Lectur.

Eular theory

$G = (V, E)$

Fundamental concept of Graph

Point

It is particular position which is located in space.

represted .

line

Two point are connected to each other

represented ———

Vertin

Vertin is a synonym of Point.

Edges

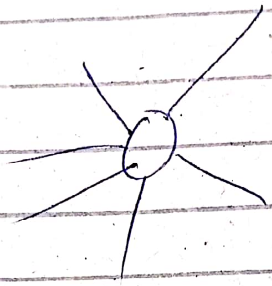
It is connected to two vertices.

Lecture 2 27/9/2023

The degree of a vertex

denoted by deg

The degree of v = 6



Isolated vertex

with there is a vertex

Regular Graph

3 - regular Graph

Complement Graph

$\bar{G} = (V, \bar{E}$

Path

~~Walk~~

Cycles

Connectivity

Connected Component

undirected Edge

Edge $\{u, v\}$

Arc (u, v)

tail head

$u \longrightarrow v$

$+$

$v \longleftarrow u$

$=$

$u \text{———} v$

The degree of vertex

Directed Path

weighted Graph

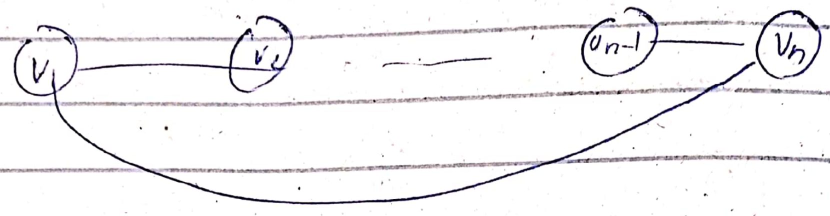
weighted Path

Lecture 3 28/9/23

PATHS GRAPH

Cycle Graph

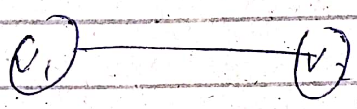
C_n $n \geq 3$

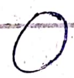


Complete Graph

K_n, $n \geq 2$ and all edges between them $(n(n-1)/2)$

for edges





$$\frac{\cancel{2}(1)}{\cancel{2}} = 1$$

Trees

Def

Tree is connected graph without cycles.

A tree is a connected graph on n vertices with n-1 edges.

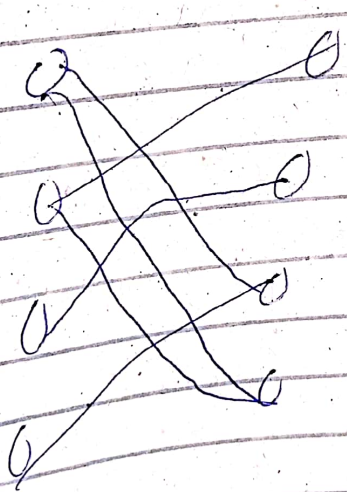
Drawing a tree

Make a tree

Connected removing cycle

Make a tree
when n-1 edges left in graph it make tree.

Binartite Graph
Bipartite
A graph G is Bipartite if its vertices can be partitioned into two disjoint



Example

Complete Bipartite $K_{2,3}$, $K_{m,n}$.

20×3

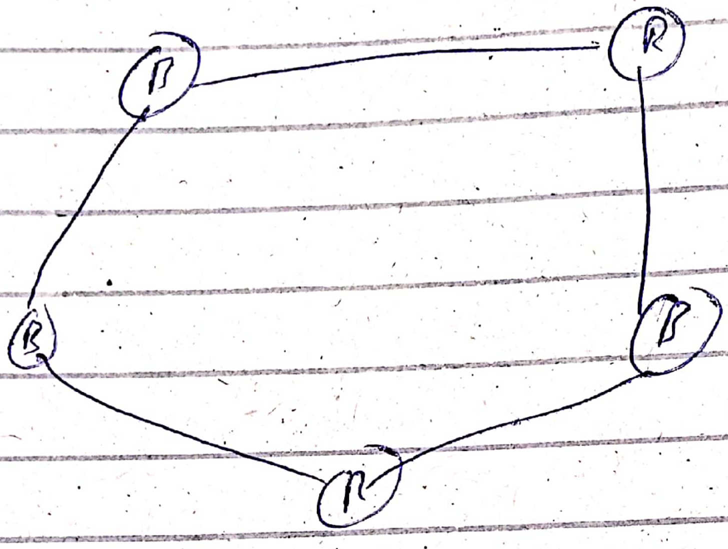
$\cancel{60}^{30} \times 5$

$\cancel{2}$

Cycle Graph

For odd $n > 2$, C_n is not Bipartite

It is apply for even, C_n is bipartite Graph



Trees are Bipartite

Lecture 4/10/2023

Handshaking Lemma

$$\sum_{v \in V} \text{degree}(v) = 2 \cdot |E|$$

Proof
induction

Bipartite Graph

Lecture 5/10/2023

Connected Graphs
Gaurini Puzzle

Lecture 5 18/10/23

Connected Components

Graph of Configuration

→ Lower Bound (Theorem)

undirected Graph $G(V, E)$
$|V| - |E|$

Proof

Heaviest ston (upper Bound)
Proof

Directed Acyclic Graph

Citation Graph

Dependancy Graph

Topological Ordering

Strongly connected component

Lecture 6 18/10/23

Eulerian Graph

Theorem
The graph is balance, it its
in degree or out degree are same.

-> traversed atleast once all the
path

-> Path instead of cycle
--> Path to cycle

Efficient Algorithm

. All string of length 3

Finding a Permutation

Overlap Graph

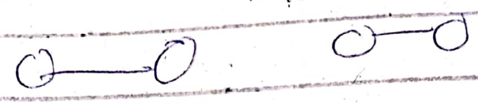
De Bruijn Graph

Lecture 7 25/10/23

Road Repair

Component

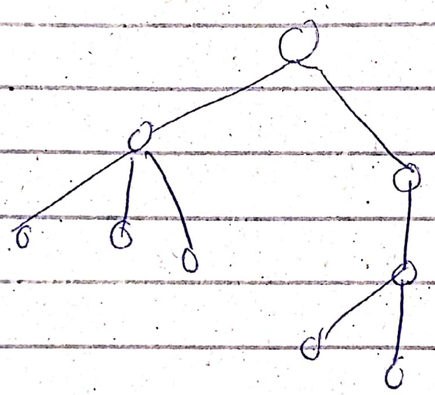
This is two component



Trees

- → A Tree is a connected graph without cycles
- → A tree is a connected graph on n vertices with $n-1$ edges
- → There is a unique path for any edge

Example



Induction hypothesis

Minimum Spanning tree

A spanning tree of a graph G is a subgraph of G which is a tree and contain all vertices of G.

Lecture 8 26/10/23

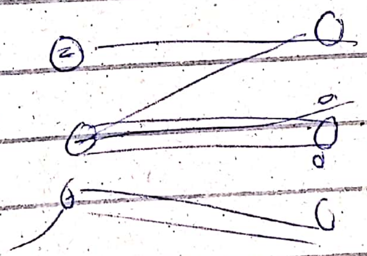
Kruskhal Algorithm

weight (f) $\geq$ weight (e_{k+1})

Job Assignment

Room Assignment

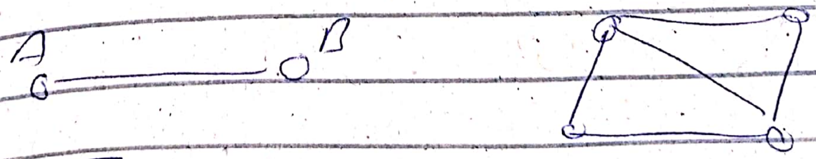
Bipartite Graph



not connected with each other

Matching

Maximal Matching



Hall's Theorem

$|S| \leq |N(S)|$

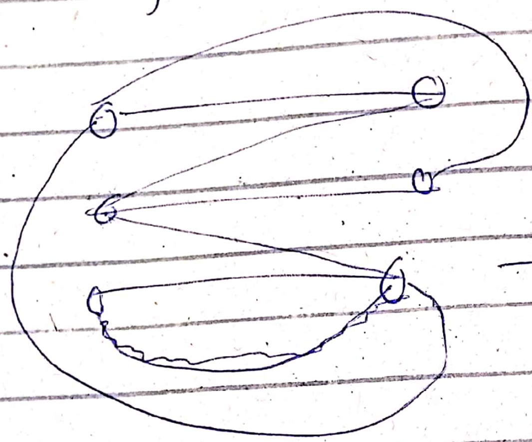
945
15.

Lecture 1/11/23

→ Linear Graph
→ Subways

→ Plannar Graph
Design of Electronic Circuits

Three Utility Problem



— not possible

Maps and Planar Graps
Every graph is planner graph

Eulers Formula

Theorem

$v - e + f = 2$

Tree is a connected graph which can't contain cycle.

Application Euler formula

Th. Number of faces

$$v - e + \frac{2e}{3} = 2$$

$2e \leq 6$

$$v - e + f = 2$$

$$f \leq \frac{2e}{3}$$

Putting value

$$e \leq 3v - 6$$

- The no of phases in Bipartite graph

Planner Graphs are sparse

$\Rightarrow$

Lecture

2/11/23

Graph Colouring

Theorem (Appel, Hakel, 1976)

Chromatic number

chromatic number $\chi(G)$

Full Graph

Cycle

Bipartite Graph

Six Colour Theorem

Graph of Bounding degree

Application

-> Scheddling problem
-> Register Allocation
->

After mids

Lecture 11 29/11/23

Frendship Graph

Complement Graph

Cliques and Independent sets.

→ Cliques

Independent sets

Clique and IS:

$$w(G) = \alpha(\bar{G})$$

Connecting to Coloring

Graph Theory

Graph Theory 13/12/23

Star Graph

Weighted Graph

Multi graph
Multiple edges

planner Graph

Non-planner Graph

Application of Graph Theory

Definition - Edge Type
-> Loop
-> Multiple Edges

Adjacent vertice
Those who have common edge.

Walk (edge repetition

Trail and circuit
The walk no edge is repeated.

Path

Lecture

Cycle 14/12/23

Distance between two vertices

Radius of Connected Graph

Diameter of Graph

Central point

Centre

Circumference

Girth
Smallest cycle of shortest edges.

Sum of degrees of vertices theorem.

Lecture 20-12-23

Adjacent matrix
Directed graph Representation
Pros and cons

Incidence Matrix
Incidence matrix representation

Adjacency list
Pros and cons

Trees and Forest

Tree (n-1) edges
undirected, connected, acyclic

forest
undirected, disconnected, acyclic graph.

Properties of Trees

Spanning tree
It is a sub graph.

Method to find spanning tree
1) Cutting down method
2) Building up Method.

Circuit Rank

Spanning tree G = n - 1 which is called circuit rank of G

m = edges.

n = 5

Connectivity

Cut Verten

A single verten whose removal disconnect graph is called cut-verten.

connect graph = n-2 cut vertices

Enample

Cut Edge

It is a single edge

Cut set

all no of edges which will to remove the graph is disconnected.

Edge Connectivity

minimum no of edges remove the graph is disconnected.

minmum two required.

Verten Connectivity

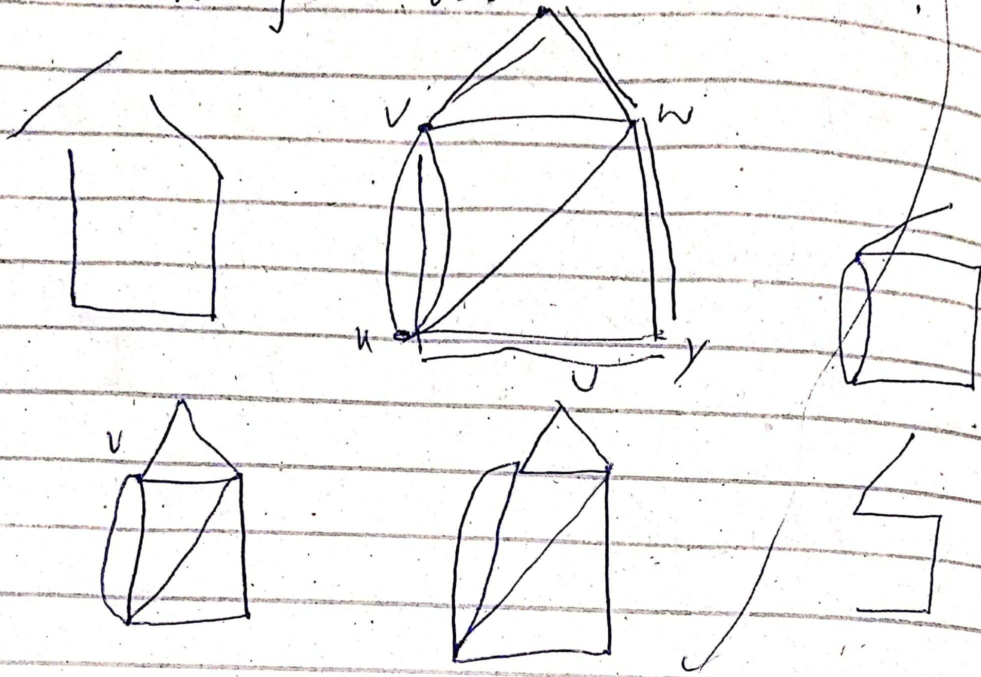
Yes. G contain an Euler circuit because every ~~edge~~ vertice has an even degree. So the possible euler circuit are

Eular ~~Path~~ Circuit

-> edge not repeat

-> start and end point are same

u - w - y - u - v - u



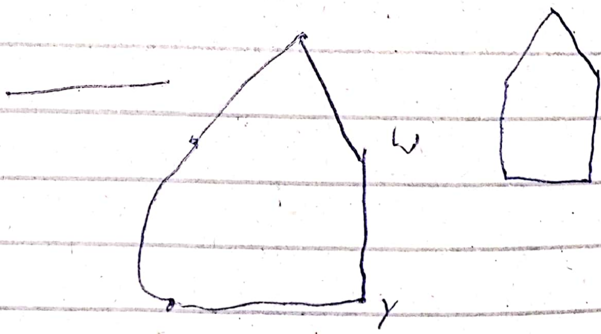
u -> v -> w -> y -> u - v - u - w - u

Humaliton graph

-> visit all vertices,

-> No repitition of vertice.

-> Initial and final is same.



w - y - u - v - u - w

Lecture

Hand shaking

Sum of degree of vertices theorem

$$\sum_{i=1}^{n} d(v_i) = 2(E)$$

$$\sum_{i=1}^{n} d(v_i) = \sum_{even} d(v_i) + \sum_{odd} d(v_i)$$

Pendant vertex (contain one neighbour)
Isolated vertex (

Degree depend upon edges not ~~on~~ on vertex

A graph has 21 edges
3 vertices has 4 degree
& all the other vertices has 2

$|E| = 21$

$\sum d(v_i) = 4$

Q if a graph has 21 edges and 3 vertex having degree 4 and all other vertex has four degree 2, find

$$\sum_{i=1}^{n} d(v_i = 2(E)$$

$$= 2(21)$$

$= 42$

$3(4) + (n-3)2 = 42$

$n = 18.$

Question

If a graph has 24 edges and find the no of each vertices 4

$\Sigma d(v) = 2(E)$

$n4 = 2(24)$

$\boxed{n = 12}$

Question

If a graph has 35 edges, ~~find~~ four vertices of degree 5, four vertices of degree 3 find vertices, find vertices with degree 2

$\Sigma d(v) = 2(35)$

$4\times5 + 4(5) + 4(3) + 2n = 70$

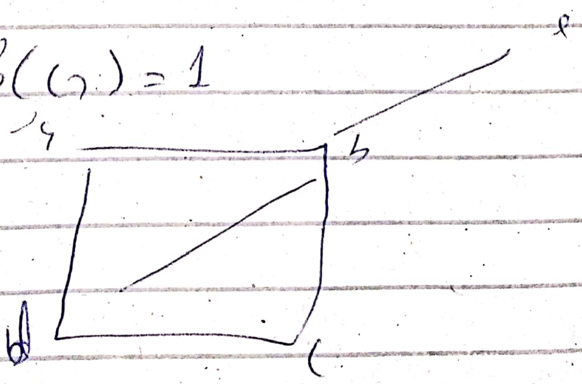
$20 + 12 + 2n = 70$

$32 + 2n = 70$

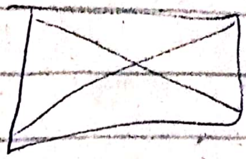
$\boxed{n = 9}$

Minimum of a Degree

(delta) $\delta(G) = 1$



$\delta(4) = 3$ $K_3 \rightarrow$ complete graph



$$\sum_{i=1}^{n} d(v_i) = 2(E)$$

$$\delta(G) \cdot |V| \le 2|E|$$

find the minimum degree with this.

<u>Maximum degree</u>

delta $\Delta(G) = 4$

$$\Delta(G) \cdot |V| \ge 2|E|$$

It is equal in case of degree are same. for both case Min/Max

Maximum degree of a vertex in a graph G with 35 edges and 7 each vertex is at least degree 3.

$$E = 35$$

$$\sum d(v) = 2E$$

$$\delta(G) \cdot v \le 2(E)$$

$$3(v) \le 2(35)$$

$$v \le \frac{70}{3}$$

total no of vertices $v \le 23$

$\sqrt{132} + \sqrt{n}$

0 -1 -1 0

Complement of a Graph

$V(G) = V(\bar{G})$

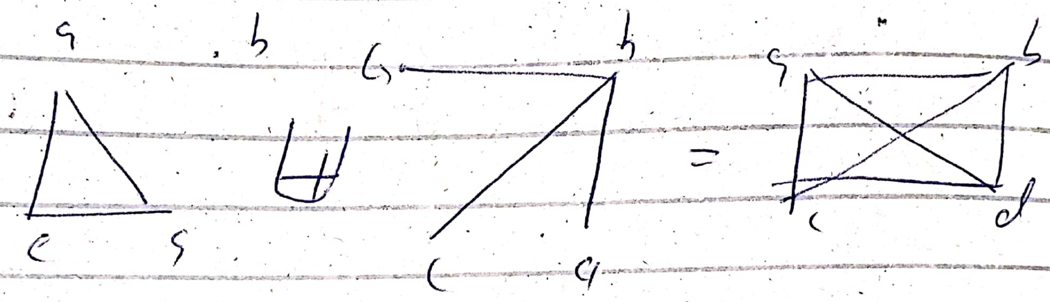
$\rightarrow E(\bar{G}) = \{(u, v) \mid (u,v) \notin E(G)\}$

$E(\bar{G}) = E(K_n) - E(G)$

$G \cup \bar{G} = K_n$

$G \cap \bar{G} =$ Null Graph

$|E(\bar{G})| + E(G) = E(K_n) = \frac{n(n-1)}{2}$



Consider a simple graph

$|E(G)| = 30, \quad E(\bar{G})| = 36$

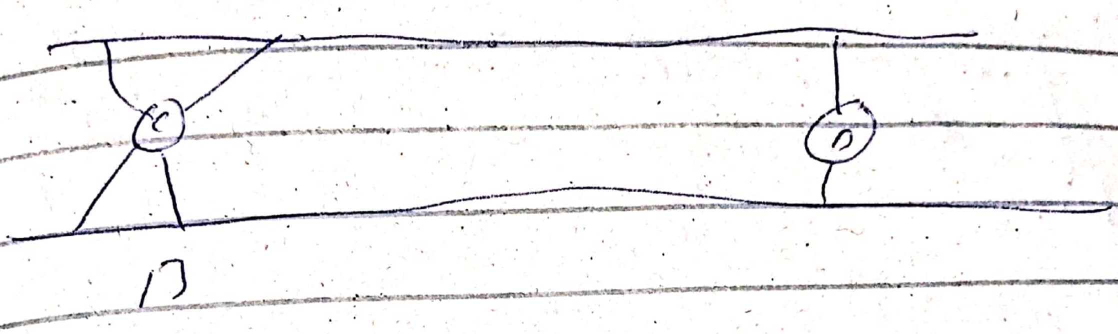
$|V(G)| = ?$

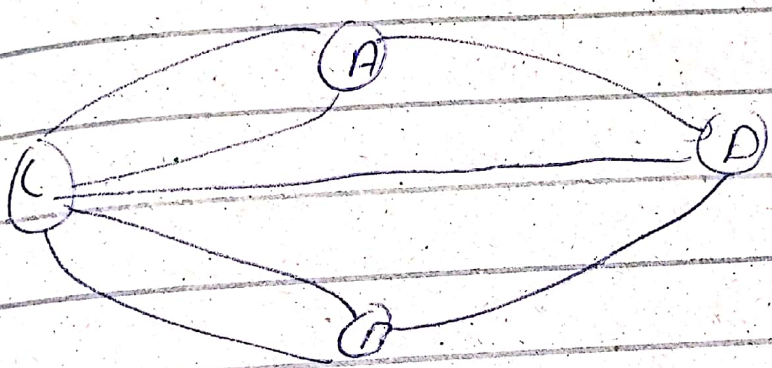
$|E(G)| + E(\bar{G})| = K_n = \frac{n(n-1)}{2}$

$30 + 36 = \frac{n(n-1)}{2}$

$66 =$

Seven bridge of kingsberg





Every vertice has even degree of to solve this.