Linear Dependency & Independency

Q. Test the linear dependency & find the relationship if it exist for:

$X_1 = (1,2,3) \quad X_2 = (3,-2,1) \quad X_3 = (1,-6,5)$

Sol. Let λ_1, λ_2 & λ_3 be three scalar.

Consider $\boxed{\lambda_1 X_1 + \lambda_2 X_2 + \lambda_3 X_3 = 0}$.

$$\lambda_1(1,2,3) + \lambda_2(3,-2,1) + \lambda_3(1,-6,5) = 0 \quad \text{--- (1)}$$

$$\lambda_1 + 3\lambda_2 + \lambda_3 = 0$$
$$2\lambda_1 - 2\lambda_2 - 6\lambda_3 = 0$$
$$3\lambda_1 + \lambda_2 + 5\lambda_3 = 0$$

$$\underbrace{\begin{bmatrix} 1 & 3 & 1 \\ 2 & -2 & -6 \\ 3 & 1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}}_{\lambda} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{B}$$

$\therefore \boxed{A\lambda = B}$ --- (2)

Now Augumented Matrix, $C = [A : B]$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 2 & -2 & -6 & 0 \\ 3 & 1 & 5 & 0 \end{array}\right]_{3\times 4}$$

$\boxed{R_2 = R_2 - 2R_1}$

$\boxed{R_3 = R_3 - 3R_1}$

$$C = \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & -8 & 2 & 0 \end{array}\right]$$

$\boxed{R_3 = R_3 - R_2}$

$$C = \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & -8 & -8 & 0 \\ 0 & 0 & 10 & 0 \end{array}\right]$$

(2) $\Rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -8 & -8 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_1 + 3\lambda_2 + \lambda_3 = 0$ --- (A)

$-8\lambda_2 - 8\lambda_3 = 0$ --- (B)

$10\lambda_3 = 0$ --- (C)

from (C) $\boxed{\lambda_3 = 0}$

(B) $-8\lambda_2 - 0 = 0$

$\boxed{\lambda_2 = 0}$

(A) $\Rightarrow \lambda_1 + 0 + 0 = 0$

$\boxed{\lambda_1 = 0}$

$\because \lambda_1 = \lambda_2 = \lambda = 0$ $\therefore$ the given Vectors X_1, X_2, X_3 are Linearly independent & there exist no solution.

Q/ Investigate the linear dependance & independance of the following vectors.

$x_1 = (1,2,4)$ $x_2 = (2,-1,3)$ $x_3 = (0,1,2)$ $x_4 = (-3,7,2)$.

Sol: Let $\lambda_1, \lambda_2, \lambda_3$ & λ_4 be any 4 vectors.

Now consider:

$$\boxed{\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 = 0} \quad \text{— (A)}$$

$$\lambda_1(1,2,4) + \lambda_2(2,-1,3) + \lambda_3(0,1,2) + \lambda_4(-3,7,2) = 0$$

$$\lambda_1 + 2\lambda_2 + 0 + 3\lambda_4 = 0$$
$$2\lambda_1 - \lambda_2 + \lambda_3 + 7\lambda_4 = 0$$
$$4\lambda_1 + 3\lambda_2 + 2\lambda_3 + 2\lambda_4 = 0$$

No agumented matrix:

$$\boxed{C = A:B}$$

$$C = \begin{bmatrix} 1 & 2 & 0 & -3 & : & 0 \\ 2 & -1 & 1 & 7 & : & 0 \\ 4 & 3 & 2 & 2 & : & 0 \end{bmatrix}$$

$\boxed{R_1 = R_2 - 2R_1}$ $\boxed{R_3 = R_3 - 4R_1}$.

$$C = \begin{bmatrix} 1 & 2 & 0 & -3 & : & 0 \\ 0 & -5 & 1 & 13 & : & 0 \\ 0 & -5 & 2 & 14 & : & 0 \end{bmatrix}$$

$\boxed{R_3 = R_3 - R_2}$

$$C = \begin{bmatrix} 1 & 2 & 0 & -3 & : & 0 \\ 0 & -5 & 1 & 13 & : & 0 \\ 0 & 0 & 1 & 1 & : & 0 \end{bmatrix}$$

consider $\boxed{A\lambda = B}$

$$\therefore \begin{bmatrix} 1 & 2 & 0 & -3 \\ 0 & -5 & 1 & 13 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_1 + 2\lambda_2 - 3\lambda_4 = 0 \quad \text{— (1)}$$
$$-5\lambda_2 + \lambda_3 + 13\lambda_4 = 0 \quad \text{— (2)}$$
$$\lambda_3 + \lambda_4 = 0 \quad \text{— (3)}$$

Let $\boxed{\lambda_3 = k}$ (non-zero constant)

(3) $\Rightarrow k + \lambda_4 = 0$, $\boxed{\lambda_4 = -k}$

(2) $\Rightarrow -5\lambda_2 + k - 13k = 0$

$$-5\lambda_2 - 12k = 0$$
$$-5\lambda_2 = 12k$$
$$\boxed{\lambda_2 = \frac{-12k}{5}}.$$

(1) $\Rightarrow \lambda_1 - 2\left(\frac{12k}{5}\right) + 3(-k) = 0$

$$\lambda_1 - \frac{24}{5}k + 3k = 0$$
$$\lambda_1 - \frac{9k}{5} = 0$$
$$\boxed{\lambda_1 = \frac{9k}{5}}$$

To check the relation we put the values of $\lambda_1, \lambda_2, \lambda_3$ & λ_4 in $\Rightarrow$ (A)

$$\frac{9k}{5} x_1 + \frac{12k}{5} x_2 + k x_3 - k x_4 = 0$$

~~Hence the s.s is possible~~

Now xing the =n by 5 we get & taking k common

$k(9x_1 - 12x_2 + 5x_3 - 5x_4) = 0$

$\Rightarrow \boxed{9x_1 - 12x_2 + 5x_3 - 5x_4 = 0}$

So this is the required relation b/w the vectors.

Home task

Q1// $A = \begin{bmatrix} 2 & 2 & 9 \\ 12 & 11 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

Q2// $A = \begin{pmatrix} -1 & 6 \\ 2 & 3 \end{pmatrix}$

Q3// $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

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