Topic: Eigen Values & Eigen Vectors

Here $A \rightarrow$ Square Matrix $\lambda \Rightarrow$ Eigen Values

① $|A - \lambda I| = 0$

Solving for λ & find λ

② Eigen Vector : X is Eigen Vector for λ

the the value of x is given by

$$(A - \lambda I)x = 0$$

means This is a System of Homogeneous linear =ns

Q1, $A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$

Sol Let λ is eigen values of A

Then $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 2 \\ 3 & 3-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(3-\lambda) - 6 = 0$$

$$12 - 4\lambda - 3\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda = 6, 1$$

t $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ ~~fore~~ be The Eigen Vector for λ

Then X can be given by

$$(A - \lambda I)X = 0$$

Now Eigen Vector for $\lambda = 1$

$$\begin{bmatrix} 4-1 & 2 \\ 3 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1 + 2x_2 = 0$$

Now Let $x_2 = 1$

$\Rightarrow 3x_1 + 2 = 0$

$\Rightarrow 3x = -2.$

$\Rightarrow x = -2/3$

So $X = \begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$

Now Eigen vector for $\lambda = 6$

Then

$$\begin{bmatrix} 4-6 & 2 \\ 3 & 3-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 = 2R_2 + 3R_1$

$$\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x_1 + 2x_2 = 0$

$\cancel{-2}x_1 = \cancel{-2}x_2$

$x_1 = x_2$

of $x_1 = 1$ Then $x_2 = 1$

So Eigen Vector for $\lambda = 6$

$X = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Find the Eigen Values & Eigen Vectors of The following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$$

Sol :: To solve or to find ~~EVa&~~ Eigen Values & Eigen vectors we have to follow 4 steps

① Characteristic Matrix

② " Equation

③ " Polynomial

④ " roots

Given Matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$

The characteristic eq^n is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{vmatrix} = 0$$

Now the $=^n$ is

$$\lambda^3 - P\lambda^2 + Q\lambda - |A| = 0 \quad \text{——— ①}$$

here $p \rightarrow$ sum of digonal elements

& $Q \rightarrow$ Sum of diagonal minors

Now $P = 1-2-3$

$= 1-5$

$P = -4$

& $Q = \begin{vmatrix} -2 & 6 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix}$

$Q = 6-3-2$

$Q = 6-5$

$Q = 1$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{vmatrix}$

$|A| = 1(6) - 2(0) + 3(0)$

$|A| = 6$

Now eqn ① becomes

$\lambda^3 + 4\lambda^2 + \lambda - 6 = 0$ ——— ②

Now solving eqn ② by Synthetic devision we get

$$\begin{array}{c|cccc} 1 & 1 & 4 & 1 & -6 \\ & 0 & 1 & 5 & 6 \\ \hline & 1 & 5 & 6 & \multicolumn{1}{|c}{0} \end{array}$$

$\therefore \boxed{\lambda = 1}$

$\therefore \lambda^2 + 5\lambda + 6 = 0$

$\lambda^2 + 3\lambda + 2\lambda + 6 = 0$

$\lambda(\lambda+3) + 2(\lambda+3) = 0$

$(\lambda+3)(\lambda+2) = 0$

$\lambda + 3 = 0$ & $\lambda + 2 = 0$

$\lambda = -3$ & $\lambda = -2$

$\therefore$ The Charateristic roots are $(1, -2, -3)$

$\therefore$ $\boxed{\text{Eigen Values are } 1, -2, -3}$

Finding Eigen Vectors :

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be The eigen vectors Corresponding to eigen values

$$[A - \lambda I] X = 0$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -2-\lambda & 6 \\ 0 & 0 & -3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A

Eigen vectors for $\lambda = 1$

$=^n$ A $\Rightarrow$ $\begin{bmatrix} 0 & 2 & 3 \\ 0 & -3 & 6 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$0x_1 + 2x_2 + 3x_3 = 0$ —— (a)

$0x_1 - 3x_2 + 6x_3 = 0$ —— (b)

$0x_1 + 0x_2 - 4x_3 = 0$ —— (c)

Solving the $=^{ns}$ by cramer's rule we have

Taking $=^n$ (a) & (b)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ -3 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 0 & 2 \\ 0 & -3 \end{vmatrix}}$$

$$\frac{x_1}{21} = \frac{-x_2}{0} = \frac{x_2}{0}$$

Dividing by 21 we get

:

$$= \frac{-x_2}{0} = \frac{x_3}{0}$$

$$\lambda = 1, \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

~~Now~~

Let $\lambda = -2$.

$$eq^n (A) \Rightarrow \begin{bmatrix} 1+2 & 2 & 3 \\ 0 & -2+2 & 6 \\ 0 & 0 & -3+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + 2x_2 + 3x_2 = 0$ — (d)

$0x_1 + 0x_2 + 6x_2 = 0$ — (e)

$0x_1 + 0x_2 - x_3 = 0$ — (f)

From eq^n (d) & (e)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 3 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 3 & 2 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{12} = \frac{-x_2}{18} = \frac{x_3}{0}$$

Dividing by 6 we get

$$\frac{x_1}{2} = \frac{-x_2}{3} = \frac{x_3}{0}$$

$$\therefore \text{for } \lambda = -2 \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

Now for $\lambda = -3$

$$eq^n (A) \Rightarrow \begin{bmatrix} 4 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$4x_1 + 2x_2 + 3x_3 = 0$ — (g)

$0x_1 + x_2 + 6x_3 = 0$ — (h)

$0x_1 + 0x_2 + 0x_3 = 0$ — (i)

$$\frac{x_1}{\begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 4 & 3 \\ 0 & 6 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 4 & 2 \\ 0 & 1 \end{vmatrix}}$$

$$\frac{x_1}{9} = \frac{-x_2}{-24} = \frac{x_3}{4}$$

$$\therefore \text{for } \lambda = -3, \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ -24 \\ 4 \end{bmatrix}$$

$\therefore$ Eigen Vectors are

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ -24 \\ 4 \end{bmatrix}$$