Monday

7/10/24

Graph Theory

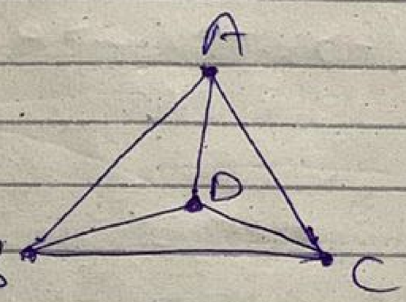
=> Properties of Graph Theory

~~Basically used for~~

~~chara~~

a) => Distance => Number of edges in Shortest possible path between vertex X & vertex Y.

Denoted by $d(X, Y)$.



Find shortest path b/w B and D.

1) B -> C -> A -> D , length = 3
2) B -> D , length = 1 (shortest)
3) B -> A -> D ,
4) B -> C -> D

b) => Eccentricity of a Vertex :

Denoted by $e(V)$

Maximum distance b/w a vertex to all other vertices.

Tuesday Purely from javatpoint

8/10/24 Graph Theory

Graph Representations (1)

→ A graph representation is a technique to store graph into the memory of a computer.
→ To represent a graph, we just need the set of vertices, and for each vertex, the neighbors of the vertex (vertices which is directly connected to it by an edge). If it is a weighted graph, then the weigth will associated with each edge.

Adjency Matrix (2)

→ A sequential representation.
→ Used to represent which nodes are adjacent to each other i.e. is there any edge connecting nodes to a graph.
→ We have to construct $n \times n$ matrix A. If there is any edge from a vertex i to vertex j, then the corresponding element of A, $a^{i,j} = 1$, otherwise $a^{i,j} = 0$.

→ Even if graph on 100 vertices contain only 1 edge, we still have a 100×100 matrix with lots of zeros.

→ If any weigthed graph, then instead of 1s and 0s, we can store weigth of th edge.

Look at the following undirected graph representation

2) Paths :-

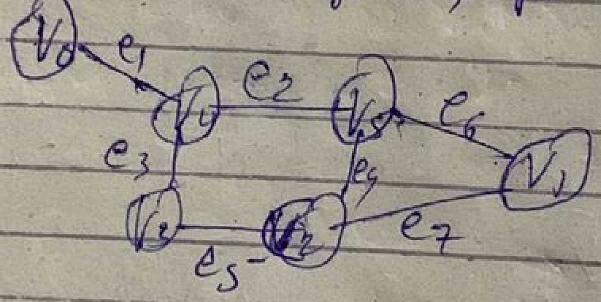
Walks :-

→ A walk in a graph is a sequence of edges such that each edge (except for the first ones) starts with a vertex where the previous edge ended.

→ The length of a walk is the no. of edges in it.

→ Path = walk where all edges are distinct

walk of length 6, path of length 4

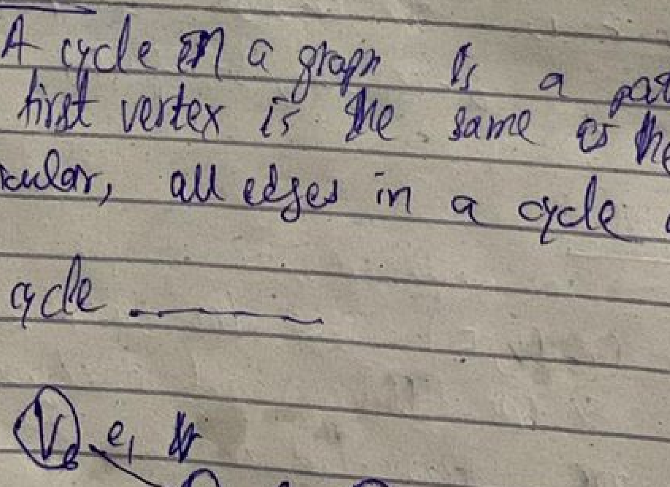


It is sometimes convenient to specify a path......

Cycles :-

A cycle in a graph is a path whose first vertex is the same as the last one. In particular, all edges in a cycle are distinct.

A simple cycle ....



5) Weigthed Graphs

(big ass triangle graph example)

- A weigthed graph associates a weight with every edge.
- The weight of a path is the sum of weights of its edges.
- A shortest path b/w two vertices is the minimum weigth.

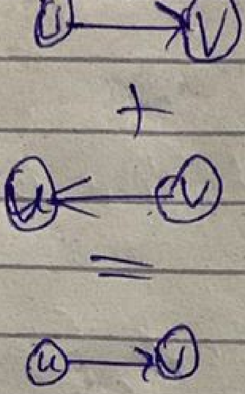
(after that example)

Basic Graphs (New Lect.)

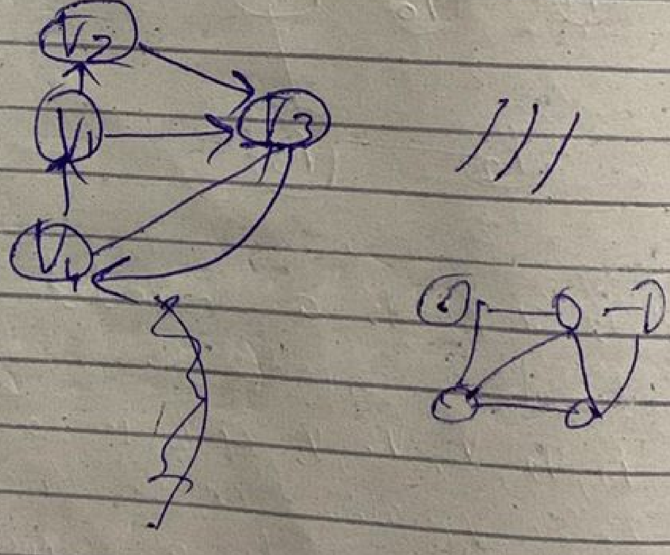
- Paths Graph $P_n, n \geq 2$ consists of n vertices $V_1, \ldots, V_n$ and $n-1$ edges $\{V_1, V_2\}, \ldots, \{V_{n-1}, V_n\}$

$$V_1 - V_2 - \cdots - V_{n-1} - V_n$$

Arc(u,v)



The indegree of a vertex v is the no. of edges ending at v.
The outdegree of a vertex v is the no. of edges leaving at v.



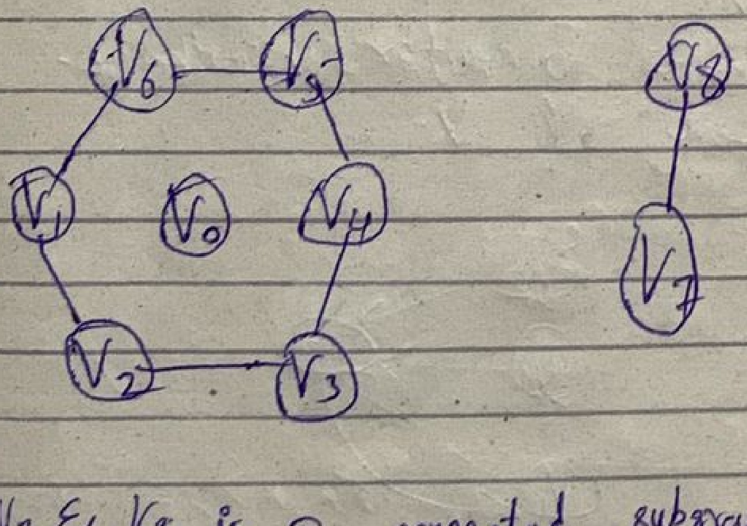
Directed Paths

same as above graph

Simple cycle of length 4: (e_5, e_4, e_2, e_3)

3) Connectivity:-

-> A graph is called connected if there is a graph b/w every pair of its vertices.
-> A connected component of a graph G is a maximal connected subgraph of G.



-> V_1, V_2 & V_3 is a connected subgraph, but not a connected component.
-> V_1, V_2, V_3, V_4, V_5 & V_6 is a connected component.
-> V_7 & V_8 also form a connected component.
V_0 is also a connected component. as each isolated vertex is a connected component.

4) Directed Graph:-

Undirected edge
Arc (Directed)

MONDAY

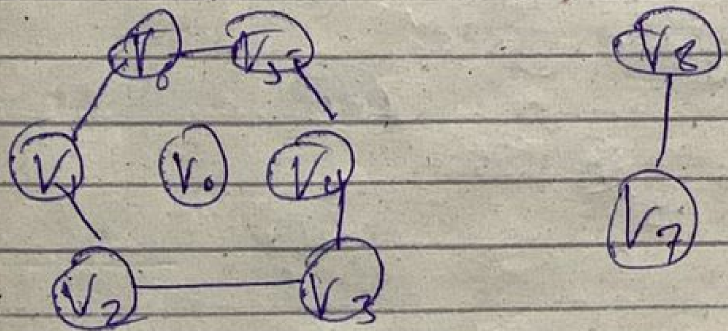
14/10/24

Graph Theory

- The degree of vertex
- Paths
- Connectivity
- Directed graph
- Weighted graph

1) The degree of a vertex

→ The degree of vertex is the no. of its incident edges.
~~[illegible]~~ isolated vertices



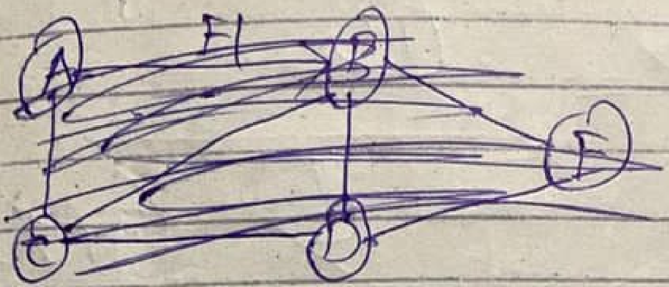
Regular graph where each vertex has same degree
A regular of graph k is also called a k-regular
e.g., 2-regular, 3-regular

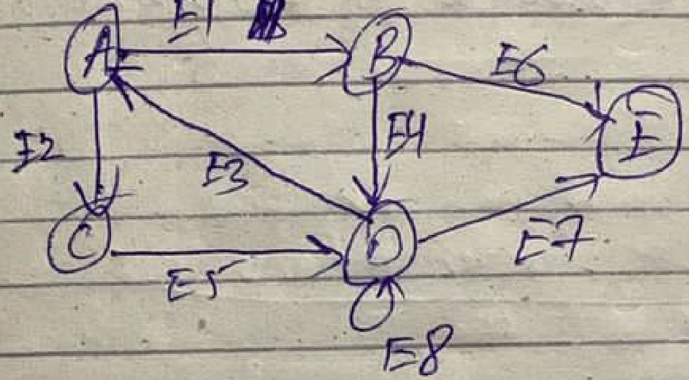
Complement Graph

The complement of graph is $G=(V,E)$ is a graph $\bar{G}=(V,\bar{E})$ on the same set of vertices V & the following set of edges
Two vertices are connected in $\bar{G}$ if & only if they are not connected in G

i.e., $(u,v) \in \bar{E}$ if and only if $(u,v) \notin E$
in G & $\bar{G}$

Total 13 slides; 1-6 (written)
9-13 (taken) (5 pics)





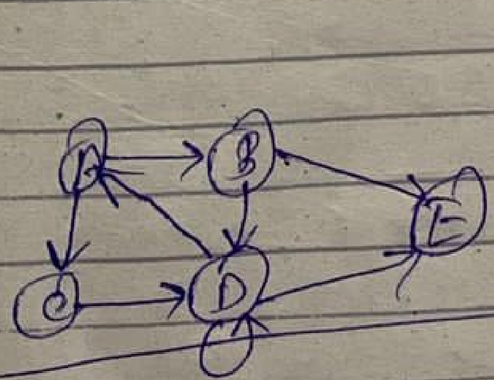
	E1	E2	E3	E4	E5	E6	E7	E8
A	1	1	-1	0	0	0	0	0
B	-1	0	0	1	0	1	0	0
C	0	-1	0	0	1	0	0	0
D	0	0	1	-1	-1	0	1	1
E	0	0	0	0	0	-1	-1	0

Adjency List

(TAKEN) 9

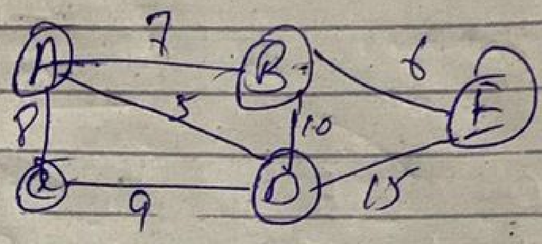
Example

10 (TAKEN)



5

Undirected weighted graph



6

Pros of adjacency matrix

Representation is easier to implement & follow.

Cons Takes alot of space & time to visit all the neighbors of a vertex, we have to transverse all the vertices in the graph, which takes quite some time.

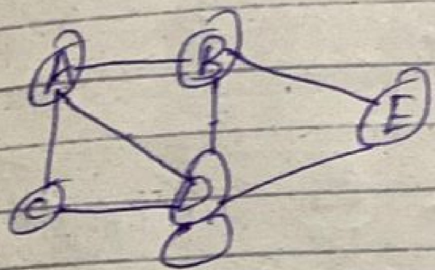
Incidence Matrix

7

→ Graph can be represented using a matrix of size.
· total no. of vertices by total no. of edges
(TAKEN)

Example: (TAKEN)

8



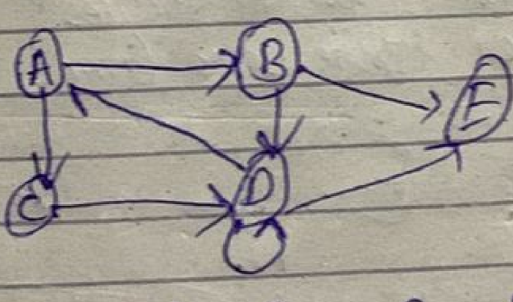
3

$$
\begin{array}{c|c}
 & \begin{matrix} A & B & C & D & E \end{matrix} \\
\begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} &
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

~~In the above exampl~~

Directed graph now ↓

4



$$
\begin{array}{c|c}
 & \begin{matrix} A & B & C & D & E \end{matrix} \\
\begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} &
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

~~I represents a row~~

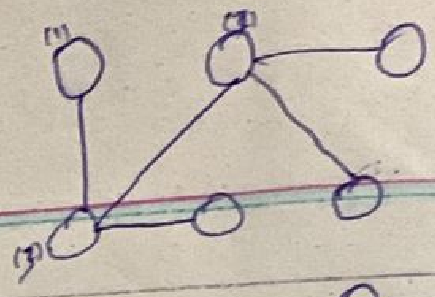
3) Corollary 3

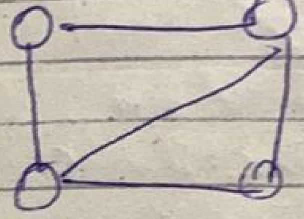
If a non-directed graph, if the degree of each vertex is k, then $k|V| = 2|E|$

4) Corollary 4

If the degree of each vertex in a non-directed graph is at least k, then
$k|V| \leq 2|E|$

5) Corollary 5 If the degree of each vertex in a non-directed graph is at most k, then
$k|V| \geq 2|E|$





$$\sum_{i=1}^{4} d(V_i) = 2|E|$$
$$= 2(5)$$
$$= 10$$

1) <u>Corollary 1</u>

For directed graph $G = (V, E)$
where, vertex set $V = \{V1, V2, V3, \ldots, Vn\}$
then,
$$\sum_{i=1}^{n} deg^{+}(V_i) = \underbrace{|E|}_{\text{(no. of edges)}} = \sum_{i=1}^{n} deg^{-}(V_i)$$

2) <u>Corollary 2</u>: The no. of vertices in any non-directed graph with odd degree is even.

Example: It is impossible to make a graph v(no. of vertices) $= 6$ where the vertices have degrees 1, 2, 2, 3, 3, 4. This is because the sum of the degrees $deg(V)$ is,
$$deg(V) = 1+2+2+3+3+4 = 15$$
$deg(V)$ is always an even no. but 15 is odd.

g) ⇒ Circumference of a graph: Total no. of edges is the longest cycle of graph G is known as the circumference of G.

Circumference is "6" (previous example)

$a \to c \to f \to g \to e \to b \to a$

or

$a \to c \to f \to d \to e \to b \to a$

f) ⇒ Girth of a graph: Total no. of edges in the shortest cycle of graph G is known as girth.

Denoted by $g(G)$.

Girth is 4 i.e. $g(G) = 4$

$a \to c \to f \to d \to a$,

$d \to f \to g \to e \to d$

or

$a \to b \to e \to d \to a$

⇒ Sum of degrees of vertices Theorem

For non-directed graph $G = (V, E)$ where vertex set $V = \{V_1, V_2, V_3, \ldots, V_n\}$, then,

$$\sum_{i=1}^{n} \deg(V_i) = 2|E|$$

In other words, for any graph, sum of degrees of vertices equal twice the no. of edges

c) => Radius of Connected Graph

The radius of a connected graph is the minimum eccentricity from all the vertices.

Denoted by $r(G)$.

$r(G) = 2$ of the example on previous page (eccentricity's example) ~~of vertex~~.

d) => Diameter of a Graph

The diameter of a ~~connected~~ graph is the maximum eccentricity from all the vertices.

Denoted by $d(G)$.

$d(G) = 3$ of the example on previous page (eccentricity's example).

e) => Central Point : If the eccentricity of a graph is equal to its radius, it is known as central point of graph.

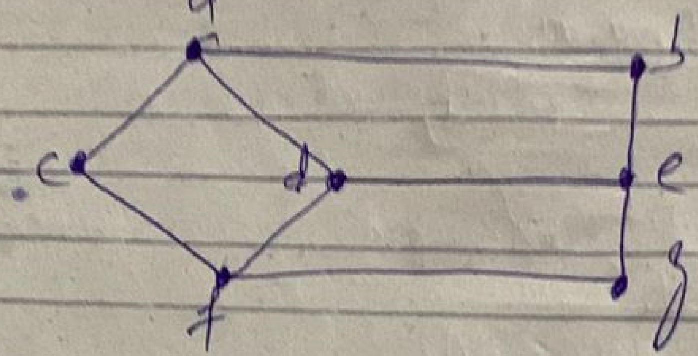
~~Denoted by~~

When $e(V) = r(G)$

f) => Centre :

Set of all central point of the graph is known as centre of the graph.

$\{d\}$ is the centre of the graph (previous page)

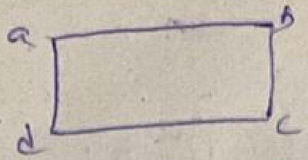


a to b = 1
a to c = 1
a to f = 2
a to d = 1
a to e = 2
a to g = 3

The eccentricity of vertex "a" is 3 i.e. $\boxed{e(a)=3}$

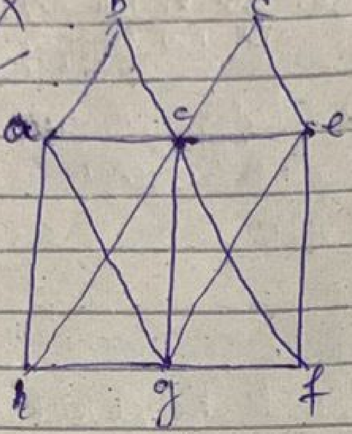
Eccentricities of the rest are,

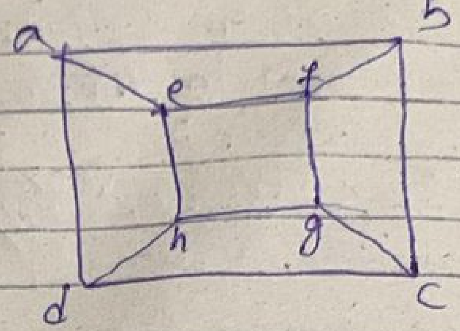
$e(b)=3$
$e(c)=3$
$e(d)=2$
$e(e)=3$
$e(f)=3$
$e(g)=3$



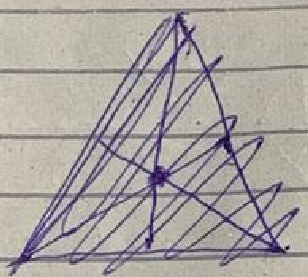
E ✓
H ✓

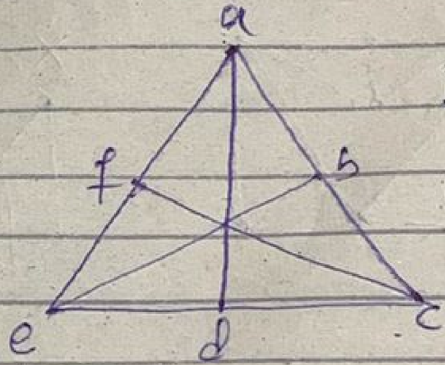
EX
H ✓



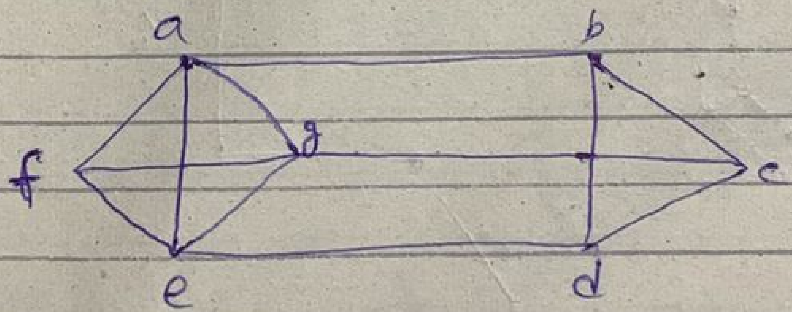


EX
H ✓

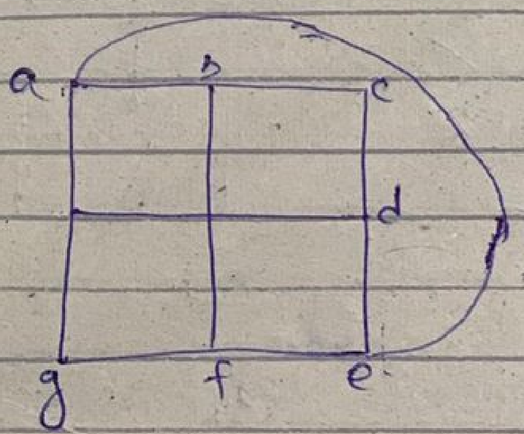




EX
H ✓



EX
H ✓



EX
H ✓

Tuesday
5/11/24.

GRAPH THEORY

PDF pede [4/11/24 => Euler Graph]: Every vertix's degree must be even. Agar aik ki bhi degree odd hui toh euler nahi hoga.

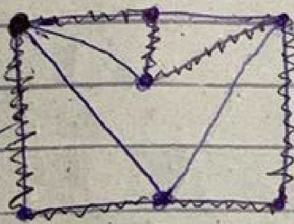
Hamiltonian Circuit/ ~~Graph~~ Cycle

In a connected graph, a closed walk that traverse every vertex of G exactly once, except of course the starting & ending vertices.

Hamiltonian Graph

A graph G is said to be hamiltonian if it has a hamiltonian circuit.

Hamiltonian ✓



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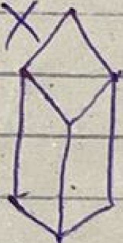
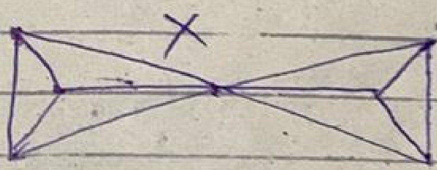
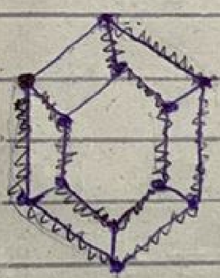
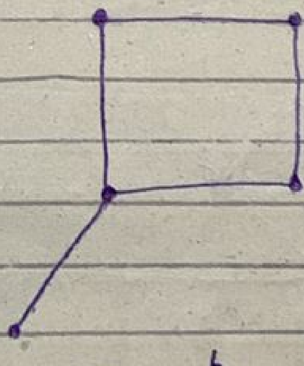


X



✓





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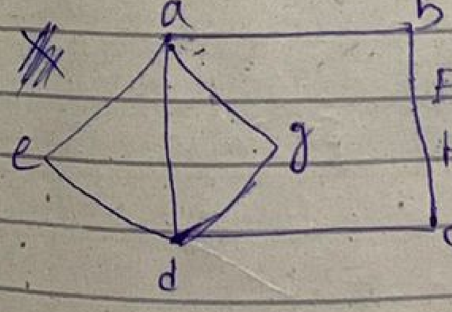
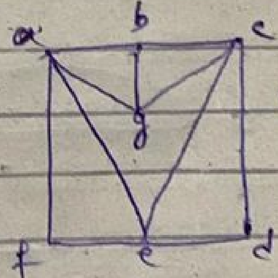
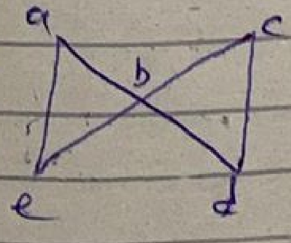


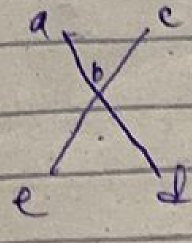
E ✓
H X



E X
H ✓

E ✓
H X



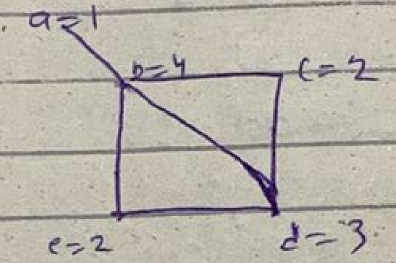


E X
H X

$\Rightarrow \{1, 3, 3, 4, 5, 6, 6\}$

because there are two vertices with n-1 edges, that is why ~~edge~~ ~~with~~ vertex with 1 edge isn't possible.

To check if a simple graph corresponding to a degree sequence is available or not, we use "Haval-Hakimi Theorem."



(get in deg. sequence)
(removing left most

1) Degree sequence: $\underline{4}, 3, 2, 2, 1$

$\underline{2}, 1, 1, 0$

$\boxed{0, 0, 0}$

as, we get 0, 0, 0 at end (all zeros), so simple graph is possible.

2) Degree sequence: $\underline{7}, 6, 5, 4, 4, 3, 2, 1$

$\underline{5}, 4, 3, 3, 2, 1, 0$

$\underline{3}, 2, 2, 1, 0, 0$

$\underline{1}, 1, 0, 0, 0$

$0, 0, 0, 0$

Monday
21/10/24 Graph Theory

Handshaking Lemma (Lect-04)

Monday
21/10/24 Database System

Database Development Process

~~[illegible]~~ Lengthy process

User requirement & detail should be known by database creator. As early you know this info as a database creator, the better it is.

→ Two strategies:
(a) Top-down approach
(b) Bottom-up approach

→ Top-down approach: Defined by upper level. Starts by a general issue, divided into small chunks are then work is done on it. Domain is defined (top management). Requirement is given at first stage.

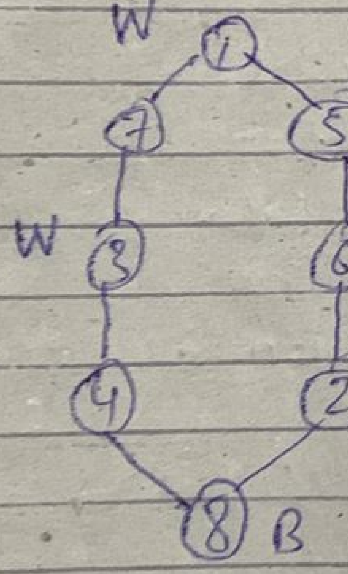
→ Bottom-up approach: Starts with a small issue, we work on it and then it is integrated further. Domain is not defined. Requirement is not given at first stage.

TUESDAY
22/10/2024

GRAPH THEORY

Lect-05: "Connected Components"

"Ask Uzair about the chess problem on p.g #3"



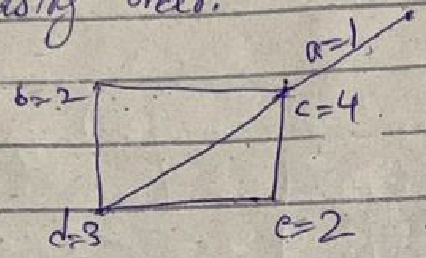
(Revisiting the Guarini Puzzle)
↓
connected graph of given scenario

MONDAY
28/10/24

GRAPH THEORY

→ **Degree Sequence**

The arrangement of degree of all vertices of a graph G, either in non-increasing or non-decreasing order.



non-decreasing ≠ decreasing
non-increasing ≠ increasing

Can a simple graph corresponding to a degree sequence exist or not? (a possible question)

☆ Sum of degrees of vertices should be even for simple graph

(invalid) ⇒ $\{2,\underbrace{3,3}_{3},5,4,4\} = 21$ (odd so invalid degree sequence) (invalid)

☆ (The no. of odd degrees should be even for simple graph

(invalid) ⇒ $\{2,\underbrace{5}_{\text{not possible(max=4)}},4,3,4\}$ (invalid)

☆ The total no. of vertices should not be equal to any degree of a vertex.

⇒ $\{3,3,3,1\}$ (invalid)

3) Degree Sequence: 6, 6, 6, 6, 3, 3, 2, 2

5, 5, 5, 2, 2, 1, 2

4) degree sequence: 7, 6, 6, 4, 4, 3, 2, 2

5, 5, 3, 3, 2, 1, 1

4, 2, 2, 1, 0, ~~0~~ 1

4, 2, 2, 1, 1, 0

1, 1, 0, 0, 0

0, 0, 0, 0