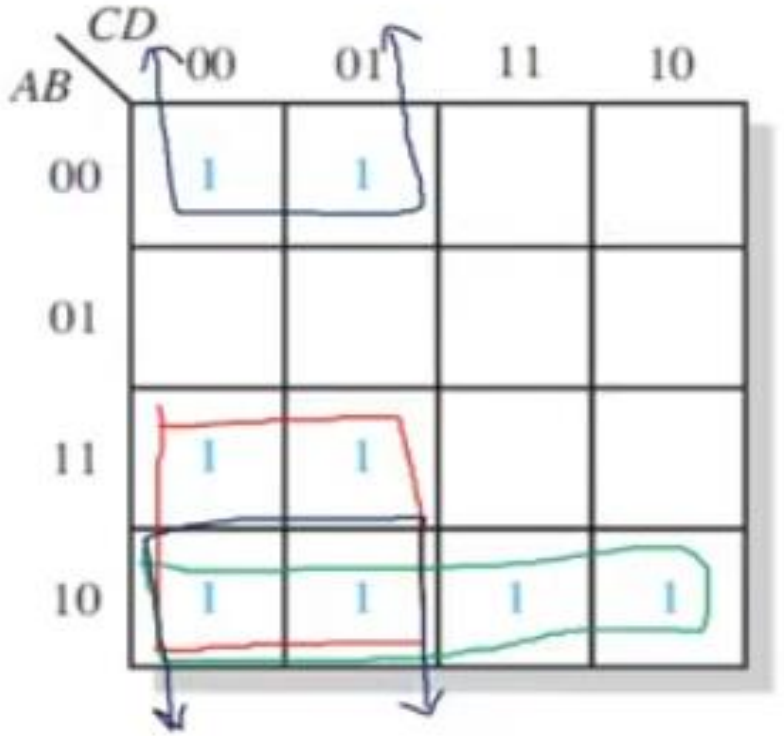
Four Variable SOP Expression

Simplification using K mapping

EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

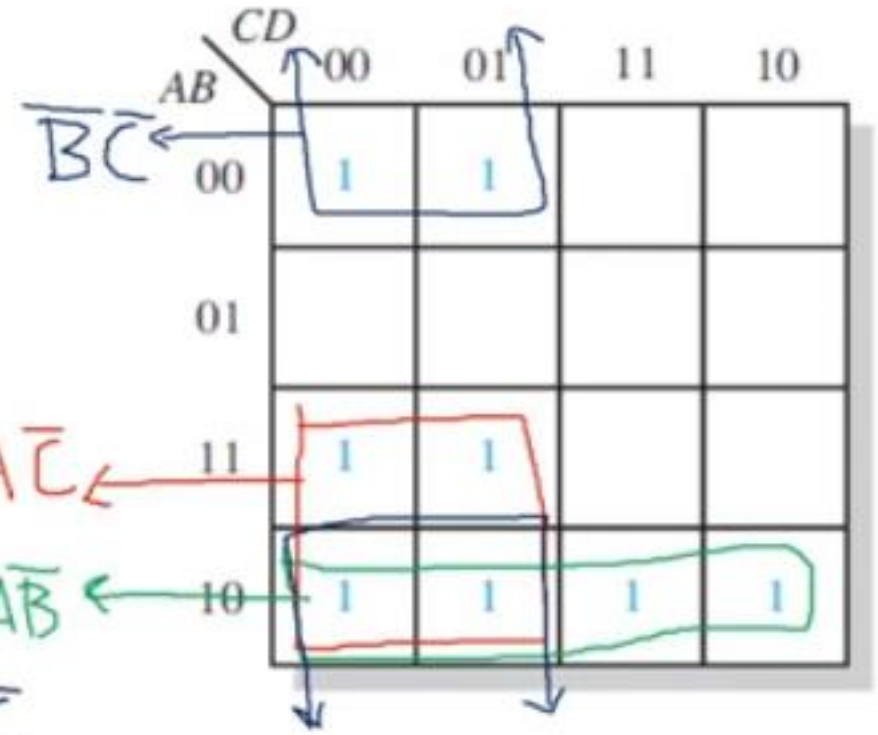
$$\overline{B}\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}CD$$



EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}B\overline{C}D + A\overline{B}CD$$



$$= A\overline{B} + A\overline{C} + \overline{B}\overline{C}$$

Example

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

$$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$$

0000 0100 1100 0011 1011 0010 0110 1110 1010
1000

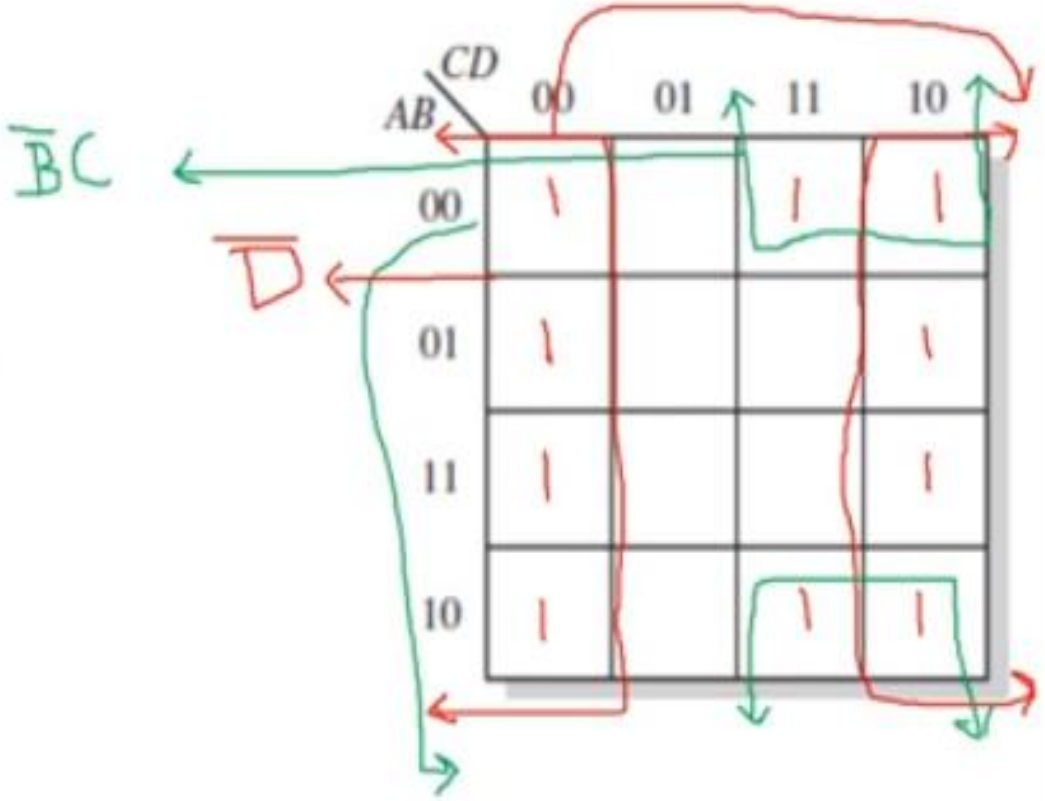
AB \ CD	00	01	11	10
00	1		1	1
01	1			1
11	1			1
10	1		1	1

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

0000 0100 1100 0011 1011 0010 0110 1110 1010
1000



$$= \overline{B}C + \overline{D}$$