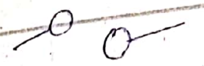
27-09-2023

Graph Theory



→ isolated vertex form a component.
→. number of incident edges of a vertex are called degree of vertex
→ number of neighbors.
→ denoted by deg (v).
→ The degree of every vertex is 2.
$\forall i$, deg (v) = 2.

Isolated vertices.

deg(v) = 0, isolated vertex, no degree.

Regular graph

→ each vertex has the same degree
→ regular graph of k degree is called k-regular. e.g:-
3-regular, 2-regular

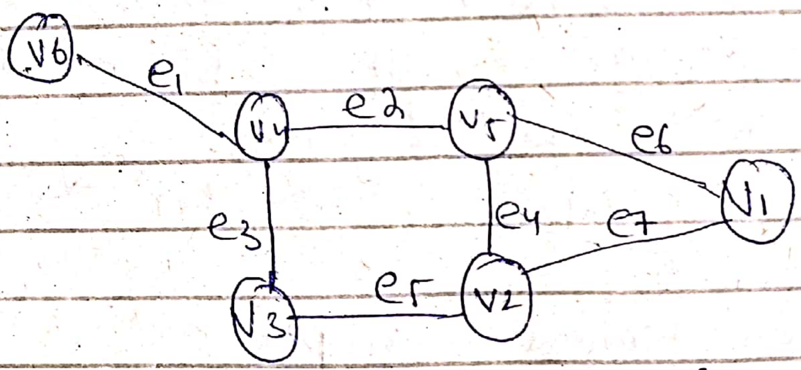
Complement graph.

→ $G = (V, E)$ → complement
$\bar{G} = (V, \bar{E})$
→ Two vertices are connected in $\bar{G}$ if and only if they are not connected in G.

Paths :-

* Walk
→ sequence of edges, such that each edge (except for the first one) starts with a vertex where the previous ended.
→ length of walk is number of edges.

Path
→ a walk where all edges are distinct.
→ simple path → a walk where all vertices are distinct.



A walk of length 6 : ($e_1, e_2, e_4, e_5, e_3, e_1$).
→ not a path : uses e_1 twice.

→ path of length 4 : (e_7, e_6, e_2, e_3)
not a simple path ; visit v_2 twice.

Cycle :-

→ a path whose first and last vertices are same.

→ all edges are distinct.

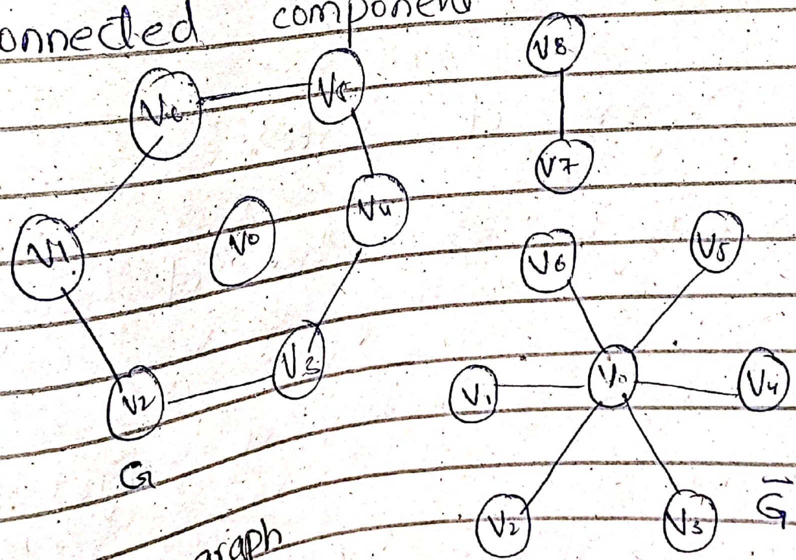
→ Simple cycle → where all vertices are taken once.

Connectivity :-

→ a graph is connected if there is a path b/w every pair of its vertices.

→ connected sub graph.

→ connected component → maximum connectivity.



G

$\vec{G}$

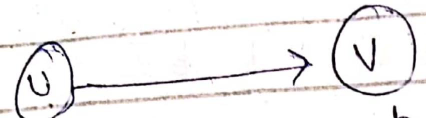
Directed graph edge

undirected edge

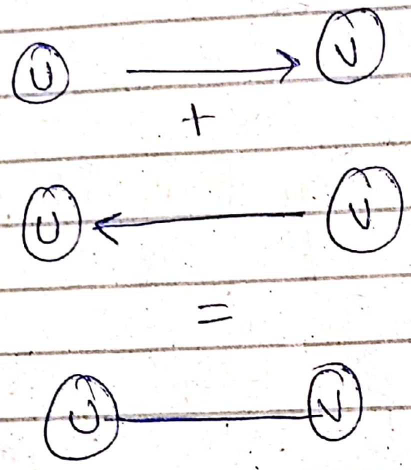
Edge {U, V}

Arc

Arc (u, v)



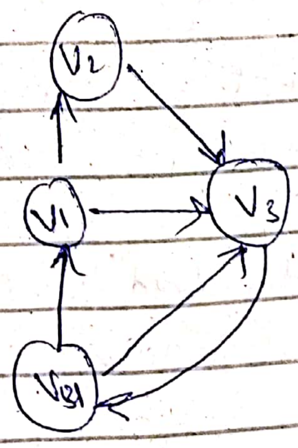
tail head.



Degree of vertex

→ indegree ⇒ number of edges ending at v.

→ out degree ⇒ number of edges leaving from v.



(v_2, v_3, v_4) a path of length 2.

Weighted Graphs.

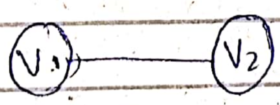
→ weight can be put on edges.
→ weight of path is sum of weights of its edges.
→ shortest path b/w two vertices is a path of minimum weight
→ distance b/w two vertices is length of shortest path between them.

Thursday 28-09-2023

Path graphs
→ represented by P_n, $n \geq 2$.

Cycle graph
→ represented by C_n, $n \geq 3$

→ complete graph (clique)
→ represented by K_n, $n \geq 2$
→ all vertices have connection to each other.
total edges → $n(n-1)/2$



$\frac{2(1)}{2} = 1$ edge

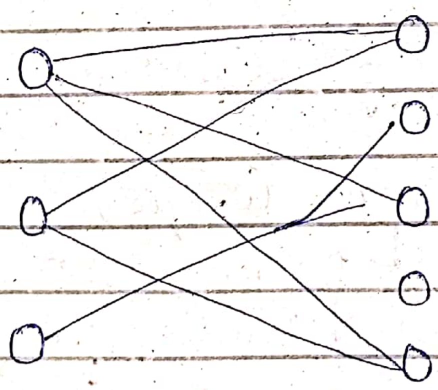
★ Trees
→ tree does not have cycle.
→ total edges $(n-1)$
→ unique path to each node.

→ a connected graph without cycle.

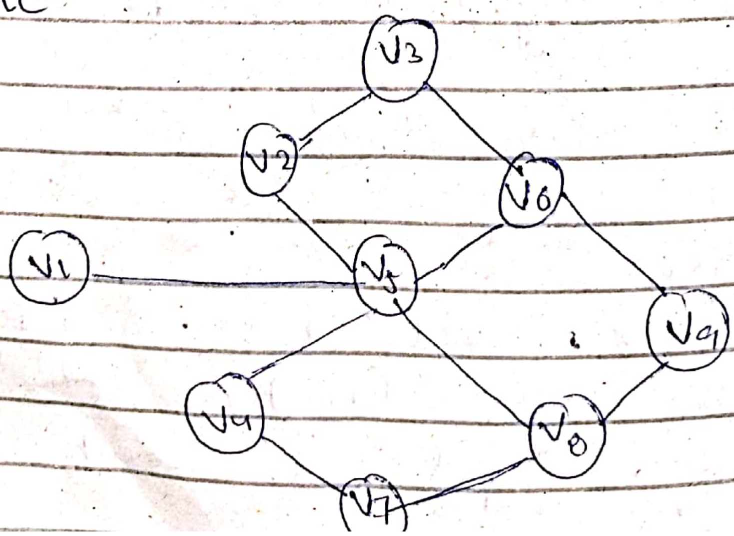
Drawing a tree.
connected ; number of edges is n-1.

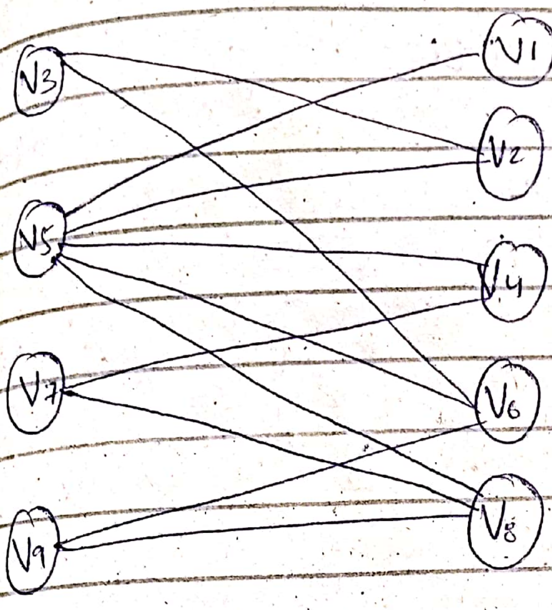
Make a Tree.
→ remove cycles until edges are n-1 without disturbing connectivity.

Bipartite Graphs.
→ two parts.

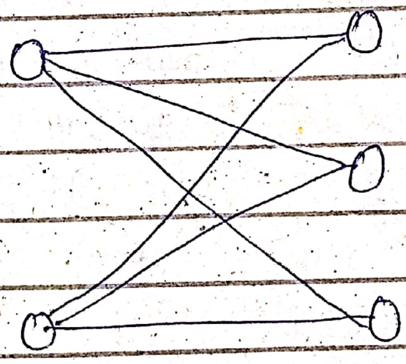


Bipartite



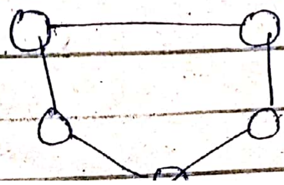


Complete Bipartite graph.

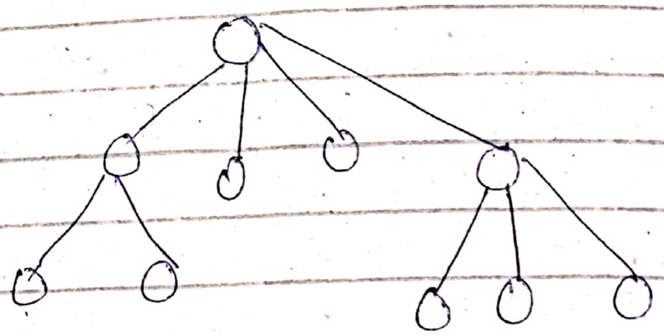


represented by.
$K_{m,n}$ e.g $K_{2,3}$

Cycle graph.
→ For even no. of nodes C_n is bipartite.
→ For odd $n > 2$, C_n is not bipartite.

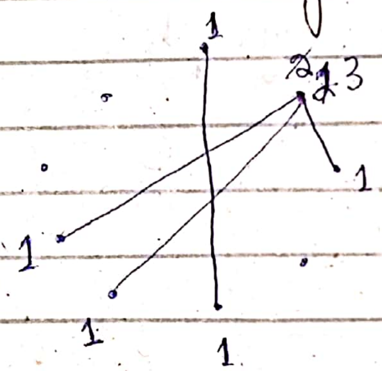


Trees are bipartite



Wednesday 04-10-2023

★ Handshaking Lemma



4

no. of odd points is always even, we will never reach situation when there are 9 points of degree 5.

→ A graph has an even number of odd nodes.

Lemma :

For any graph $G(V, E)$, the sum of degrees of all its nodes is twice the number of edges.

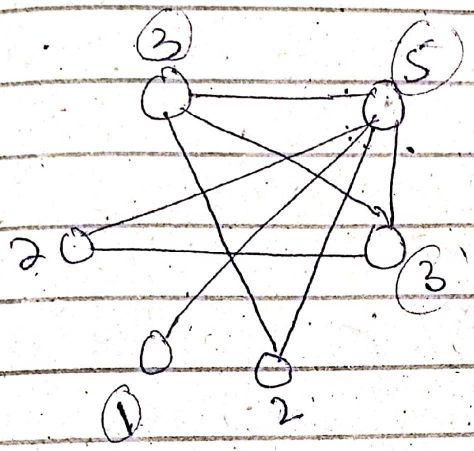
$$\sum_{v \in V} \text{degree}(v) = 2 \cdot |E|$$

$$\underline{2 \cdot 4} = \underline{8}$$

Handshaking lemma

a graph had an odd no. of nodes, then the sum of degrees also be odd.

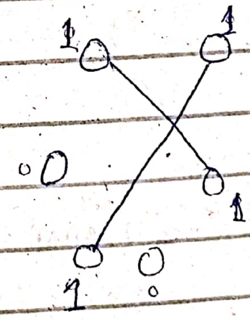
e.g.-



no. of edges = 8.
sum of degrees = 16
according to handshaking lemma
$2 \cdot 8 = 16$

$2 \cdot 0 = 0$
$2 \cdot 1 = 2$

Proof:-



→ remove all edges.
theorem holds for degree 0
start adding them back,
one by one.
$2.1 = 2$
as well as to twice the number of edges.
→ we proved formula by induction on no. of edges.

sum of degrees increase by 2.

20, 60

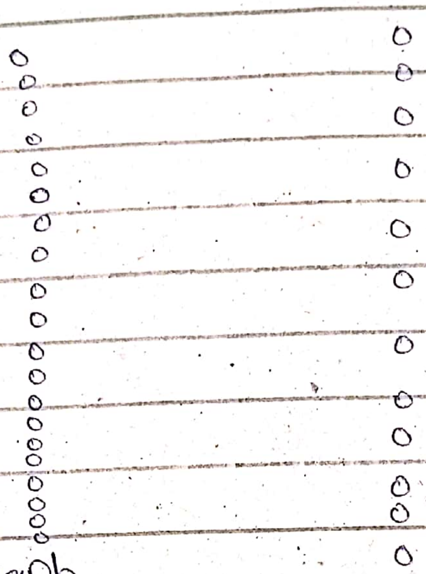
total students = 20, problems = 3, 1 prob solved by 5 students.

In graph :-

20 students

20 x 3

= 60 edges.



K problem

→ bipartite graph.

→ double counting technique is used

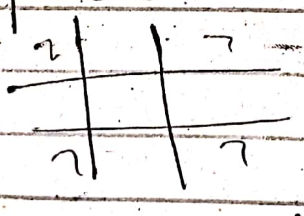
Thursday 05-10-2023

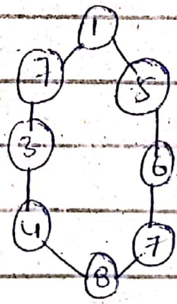
⟹ Connected component.

* The heviest stone

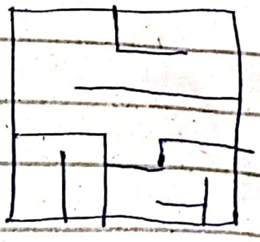
→ no. of comparisons to show how many heavy stones are there?

Guarini puzzle.





Connected components in a maze



three connected components.

connected graph. an undirected graph

→ Two nodes are connected, if there is path b/w them.

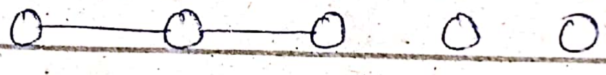
→ it is transitive.

graph is connected if any two nodes are connected / there is a path b/w them

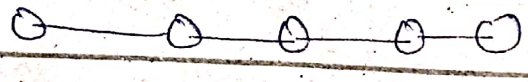
Connected components

→ subset of undirected graph

5 components.



3 connected components.



1 connected component.

sday 19-10-2023

imp.

Eulerian and Hamiltonian Cycles.

Toy genome assembly problem.

* Find a string whose all substrings have length 3. , AGC, ATC, CAG, CAT, CCA, GCA, TCA, TCC

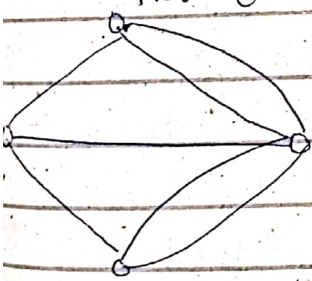
* Eulerian cycle (or path) visits every edge exactly once.

→* for both directed & undirected graphs.

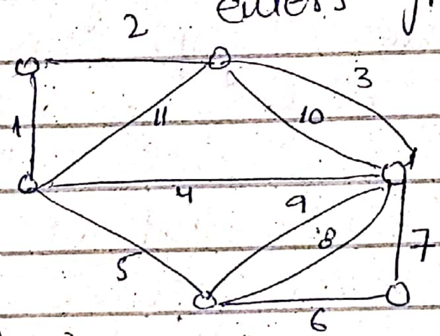
→ a cycle must have the same starting and ending nodes.

→ in path starting & ending nodes should not necessarily be equal.

not euler



euler's graph.



Criteria

Theorem

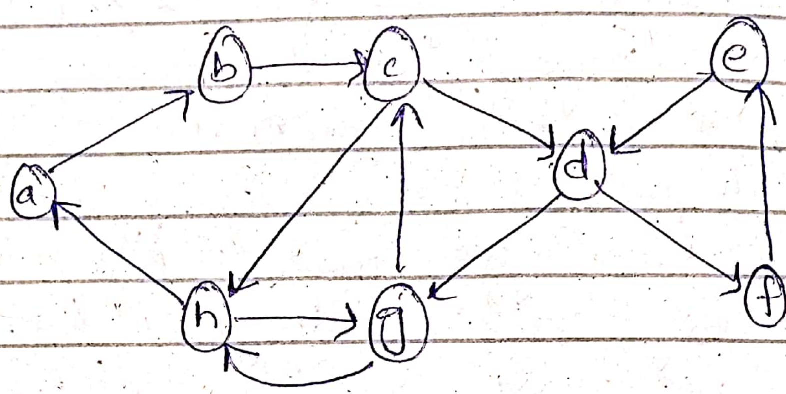
A connected undirected graph contains eulerian cycle if the degree of every node is even.

Theorem

A strongly connected directed graph contains eulerian cycle, if and only if, for every node, its in-degree is equal to its out-degree.

Proof (directed case)

if some node is imbalanced, there is clearly no Eulerian cycle



- assume that the graph is balanced.
- start walking from any node
- since graph is balanced, we'll come back to starting node.

Path instead of cycle.

- similar criteria for path.
- A graph is allowed to contain two imbalanced nodes: starting & ending nodes.
- adding one edge to the nodes give the eulerian cycle.
- starting node has one less in-degree, ending node has one less out degree.

Efficient Algorithm

* Hamiltonian cycle

→ A hamiltonian cycle visits every node of the graph exactly once!

Simple criteria

→ no simple criteria is known for hamiltonian cycle

→ no polynomial time algorithm

Toy Genome Assembly Problem.

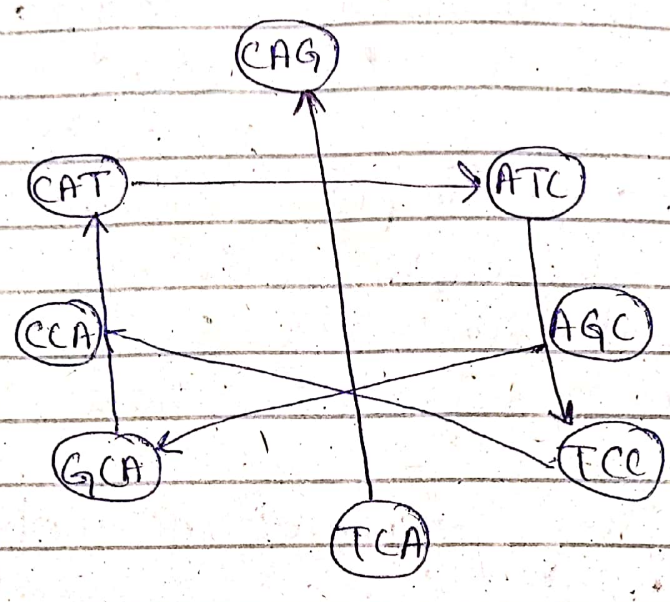
e.g:-

DISCRETE
DIS
 ISC
  SCR
   CRE
    RET
     ETE

every two neighbor substrings have common part, called overlap of length 2.

→ we need to find order of substrings.

Overlap graph of Toy genome

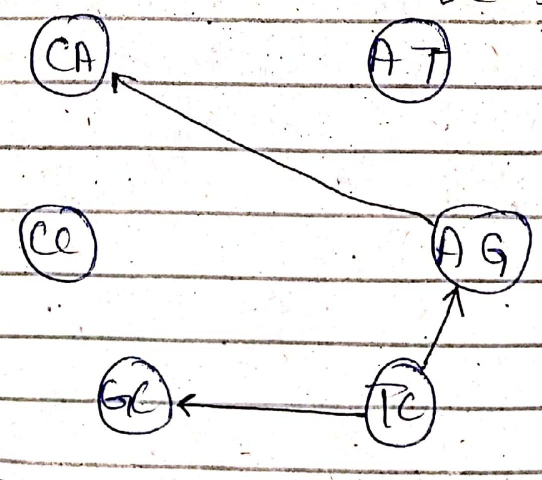


we reduced the problem of genome assembly to Hamiltonian cycle problem.

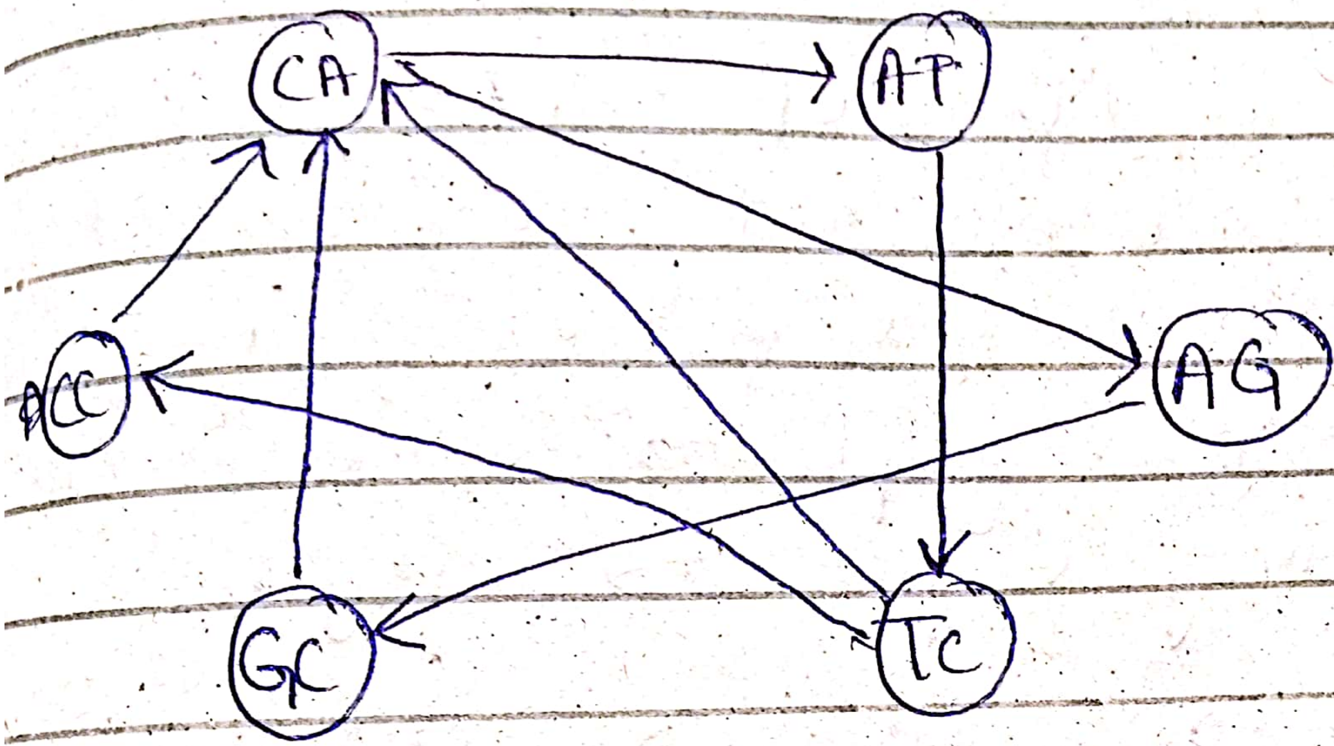
But we don't have efficient algorithm for Hamiltonian cycle problem.

Speculating.

De-Brujin graph



TCAGCATCCA.


CA
AT
ACC
AG
GC
TC

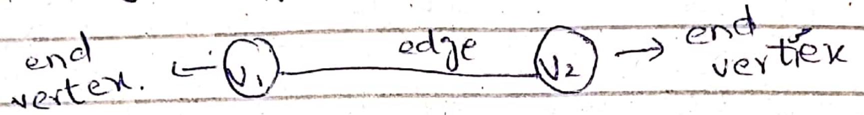
Graph

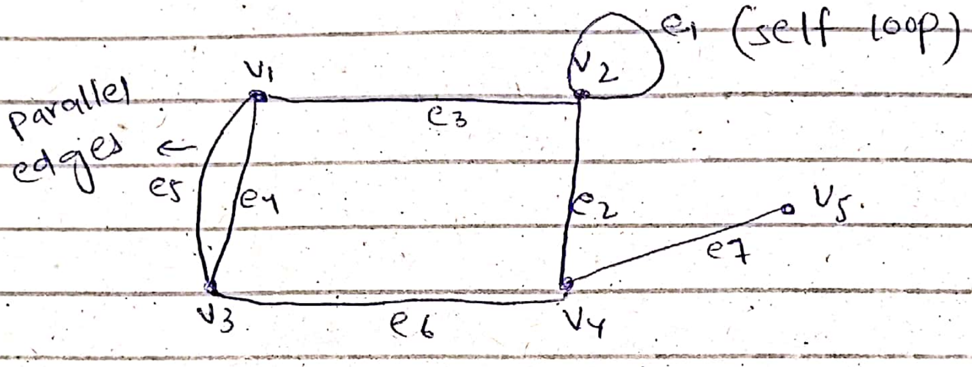
A graph consists of ordered pairs of set of vertices and set of edges.

$G(V, E)$, $V = \{v_1, v_2, v_3, \ldots\}$, $E = \{e_1, e_2, e_3 \ldots\}$.

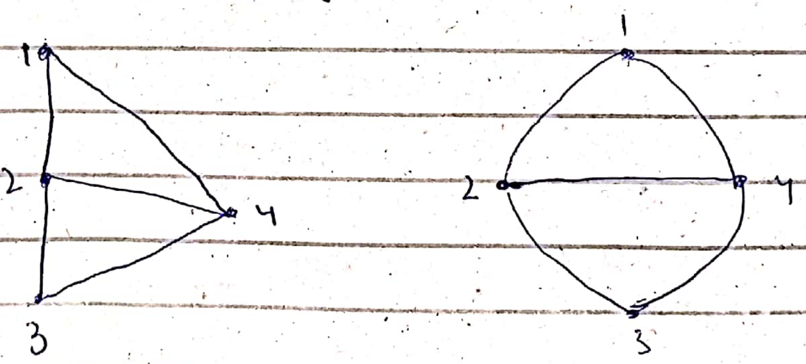
points in the graph → nodes, vertices

lines joining vertices → edges.





→ A graph that has neither self-loop nor parallel edges is called simple graph.



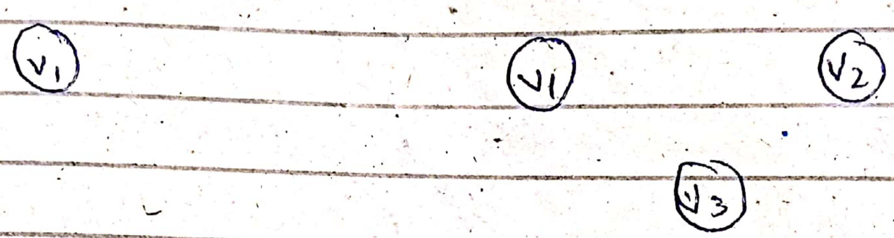
same graphs b/c incidence b/w edges and vertices are same.

Finite Graph

A graph with finite number of vertices as well as finite no. of edges is called a finite graph.

otherwise it is infinite.

null graph :- A graph having no edge. But should contain at least one vertex.



Every vertex in null graph is isolated vertex.

isolated vertex :- A vertex with degree zero. $deg(v) = 0$

pendant vertex :- A vertex with degree one. $deg(v) = 1$.

Handshaking Theorem.
The sum of the degrees of all vertices in a graph is twice the number of edges in it.

$$\sum_{i=1}^{n} d(v_i) = 2e$$

→ The number of vertices of odd degree in a graph is always even.

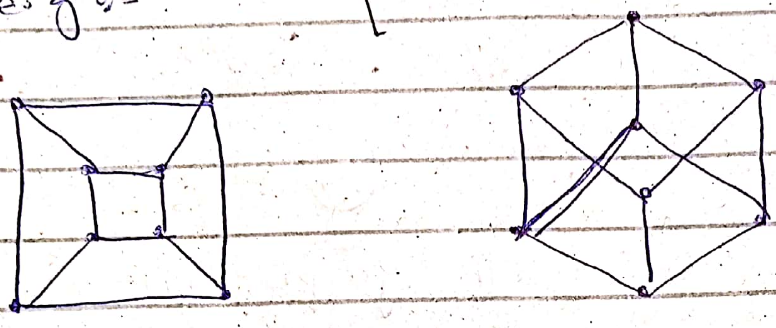
Isomorphic Graphs.

→ Same graphs drawn differently.

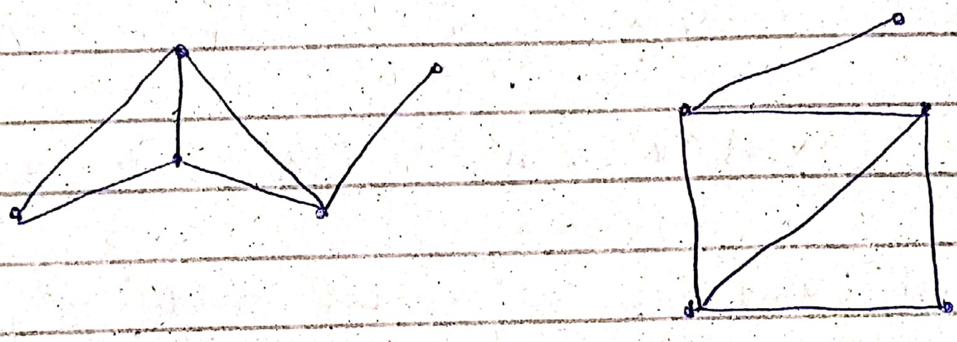
→ have one-to-one correspondance in vertices and edges.

e.g.-

(1)



(2)



→ same number of vertices & edges