DLD
Quine-McCluskey Method

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The Quine-McCluskey Method

- To apply the Quine-McCluskey method, first write the function in standard **minterm**
- (SOP) form.
- To illustrate, we will use the expression

$$X = \underset{0001}{\overline{A}\,\overline{B}\,\overline{C}D} + \underset{0011}{\overline{A}\,\overline{B}CD} + \underset{0100}{\overline{A}B\overline{C}\,\overline{D}} + \underset{0101}{\overline{A}B\overline{C}D} + \underset{1010}{A\overline{B}C\overline{D}} + \underset{1100}{AB\overline{C}\,\overline{D}} + \underset{1101}{AB\overline{C}D} + \underset{1111}{ABCD}$$

- represent it as binary numbers on the truth table shown in Table 4–9. The minterms that appear in the function are listed in the right column.

The Quine-McCluskey Method

TABLE 4–9

ABCD	*X*	Minterm
0000	0	
0001	1	m_1
0010	0	
0011	1	m_3
0100	1	m_4
0101	1	m_5
0110	0	
0111	0	
1000	0	
1001	0	
1010	1	m_{10}
1011	0	
1100	1	m_{12}
1101	1	m_{13}
1110	0	
1111	1	m_{15}

∑(m1,m3,m4,m5,m10,m12,m13,m15)

The Quine-McCluskey Method

TABLE 4–9

$ABCD$	X	Minterm
0000	0	
0001	1	m_1
0010	0	M_2
0011	1	m_3
0100	1	m_4
0101	1	m_5
0110	0	
0111	0	
1000	0	
1001	0	
1010	1	m_{10}
1011	0	
1100	1	m_{12}
1101	1	m_{13}
1110	0	
1111	1	m_{15}

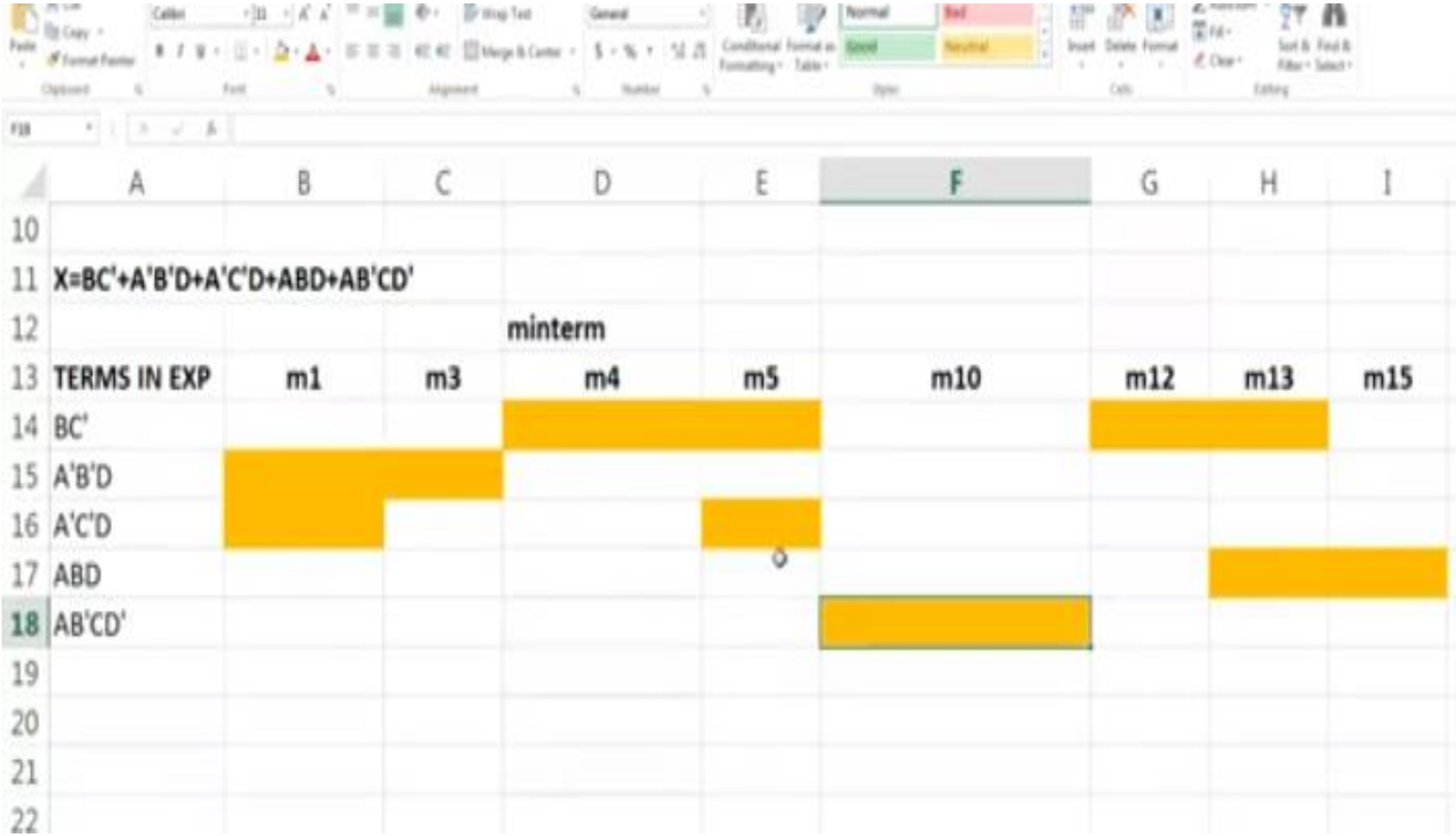
$X = \Sigma(m1, m3, m4, m5, m10, m12, m13, m15)$

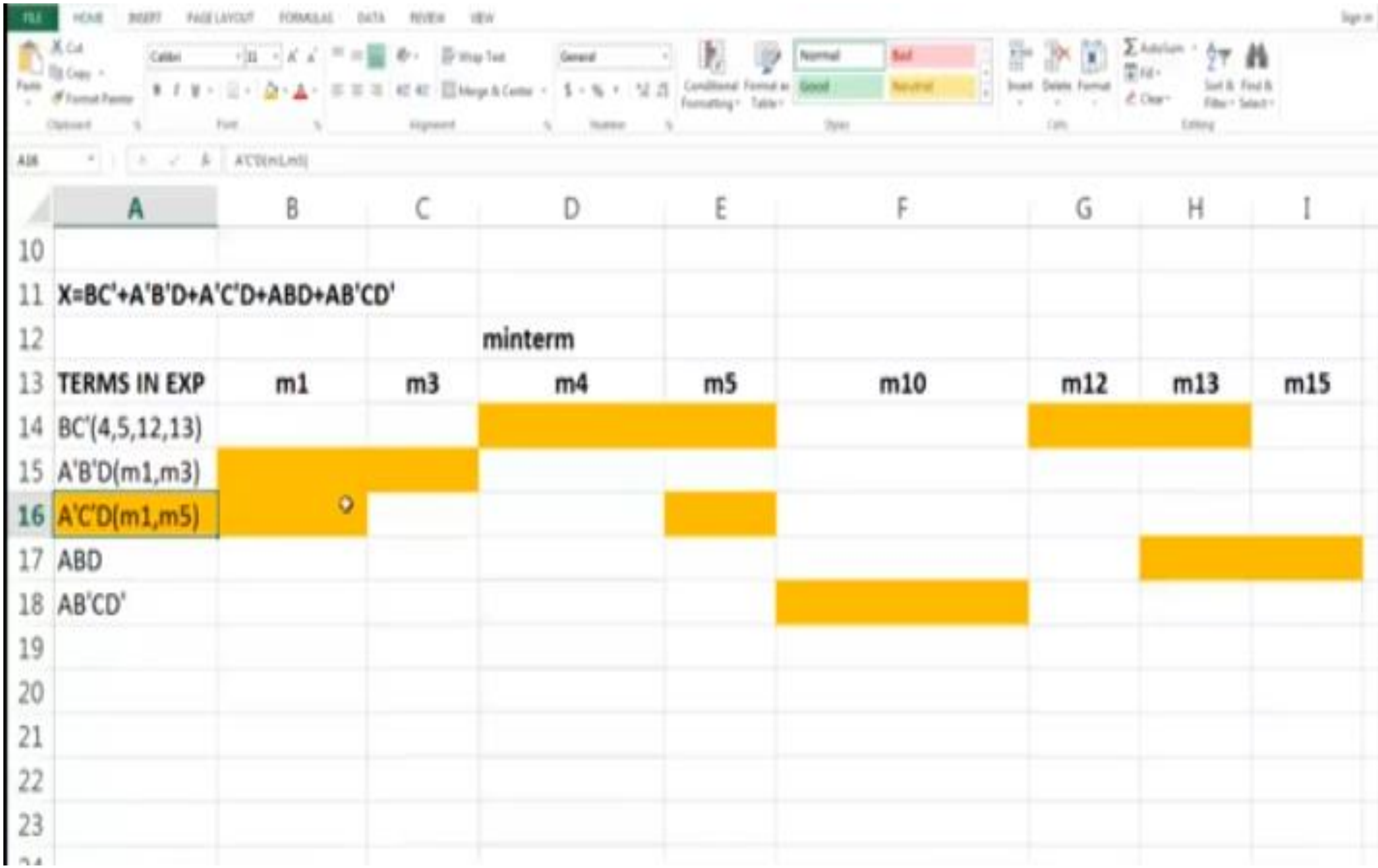
$= \Pi\ (M_0 - -$

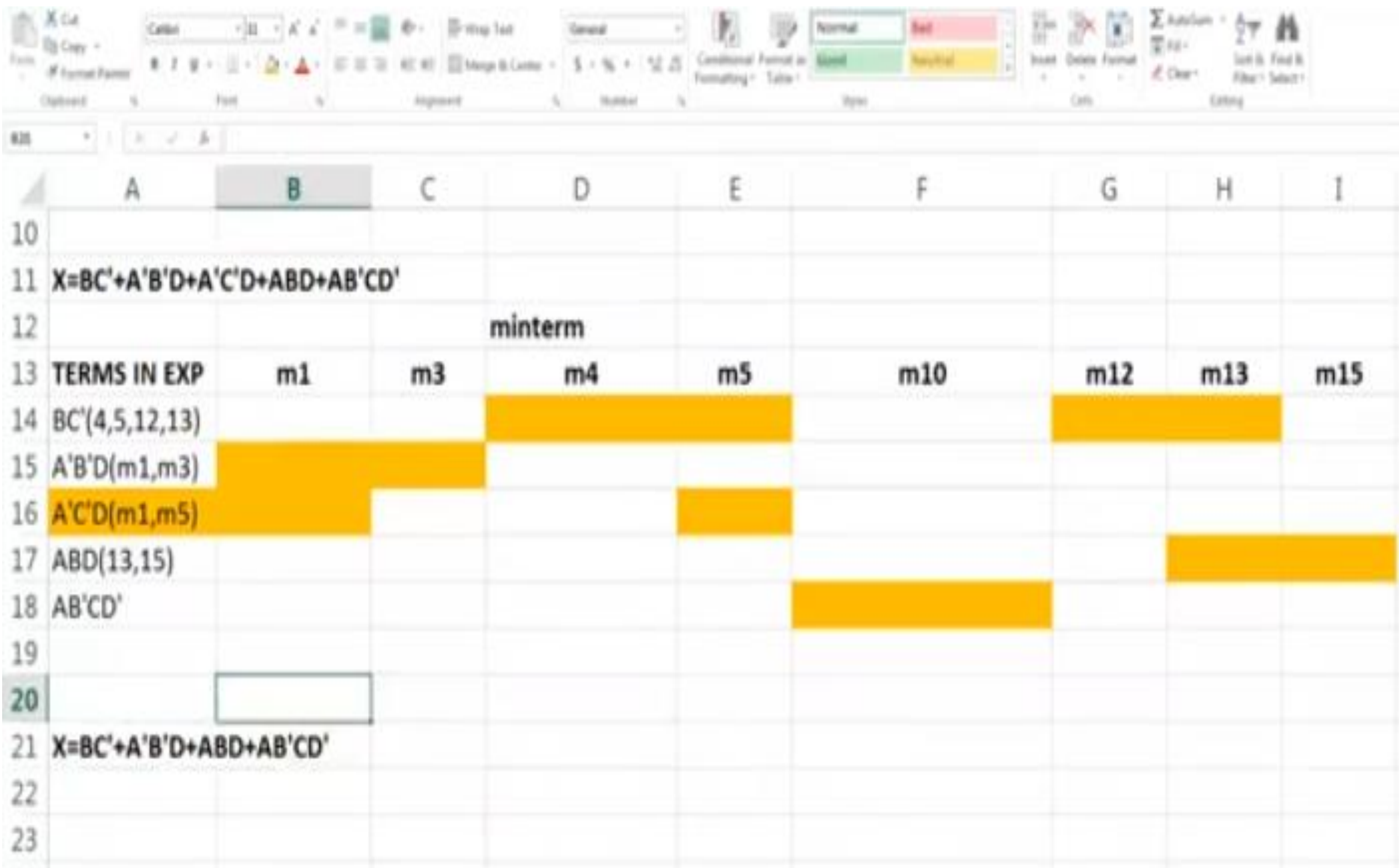
	A	B	C	D	E	F
1	No. of 1s	Minterm(PT)	ABCD	1st level	No. of 1s	2nd level
2	1	m1	0001	(m1,m3)00x1	1	(m4,m5,m12,m13)x10x
3		m4	0100	(m1,m5)0x01		
4	2	m3	0011	(m4,m5)010x		
5		m5	0101	(m4,m12)x100		
6		m10	1010	(m5,m13)x101	2	
7		m12	1100	(m12,m13)110x		
8	3	m13	1101	(m13,m15)11x1	3	
9	4	m15	1111			
10						
11						
12						
13						

No. of 1s	Minterm(PT)	ABCD	1st level	No. of 1s	2nd level
1	m1	0001	(m1,m3)00x1		(m4,m5,m12,m13)x10x
	m4	0100	(m1,m5)0x01	1	
2	m3	0011	(m4,m5)010x		
	m5	0101	(m4,m12)x100		
	m10	1010	(m5,m13)x101	2	
	m12	1100	(m12,m13)110x		
3	m13	1101	(m13,m15)11x1	3	
4	m15	1111			

X=BC'+A'B'D+A'C'D+ABD+AB'CD'


A
B
C
D
E
F
G
H
I
X=BC'+A'B'D+A'C'D+ABD+AB'CD'
minterm
TERMS IN EXP
m1
m3
m4
m5
m10
m12
m13
m15
BC'
A'B'D
A'C'D
ABD
AB'CD'


X=BC'+A'B'D+A'C'D+ABD+AB'CD'
minterm
TERMS IN EXP
m1
m3
m4
m5
m10
m12
m13
m15
BC'(4,5,12,13)
A'B'D(m1,m3)
A'C'D(m1,m5)
ABD
AB'CD'



	A	B	C	D	E	F	G	H	I
10									
11	X=BC'+A'B'D+A'C'D+ABD+AB'CD'								
12				minterm					
13	TERMS IN EXP	m1	m3	m4	m5	m10	m12	m13	m15
14	BC'(4,5,12,13)								
15	A'B'D(m1,m3)								
16	A'C'D(m1,m5)								
17	ABD(13,15)								
18	AB'CD'								
19									
20									
21	X=BC'+A'B'D+ABD+AB'CD'								
22									
23									

The Quine-McCluskey Method

- The second step in applying the Quine-McCluskey method is to arrange the minterms in the original expression in groups according to the number of 1s in each minterm.

TABLE 4–10

Number of 1s	Minterm	ABCD
1	m_1	0001
	m_4	0100
2	m_3	0011
	m_5	0101
	m_{10}	1010
	m_{12}	1100
3	m_{13}	1101
4	m_{15}	1111

The Quine-McCluskey Method

- compare adjacent groups, looking to see if any minterms are the same in every position *except one*. If they are, place a check mark by those two minterms, as shown in Table 4–11.

TABLE 4–11

Number of 1s in Minterm	Minterm	ABCD	First Level
1	m_1	0001 ✓	(m_1, m_3) 00x1
	m_4	0100 ✓	(m_1, m_5) 0x01
2	m_3	0011 ✓	(m_4, m_5) 010x
	m_5	0101 ✓	(m_4, m_{12}) x100
	m_{10}	1010	(m_5, m_{13}) x101
	m_{12}	1100 ✓	(m_{12}, m_{13}) 110x
3	m_{13}	1101 ✓	(m_{13}, m_{15}) 11x1
4	m_{15}	1111 ✓	

The Quine-McCluskey Method

- The terms listed in the *First Level* have been used to form a reduced table (Table 4–12) with one less group than before

TABLE 4–12

First Level	Number of 1s in First Level	Second Level
(m_1, m_3) 00x1	1	$(m_4, m_5, m_{12}, m_{13})$ x10x
(m_1, m_5) 0x01		$(m_4, m_5, m_{12}, m_{13})$ x10x
(m_4, m_5) 010x ✓		
(m_4, m_{12}) x100 ✓		
(m_5, m_{13}) x101 ✓	2	
(m_{12}, m_{13}) 110x ✓		
(m_{13}, m_{15}) 11x1	3	

The Quine-McCluskey Method

$$X = B\overline{C} + \overline{A}\overline{B}D + \overline{A}\overline{C}D + ABD + A\overline{B}C\overline{D}$$

TABLE 4–13

	Minterms							
Prime Implicants	m_1	m_3	m_4	m_5	m_{10}	m_{12}	m_{13}	m_{15}
$B\overline{C}$ $(m_4, m_5, m_{12}, m_{13})$			✓	✓		✓	✓	
$\overline{A}\overline{B}D$ (m_1, m_3)	✓	✓						
$\overline{A}\overline{C}D$ (m_1, m_5)	✓			✓				
ABD (m_{13}, m_{15})							✓	✓
$A\overline{B}C\overline{D}$ (m_{10})					✓			

The Quine-McCluskey Method

$$X = B\overline{C} + \overline{A}\overline{B}D + \overline{A}\overline{C}D + ABD + A\overline{B}C\overline{D}$$

TABLE 4–13

	Minterms							
Prime Implicants	m_1	m_3	m_4	m_5	m_{10}	m_{12}	m_{13}	m_{15}
$B\overline{C}$ (m_4, m_5, m_{12}, m_{13})			✓	✓		✓	✓	
$\overline{A}\overline{B}D$ (m_1, m_3)	✓	✓						
$\overline{A}\overline{C}D$ (m_1, m_5)	✓			✓				
ABD (m_{13}, m_{15})							✓	✓
$A\overline{B}C\overline{D}$ (m_{10})					✓			

Notice that the two minterms in $\overline{A}\overline{C}D$ are covered by the prime implicants in the first two rows, so this term is unnecessary. The final reduced expression is, therefore,

$$X = B\overline{C} + \overline{A}\overline{B}D + ABD + A\overline{B}C\overline{D}$$