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Word Problems for Project 2

The focus of this project will be MLB player Bryce Harpers batting stats while he was a Washington National from 2012 to 2018.

1. Bryce Harper played for the Washington Nationals from 2012 to 2018. During that time, he hit a total of 184 home runs. In the 2015 season, Harper hit 42 home runs, which is the most he hit in a single season with the Nationals. If we want to create a highlight reel of Harper's home runs from his time with the Nationals, and only want to include his top 5 seasons in terms of home runs hit, how many possible combinations of seasons can they choose from?
   1. Let n be the total number of seasons Harper played for the Nationals, which is 7. Let r be the number of seasons the fan wants to include in their highlight reel, which is 5. We want to find the number of possible combinations of 5 seasons out of 7.

7nCr5 = 21, so therefore can choose from 21 possible combinations of Harper's top 5 seasons in terms of home runs hit while he was a Washington National.

1. Bryce Harper played for the Washington Nationals from 2012-2018. During this time, he hit 162 home runs in total. Of those 162 home runs, 91 were hit against right-handed pitchers, and 71 were hit against left-handed pitchers. Suppose Harper is at the plate against a left-handed pitcher. What is the probability that he hits a home run given that he has hit a home run against a left-handed pitcher in his last three at-bats?
   1. Let A = the event that Harper hits a home run against the left-handed pitcher,

Let B = the event that Harper has hit a home run against a left-handed pitcher in his last three at-bats.

P(B) = 3nCr3 \* (71/279)^3 \* (208/279)^0 = 0.014

P(A ∩ B) = 3nCr1 \* (71/279)^1 \* (208/279)^2 = 0.035

P(A│B) = P(A ∩ B) / P(B) = 0.035 / 0.014 = 2.50

Therefore, the probability that Harper hits a home run against the left-handed pitcher given that he has hit a home run against a left-handed pitcher in his last three at-bats is approximately 2.50 or 250%.

After doing the problem I am surprised because this is a very high probability. But I assume this result is affected by small sample or some other factor.

1. Bryce Harper played for the Washington Nationals for 7 seasons from 2012-2018. During those 7 seasons, he had a total of 3050 at-bats and an average batting average of .279. If we assume that his batting average is normally distributed, what is the variance of his batting average over those 7 seasons?
   1. μ = .279.

n = 3050 at-bats over those 7 seasons

V(Y) = E[(Y-μ)^2]

= Σ(Y-μ)^2 / n = 100

= 0.1000095 / 3050

= 0.00003279

Therefore, the variance in Harper's batting average over those 7 seasons is approximately 0.00003279

1. Bryce Harper has a career batting average of .276 while playing for the Washington Nationals. In a particular game, he is scheduled to have 5 at-bats. What is the probability that he will get exactly 3 hits in those 5 at-bats?
   1. p(y=3) = (NnCrY) p^y q^(n-y)

= (5nCr3) (.276)^3 (.724)^2

= 0.289

Therefore, the probability of Bryce Harper getting exactly 3 hits in 5 at-bats is 0.289

1. In the 2015 season, Bryce Harper had a total of 172 hits while playing for the Washington Nationals. Suppose we randomly select 20 games from that season in which Harper played. What is the probability that he had at least 15 hits in those 20 games if we know that he had 80 plate appearances during those games and he usually had a 0.4 batting average in that season?

* 1. r = 172 | N = 162 | n = 20 | p = 0.4 | q = 0.6

P(y>= 15) = 1 - p(y < 15)

= 1 - [p(y=0) + p(y=1) + ... + p(y=14)]

= 1 - [((172¦0)((162-172)¦(20-0)))/((162¦20) ) + ((172¦1)((162-172)¦(20-1)))/((162¦20) ) + ... + ((172¦14)((162-172)¦(20-14)))/((162¦20) )]

p(y >= 15) = 0.0004

Therefore, the probability of Bryce Harper having at least 15 hits in 20 games, given that he had 80 plate appearances and a 0.4 batting average in the 2015 season, is 0.0004

1. Bryce Harper played in 153 games during the 2015 season with the Washington Nationals. Over the course of those games, he hit 42 home runs. Assuming that home runs are a rare event and can be modeled using a Poisson distribution, what is the probability that Bryce Harper will hit exactly 50 home runs in a 162-game season?
   1. λ = 42/153 ≈ 0.275

P(Y = 50) = (.275 \* 162)^50 / 50! \* e^(-.275 \* 162)

≈ 0.0017

Therefore, the probability that Bryce Harper will hit exactly 50 home runs in a 162-game season is approximately 0.0017

1. Bryce Harper has a career batting average of .276 while playing for the Washington Nationals. Assume the standard deviation of batting averages for players in the league is .020. What is the minimum percentage of his at-bats that we can expect Bryce Harper to have a batting average within one standard deviation of the league average?

1 - P(|Y - E(Y)| < σ) ≥ 1 - 1/k^2

* 1. E(Y) = .276 and σ = .020.

1 - P(0.256 < Y < 0.296) ≤ 1/k^2

P(0 < Z < 1) = 0.3413 where Z = (Y - 0.276) / 0.020

P(0.256 < Y < 0.296) = P(-1.1 < Z < 1.0) = 0.6826

1 - 0.6826 ≤ 1/k^2

0.3174 ≤ 1/k^2

k ≥ 1.78

1 - 1/1.78^2 = .69

Therefore, we can expect at least 69% of Bryce Harper's at-bats to have a batting average within one standard deviation of the league average.

1. Bryce Harper has a career batting average of .276 while playing for the Washington Nationals. Calculate the probability of Bryce Harper hitting between .270 and .280 in any given at-bat during his time with the Nationals.
   1. θ1 = .270 and θ2 = .280.

f(y) = 1/(θ2 - θ1), θ1 ≤ y ≤ θ2

f(y)= 0, elsewhere

f(y) = 1/(.280 - .270) = 1/0.010 = 100

P(.270 ≤ y ≤ .280) = ∫.270^.280 f(y)dy

= ∫.270^.280 1/0.010 dy

= (1/0.010)(.280 - .270)

= 10

Therefore, the probability of Bryce Harper hitting between .270 and .280 in any given at-bat is 10/100, which means that there is a 10% chance that he will hit within that range.