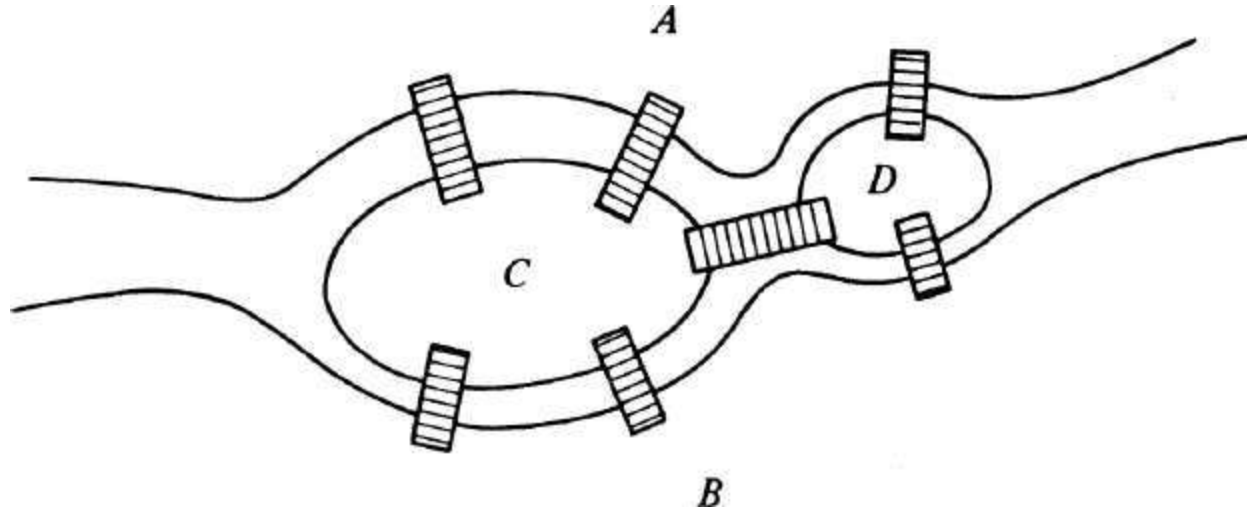# Introduction to Graph Theory
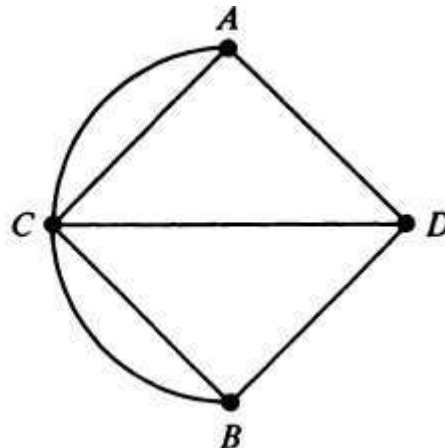
Why Graphs?

By

Hizbullah Khattak

Graph Theory

- Konigsberg Bridge Problem
  - Leonhard Euler (1707-1983) in 1736
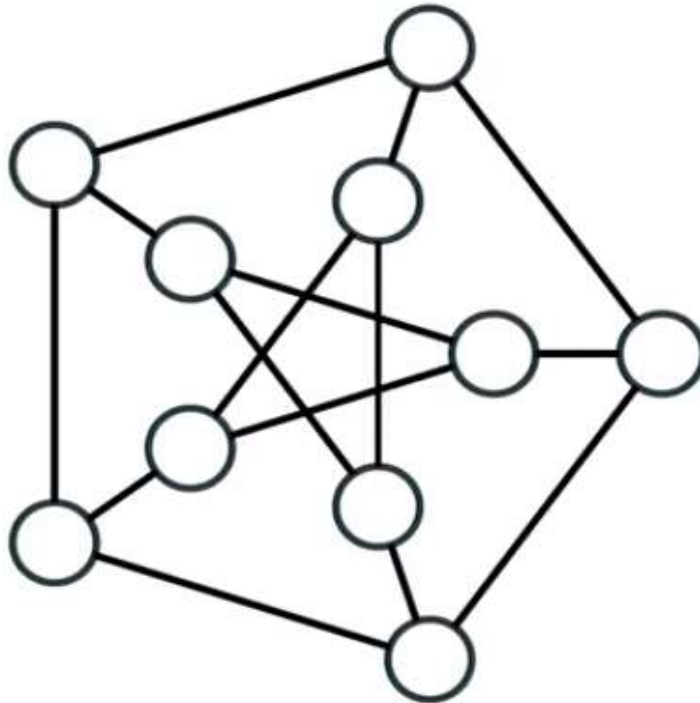
Seven Bridges of Konigsberg

Königsberg bridge problem

Graph of Königsberg bridge problem.

Outlines

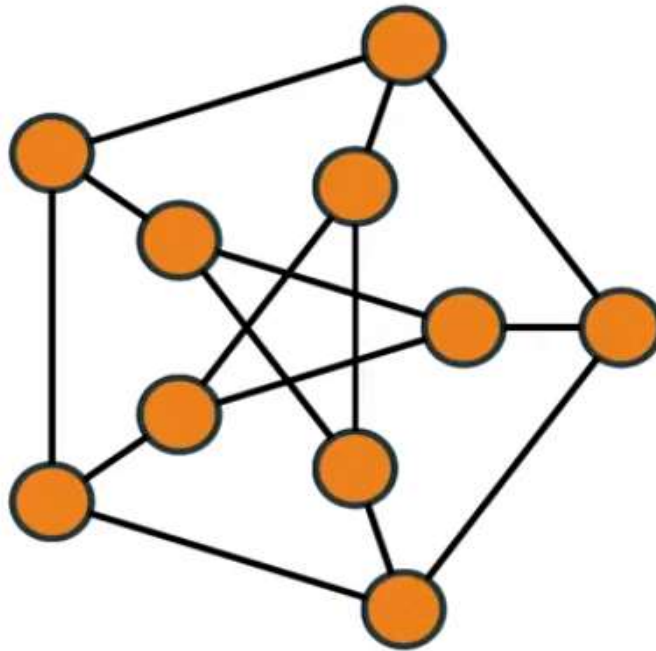- **What is a Graph?**

- Graph Examples

- Graph Applications

Graph

- A set of Objects.
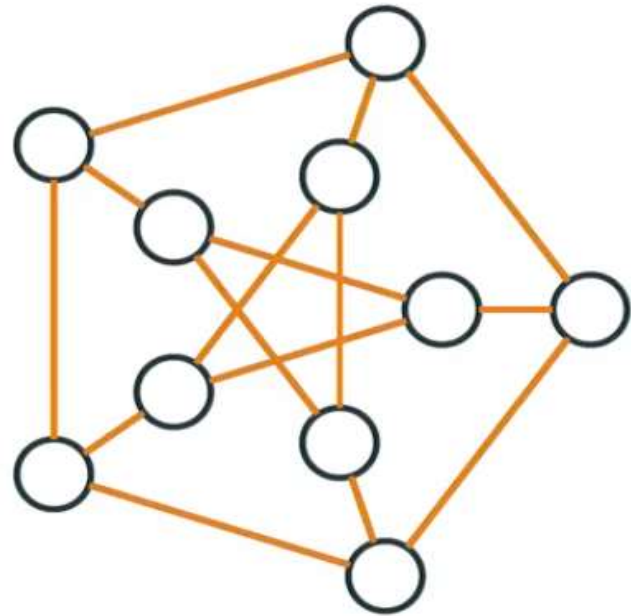- Relations between pairs of objects.

Definition

- A Graph *G = (V, E)*
- A set *V* of Vertices/Nodes

Definition….
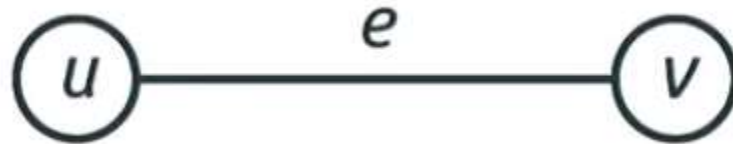
- A Graph *G = (V, E)*
- A set *V* of Vertices/Nodes
- A set *E* of Edges

Vocabulary

- We can name individual vertices and edges

- *e* Connects *u* and *v.*

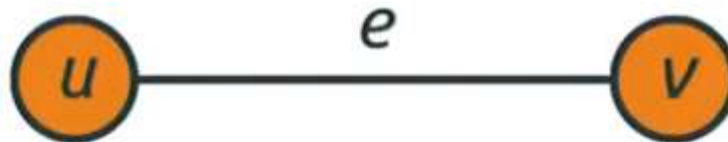- *u* and *v* are End Points of *e*
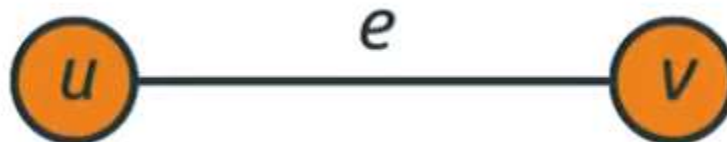
Vocabulary…

- *u* and *e* are Incident
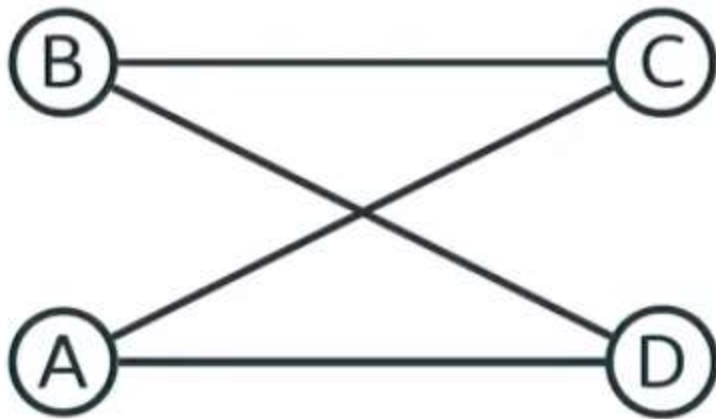
- *u* and *v* are Adjacent

- *u* and *v* are Neighbors

Drawing a Graph

Objects: {A, B, C, D}

Relations: {{A, C},{D, A},{B, D},{C, B}}

Directed Graph

- It is often convenient to consider Directed Edges (Arcs).

- They describe asymmetric relations.

- There is a flight from A to B, but not the other way around.

- Such a graph is called Directed

Drawing a Directed Graph

Objects: {A, B, C, D}

Relations: {(A, C),(D, A),(B, D),(C, B)}

Drawing a Directed Graph...

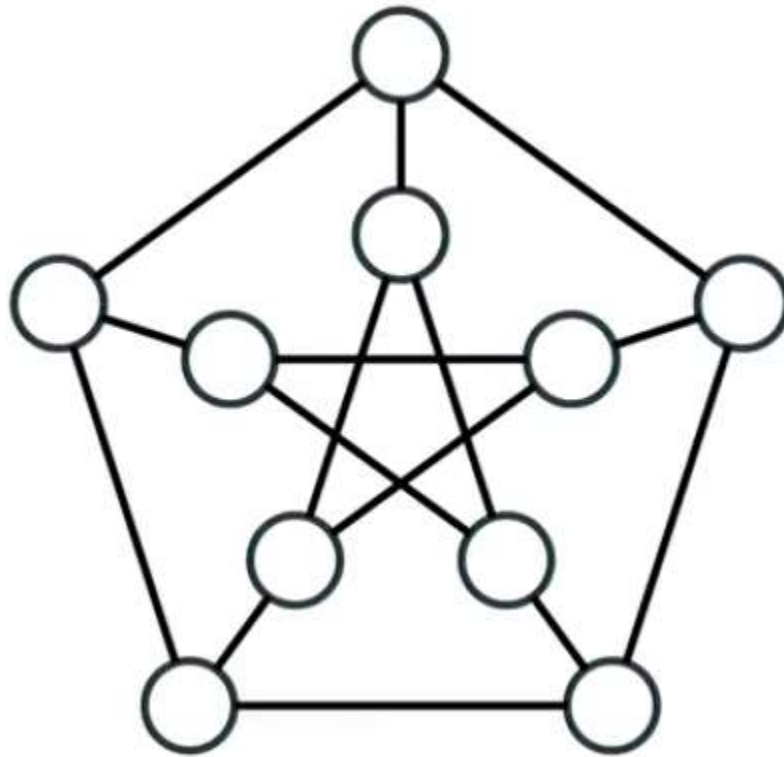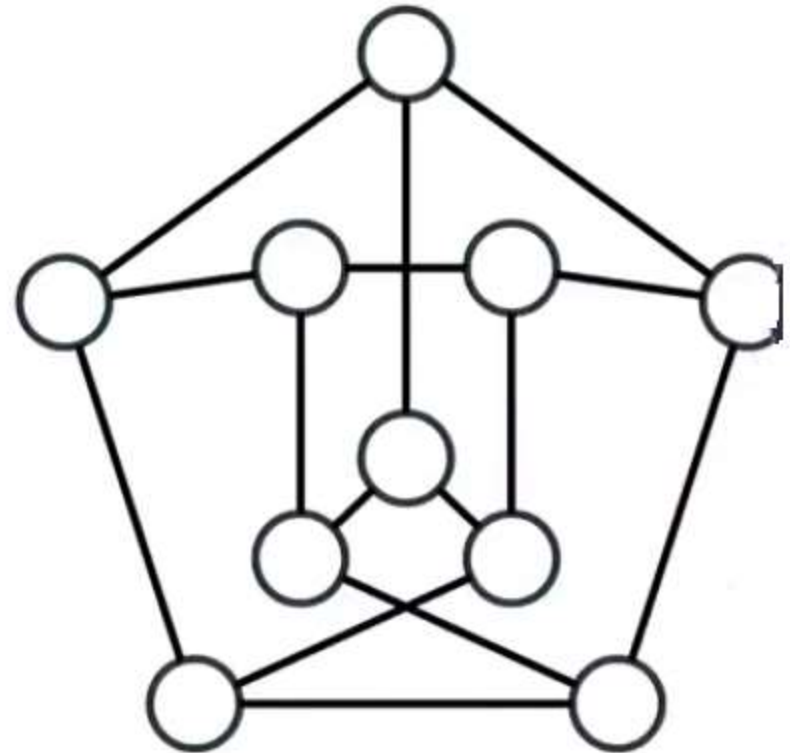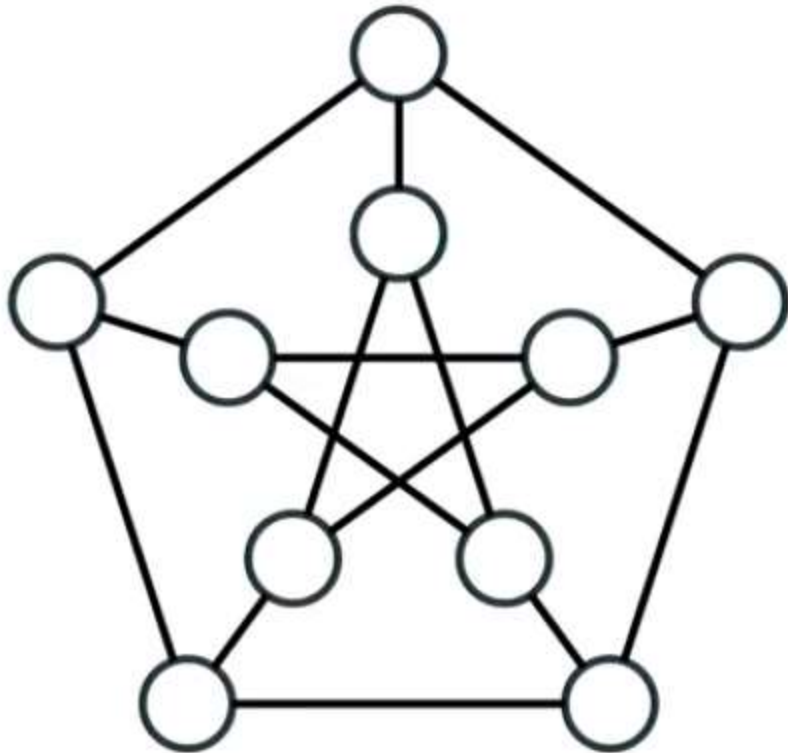Objects: {A, B, C, D}

Relations: {(A, C),(D, A),(B, D),(C, B)}

Drawing a Directed Graph…

Objects: {A, B, C, D}

Relations: {{C, A},{D, A},{B, D},{C, B}}
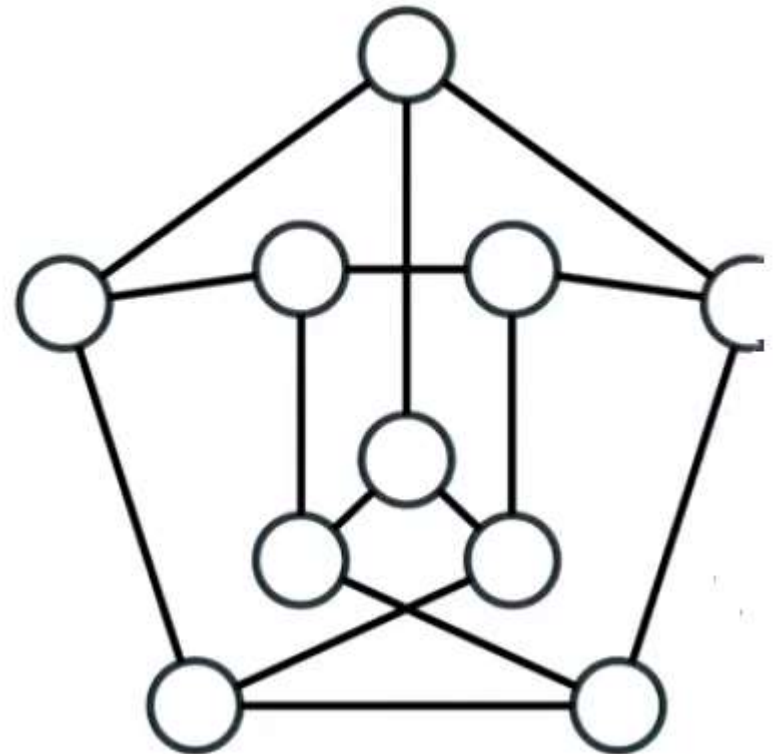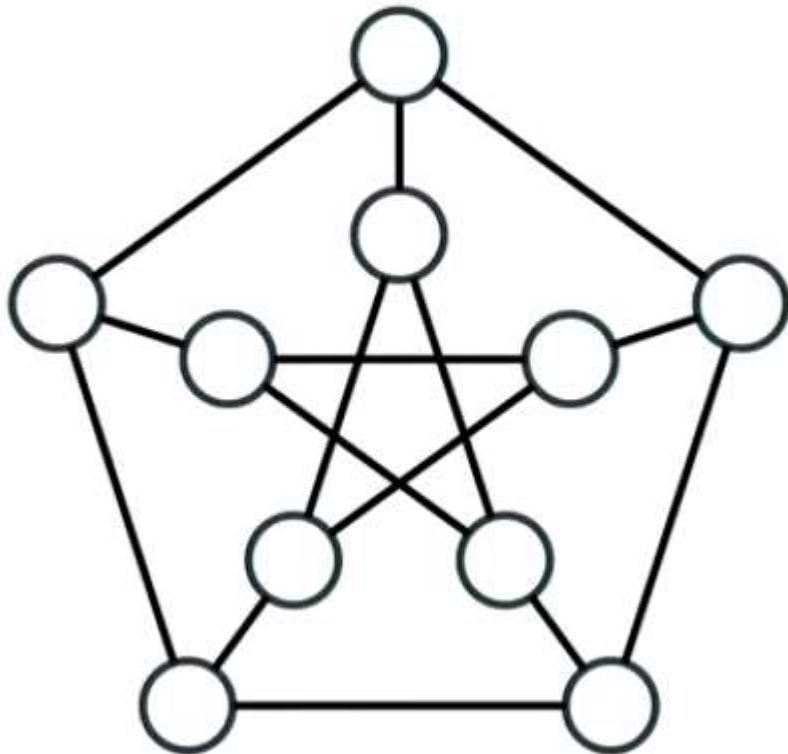
Many Ways to Draw a Graph

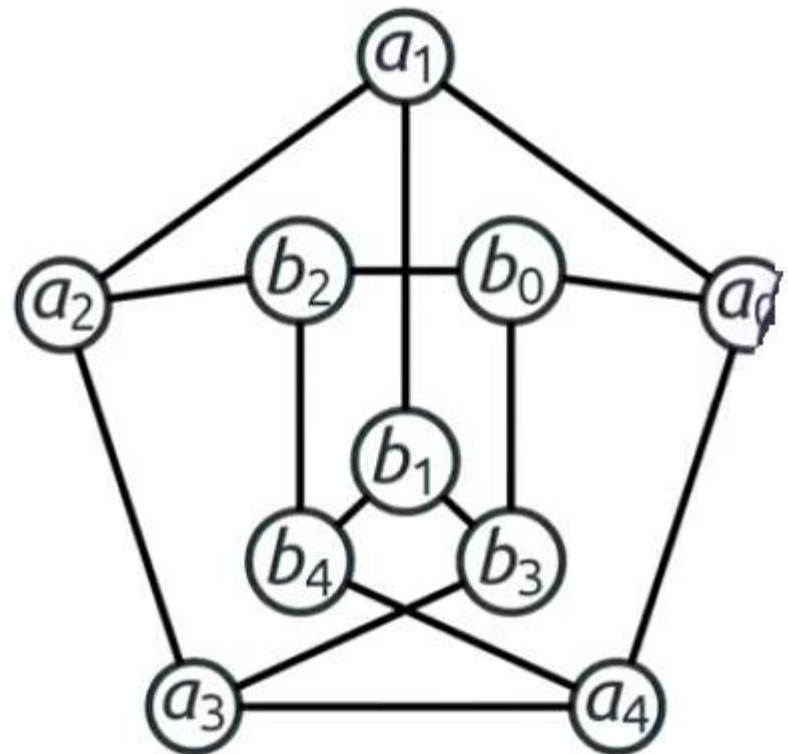# Many Ways to Draw a Graph..
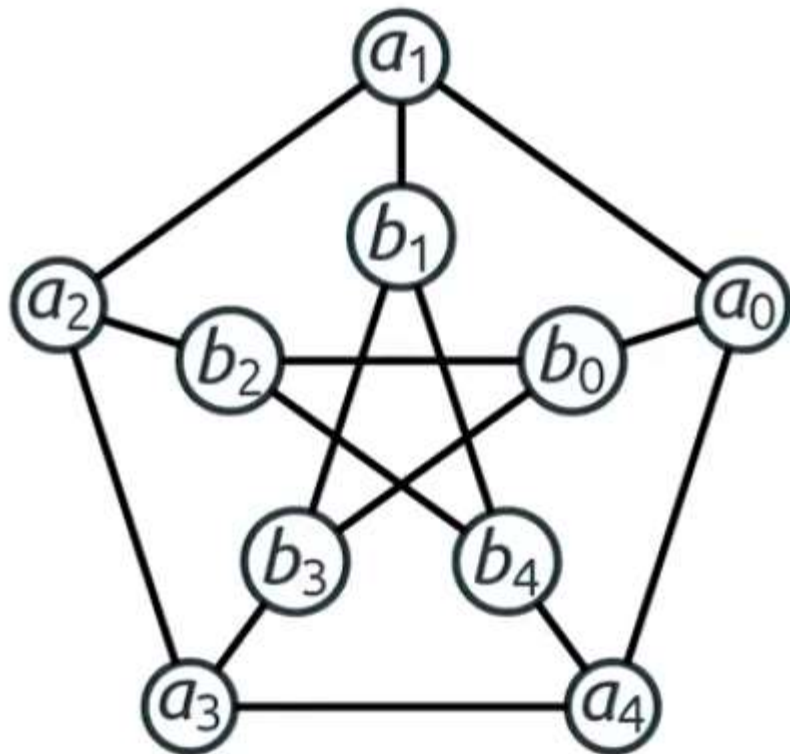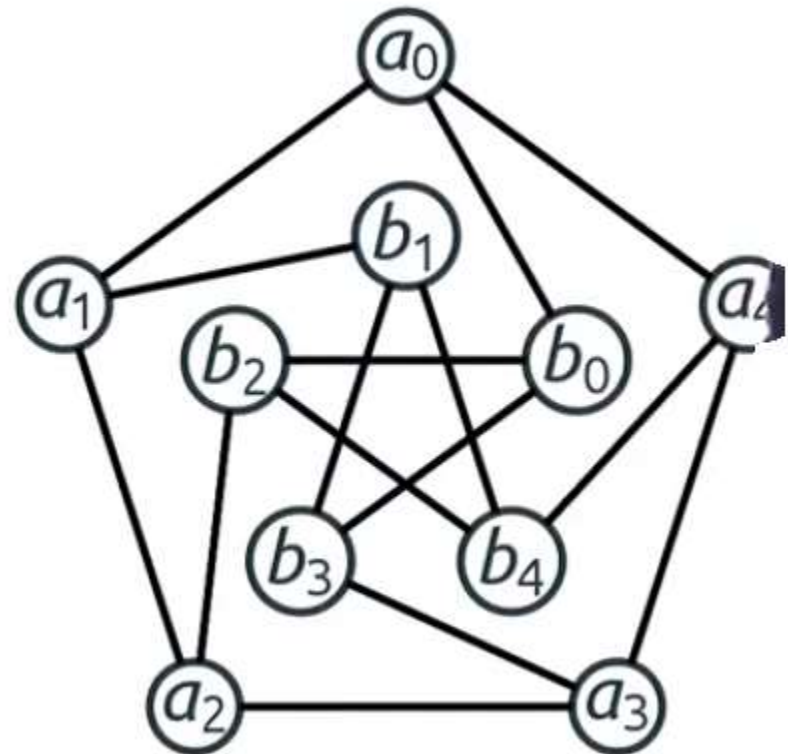
- Are these graphs the same?

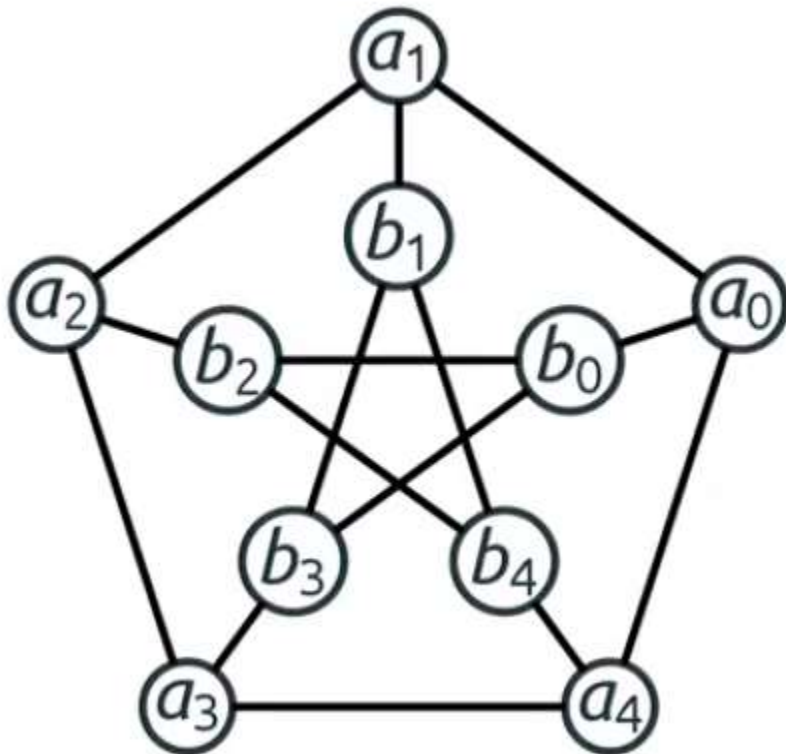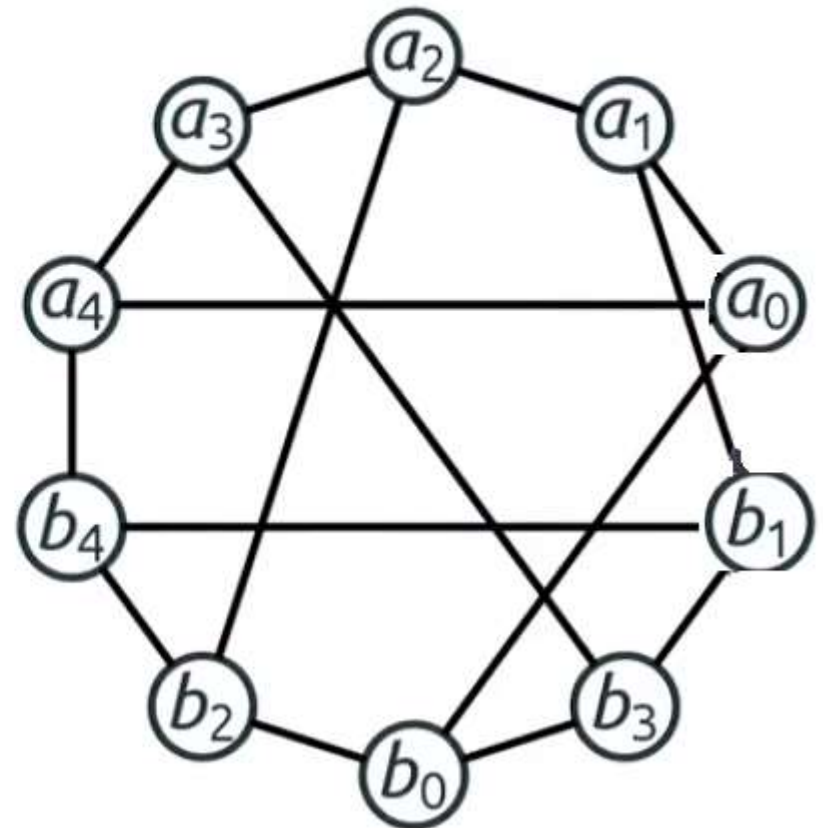# Many Ways to Draw a Graph..

- 10 vertices and 15 edges?

Many Ways to Draw a Graph..
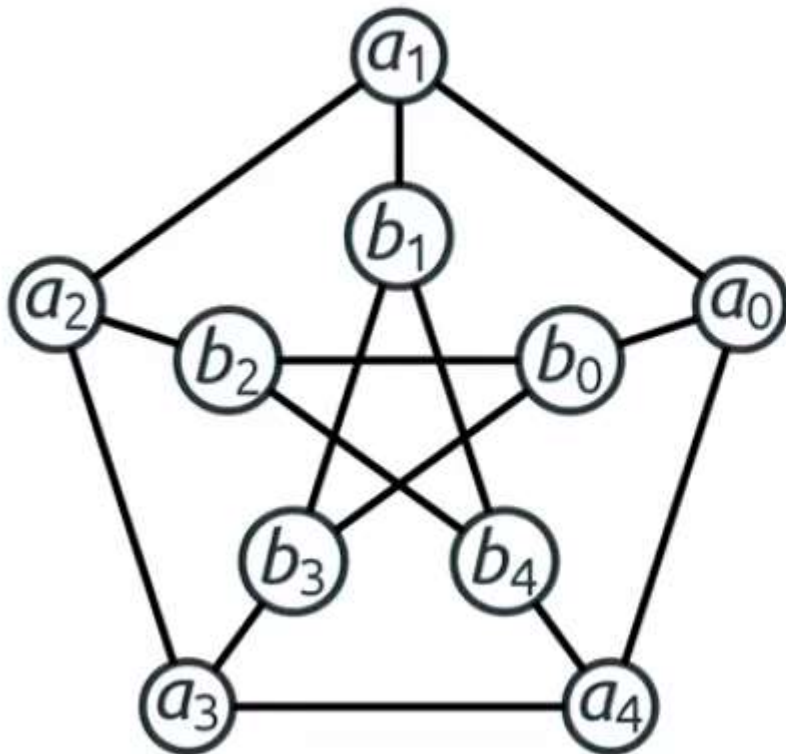
- Are these graphs the same?

Many Ways to Draw a Graph..
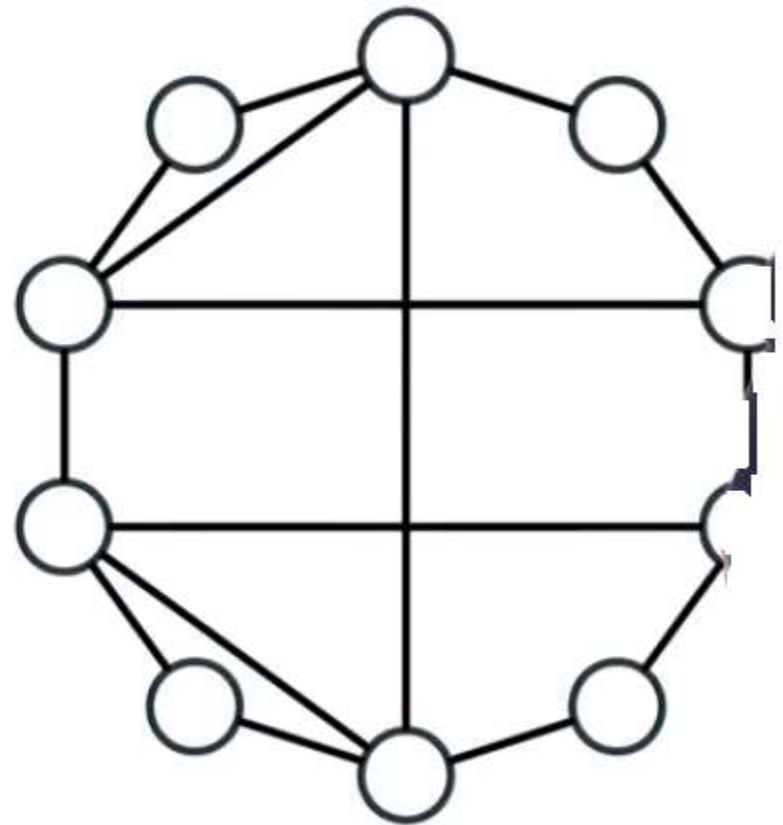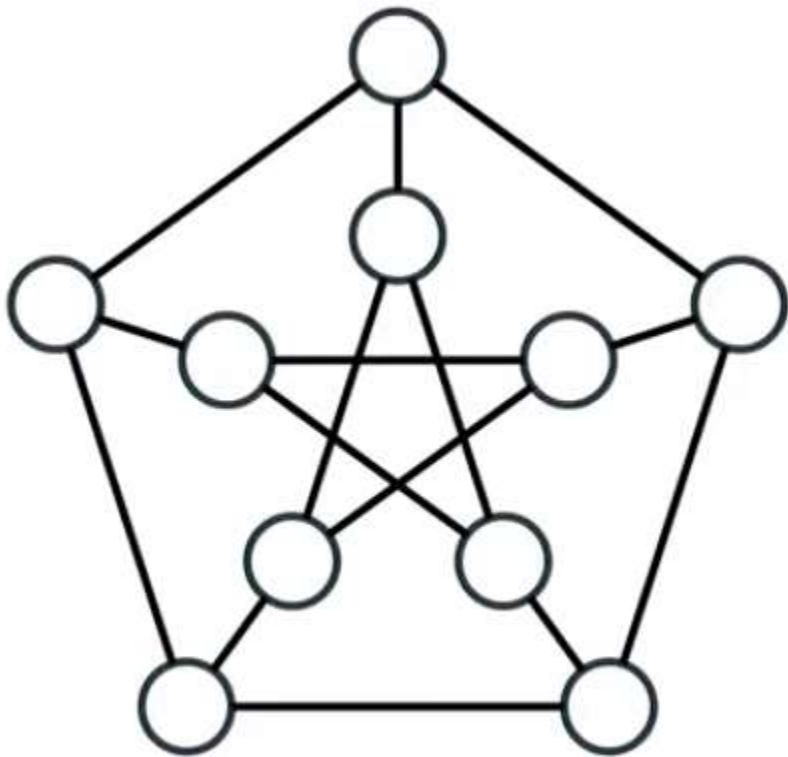
- Are these graphs the same?

Many Ways to Draw a Graph..
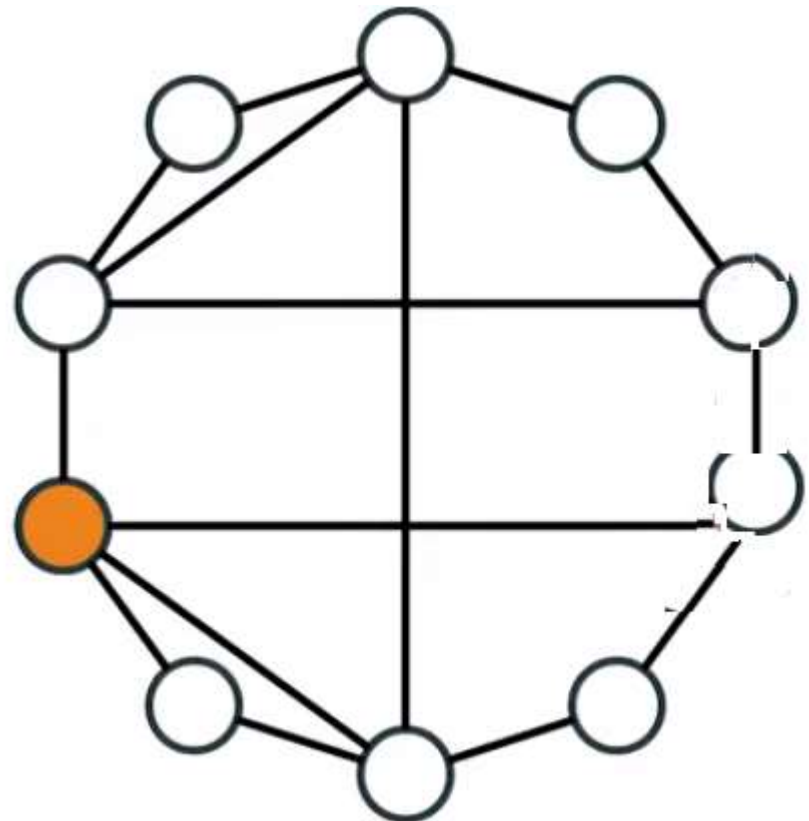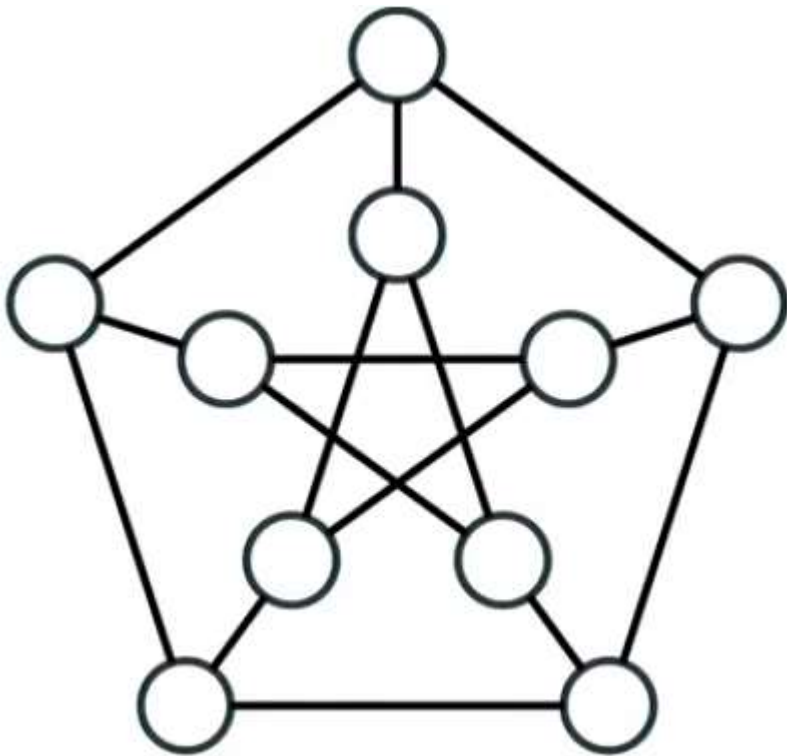
Are these graphs the same?
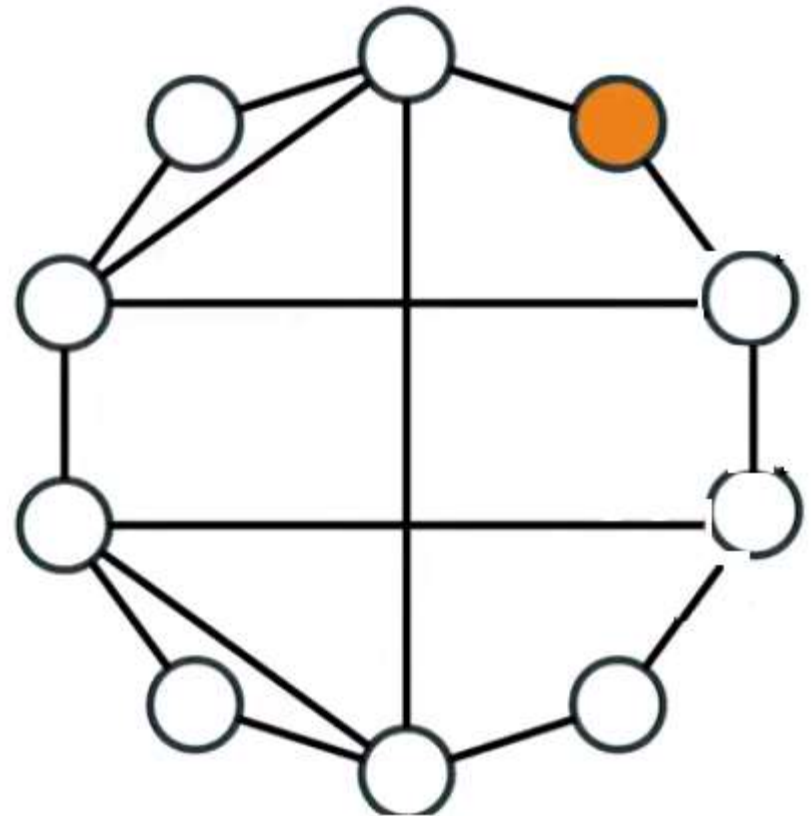
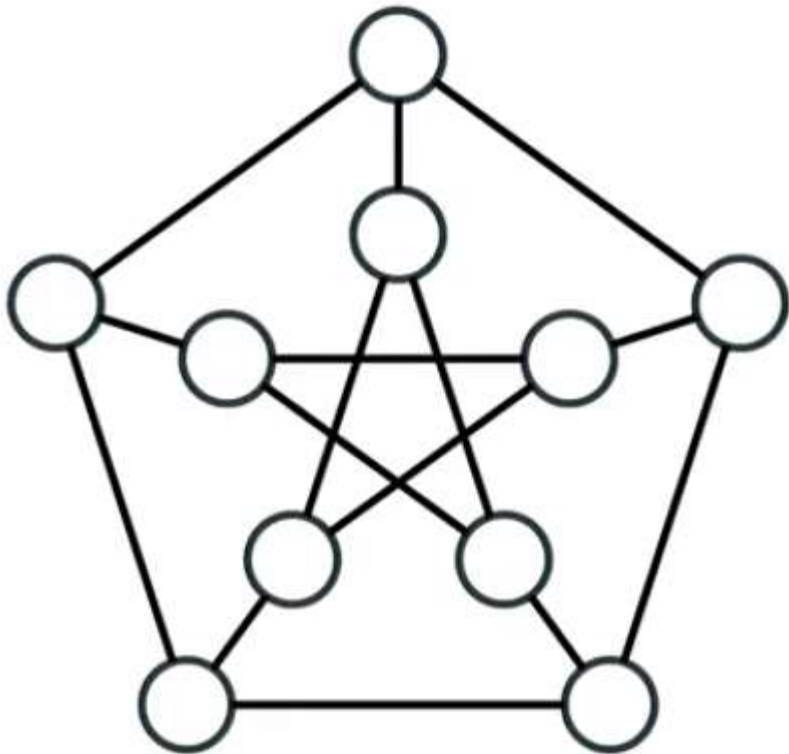# Are these Graph the same?

- 10 vertices and 15 edges?

Are these Graph the same?..

- 10 vertices and 15 edges?

Are these Graph the same?..

- 10 vertices and 15 edges?

Graph Drawing is beautiful

- Graph drawing is the best possible field I can think of: It merges aesthetics, mathematical beauty and wonderful algorithms.

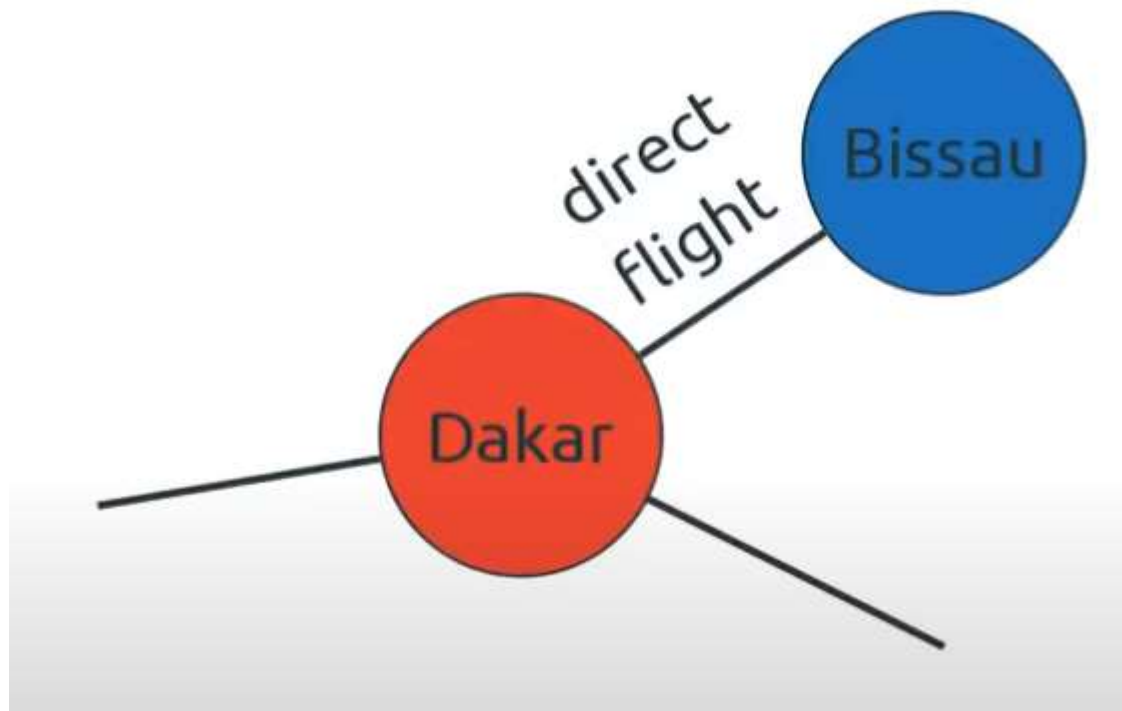 It therefore provides a harmonic balance between the left and right brain parts. (Donald E. Knuth)

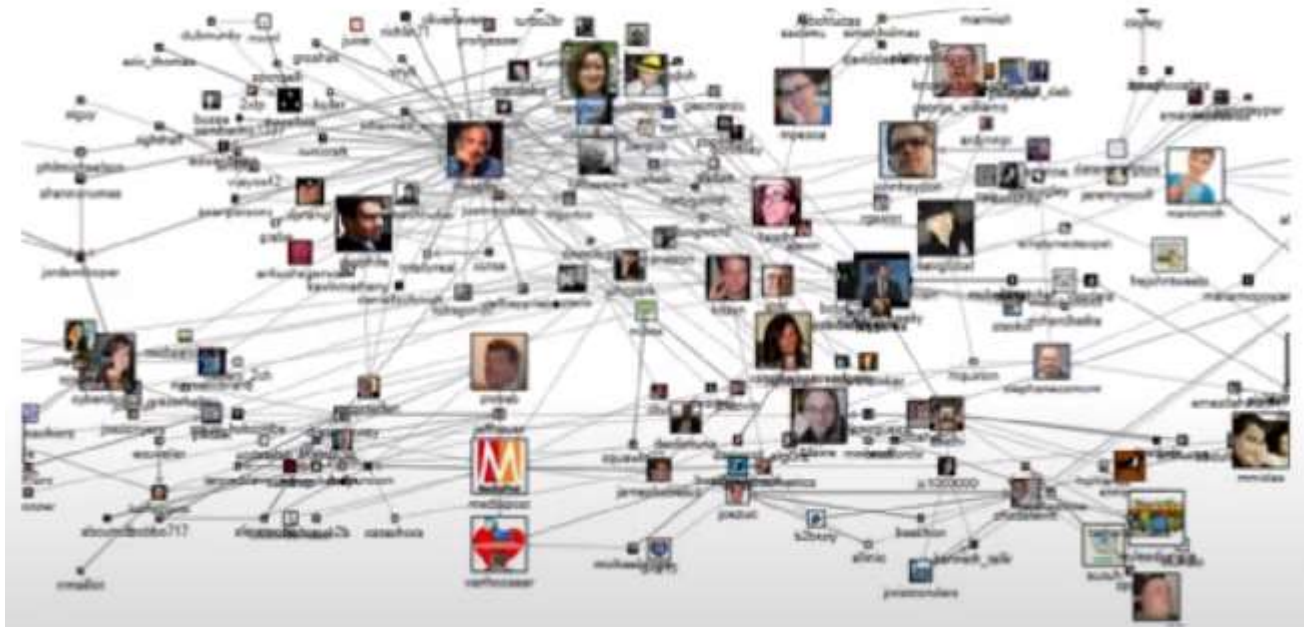# Graph Examples

Airlines Graph
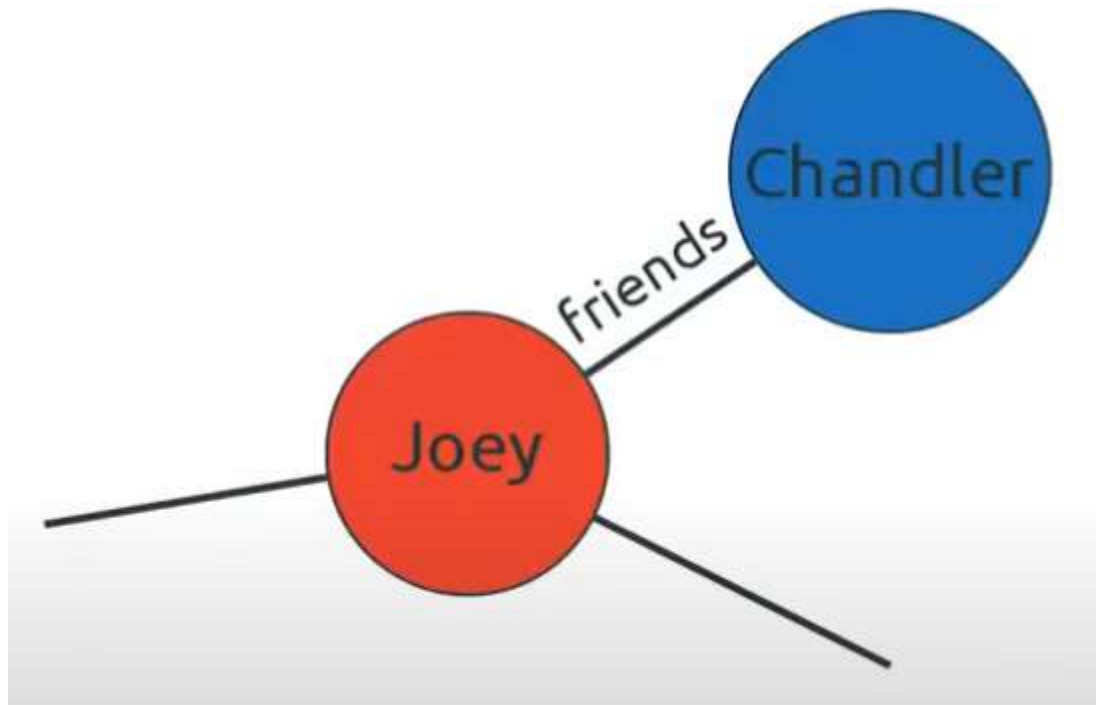
Graph Examples…

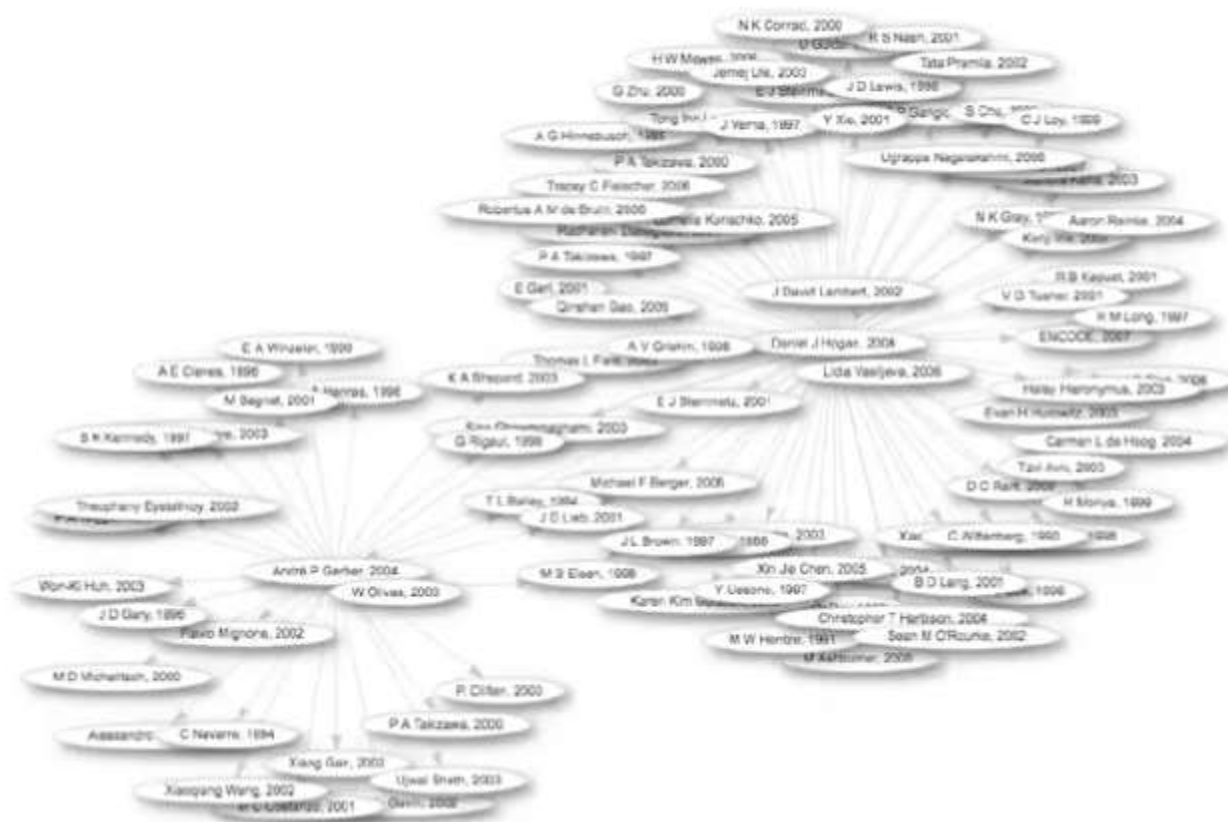Airlines Graph

Graph Examples…

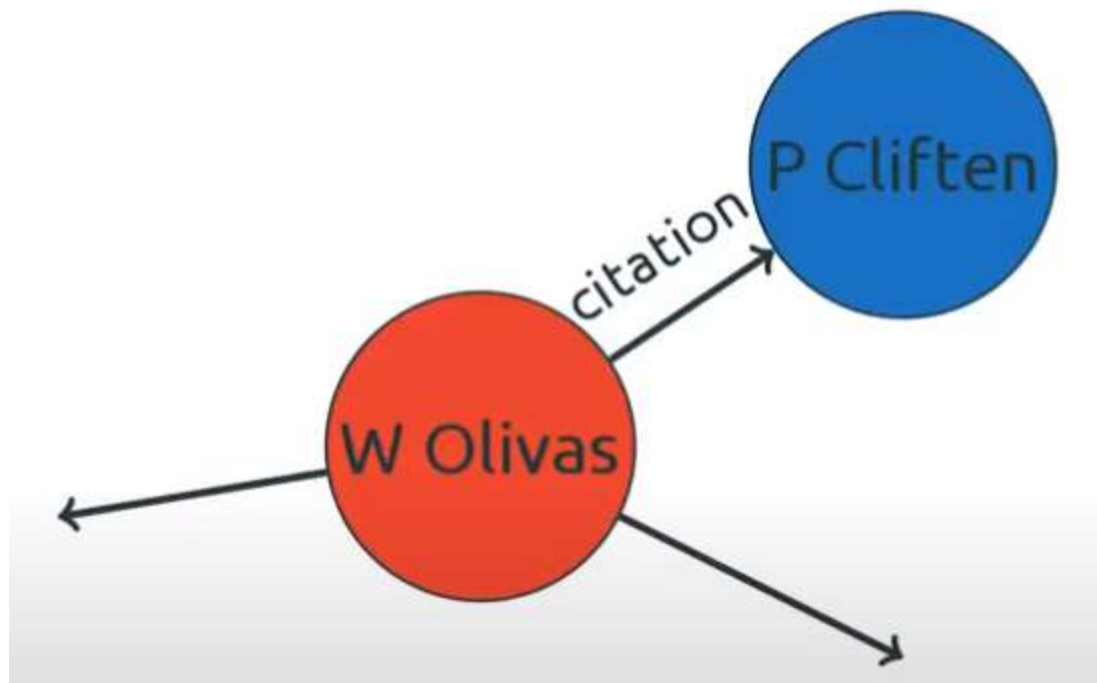Facebook Graph

Graph Examples…

Facebook Graph

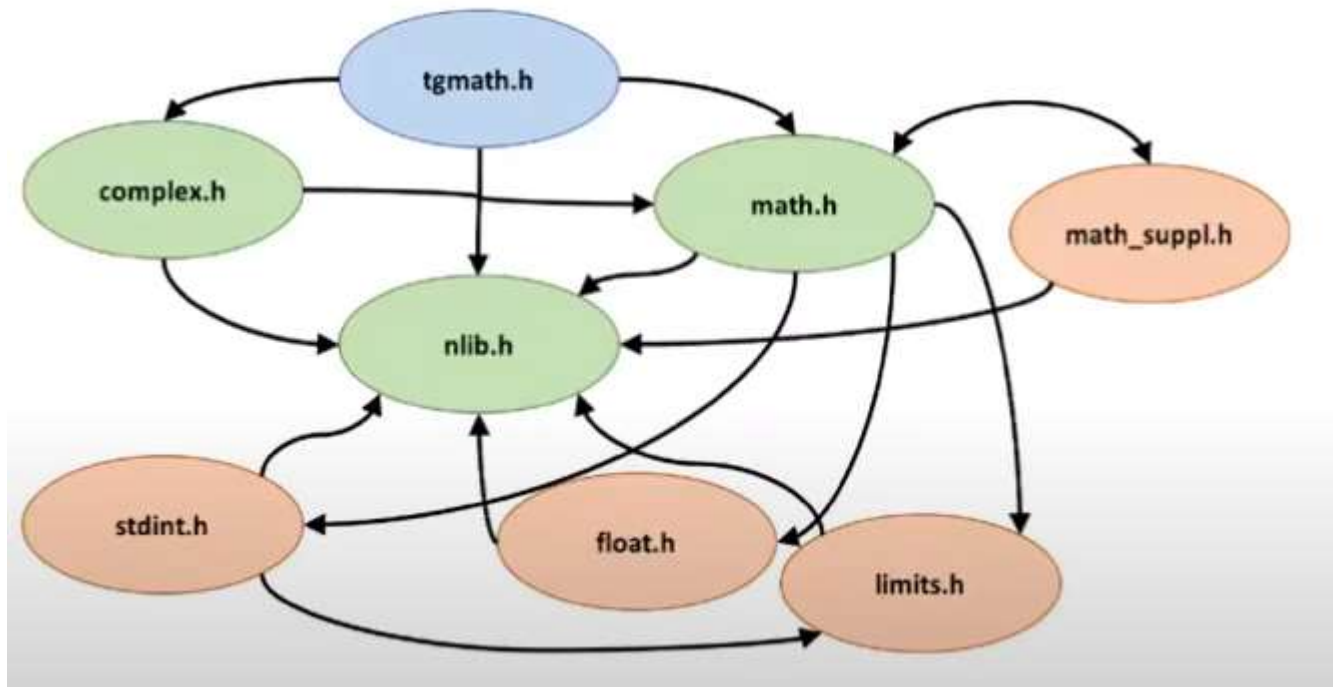# Graph Examples...

Citation Graph for a Paper

Graph Examples…

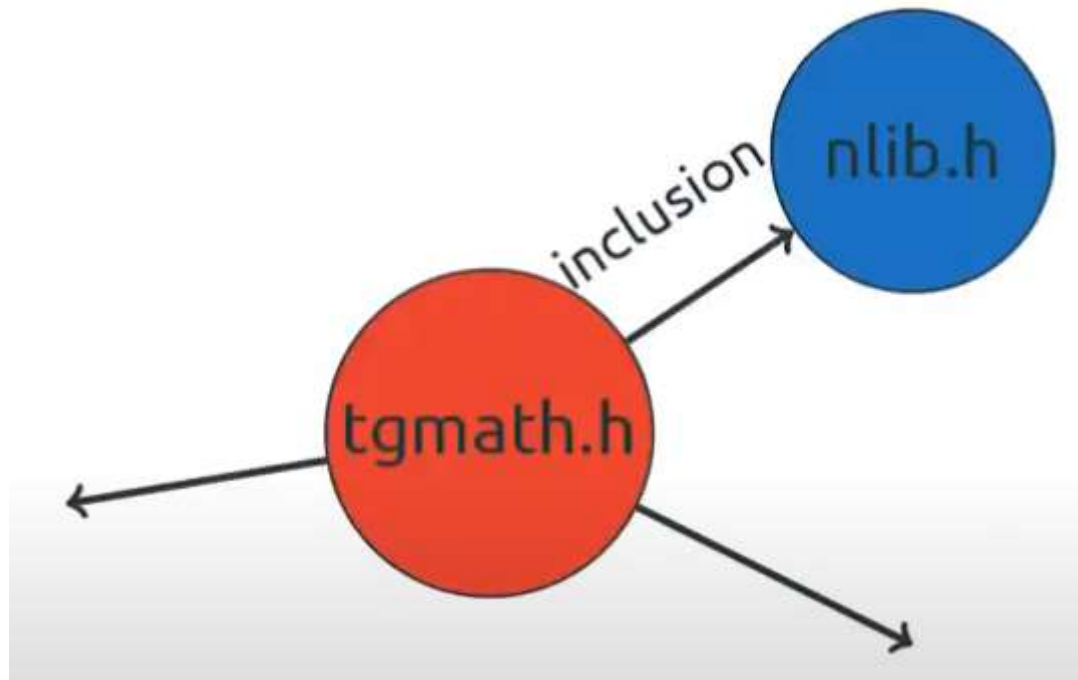Citation Graph for a Paper

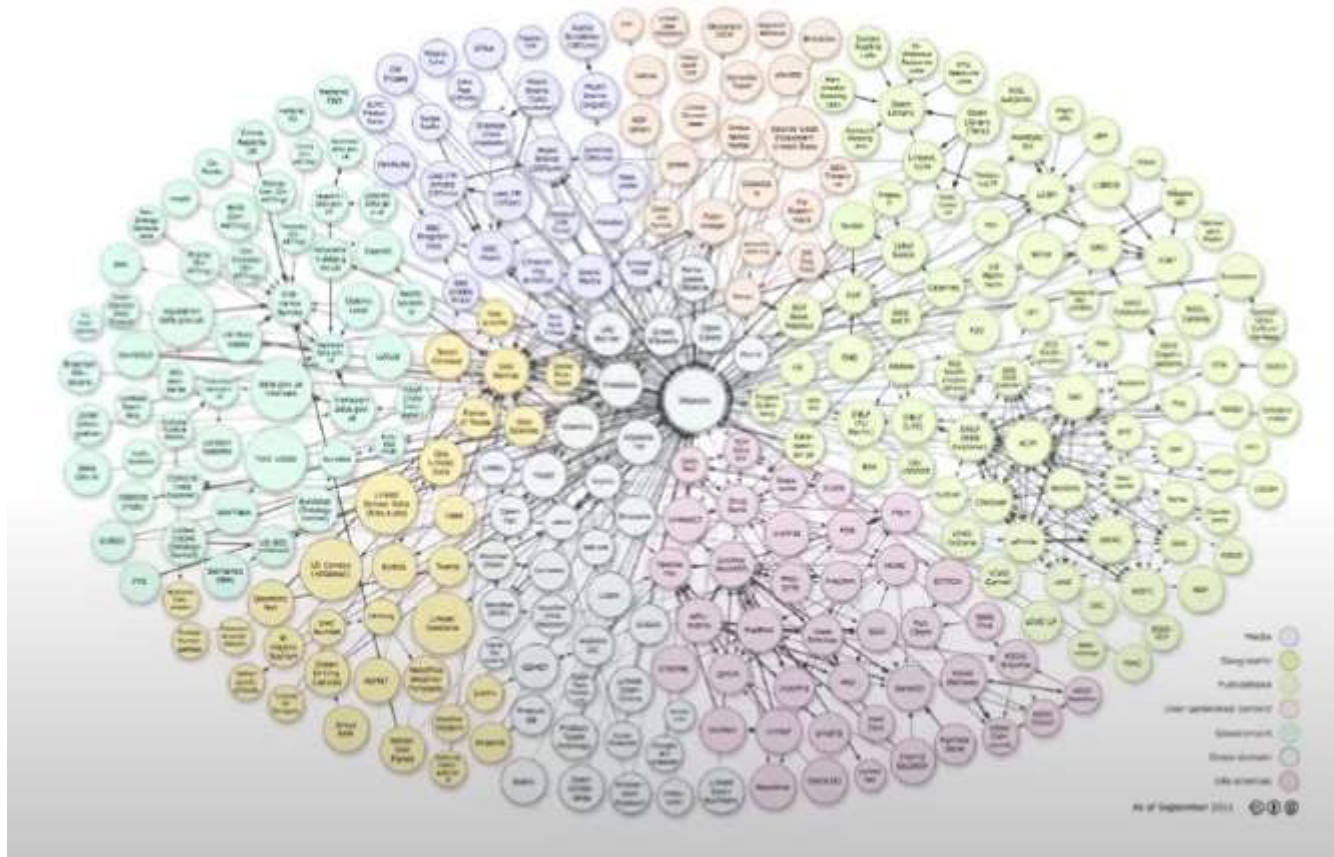# Graph Examples…

Dependency Graph

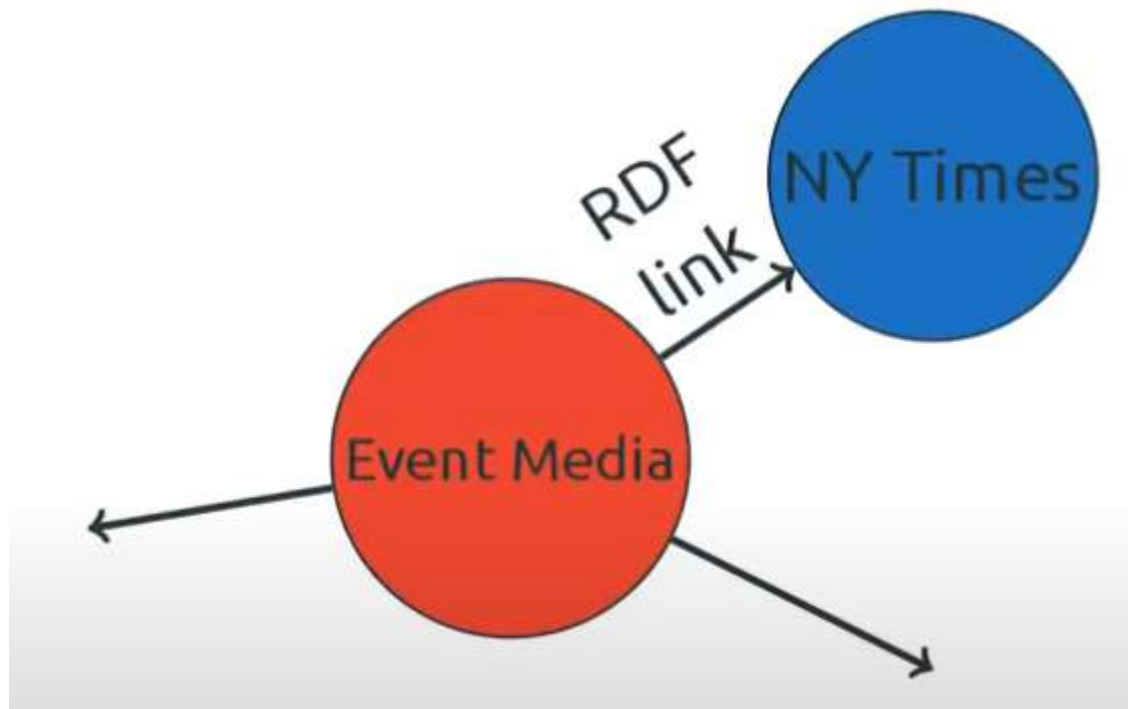# Graph Examples...

Dependency Graph

Graph Examples...
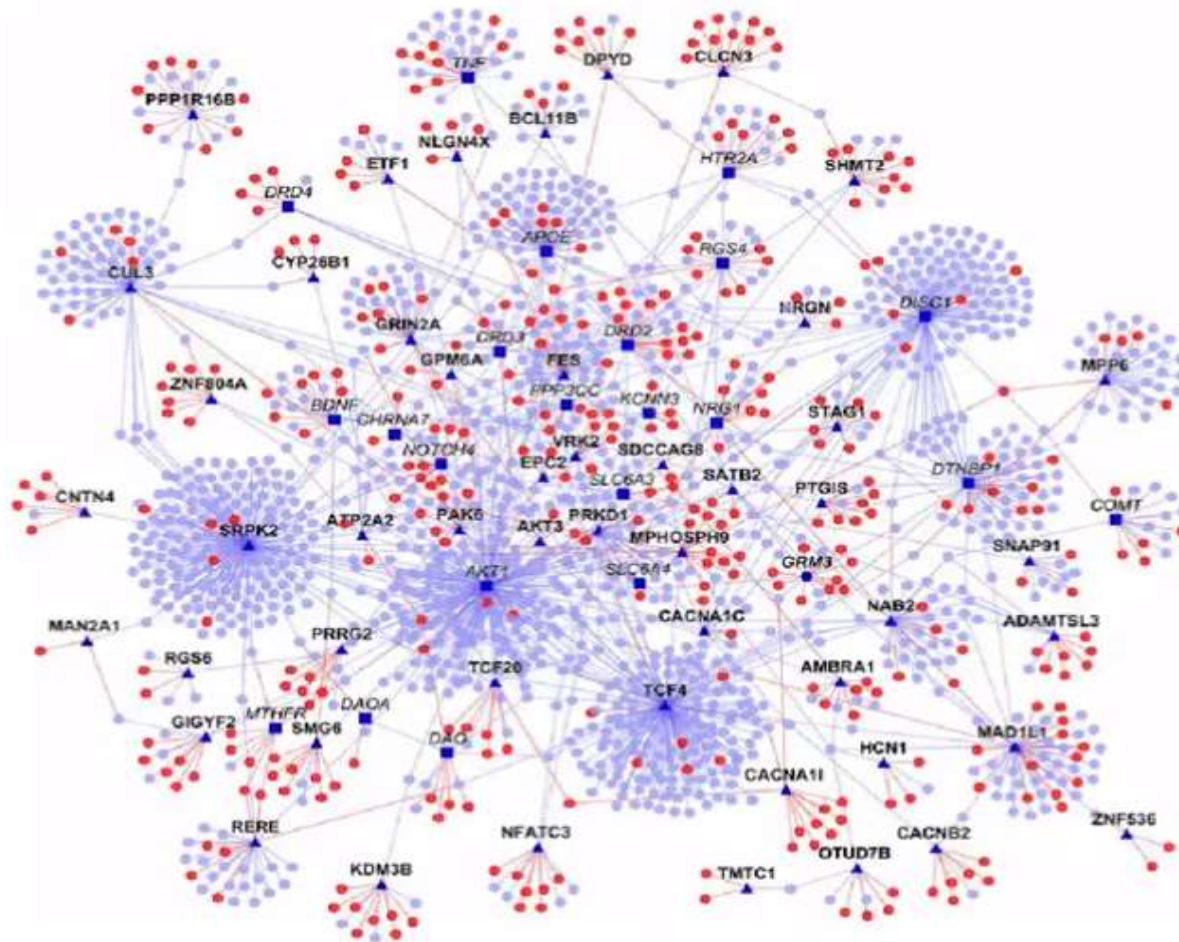
Linked Open Data diagram

Graph Examples…
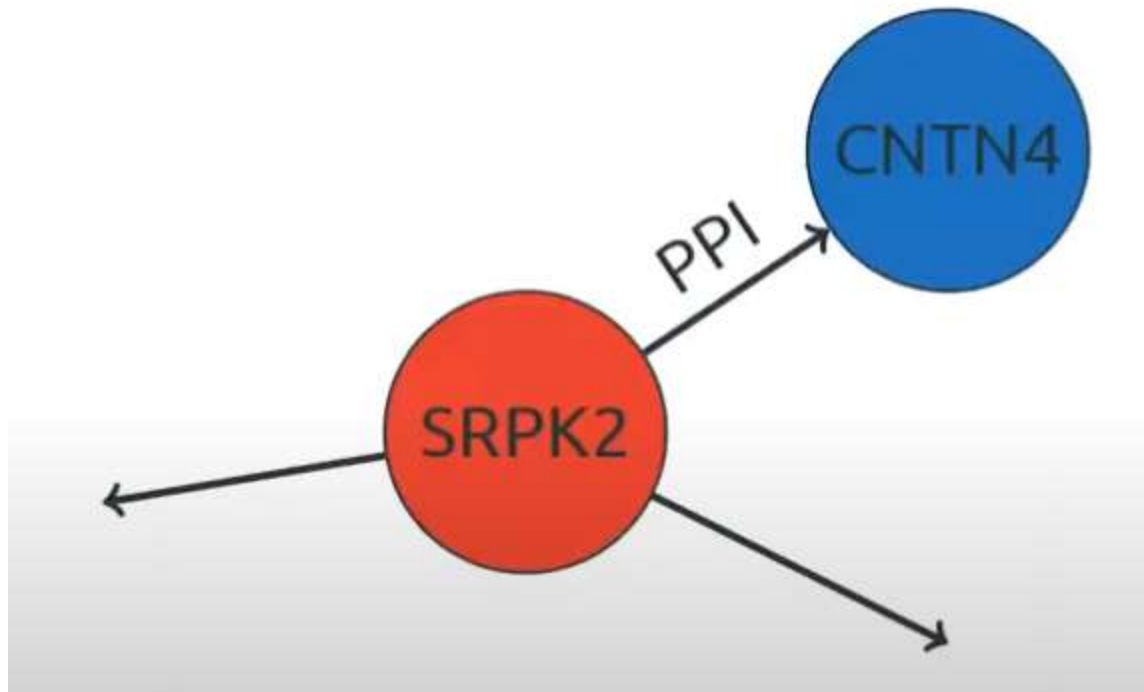
Linked Open Data diagram

Graph Examples…

Schizophrenia Protein-Protein Interaction

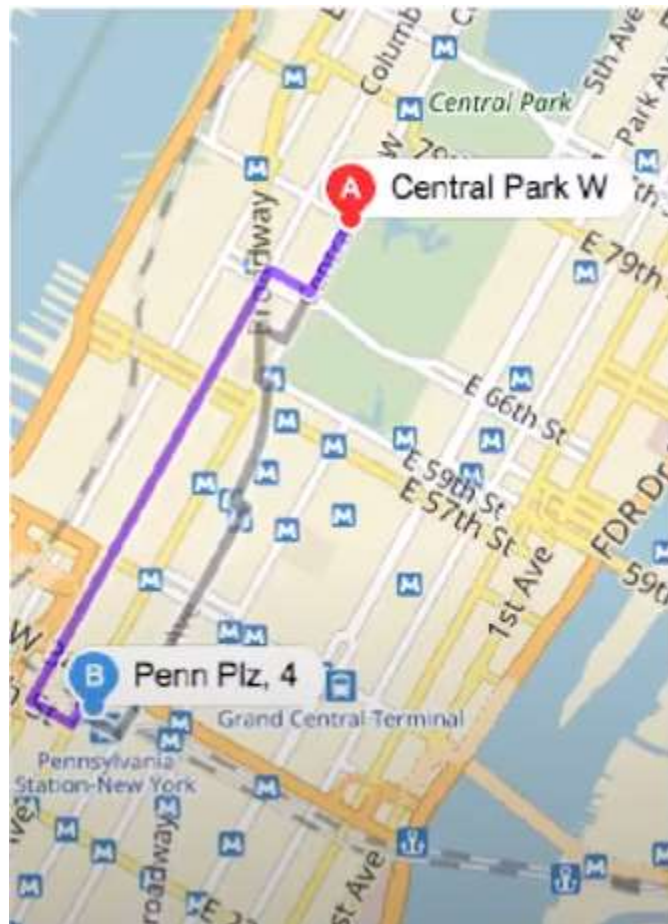# Graph Examples...

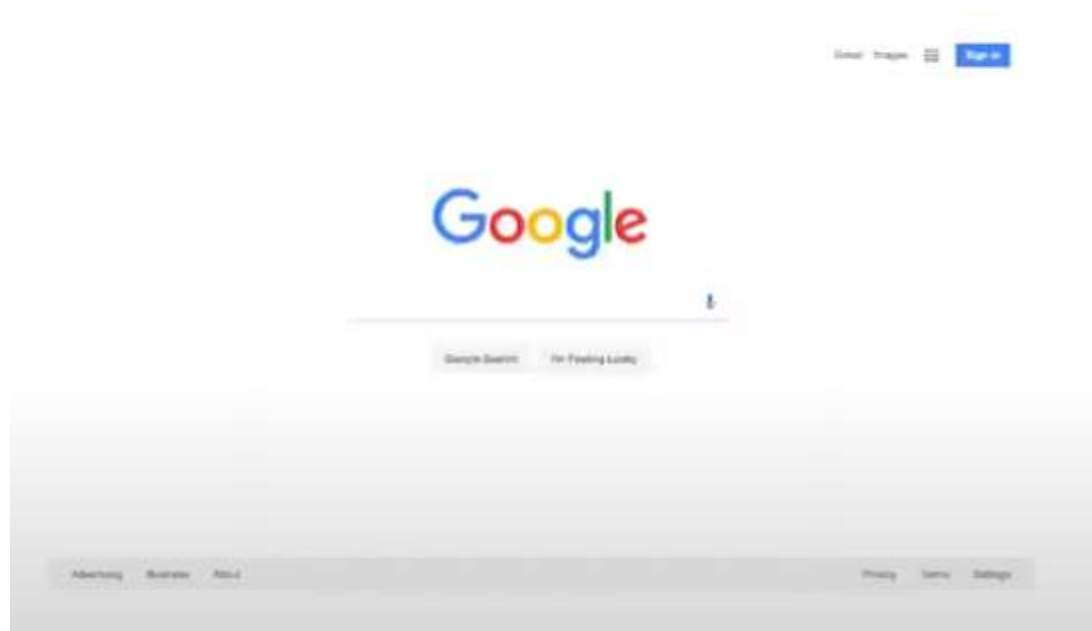Schizophrenia Protein-Protein Interaction

Graph Applications

Navigation

Graph Applications…

Navigation

Graph Applications…

Page Rank

Graph Applications…

Page Rank
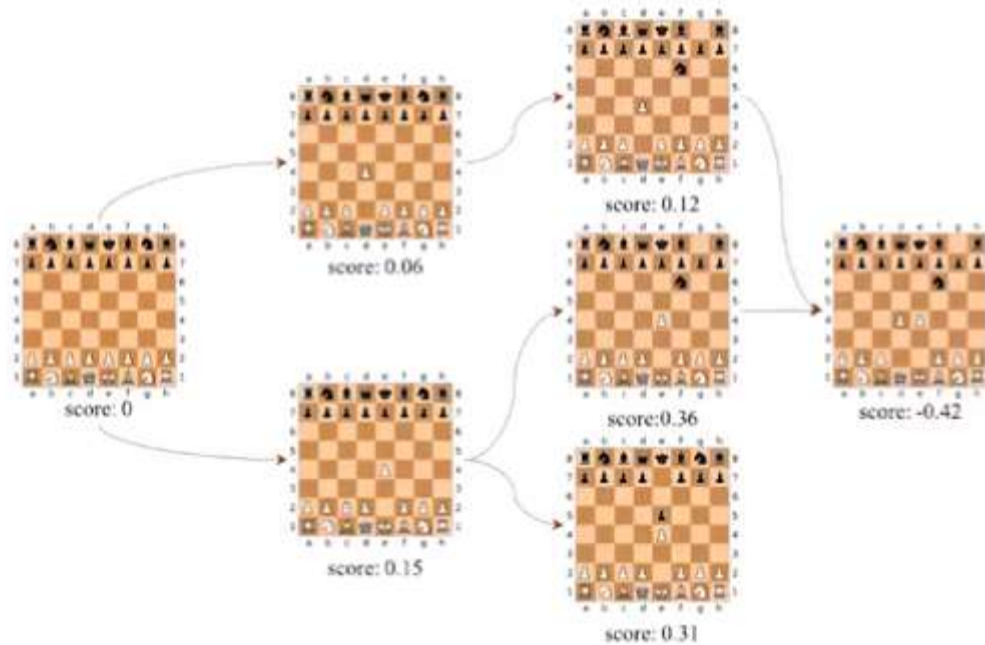
Graph Applications…

Page Rank

Graph Applications…
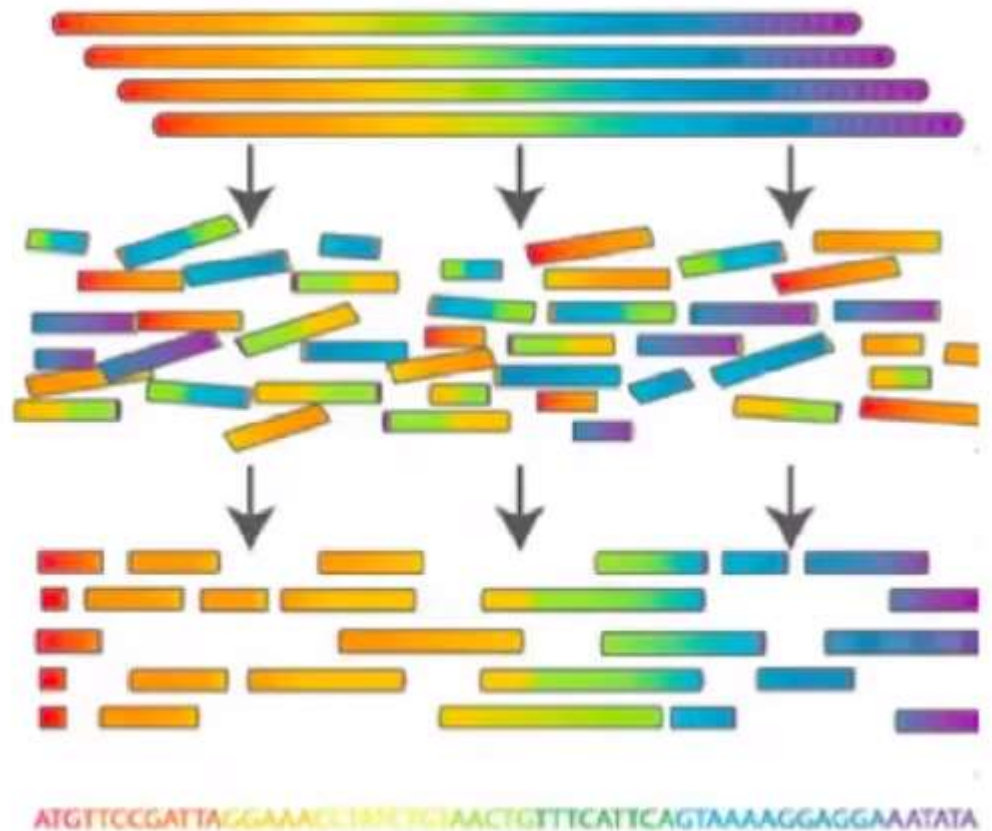
Game Strategies

Graph Applications…

Game Strategies

Graph Applications…

Genome Assembly

Graph Applications...

GSM

4 Frequency
Ranges

Graph Applications...

GSM

4 Frequency
Ranges

Graph Applications…

Computer Chips

Graph Applications…

Computer Chips