Eight Puzzle

Using Informed and Uninformed Search Algorithms

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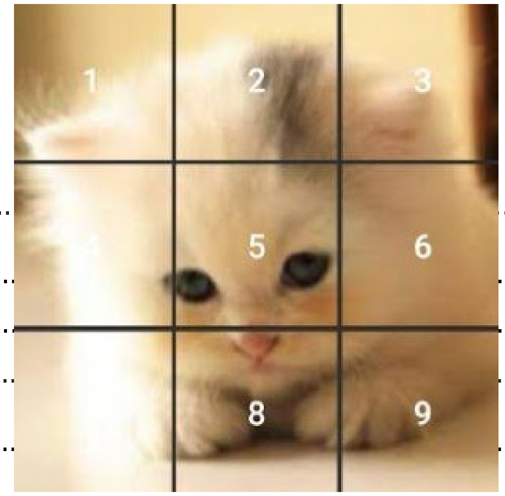
Table of Contents



Introduction

The puzzle consists of an area divided into a grid, 3 by 3 for the 8-puzzle. On each grid square is a tile, expect for one square which remains empty. Thus, there are eight tiles in the 8-puzzle. A tile that is next to the empty grid square can be moved into the empty space, leaving its previous position empty in turn. Tiles are numbered, 1 thru 8 for the 8-puzzle, so that each tile can be uniquely identified.

The aim of the puzzle is to achieve an ordered configuration of tiles from a given (different) configuration by sliding the individual tiles around the grid as described above.

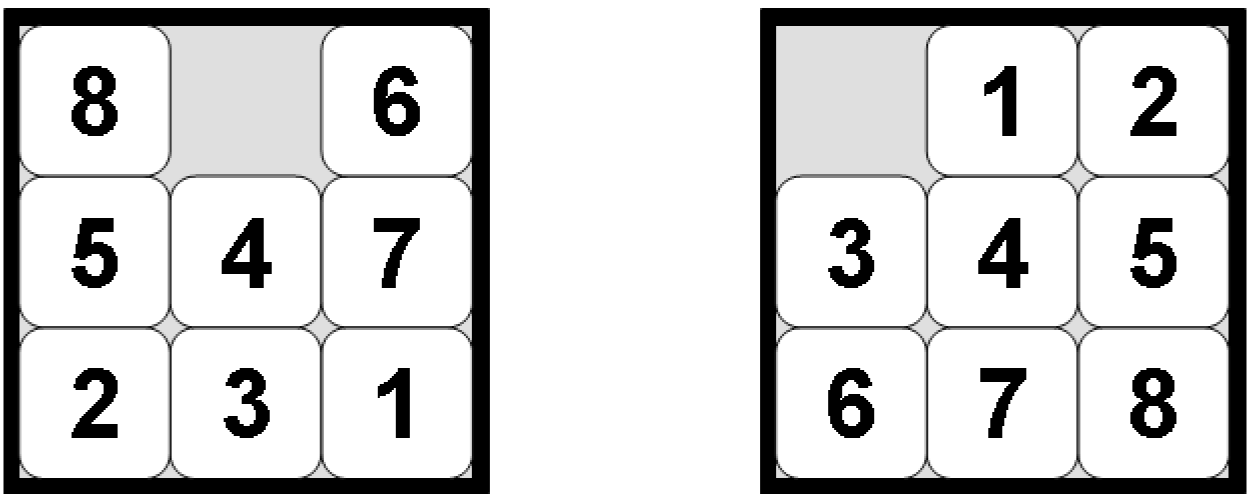
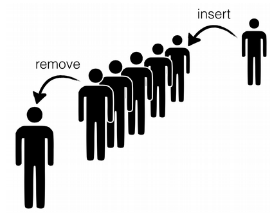


Figure 1: Configurations of the 8-Puzzle: worst-case initial state (left) for goal state (right)

Uninformed Search



Data structures used:

- To implement BFS we use a *queue* to simulate horizontal expansion of nodes, to explore the breadth of a vertex depth before moving on. This behavior guarantees that the first path located is one of the shortest-paths present.

- To implement DFS we use a *stack* to simulate depth traversal, to explore possible vertices (from a supplied root) down each branch before backtracking. Below is a listing of the actions performed upon each visit to a node:
    - Mark the current vertex as being visited.
    - Explore each adjacent vertex that is not included in the visited set.

Algorithms used:

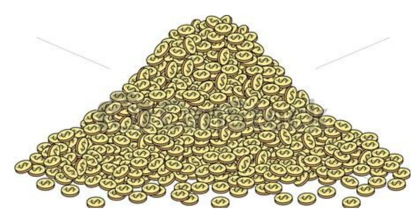
- Breadth-First Search

Starts at the tree root and explores the neighbor nodes first, before moving to the next level neighbors.

- Depth-First Search

Starts at the root and explores as far as possible along each branch before backtracking.

Informed Search



- To implement A*, at each step of the algorithm, the node with the lowest $f(x)$ value is removed from the *heap*, the f and g values of its neighbors are updated accordingly, and these neighbors are added to the heap. The algorithm continues until a goal node has a lower f value than any node in the heap (or until the heap is empty).The f value of the goal is then the length of the shortest path, since h at the goal is zero in an admissible heuristic.

$$f(n) = g(n) + h(n)$$

where n is the last node on the path, $g(n)$ is the cost of the path from the start node to n, and $h(n)$ is a heuristic that estimates the cost of the cheapest path from n to the goal. The heuristic is problem-specific.

Highlights of the code

The generation of all the possible scenarios (either up, down, left or right).

```
def generate_neighbours(self):  # creates a list of neighbouring states

    actions = ((0, 1, 2, -3), (6, 7, 8, 3), (0, 3, 6, -1), (2, 5, 8, 1))  # up,down,left and right
    neighbours = []
    empty_position = self.value.index(0)

    for position in actions:
        if empty_position not in position[:3]:
            temp_list = self.value[:]
            temp_list[empty_position + position[3]], temp_list[empty_position] = temp_list[empty_position], temp_list[empty_position +
            neighbours.append(State(temp_list, self))

    return neighbours
```

Definition of the root (The node that has no parent), this is the initial state of the board.

```
class State:

    def __init__(self, initial_values, parent=None):
        self.value = initial_values
        self.parent = parent
        if self.parent:
            self.depth = self.parent.depth + 1
        else:
            self.depth = 0
```

The tracing of the path, prints all the movements taken to reach the goal.

```
#   This function traces the moves made along the path to reach the goal state.
def trace_path(state):
    path = []
    while state.parent:
        child = state
        state = state.parent
        child_index = child.value.index(0)
        parent_index = state.value.index(0)
        if child_index is not parent_index:
            difference = child_index - parent_index
            if difference == -3:
                path.append('up')
            elif difference == 3:
                path.append('down')
            elif difference == -1:
                path.append('left')
            else:
                path.append('right')
    path.reverse()
    print('path:', path)
```

The class *solver* performs all the search operations. BFS, DFS and A* with both Manhattan's and Euclidean`s heuristic.

```
class Solver:

    def __init__(self, state):
        self.state = state
        self.puzzle = self.state.value

    def bfs(self):
        queue = deque()
        frontier = set()
        queue.append(self.state)
        frontier.add(self.state.hash_value)
        explored = set()
        nodes_expanded = depth = 0
        while queue:
            position = queue.popleft()
            frontier.remove(position.hash_value)
            if position.value == goal:
                print_goal_from_parent(position)
                print('nodes expanded:', nodes_expanded)
                trace_path(position)
                print('max depth:', depth)
                return 'success'

            explored.add(position.hash_value)
            neighbours = position.generate_neighbours()
            nodes_expanded += 1

            for neighbour in neighbours:
                if not (neighbour.hash_value in frontier or neighbour.hash_value
                    depth = max(depth, neighbour.depth)
                    queue.append(neighbour)
                    frontier.add(neighbour.hash_value)
```

```
    def dfs(self):
        stack = deque()
        frontier = set()
        stack.append(self.state)
        frontier.add(self.state.hash_value)
        explored = set()
        nodes_expanded = depth = 0
        while stack:
            position = stack.pop()
            frontier.remove(position.hash_value)
            if position.value == goal:
                print_goal_from_parent(position)
                print('nodes expanded:', nodes_expanded)
                print('max depth:', depth)
                trace_path(position)
                return 'success'

            explored.add(position.hash_value)
            neighbours = position.generate_neighbours()
            neighbours.reverse()
            nodes_expanded += 1
            for neighbour in neighbours:
                if not (neighbour.hash_value in frontier or neighbour.hash_value
                    depth = max(depth, neighbour.depth)
                    stack.append(neighbour)
                    frontier.add(neighbour.hash_value)
```

```
def astar_manh(self):
    heap = []
    heappush(heap, (manhattan_distance(self.state), self.state))
    keys = dict()
    keys[self.state.hash_value] = self.state
    nodes_expanded = 0
    explored = set()

    while heap:
        score, position = heappop(heap)
        del keys[position.hash_value]
        explored.add(position.hash_value)

        if position.value == goal:
            print_goal_from_parent(position)
            print('nodes expanded:', nodes_expanded)
            trace_path(position)
            return 'success'
        neighbours = position.generate_neighbours()
        nodes_expanded += 1
        for neighbour in neighbours:
            if not (neighbour.hash_value in keys or neighbour.hash_value in
                heappush(heap, (manhattan_distance(neighbour) + neighbour.de
                keys[neighbour.hash_value] = neighbour
            elif neighbour.hash_value in keys:
                node = keys[neighbour.hash_value]
                if manhattan_distance(neighbour) < manhattan_distance(node):
                    node.parent = position
                    node.depth = neighbour.depth
```

```
def astar_euc(self):
    heap = []
    heappush(heap, (euclidean_distance(self.state), self.state))
    keys = dict()
    keys[self.state.hash_value] = self.state
    nodes_expanded = 0
    explored = set()

    while heap:
        score, position = heappop(heap)
        del keys[position.hash_value]
        explored.add(position.hash_value)

        if position.value == goal:
            print_goal_from_parent(position)
            print('nodes expanded:', nodes_expanded)
            trace_path(position)
            return 'success'
        neighbours = position.generate_neighbours()
        nodes_expanded += 1
        for neighbour in neighbours:
            if not (neighbour.hash_value in keys or neighbour.hash_value in
                heappush(heap, (euclidean_distance(neighbour) + neighbour.de
                keys[neighbour.hash_value] = neighbour
            elif neighbour.hash_value in keys:
                node = keys[neighbour.hash_value]
                if euclidean_distance(neighbour) < euclidean_distance(node):
                    node.parent = position
                    node.depth = neighbour.depth
```

The heuristics of A* (Both Manhattan and Euclidean)

```
#   calculates manhattan's distance h(n)
def manhattan_distance(state):
        puzzle = state.value
        dist = 0
        for i, val in enumerate(puzzle):
            if val == 0:
                continue

            goal_y, goal_x = val // 3, val % 3
            y, x = i // 3, i % 3
            dist += abs(goal_y-y) + abs(goal_x-x)
        return dist

#   calculates euclidean's distance h(n)
def euclidean_distance(state):
        puzzle = state.value
        dist = 0
        for i, val in enumerate(puzzle):
            if val == 0:
                continue

            goal_y, goal_x = val // 3, val % 3
            y, x = i // 3, i % 3
            dist += math.sqrt(abs(goal_y-y)**2 + abs(goal_x-x)**2)
        return dist
```

Sample Runs

```
import time

#   input puzzles to solve
puzzle = [1, 2, 0, 3, 4, 5, 6, 7, 8]
puzzle0 = [1, 3, 4, 8, 0, 5, 7, 2, 6]
puzzle1 = [1, 2, 3, 4, 5, 0, 6, 7, 8]
puzzle2 = [1, 2, 5, 3, 4, 0, 6, 7, 8]
puzzle3 = [0, 1, 3, 4, 2, 5, 7, 8, 6]
puzzle4 = [8, 3, 5, 4, 1, 6, 2, 7, 0]    # unsolvable
puzzle5 = [2, 8, 3, 1, 0, 5, 4, 7, 6]
puzzle6 = [8, 0, 6, 5, 4, 7, 2, 3, 1]
puzzle7 = [1, 2, 3, 4, 8, 0, 7, 6, 5]
puzzle8 = [1, 2, 0, 3, 4, 5, 6, 7, 8]
puzzle9 = [1, 2, 3, 4, 5, 0, 6, 7, 8]
puzzle10 = [1, 2, 3, 4, 0, 5, 6, 7, 8]
puzzle11 = [1, 2, 3, 0, 4, 5, 6, 7, 8]
puzzle12 = [3, 1, 2, 6, 4, 5, 0, 7, 8]
puzzle13 = [6, 1, 8, 4, 0, 2, 7, 3, 5]
game = State(puzzle2)
search = Solver(game)
print('\nBFS:')
```

Puzzle = [1, 2, 5, 3, 4, 0, 6, 7, 8]

```
shahenda@Lenovo ~/Downloads/EightPuzzle-AI $ python Main.py

BFS:
=======================================
[[1, 2, 5, 3, 4, 0, 6, 7, 8], [1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 3)
('moves:', 3)
('nodes expanded:', 10)
('path:', ['up', 'left', 'left'])
('max depth:', 4)
('time taken:', 0.00026702880859375)

DFS:
=======================================
[[1, 2, 5, 3, 4, 0, 6, 7, 8], [1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 3)
('moves:', 3)
('nodes expanded:', 181437)
('max depth:', 66125)
('path:', ['up', 'left', 'left'])
('time taken:', 1.8108329772949219)

A* by Manhattan heuristic:
=======================================
[[1, 2, 5, 3, 4, 0, 6, 7, 8], [1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 3)
('moves:', 3)
('nodes expanded:', 3)
('path:', ['up', 'left', 'left'])
('time taken:', 0.0002579689025878906)

A* by Euclidean heuristic:
=======================================
[[1, 2, 5, 3, 4, 0, 6, 7, 8], [1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 3)
('moves:', 3)
('nodes expanded:', 3)
('path:', ['up', 'left', 'left'])
('time taken:', 0.00015401840209960938)
shahenda@Lenovo ~/Downloads/EightPuzzle-AI $
```

puzzle = [1, 2, 0, 3, 4, 5, 6, 7, 8]

```
BFS:
========================================
[[1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 2)
('moves:', 2)
('nodes expanded:', 6)
('path:', ['left', 'left'])
('max depth:', 3)
('time taken:', 0.00037598609924316406)

DFS:
========================================
[[1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 2, 5, 3, 4, 0, 6, 7, 8], [1, 2, 5, 3, 4, 8, 6, 7, 0], [1, 2, 5, 3, 4, 8, 6, 0, 7], [1, 2, 5, 3, 0, 8, 6, 4, 7], [1, 0, 5, 3, 2, 8, 6,
4, 7], [0, 1, 5, 3, 2, 8, 6, 4, 7], [3, 1, 5, 0, 2, 8, 6, 4, 7], [3, 1, 5, 6, 2, 8, 0, 4, 7], [3, 1, 5, 6, 2, 8, 4, 0, 7], [3, 1, 5, 6, 0, 8, 4, 2, 7], [3, 0, 5, 6, 1,
8, 4, 2, 7], [0, 3, 5, 6, 1, 8, 4, 2, 7], [6, 3, 5, 0, 1, 8, 4, 2, 7], [6, 3, 5, 4, 1, 8, 0, 2, 7], [6, 3, 5, 4, 1, 8, 2, 0, 7], [6, 3, 5, 4, 0, 8, 2, 1, 7], [6, 0, 5,
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('moves:', 294)
('nodes expanded:', 181203)
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('time taken:', 1.8039100170135498)

A* by Manhattan heuristic:
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('moves:', 2)
('nodes expanded:', 2)
('path:', ['left', 'left'])
('time taken:', 0.00015401840209960938)

A* by Euclidean heuristic:
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[[1, 2, 0, 3, 4, 5, 6, 7, 8], [1, 0, 2, 3, 4, 5, 6, 7, 8], [0, 1, 2, 3, 4, 5, 6, 7, 8]]
('depth:', 2)
('moves:', 2)
('nodes expanded:', 2)
('path:', ['left', 'left'])
('time taken:', 0.00010895729064941406)
shahenda@Lenovo ~/Downloads/EightPuzzle-AI $
```