

Quiz No. 5 is
NEXT WEEK

Lecture 23

EE 421 / CS 425

Digital System Design

Fall 2025

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Quiz No. 6 is
In Last
Lecture

Topics

- **Recap** – Path Sensitization Method for Fault Test Generation
- Examples of XOR based test set generation
- Design for testability
- BIST and SCAN technique
- Pseudorandom test vector generation through ALFSR

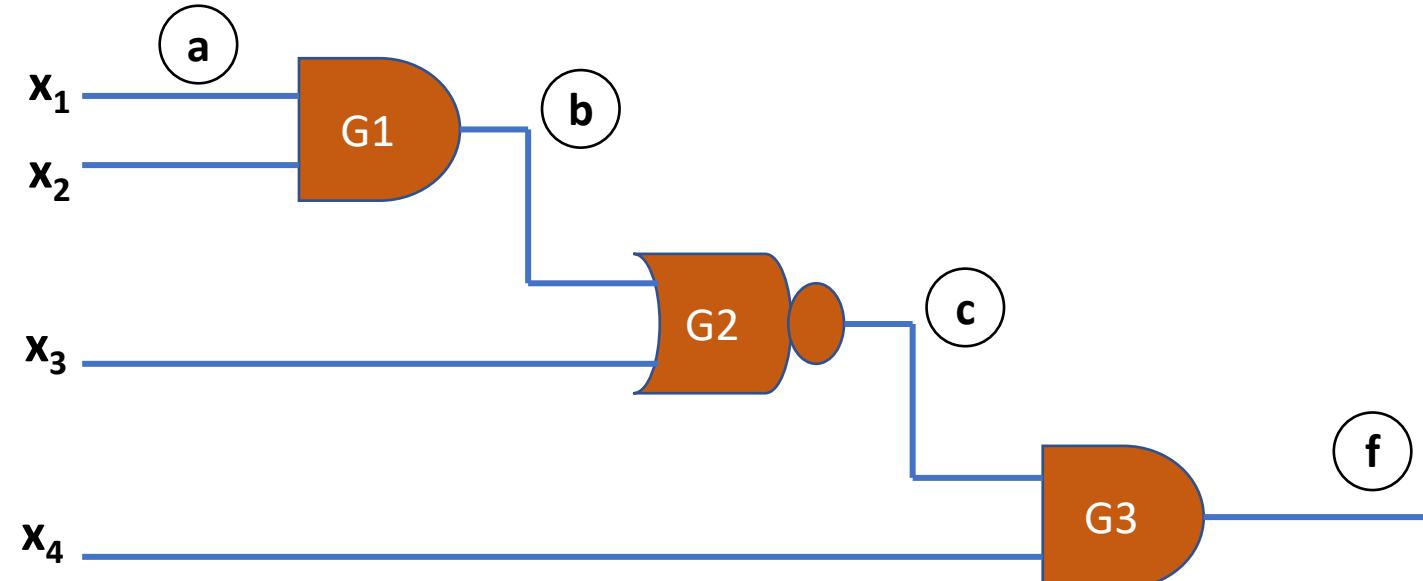
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Three Steps in Path Sensitization Method

- **Fault Excitation:** Which vector to be induced to detect the suspected SA0 or SA1 fault at the suspicious path
- **Fault Propagation:** Identify path/s through which fault can be propagated to the observable output
- **Back tracking:** Move back from output towards all inputs and assign appropriate test values

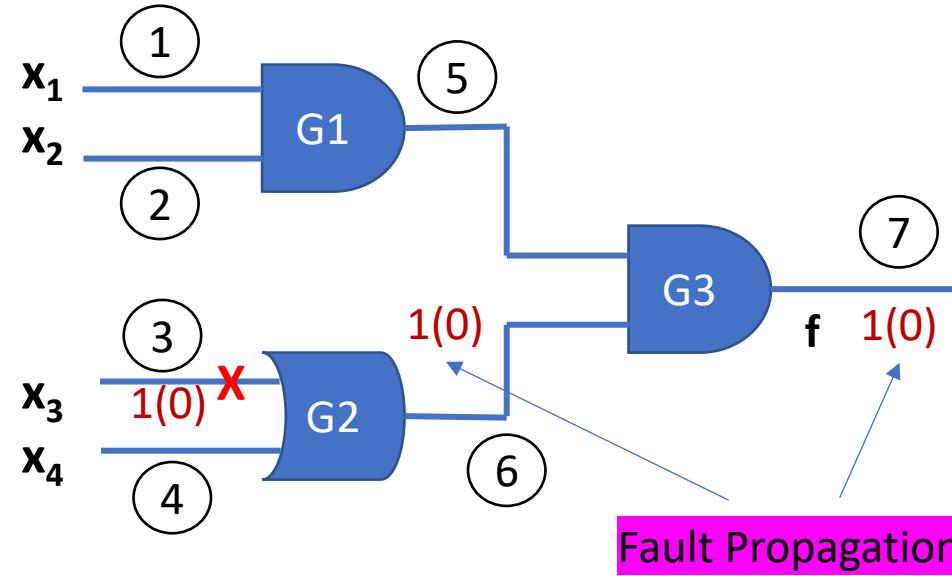
Path Sensitization – how to



To sensitize a path through an input of AND gate or NAND gate, all other inputs must be set to '1'

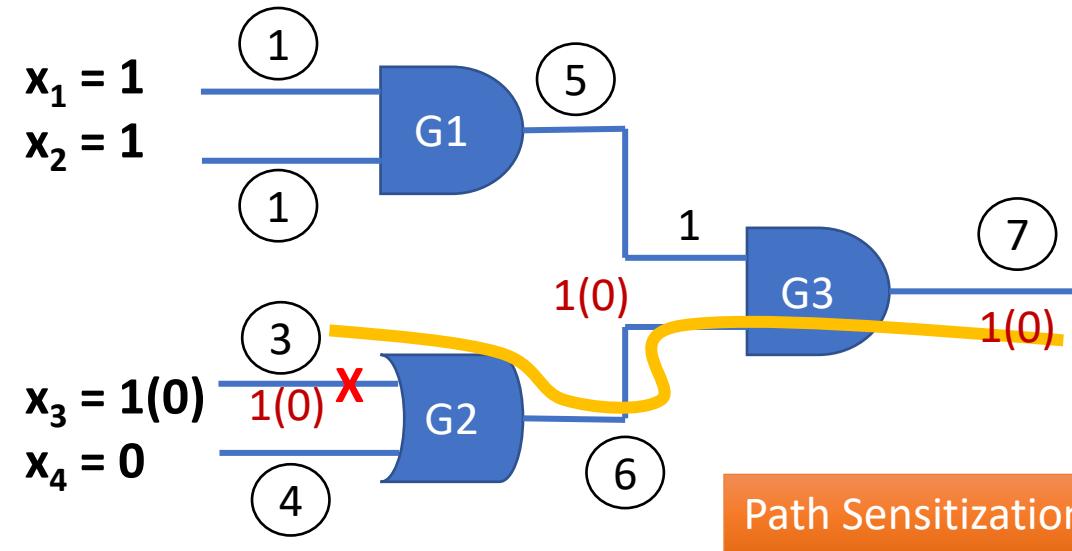
To sensitize a path through an input of OR gate or NOR gate, all other inputs must be set to '0'

Path Sensitization – Example 1



Purpose: To detect **SA0** fault at **wire 3** connected to input of OR gate
Input x_3 is selected opposite to Stuck-At fault (eg. SA0), written as **$x_3=1(0)$**
This is fault generation or excitation

Test Vectors for Example 1



Sensitized path = 3 → 6 → 7

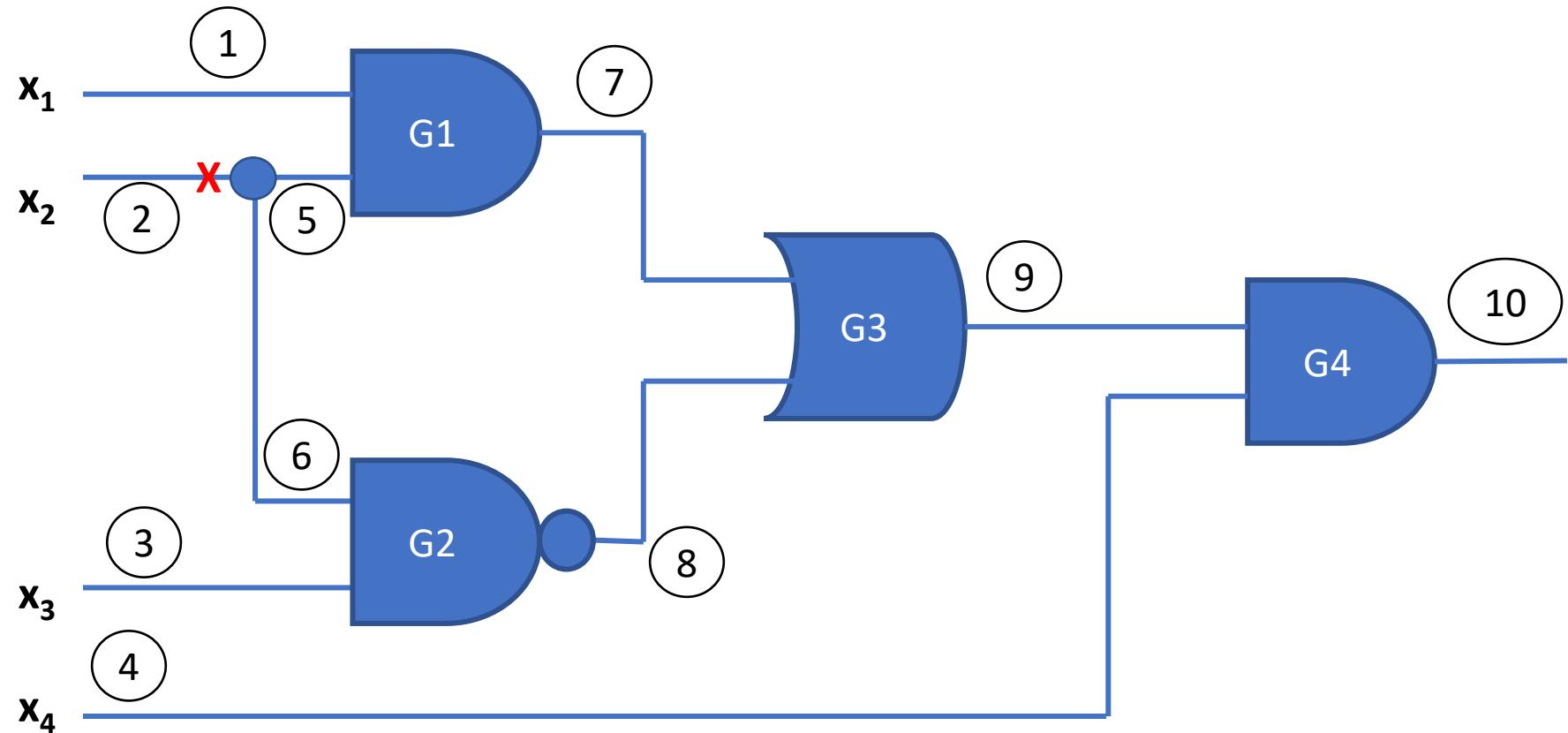
Back tracing reveals inputs to all gates to ensure fault propagation

Required test vector to detect SA0 at wire 3 is “1110”

Path Sensitization – Example 2

To detect SA1 fault at wire 2

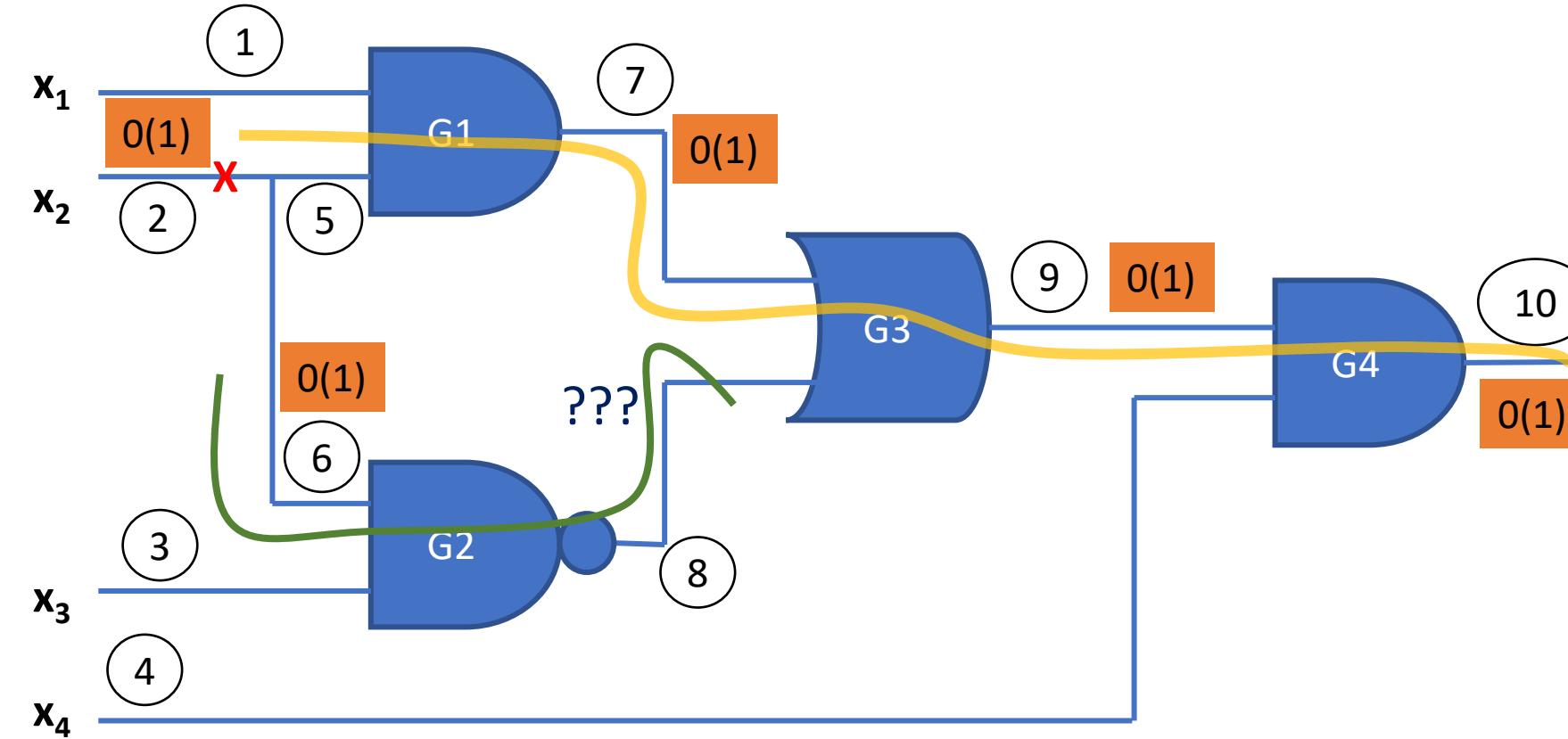
Two possible paths can be excited



continued

Detect SA1 at path 2

Fault propagation by supplying input $x_2=0(1)$



Case 1:

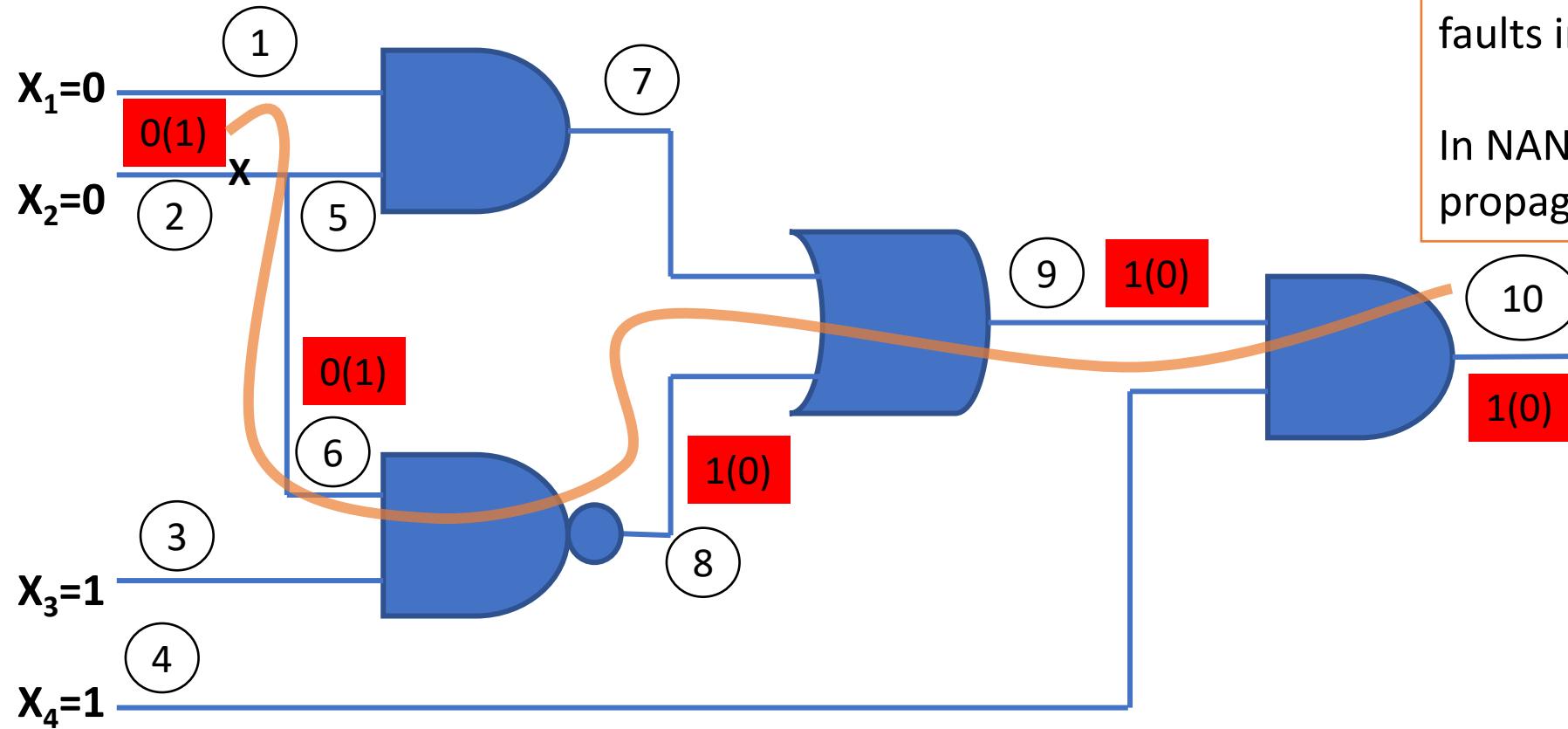
Back tracking reveals:

First Selected path = $2 \rightarrow 7 \rightarrow 9 \rightarrow 10$

But path 8=1 due to path 2 input x_2
This is not correct to have '1' at
Path 9. Hence '0' cannot be justified
at line 8 and line 2 simultaneously

This situation is '**Inconsistent**' hence
Some other path is thus required

Continued – final test vectors



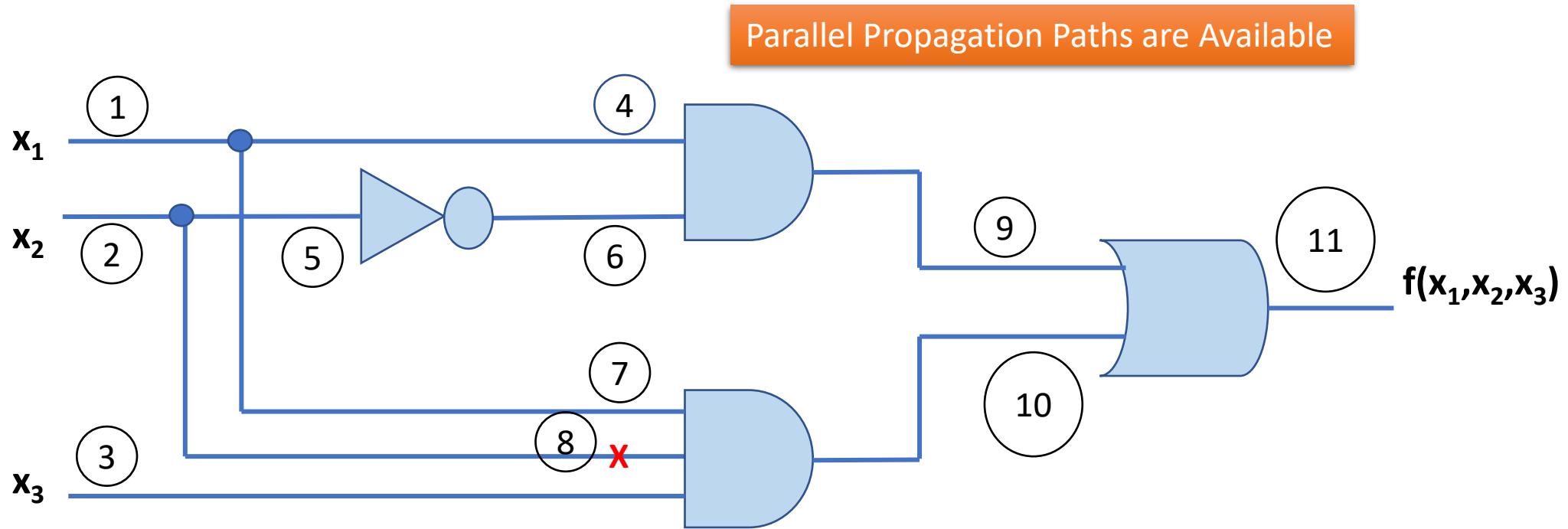
Selected path = 2 → 6 → 8 → 9 → 10

Test Vector = "0011"

This test vector can also reveal other faults in wires 6, 8 and 9

In NAND and NOR, reverse fault is propagated

Untestable Fault



Look at SA1 fault on path 8

This fault cannot be distinguished (sensitized) by changing inputs x_1 to x_3

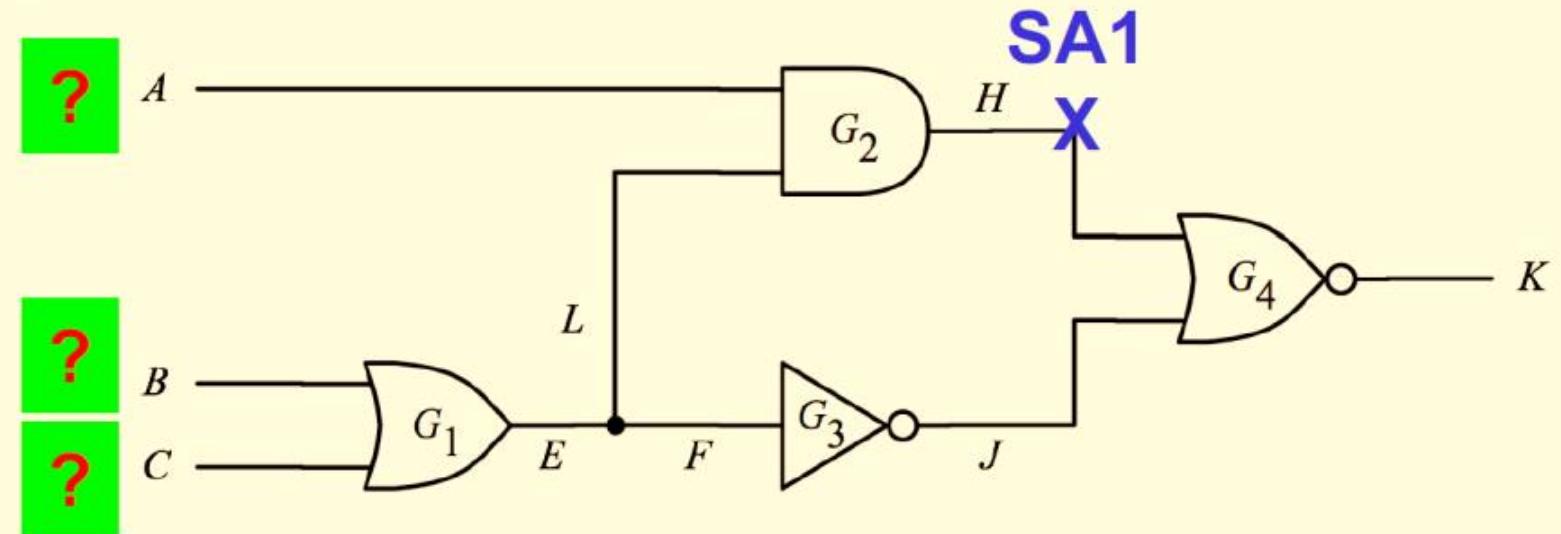
Mathematically:

An untestable fault exists when $f^{8/1} \oplus f^8 = 0$

This condition means it is not possible to test this path

Find test vector - example

- The basic principle involved in “path sensitizing” is to choose some path from the origin of the failure to the circuit output
- The path is said to be “sensitized” if the inputs to the gates along the path are assigned values so as to propagate the fault along the chosen path to the output.
- We do not need to know Boolean expression
- we can just find a test from circuit netlist



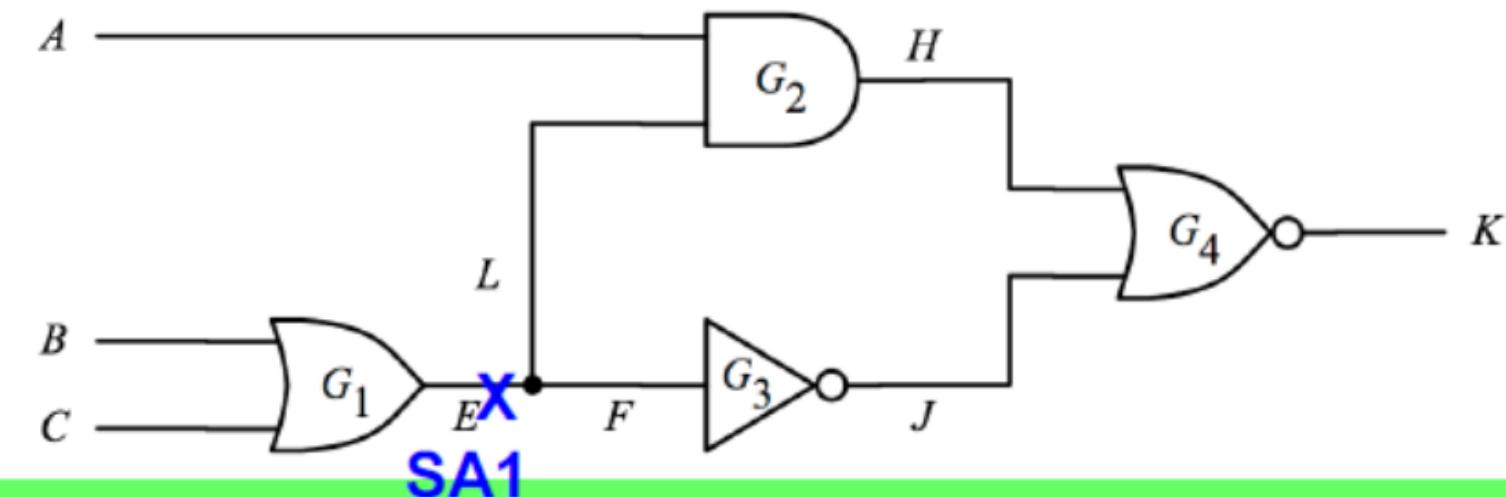
Find the test vectors – another example

Q1: Generate a test pattern for E SA1 fault. Choose path ***ELHK***.

A:

Q2: (Cont'd) Backtrack to another path ***EFJK***.

A:



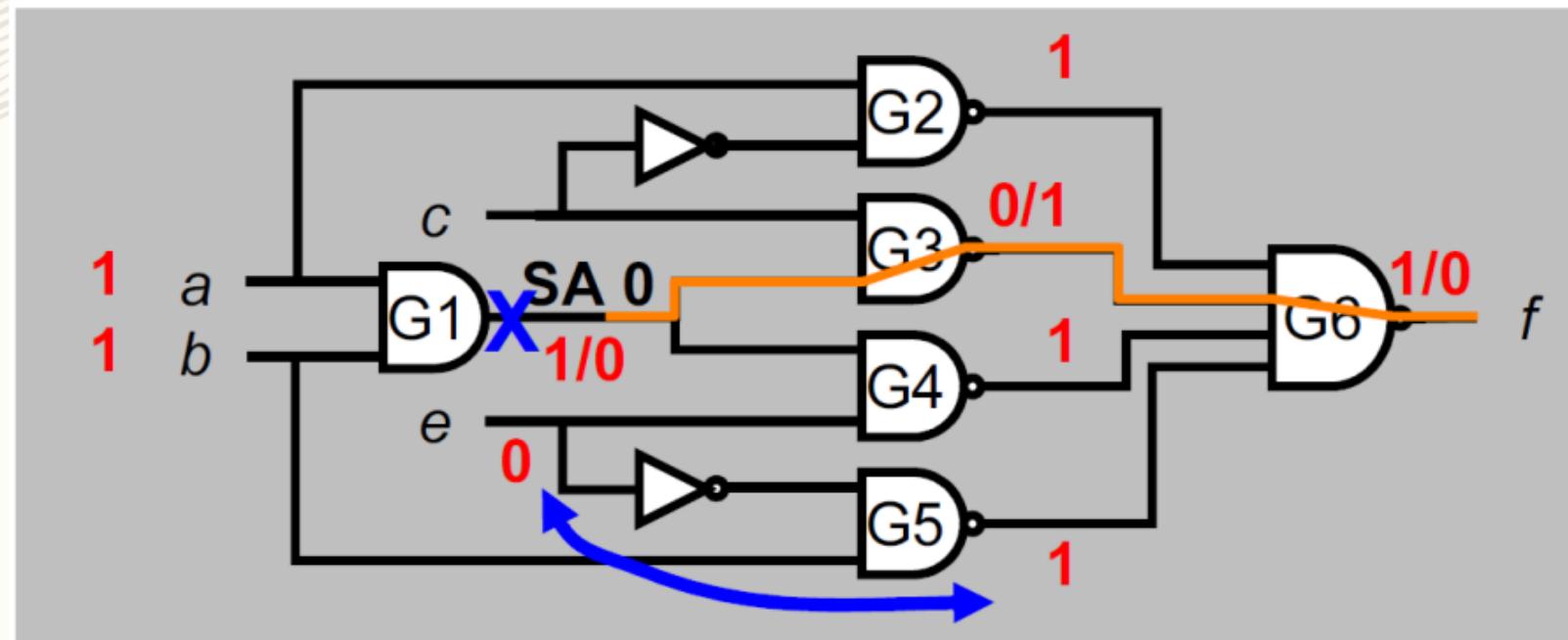
Multipath - 1

Single Path Not Enough

➤ Example :

1. Fault activation: $a = b = 1$
2. Fault propagation: Choose path {G3-G6}. want $G2 = G4 = G5 = 1$
3. $G4 = 1 \ e = 0 \ G5 = 0$ justification fails
4. Choose another path {G4-G6}. Justification also fails.

➤ SPS algorithm fails

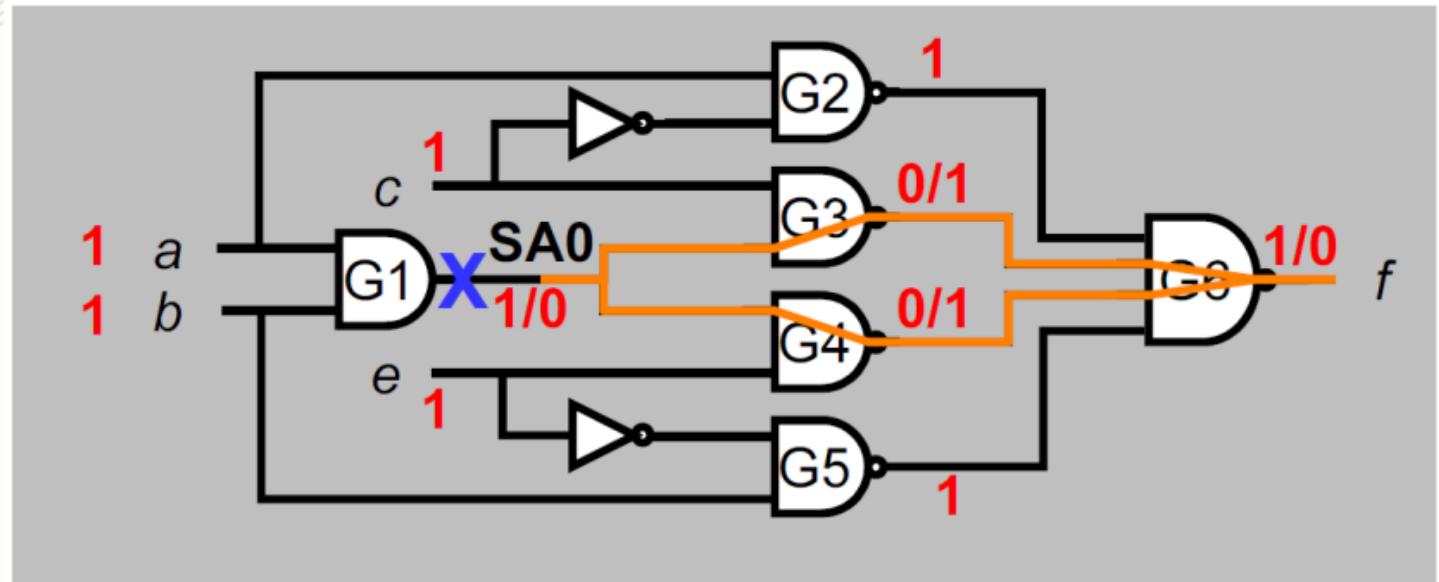


Multipath – 2

Multiple Path Sensitization

- This fault requires *multiple path sensitization*
- Both two paths {G3-G6} and {G4-G6} are sensitized
 - Error is propagated along both paths simultaneously
 - $G_2 = G_5 = 1$, $c = e = 1$
 - Test generated successfully.

<https://potharajuviyasagar.wordpress.com>



EXOR Method for Fault Test Generation

EXOR Method for Fault Test Generation

Define faulty and fault-free circuit as Boolean expressions

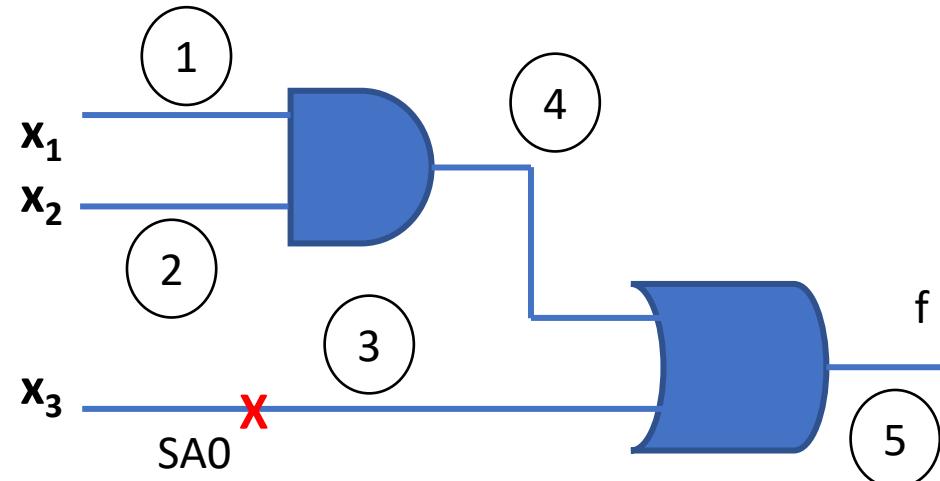
The **Exclusive OR of the two functions should be ZERO if they are same** i.e. NO FAULT

The **Exclusive OR of the two functions should be ONE if there is fault**, means different fault propagation

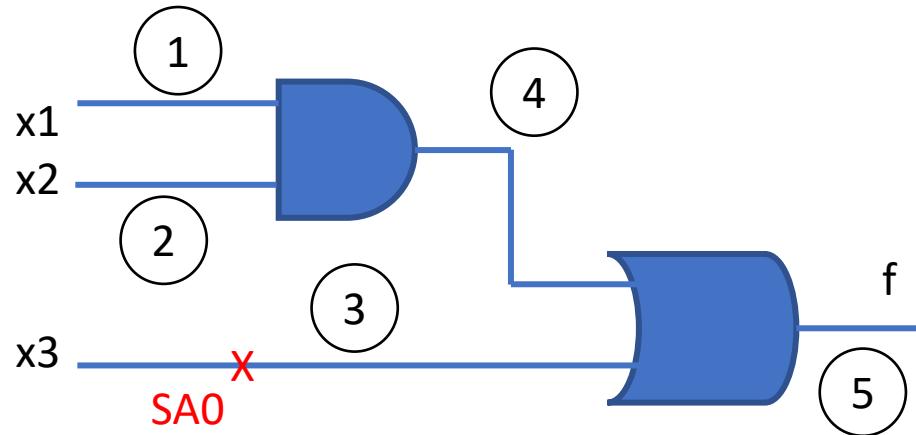
Premise:

Faulty Circuit must produce a different response from a fault-free circuit

Example SA0 fault on wire 3, input to OR gate in circuit below:



EXOR method using Boolean Algebra



$$\begin{aligned}
 F^{1/0} &= f \oplus f^{1/0} \\
 &= (x_1 \cdot x_2 + x_3) \oplus (x_3) \\
 &= x_1 \cdot x_2 \cdot x_3' \\
 \Rightarrow \text{test vector} &= 110
 \end{aligned}$$

Similarly, test for 3/0 is:

$$\begin{aligned}
 F^{3/0} &= (x_1 \cdot x_2 + x_3) \oplus (x_1 \cdot x_2) \\
 &= (x_1' + x_2') \cdot x_3 \\
 &= (x_1' \cdot x_3 + x_2' \cdot x_3)
 \end{aligned}$$

Thus tests are: 001, 011, 101

Solution

$$\begin{aligned}
 f \oplus f^3 &= (x_1 x_2 + x_3) \oplus (x_1 x_2) \\
 &= (x_1 x_2 + x_3)' \cdot (x_1 x_2) + (x_1 x_2 + x_3) \cdot (x_1 x_2)' \\
 &= \cancel{x_1} \cancel{x_2} \left(\underbrace{(x_1 x_2)' \cdot (x_3)'}_{=} \right) \left(\underbrace{x_1 x_2}_{=} \right) + \underbrace{(x_1 x_2 + x_3)}_{=} \left(\underbrace{x_1 + x_2}_{=} \right) \\
 &= 0 + \boxed{\overline{x_1} x_3 + \overline{x_2} \cdot x_3}
 \end{aligned}$$

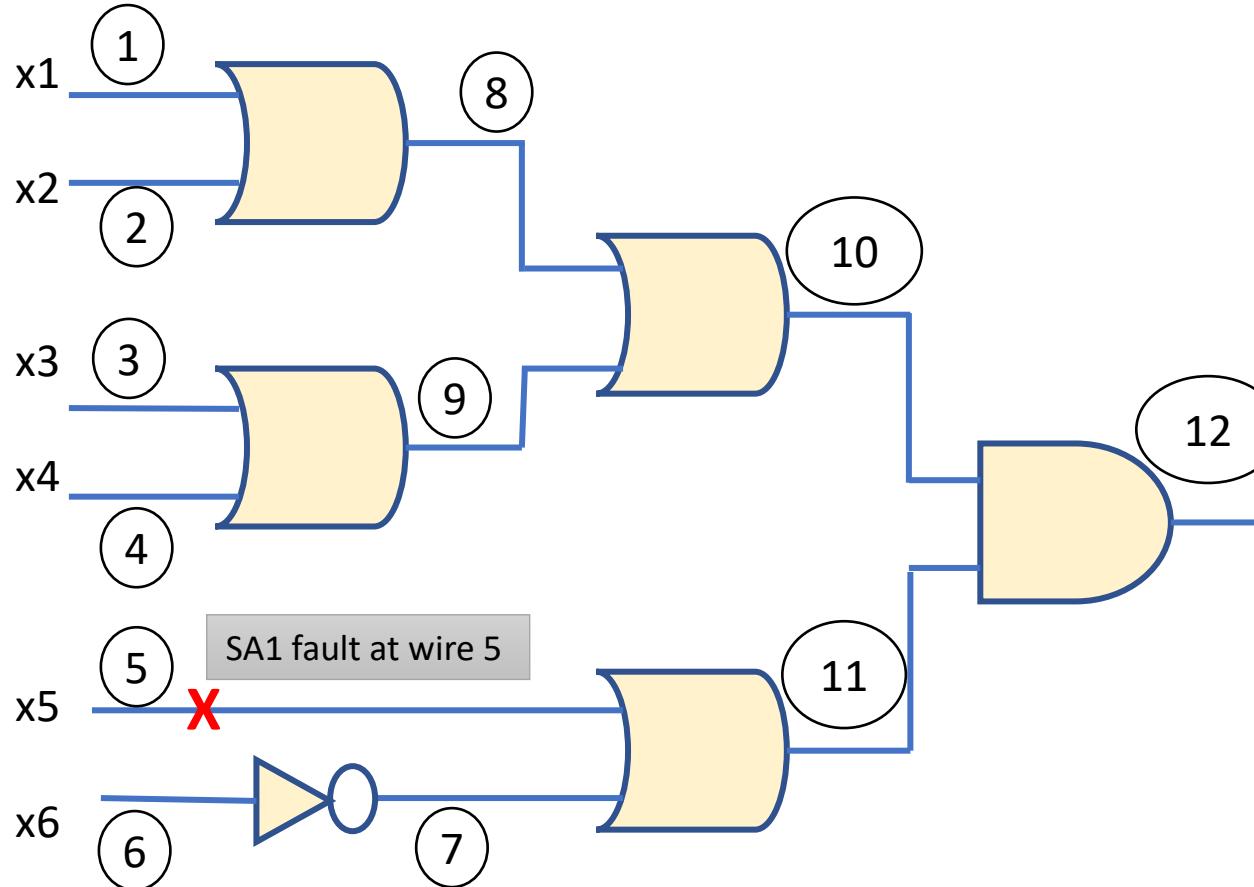
EXOR Method – The steps involved

- Step 1: Construct truth table of fault-free f and faulty $f_{p/d}$
- Step 2: Compute $f \oplus f_{p/d}$ for each row of the truth table
- Step 3: Tests (input vectors) for fault p/d are indicated by the ones in the columns corresponding to $f \oplus f_{p/d}$
- Step 4: By expressing f and $f_{p/d}$ in Boolean algebra, an expression that gives all tests for p/d can be determined

Draw Fault Table – shows a set of faults and a set of inputs

Tests (inputs)			f output	$f^{1/0}$	$f^{2/1}$	$f^{3/0}$	$f \oplus f^{1/0}$	$f \oplus f^{2/1}$	$f \oplus f^{3/0}$
x_1	x_2	x_3		SA0 at 1	SA1 at 2	SA0 at 3			
0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0	0	1
0	1	0	0	0	0	0	0	0	0
0	1	1	1	1	1	0	0	0	1
1	0	0	0	0	1	0	0	1	0
1	0	1	1	1	1	0	0	0	1
1	1	0	1	0	1	1	1	0	0
1	1	1	1	1	1	1	0	0	0

Checking Testability through EXOR Equations



$$f = (x_1 + x_2 + x_3 + x_4)(x_5 + x_6')$$

$$f^{6/0} = (x_1 + x_2 + x_3 + x_4).1$$

Test vector $F^{6/0} = f \oplus f^{6/0}$

$$= [(x_1 + x_2 + x_3 + x_4).(x_5 + x_6')] \oplus [(x_1 + x_2 + x_3 + x_4).1]$$

$$= A \cdot B + A \cdot B'$$

Note: $(x_1 + x_2 + \dots) \cdot (x_5 + \dots) \cdot (x_1 + x_2 + \dots)' = 0$

$$= (x_1 + x_2 + x_3 + x_4)(x_5' \cdot x_6)$$

$$= x_1 x_5' x_6 + x_2 x_5' x_6 + x_3 x_5' x_6 + x_4 x_5' x_6$$

Many test vectors are there for $f^{6/0}$
Eg. 100001, 010001, 001001, 000101

Solving – find test set

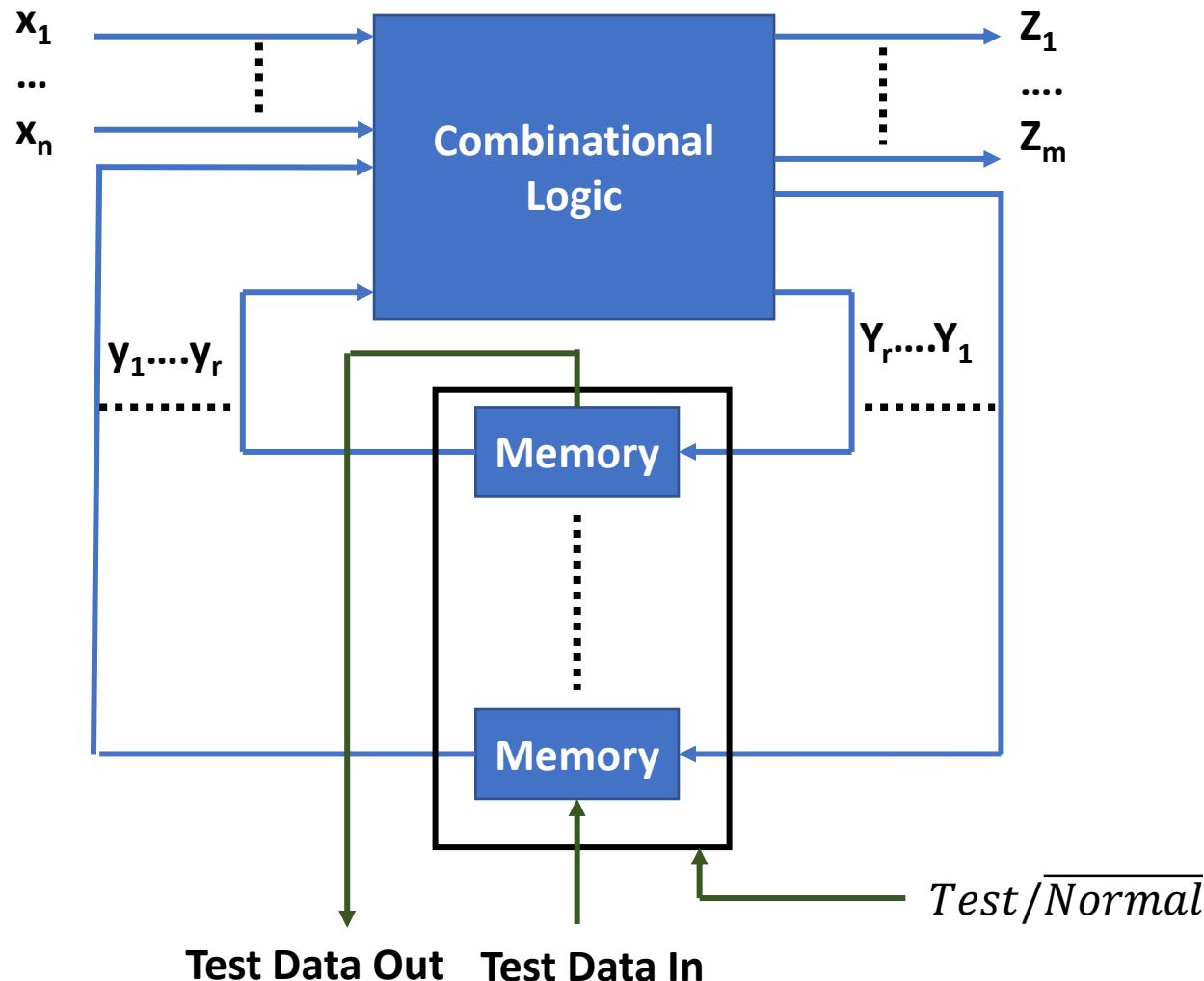
$$\begin{aligned}
 F^{6/0} &= f \oplus f^{6/0} = [(x_1 + x_2 + x_3 + x_4)(x_5 + \bar{x}_6)] \oplus [(x_1 + x_2 + x_3 + x_4)] \\
 &= [(x_1 + x_2 + x_3 + x_4)(x_5 + \bar{x}_6)]' \cdot [x_1 + x_2 + x_3 + x_4] \\
 &\quad + [(x_1 + x_2 + x_3 + x_4)(x_5 + \bar{x}_6)]. \underbrace{(x_1 + x_2 + x_3 + x_4)'}_{\text{underlined}} \\
 &= \underbrace{[(x_1 + x_2 + x_3 + x_4)' + (x_5 + \bar{x}_6)']}_{\text{underlined}} \cdot \underbrace{(x_1 + x_2 + x_3 + x_4)}_{\text{underlined}} \\
 &\therefore (x_5 + \bar{x}_6)' \cdot (x_1 + x_2 + x_3 + x_4) \\
 &= (x_5' \cdot x_6) (x_1 + x_2 + x_3 + x_4) = x_1 \bar{x}_5 x_6 + x_2 \bar{x}_5 x_6 \\
 &\quad + x_3 \bar{x}_5 x_6 + x_4 \bar{x}_5 x_6
 \end{aligned}$$

Design for Testability - Built-In Self Test BIST

Design for Testability – Built-In Self Test BIST

- Design for testability means that the **designer** introduces such features in design that can be later used for testing and debugging of circuits inside the chip
- Special circuit elements that participate in the application of test vectors and capturing of results are **integrated into the design**
- Both sequential and combinational circuits can be tested

Sequential Machines with Scan Circuits



- 'Test/Normal' isolates flipflops from internal Combinational logic and inputs $x_1 \dots x_n$
- Special I/O Test Data In and Test Data out Provide a known signature pattern that is Driven into the scan chain formed by connecting Inputs and outputs of all flipflops
- The received data at Test Data Out is compared With string provided at Test Data In after r clock cycles

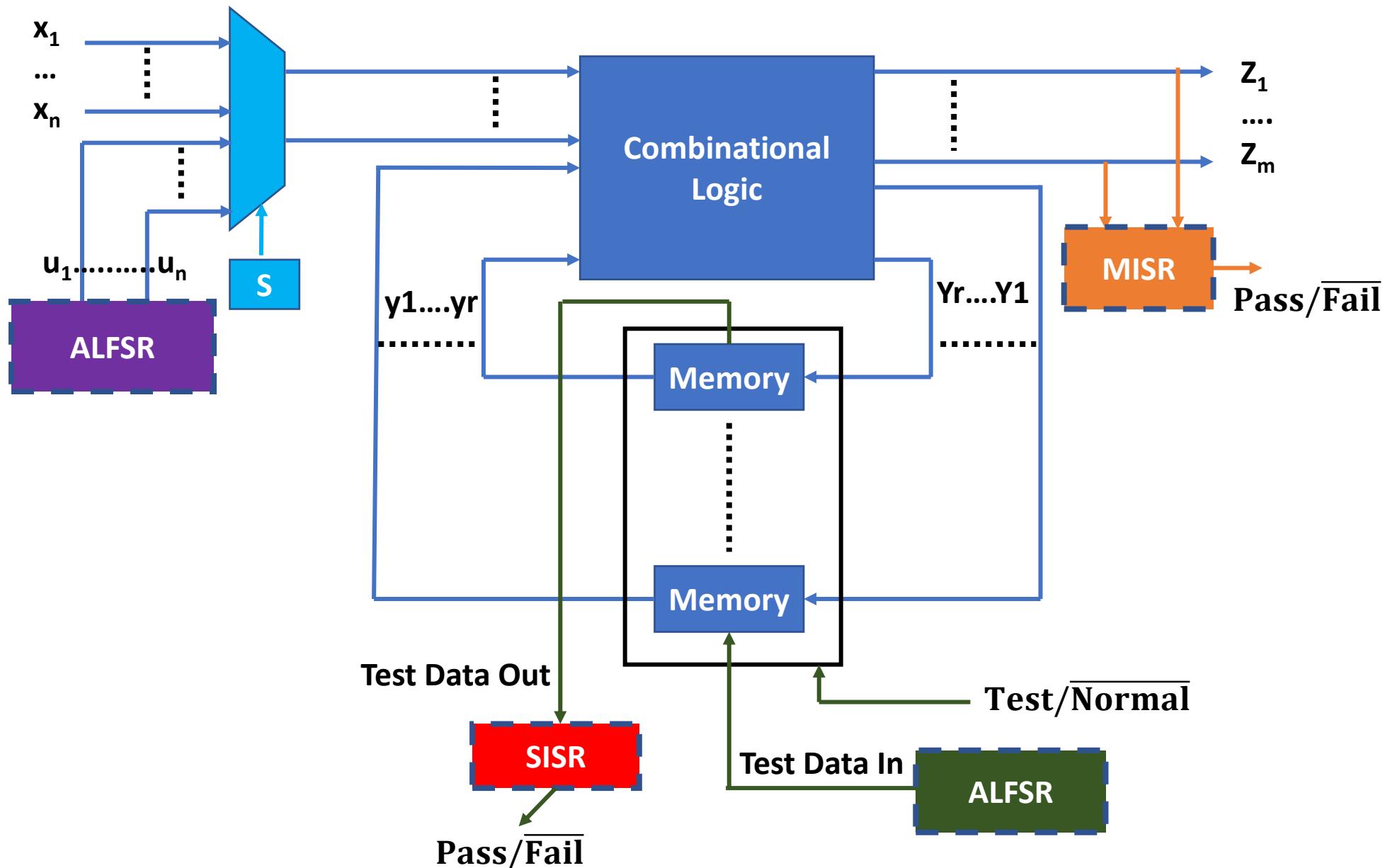
Scan Path operation for testing state machines

- Allows testing of combinational and sequential logic
- Set *Test/Normal* mode to Test
- Shift pattern into y_1, y_2, \dots, y_4 into flip flops through Scan Input to put them into known defined state
- Apply input pattern through primary inputs x_1, x_2, \dots, x_n
- Observe primary outputs z_1, z_2, \dots, z_m
- Set *Test/Normal* to Normal mode
- Clock the circuit to force one transition,
- And so on

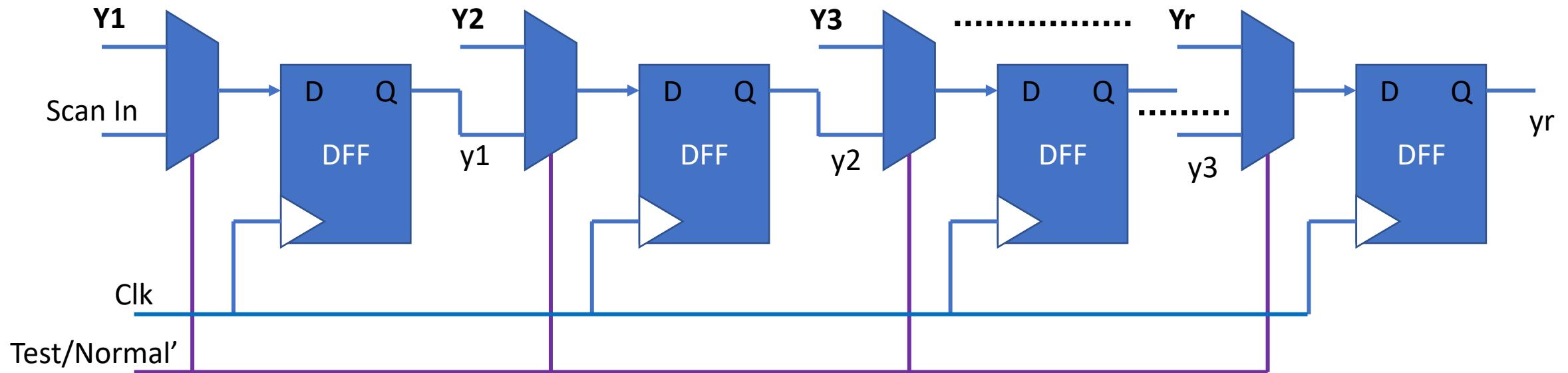
BIST – Built In Self Test

- Test pattern generator at Scan Input port as well as response capture and comparison **circuitry is placed inside the chip**. Thus, testing can be done without any other external mechanism, setting appropriate test mode on chip
- Pseudorandom test patterns are generated using ALFSR (**Autonomous Linear Feedback Shift Register**) circuits. These are applied at Scan In as well as inputs to combinational logic blocks
- Data from primary output is captured and compressed by MISR (**Multi Input Signature Register**)
- Scan Output is captured and compressed by SISR (**Serial Input Shift Register**)

Designing with BIST Circuits



Scan Path Register Design



- y_1, y_2, \dots, y_r are inputs to combinational logic
- A 2:1 Multiplexer is added to the input of each flip flop
- In Normal' mode, the Y_1, Y_2, \dots, Y_r inputs are connected to the D input of flip flops
- In Test mode, each flip flop is connected to the output of previous flip flop, forming scan chain
- Serial shift register formed in Test mode is called 'Scan Path'

Pseudorandom Test Pattern Generation

- Uses ALFSR – Autonomous Linear Feedback Shift Register
- An n-stage ALFSR produces a periodic pseudorandom sequence of n-bit binary numbers according to a special generating function that is realized through feedback lines and XOR gates within the ALFSR
- $a_{n-1}, a_{n-2}, \dots, a_0$ are the outputs of the n flip flops of the n-bit shift register with 'an' the input to the shift register equal to the exclusive OR of the feedback signals:

$$\text{i.e. } a_n = \sum_{i=0}^{n-1} a_i c_i$$

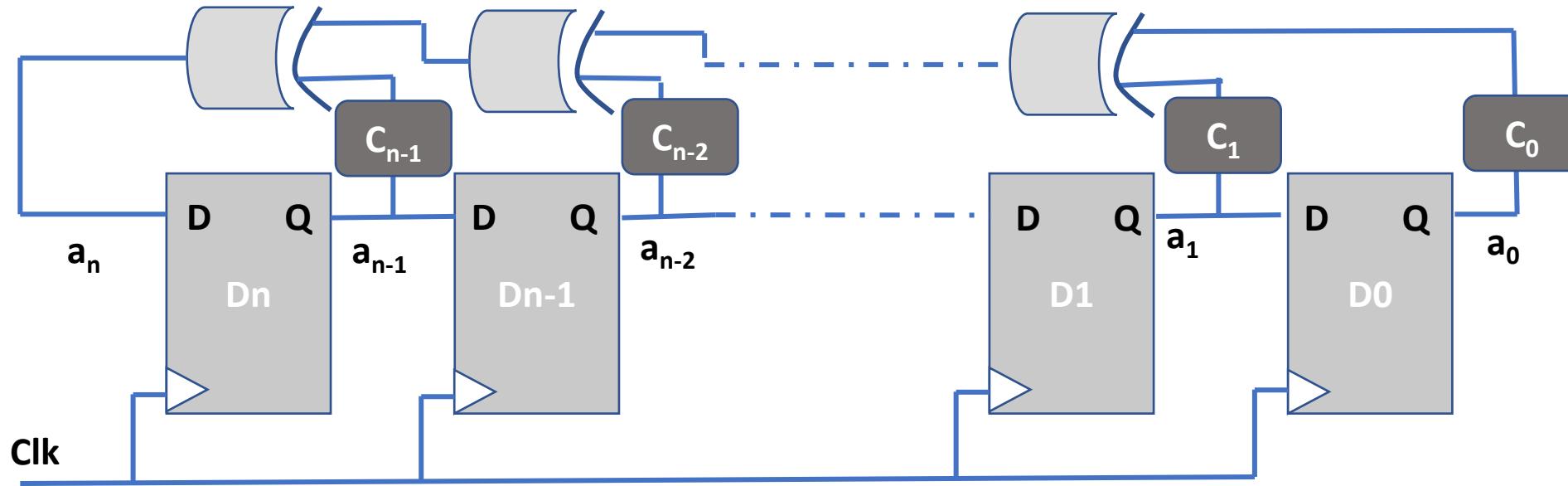
$$= a_0 c_0 \oplus a_1 c_1 \oplus a_2 c_2 \oplus \dots \oplus a_{n-1} c_{n-1}$$

The coefficients c_0, c_1, \dots, c_{n-1} represent the primitive polynomial $p(x)$

$C_i = 1$ means flip flop output a_i is connected to flip flop input through the Exclusive OR gate

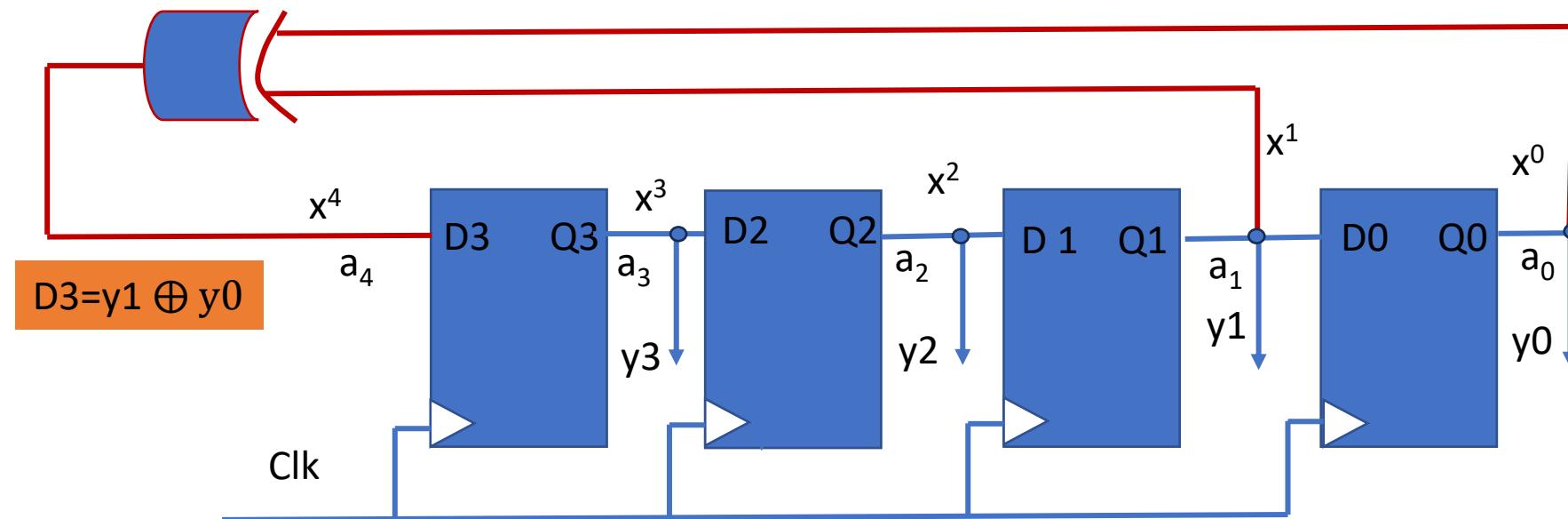
$C_i = 0$ means flip flop output is NOT connected to the feedback circuit and there is no exclusive or gate at this point

General Structure of ALFSR



$$f(x) = a_n x^n + \dots + a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0$$

Check ALFSR Sequence with given circuit or polynomial and seed value



Polynomial

$$P(x) = x^4 + x + 1$$

$$\text{Or } f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x^1 + a_0x^0$$

Use seed value $y_3y_2y_1y_0 = 1000$ that is preset at start

Generated sequence from this ALFSR circuit

D3=y1 \oplus y0

Clock	D3=(y1 \oplus y0)	y3	y2	y1	y0
1	0	1	0	0	0
2	0	0	1	0	0
3	1	0	0	1	0
4	1	1	0	0	1
5	0	1	1	0	0
6	1	0	1	1	0
7	0	1	0	1	1
8	1	0	1	0	1
9	1	1	0	1	0
10	1	1	1	0	1
11	1	1	1	1	0
12	0	1	1	1	1
13	0	0	1	1	1
14	0	0	0	1	1
15	1	0	0	0	1
16	REPEAT	1	0	0	0

Seed Value 1000