LTI System Responses and Convolution

IBEHS 3A03 Assignment 2

S. Hassan, A. Wang

October 27th, 2020

Motivation and Assumptions

Having explored the concepts of *impulse response, step response, and convolution* (all with respect to linear, time-invariant systems), we now demonstrate their mathematical relationships using MATLAB software. Namely, the impulse response as a *fundamental building block* for system output given any input signal.

Three "black-box" discrete-time LTI systems shall be tested and observed under theory, with sample physiological signals of respiration and cardiac electrical activity.

We assume zero initial conditions for each system.

I. System Unit Impulse Responses

Plots

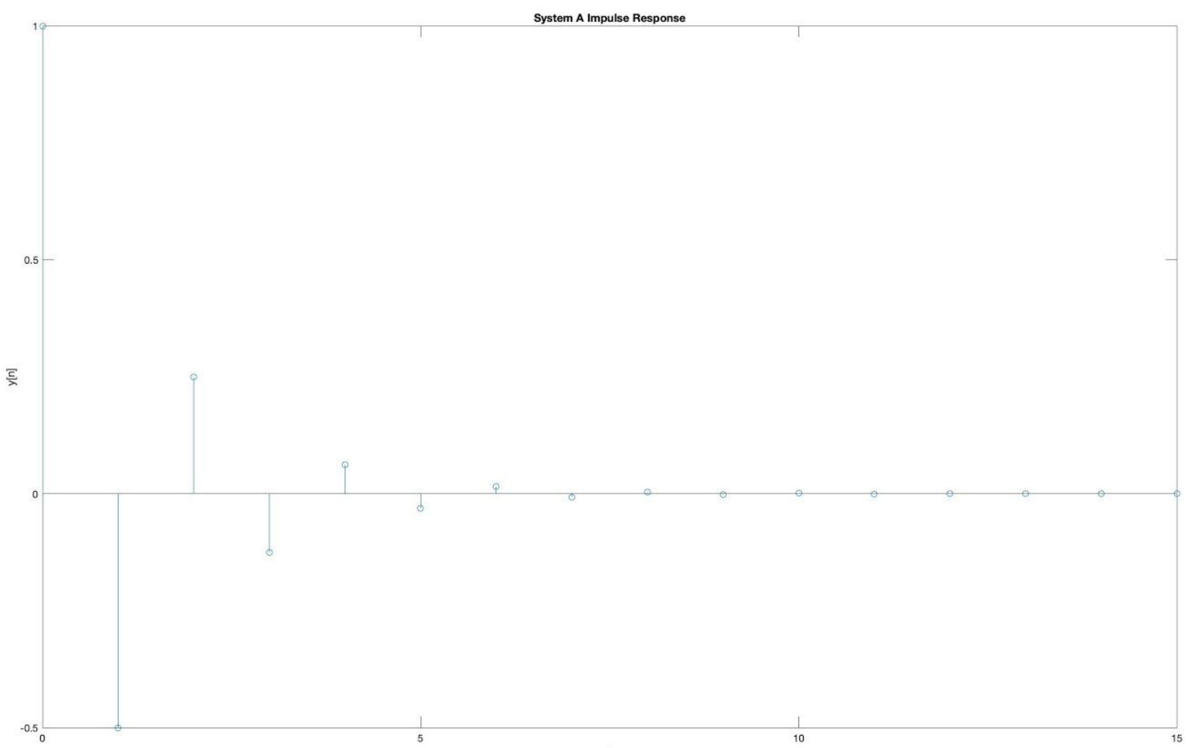


Figure 1: System A Output when input is the unit impulse function δ[n]

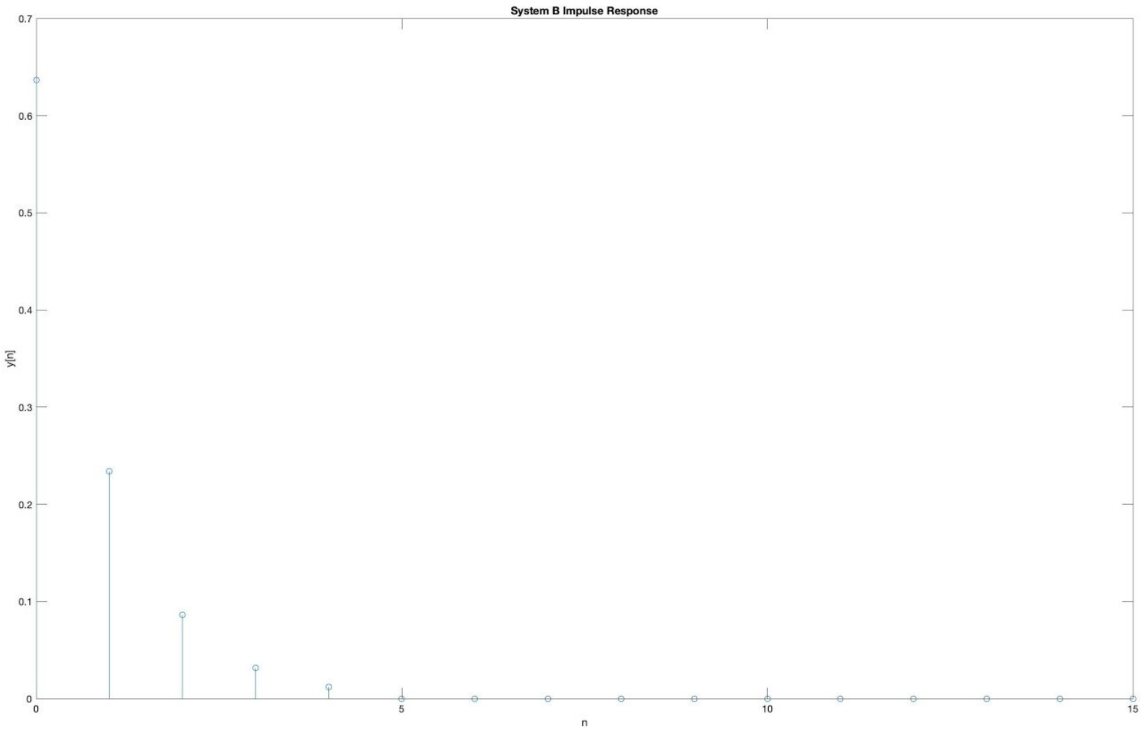


Figure 2: System B Output when input is the unit impulse function δ[n]

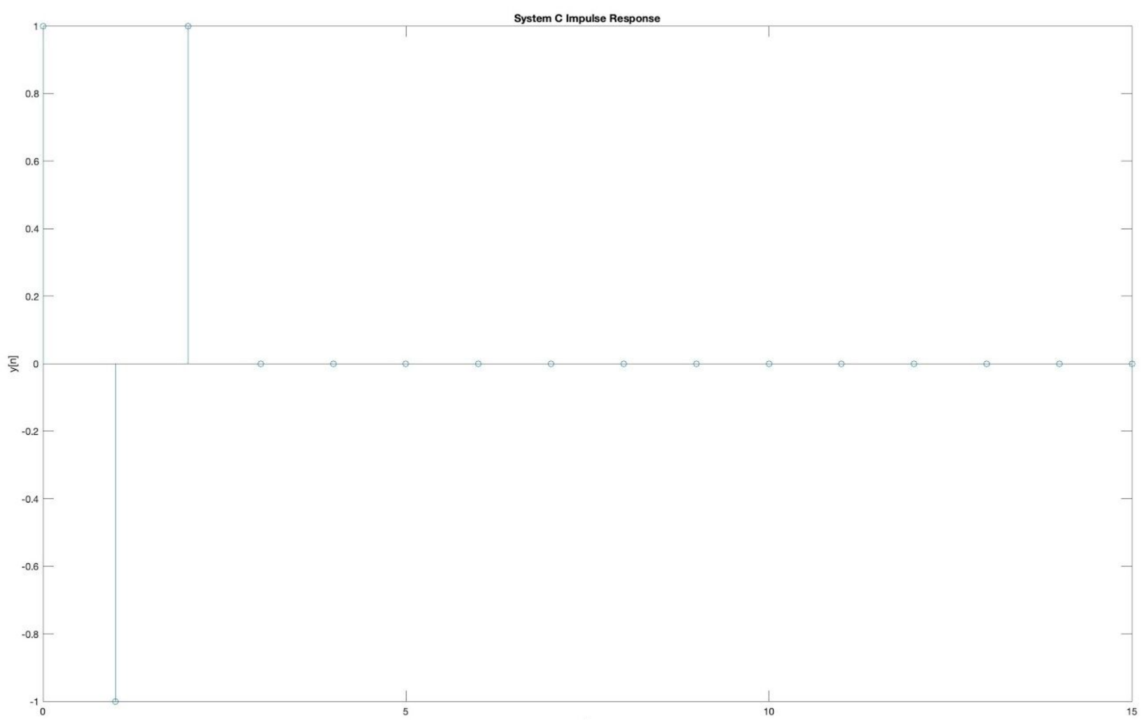


Figure 3: System C Output when input is the unit impulse function δ[n]

II. System Unit Step Responses

Plots

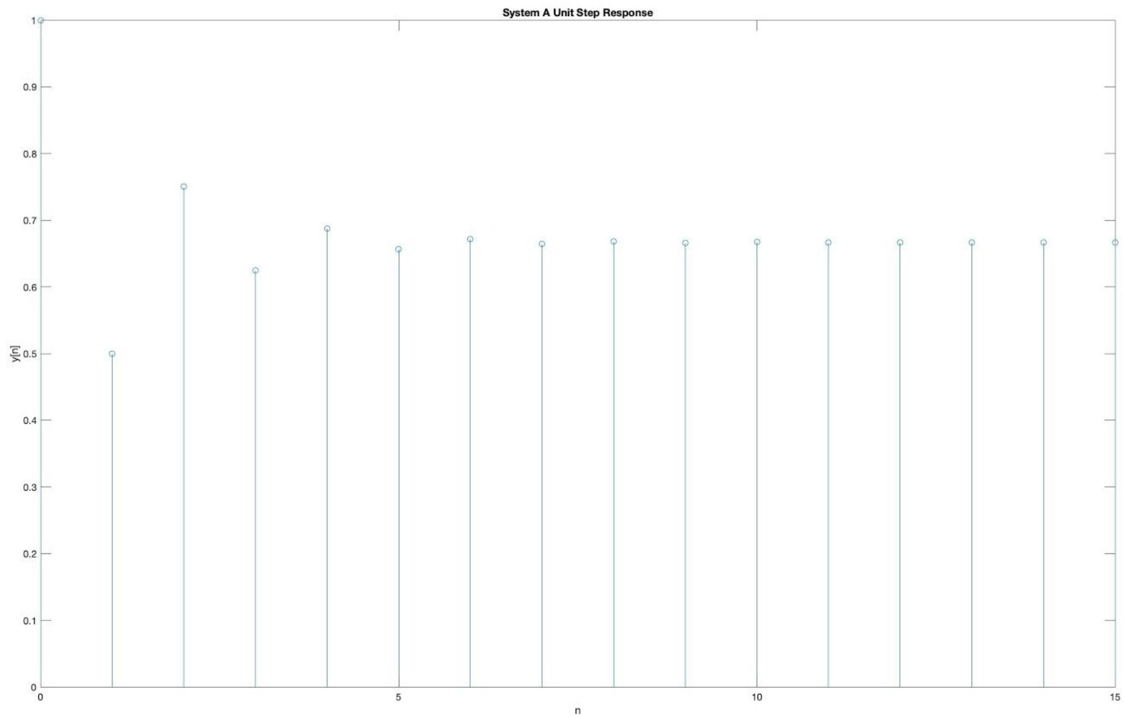


Figure 4: System A Output when the input is the unit step function u[n]

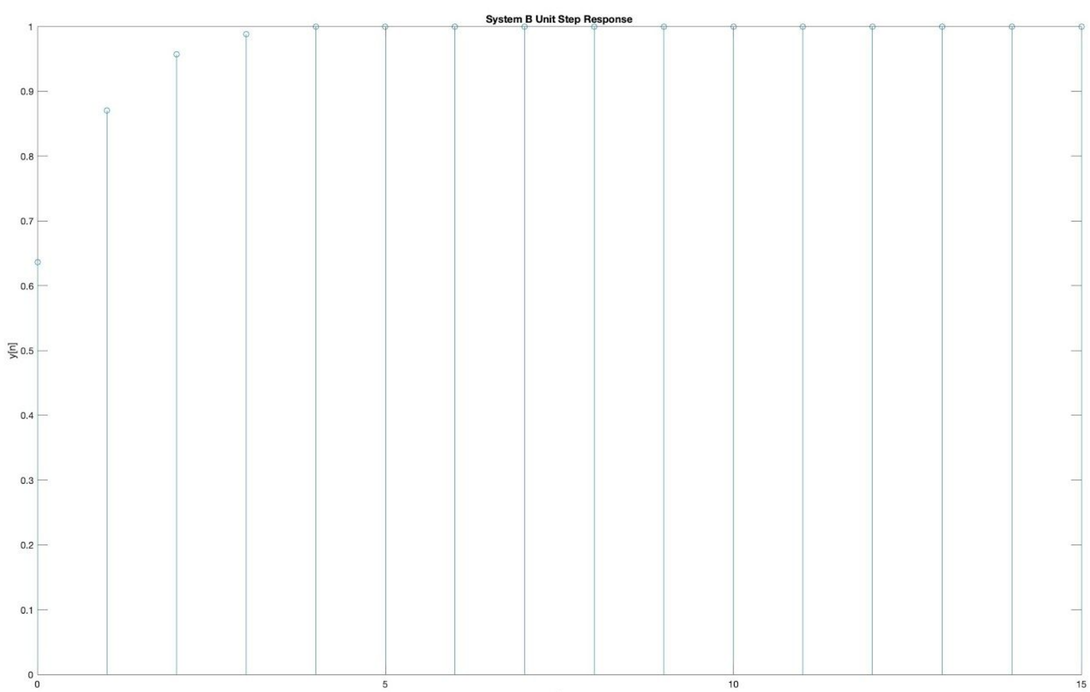


Figure 5: System B Output when the input is the unit step function u[n]

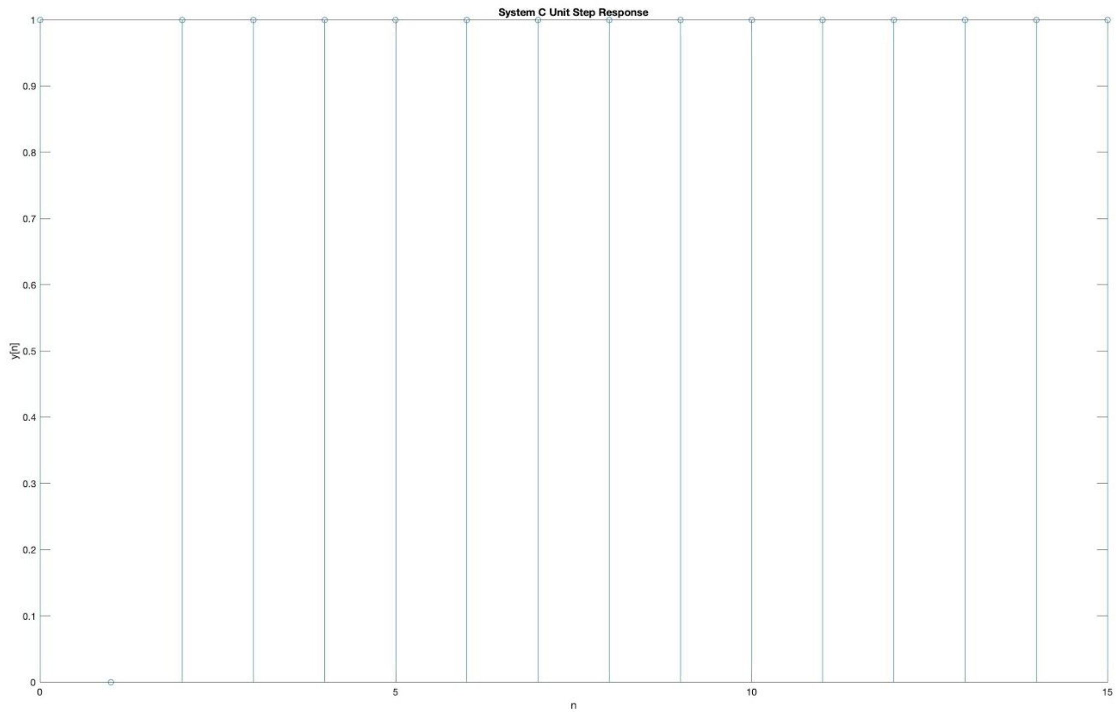


Figure 6: System C Output when the input is the unit step function u[n]

III. Cumulative Impulse Response to Step Response

Plots

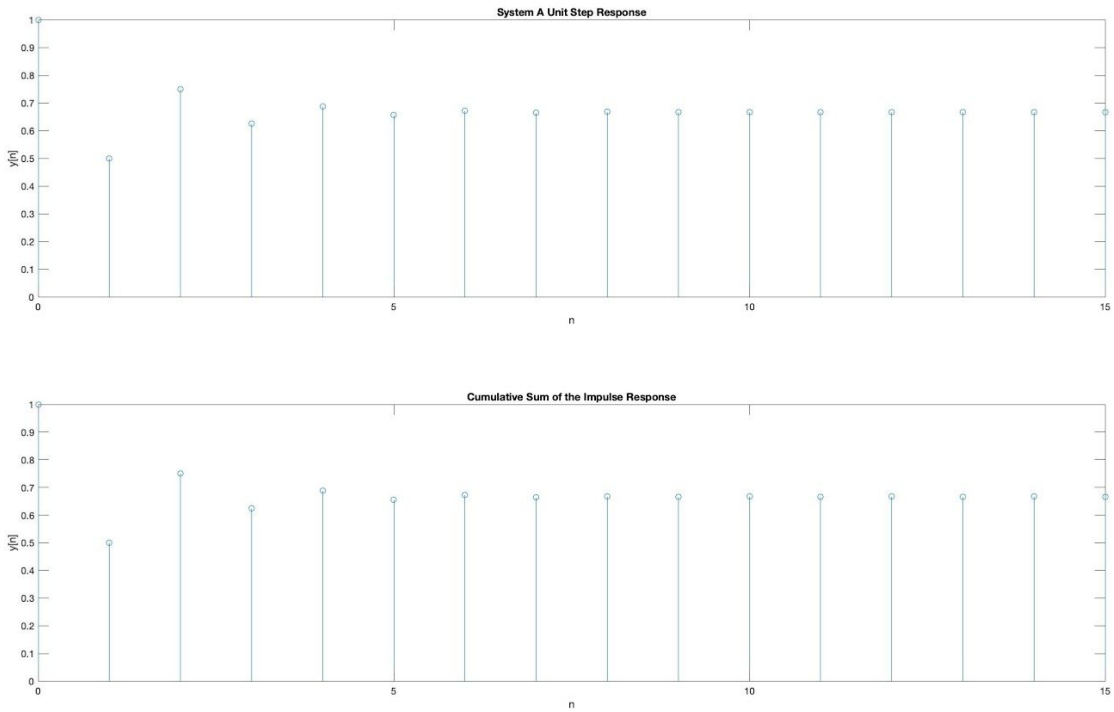


Figure 7: Demonstration that the cumulative sum of the unit impulse response is equal to the unit step function output for System A

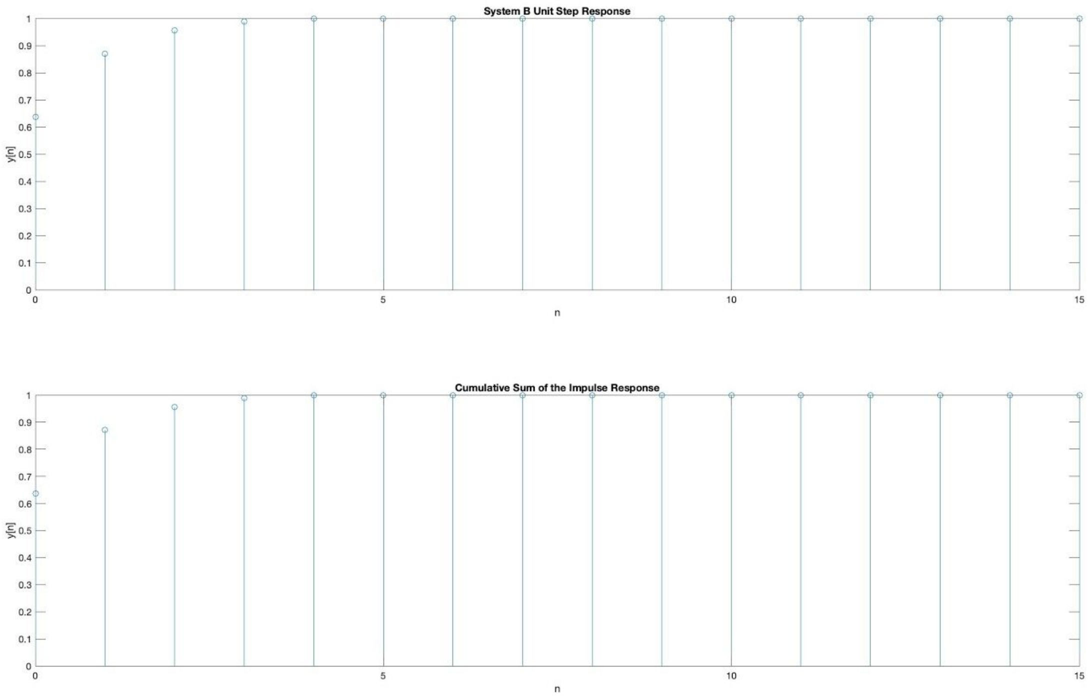


Figure 8: Demonstration that the cumulative sum of the unit impulse response is equal to the unit step function output for System B

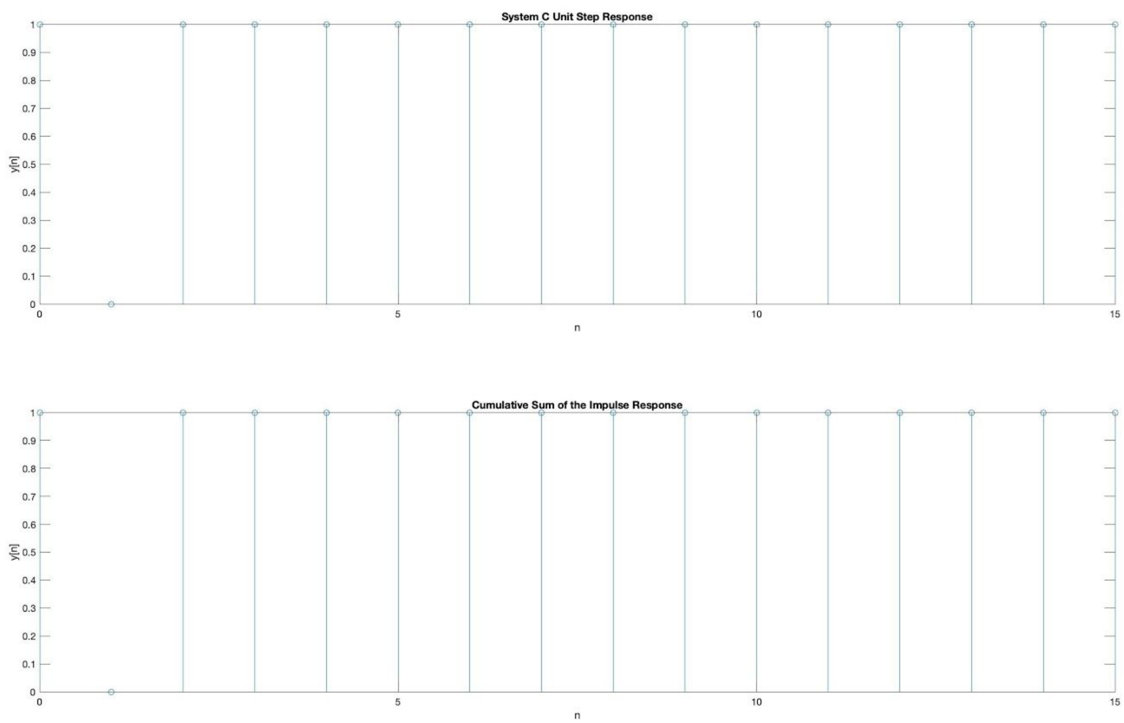


Figure 9: Demonstration that the cumulative sum of the unit impulse response is equal to the unit step function output for System C

IV. Step Response First Difference to Impulse Response

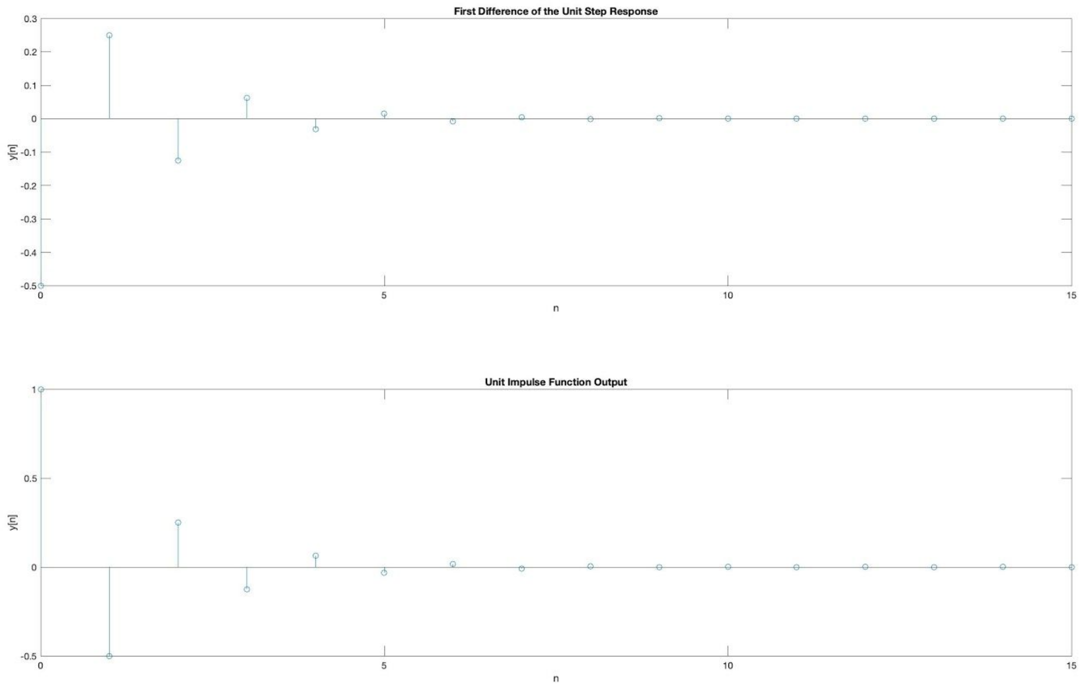


Figure 10: Demonstration that the First Difference of the unit step response is equal to the unit impulse function output for System A

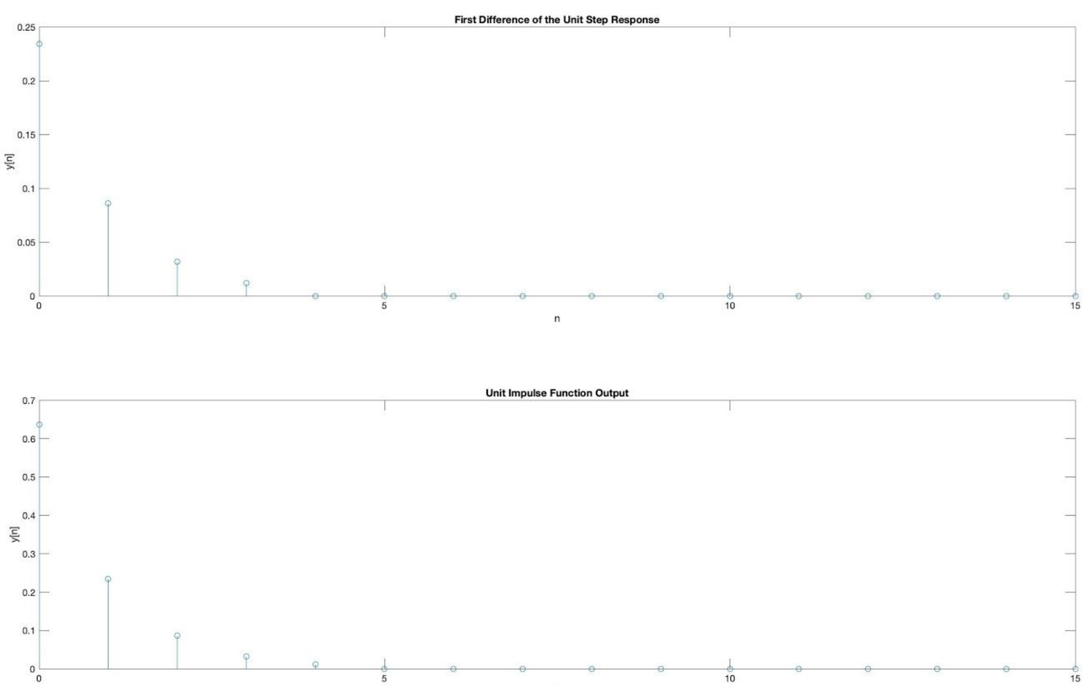


Figure 11: Demonstration that the First Difference of the unit step response is equal to the unit impulse function output for System B

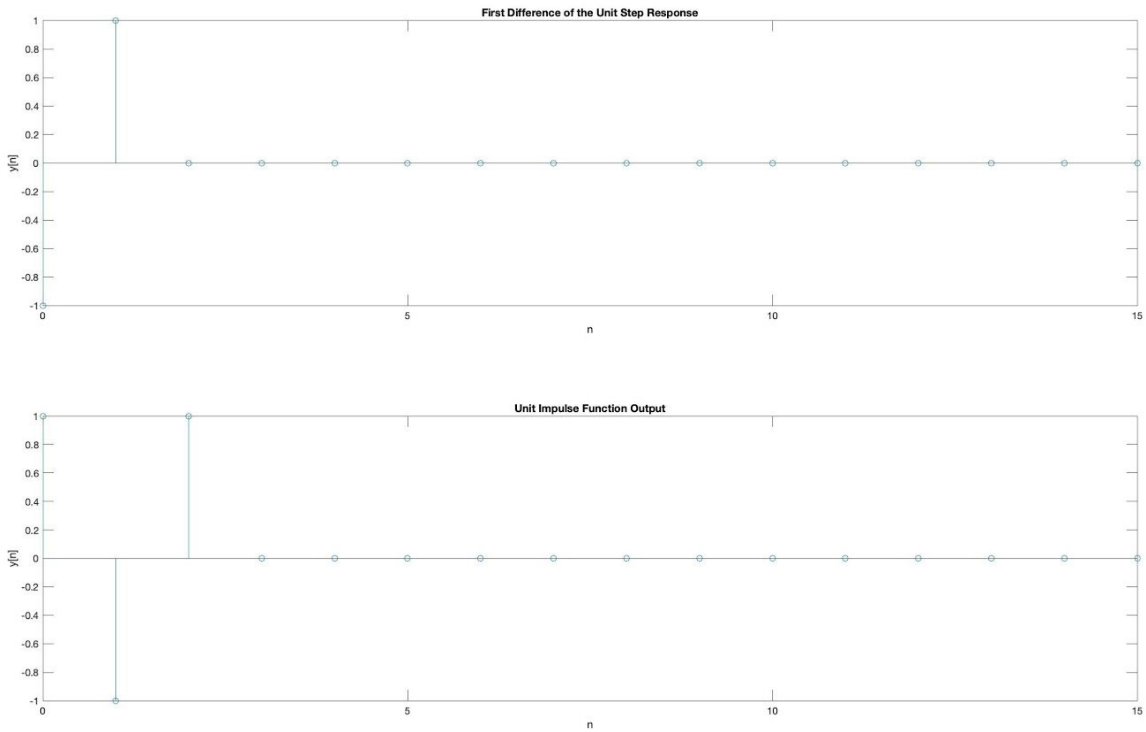
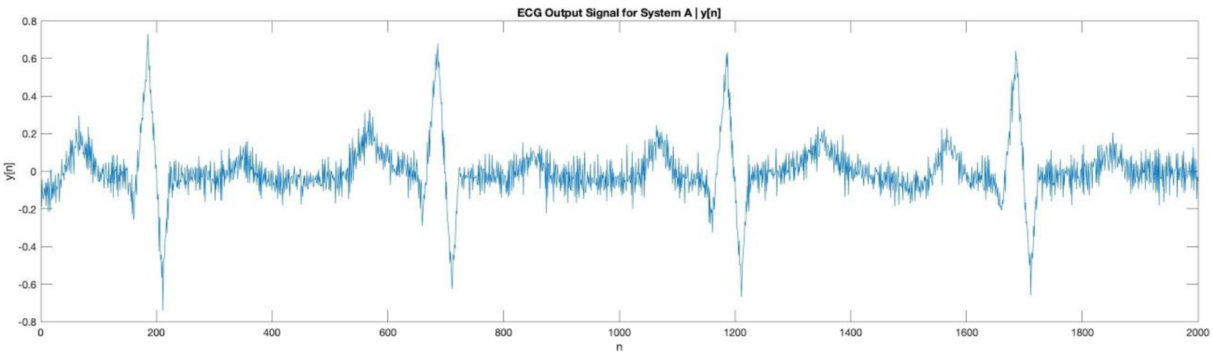


Figure 12: Demonstration that the First Difference of the unit step response is equal to the unit impulse function output for System C

V. Electrocardiogram (ECG) Sample & Respiratory Sample

Plots



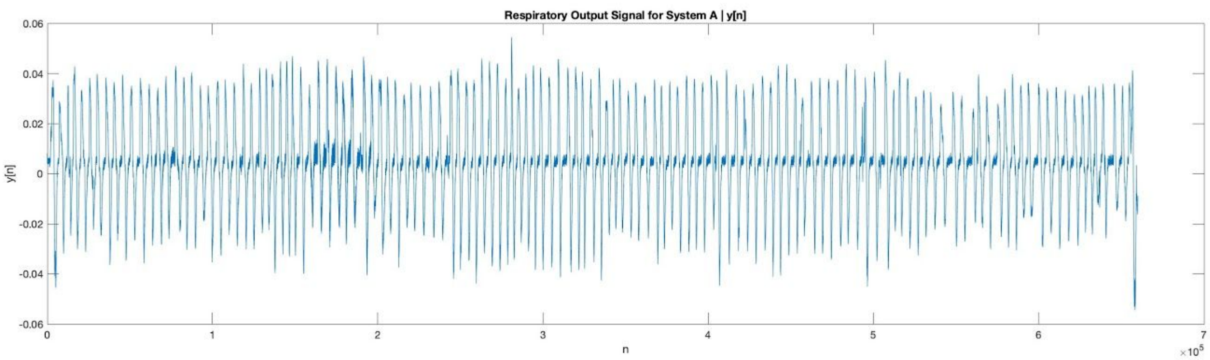


Figure 13: ECG and Respiratory Output using System A

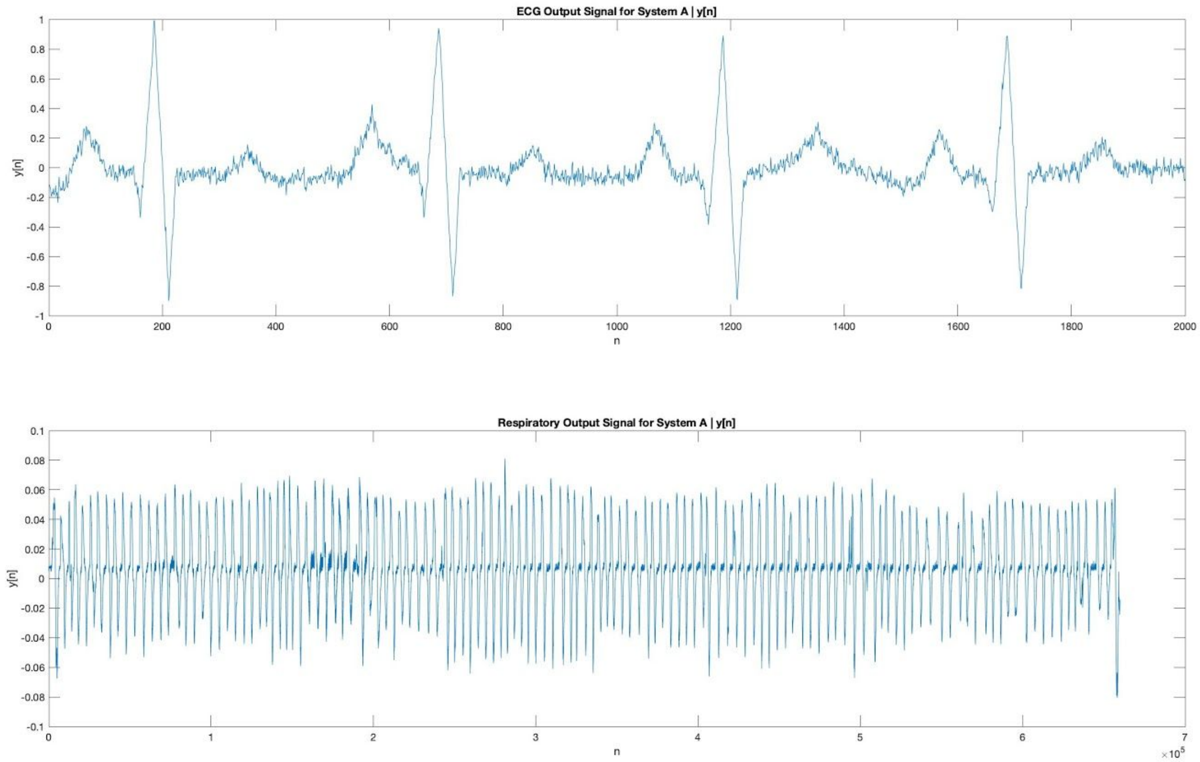


Figure 14: ECG and Respiratory Output using System B

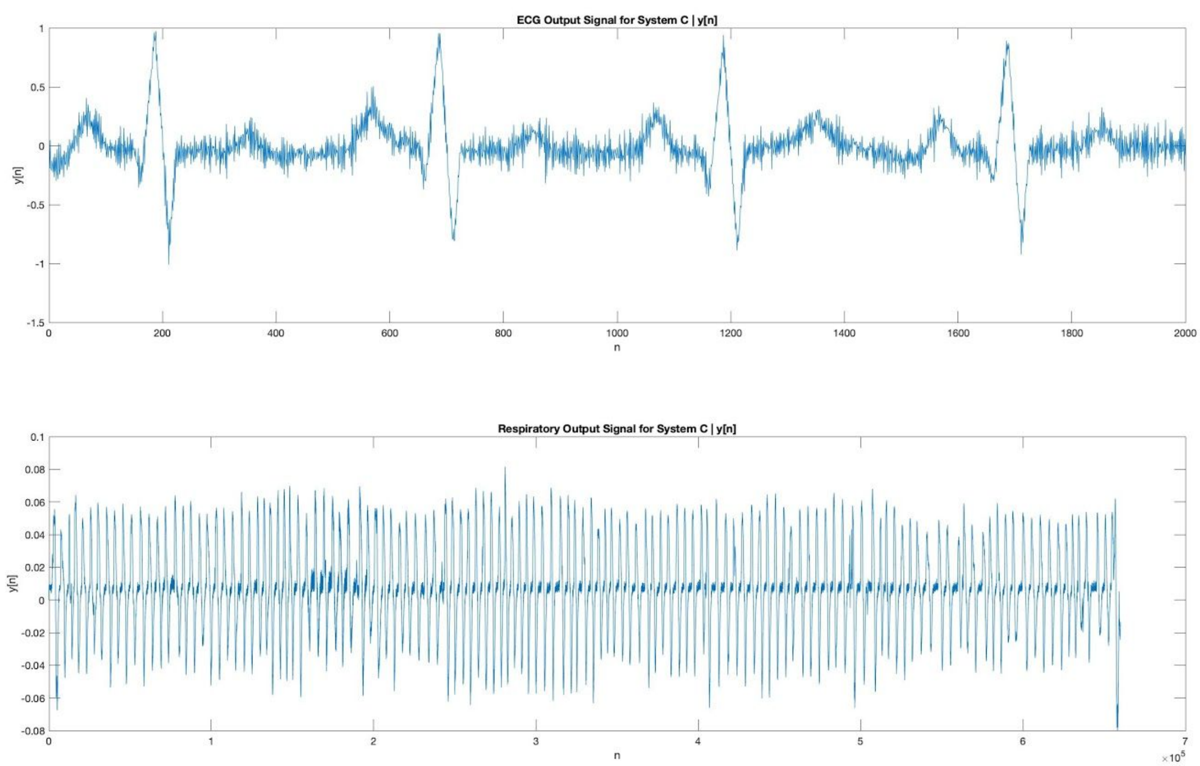


Figure 15: ECG and Respiratory Output using System C

VI. Direct Sample Output to Impulse Convolution

Plots

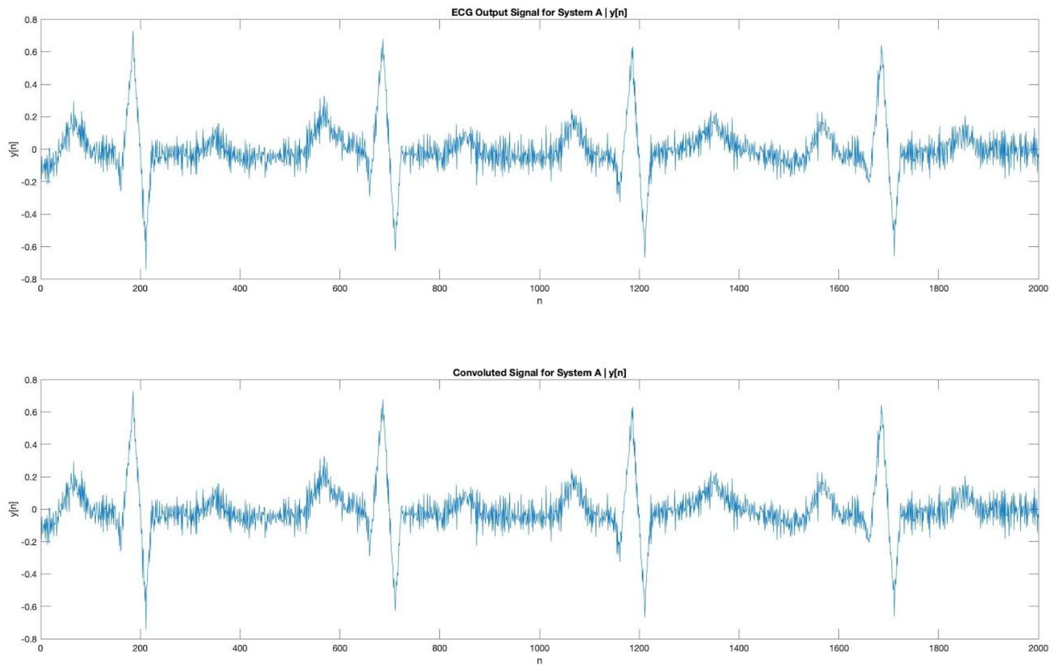


Figure 16: Demonstration that the output of the ECG is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System A

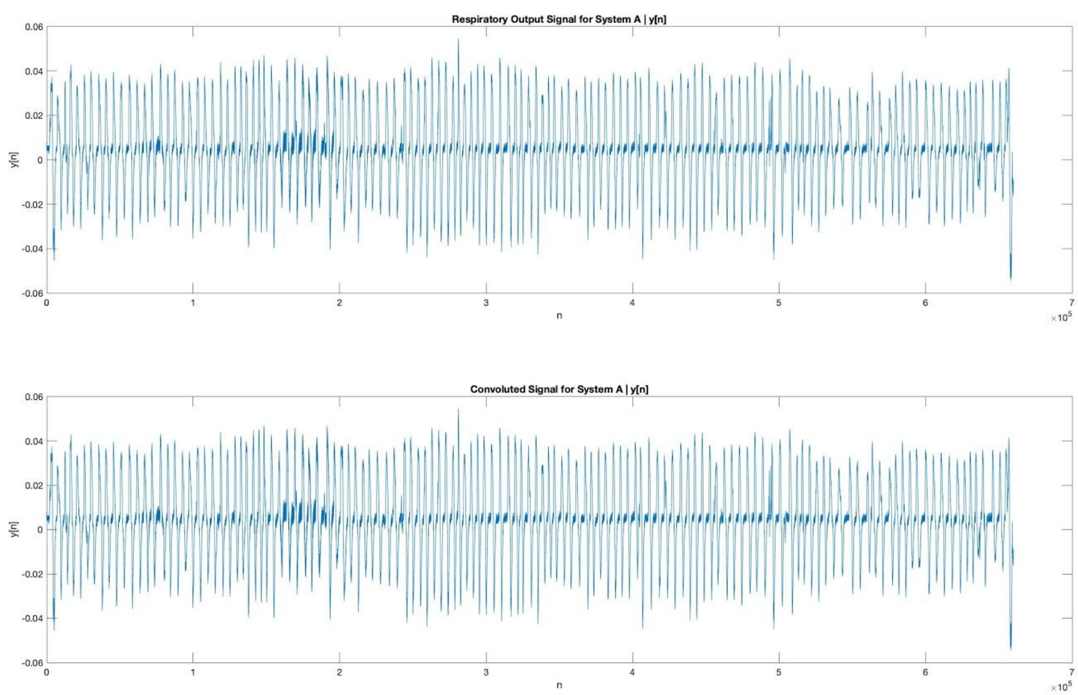


Figure 17: Demonstration that the output of the Respiratory signal is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System A

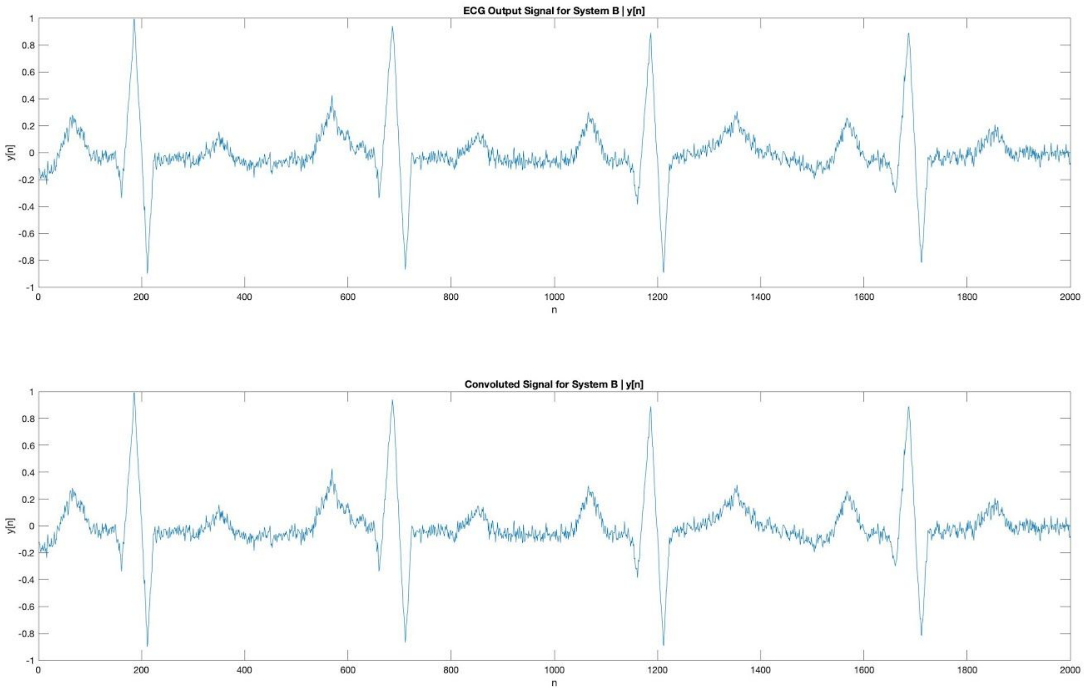


Figure 18: Demonstration that the output of the ECG is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System B

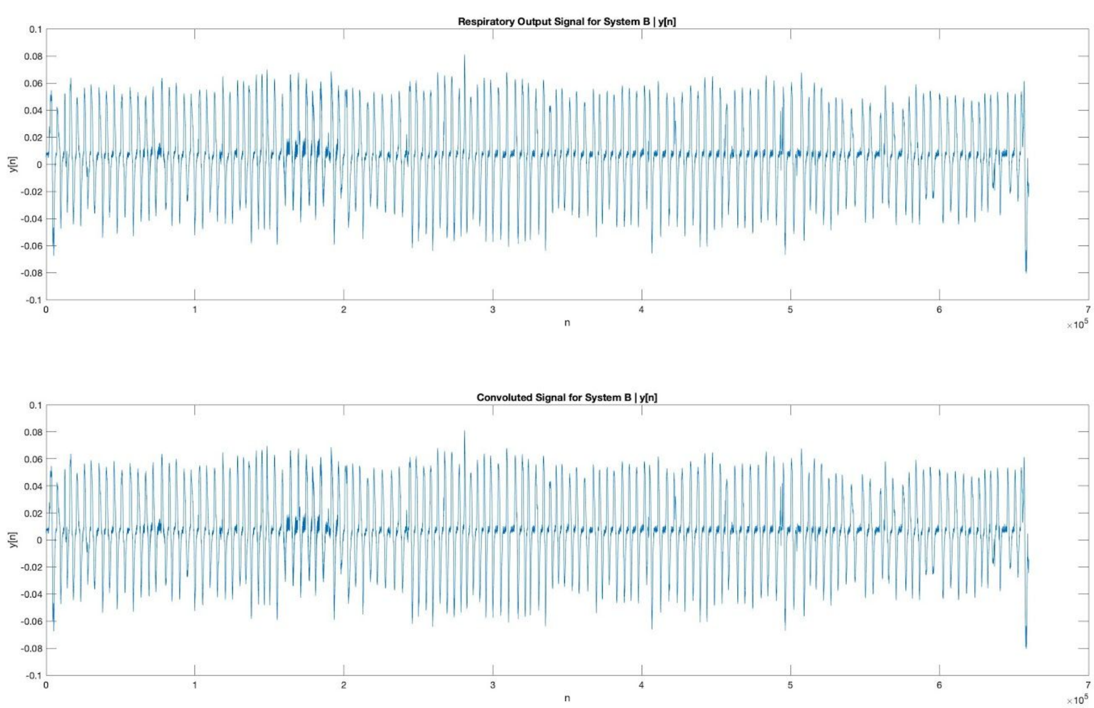


Figure 19: Demonstration that the output of the Respiratory is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System B

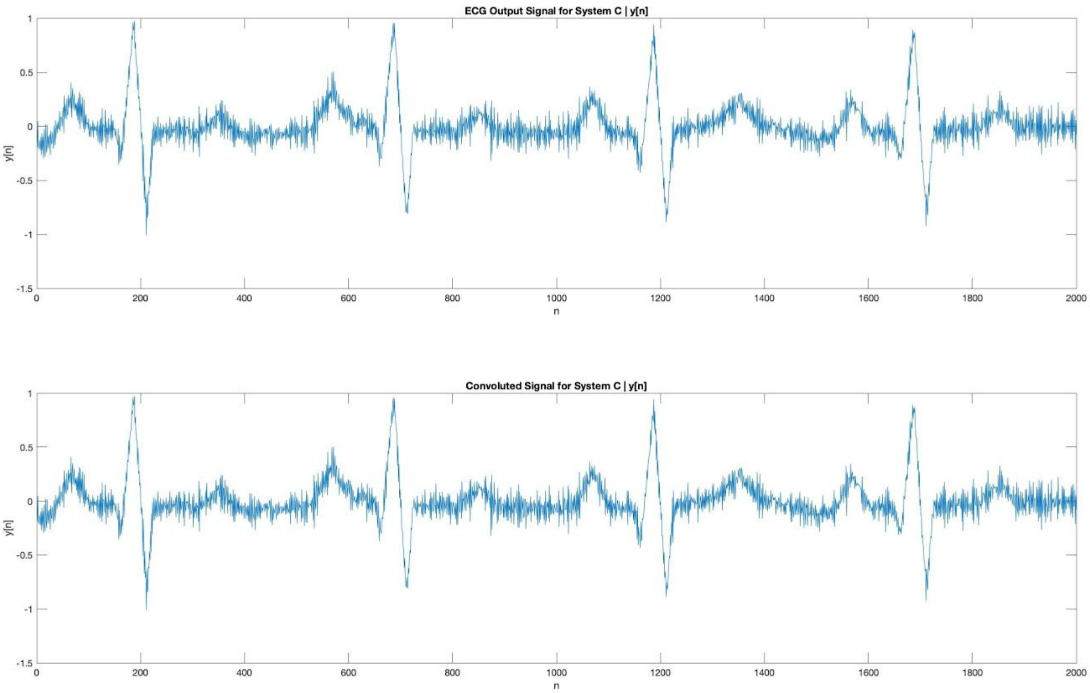


Figure 20: Demonstration that the output of the ECG is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System C

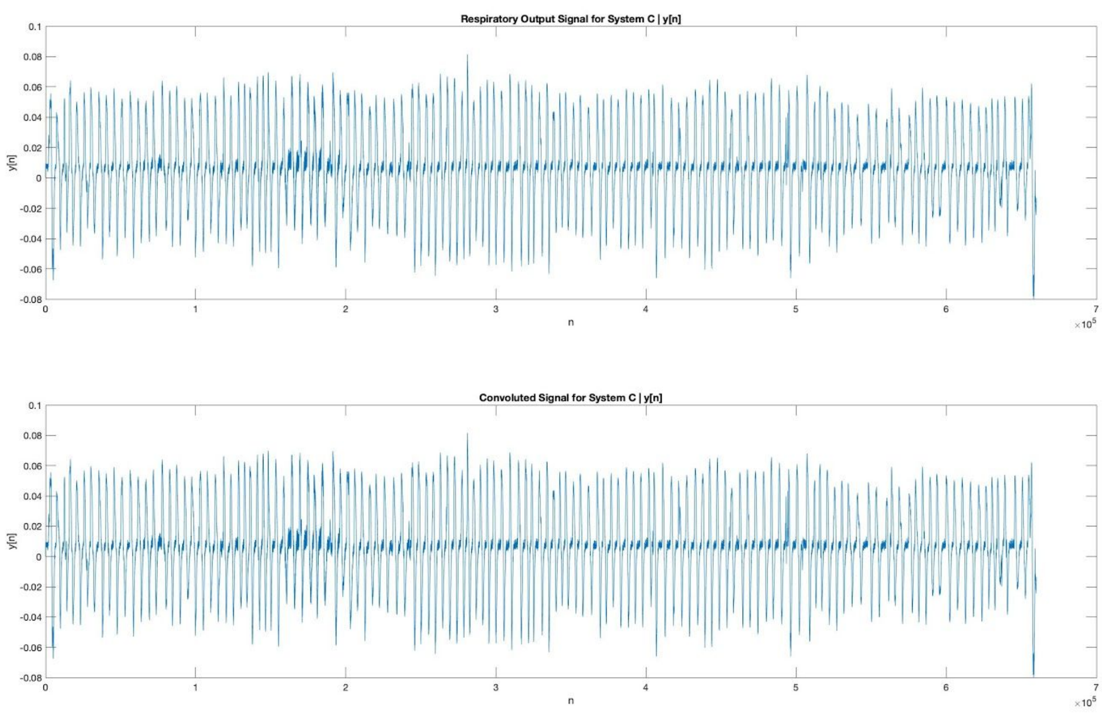


Figure 21: Demonstration that the output of the Respiratory is equal to the convolution of those input signals with the impulse response h[n] of the system computed in part I for System C

Analysis of System Impulse Responses

There is this notion of impulse response characteristics, which at the highest level separate them into two categories: finite impulse response (FIR) and infinite impulse response (IIR) (Oppenheim, 2011).

Their definitions follow their terminology fairly intuitively. In an FIR, a system's response to an impulse signal will rest to null in a finite time (Oppenheim, 2011). On the flip side, an IIR will have non-zero value(s) endlessly (Oppenheim, 2011). In Layman's terms: can the complete impulse response be contained in a plot, or not?

Here (visually, section I): A, B, and C, appear to have finite impulse response. However for more robust evidence, we can use MATLAB to look at output values at an arbitrarily large index.

We find:
- System A has non-zero outputs and indices exceeding 1000.
- System B has non-zero outputs that appear to end at 5.
- System C has non-zero outputs that appear to end at 3.

This was done by iterating through the unit impulse responses for 1000 time steps, and storing the value of the last non-zero output. From these observations we are led to believe that in truth: system A has IIR, while systems B and C have FIR.

Discussion and Conclusions

Part III

The unit step and unit impulse are closely related. In discrete time, the unit step is the running sum (cumulative sum) of the unit impulse response (Kamen, 2014).

As seen in the stem plots shown in figures seven through nine, the unit step response is then equal to the cumulative sum of the impulse response for all three systems. This follows system linearity. To confirm the equivalence, a data table comparing the outputs for System A is shown below. The data is obtained from the MATLAB code, SystemA.mat. Equalities were also seen in systems B and C

0	1	2	3	4	5	6	7
1.0	0.50	0.750	0.6250	0.6875	0.6562	0.6719	0.6641

8	9	10	11	12	13	14	15
0.6680	0.666	0.6670	0.6665	0.6667	0.6666	0.6667	0.6667

Table 1: Unit Step Response Output for System A (where n is in grey, and y[n] in white)

0	1	2	3	4	5	6	7
1.0	0.50	0.750	0.6250	0.6875	0.6562	0.6719	0.6641

8	9	10	11	12	13	14	15
0.6680	0.666	0.6670	0.6665	0.6667	0.6666	0.6667	0.6667

Table 2: Cumulative Sum of the Impulse Response for System A

Part IV

In discrete time, the unit impulse is equal to the first difference of the unit step function (Oppenheim, 2011). This can be seen with the mathematical relationship $\delta[n] = u[n] - u[n-1]$ (Oppenheim, 2011). This law is a corollary of the relationship examined in part III. The mathematical property can be qualitatively observed in figures ten through twelve. Here, it is important to note that the first difference outputs have been shifted by an index variable of one to the right (necessarily as there are $n-1$ deltas in a sequence of n values). Hence the mismatch observed in the aforementioned figures. Furthermore, this property can be confirmed via the data tables down below, comparing the outputs for System A. The data is obtained from the Matlab code, SystemA.mat.

0	1	2	3	4	5	6	7
-0.5	0.25	-0.125	0.0625	-0.0312	0.0156	-0.0078	0.0039

8	9	10	11	12	13	14	15
-0.0020	0.0010	-0.0005	0.0002	-0.0001	0.0001	-0.0000	0.0000

Table 3: First Difference of the Unit Step Output for System A

0	1	2	3	4	5	6	7
1	-0.5	0.25	-0.125	0.0625	-0.0312	0.0156	-0.0078

8	9	10	11	12	13	14	15
0.0039	-0.0020	0.0010	-0.0005	0.0002	-0.0001	0.0001	-0.0000

Table 4: Unit Impulse Response for System A

Part VII

Considering a linear time-invariant discrete-time system with *x[n] = 0*, the output *y[n]* is equal to the discrete-time convolution. Mathematically this is represented by *y[n] = h[n]*x[n], n* >= 0, where h[n] is the unit impulse response and *x[n]* is the input vector (Kamen, 2014). Knowing this, it can be proven that the direct system output to any input signal is equivalent to the convolution of that signal with the unit impulse response. The premise here is that the unit impulse and its response (in LTI systems) are fundamental building blocks. Via LTI characteristics, we can scale and distribute (convolve) the unit impulse response with any other signal to construct the correct system output without having to directly pass the signal through the system itself (Kamen, 2014).

This property was tested for systems A, B, and C, and can be qualitatively confirmed in figures sixteen through twenty-one. As seen, the ECG and respiratory outputs are equal to the convolution of the ECG and respiratory signals and system unit impulse responses.

Citations

[1] Kamen, Edward & Heck, Bonnie. (2014). Fundamentals of Signals and Systems using the Web and MATLAB (4rd ed.). United Kingdom.

[2] Oppenheim, Alan. *RES.6-007 Signals and Systems.* Spring 2011. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu. License: Creative Commons BY-NC-SA.