Lecture 2
EE 421 / CS 425
Digital System Design

Fall 2024

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Topics in Lecture 2

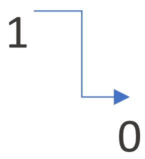
- Background to Digital Systems
- Useful number systems, conversion
- Introducing RTL Design
- The Language and Abstraction of Digital Systems
- Review Boolean Algebra
- Review K-Maps, 5 and 6 variables ??
- Logic Minimization and Synthesis
- Introducing WinLogiLab tool

Review material, use reference books:

1. Digital Systems Principles and Applications, 12th edition, Widmer, Moss, Tocci
2. Digital Design, Mano and Ciletti

Useful Number Systems

- Initial electro-mechanical computers were based on analog and decimal numbers (base 10). The computers were difficult to scale, complex, and expensive.
- Binary number system (base 2) along with Boolean Algebra was found to be the most efficient (area↔speed ↔power Δ) mechanism for computer implementation in microelectronics technology.
- Switching square waveforms in electronic circuits directly map to ‘1’ and ‘0’ in binary number system.



- In computer systems, Hex numbers (base 16) and 2’s Complement numbers are used to represent information in a compact way.

Review of Useful Number Systems

- **Decimal to Binary**

Eg. Convert 37 (base 10) to a binary number (base 2)

37 ÷ 2 = 18.5 (whole number is 18, remainder is 0.5, so put a 1 here) → 1 (LSB)
18 ÷ 2 = 9 (whole number is 9, remainder is 0 so put a zero here) → 0
9 ÷ 2 = 4.5 (whole number is 4, remainder is 0.5, put a 1 here) → 1
4 ÷ 2 = 2 (whole number is 2, remainder is 0, put a 0 here) → 0
2 ÷ 2 = 1 (whole number is 1, remainder is 0, put a 0 here) → 0
1 ÷ 2 = 0.5 (whole number is 0, remainder is 0.5, put a 1 here) → 1 (MSB)

Write MSB to LSB → 1 0 0 1 0 1
Hence $(37)_{10} = (100101)_2$

Binary to Decimal conversion

- **Convert Binary number $(1001101)_2$ to Decimal number system**
- Each bit position has a proportional weight in the power of 2

1	0	0	1	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

MSB LSB

Add up all the numbers with proportional weights

$= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 64 + 8 + 4 + 1 = \mathbf{(77)_{10}}$

Signed Binary Numbers – 2's Complement

- 2's complement of $-(0110010)_2$ Keep one or more extra '0' bit in MSB

Step 1: Take 1's complement → (1001101)

Step 2: Add '+1' to the 1's Complement

$$\begin{array}{r} 1\,0\,0\,1\,1\,1\,0\,1 \\ +\qquad\qquad\quad 1 \\ \hline (1\,0\,0\,1\,1\,1\,1\,0)_2 \\ \hline \end{array}$$

1's Complement means invert all bits

negative numbers will always have MSB = 1

Hexadecimal Number System (group of 4 bits)

Hexadecimal	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	**10**	**1010**
B	**11**	**1011**
C	**12**	**1100**
D	**13**	**1101**
E	**14**	**1110**
F	**15**	**1111**

Binary numbers up to 4 bits can be represented as single Hex digit

7-Segment Display can show Hex Digits

Converting from Hex Number System

Hex to Decimal Conversion:

$(356)_{16} = 3x16^2 + 5x16^1 + 6x16^0$

$= 768 + 80 + 6$

$\mathbf{=(854)_{10}}$

Hex to Decimal Conversion:

$(2AF)_{16} = 2x16^2 + 10x16^1 + 15x16^0$

$= 512 + 160 + 15$

$\mathbf{= (687)_{10}}$

Hex to Binary Conversion:
Straightforward

$(9F2)_{16} = \quad 9 \qquad F \qquad 2$

$\mathbf{= (1001 \;\; 1111 \;\; 0010)_2}$

Converting into Hex Number System

Binary to Hex Conversion:
Straightforward

$(1110100110)_2 =$ 0011 1010 0110
= 3 A 6

Answer = $(3A6)_{16}$

Decimal to Hex Conversion:

$(378)_{10} =$

$378 \div 16 = 23 +$ remainder of $10_{10} = \mathbf{A_{16}}$

$23 \div 16 = 1 +$ remainder of **7**

$1 \div 16 = 0 +$ remainder of **1**

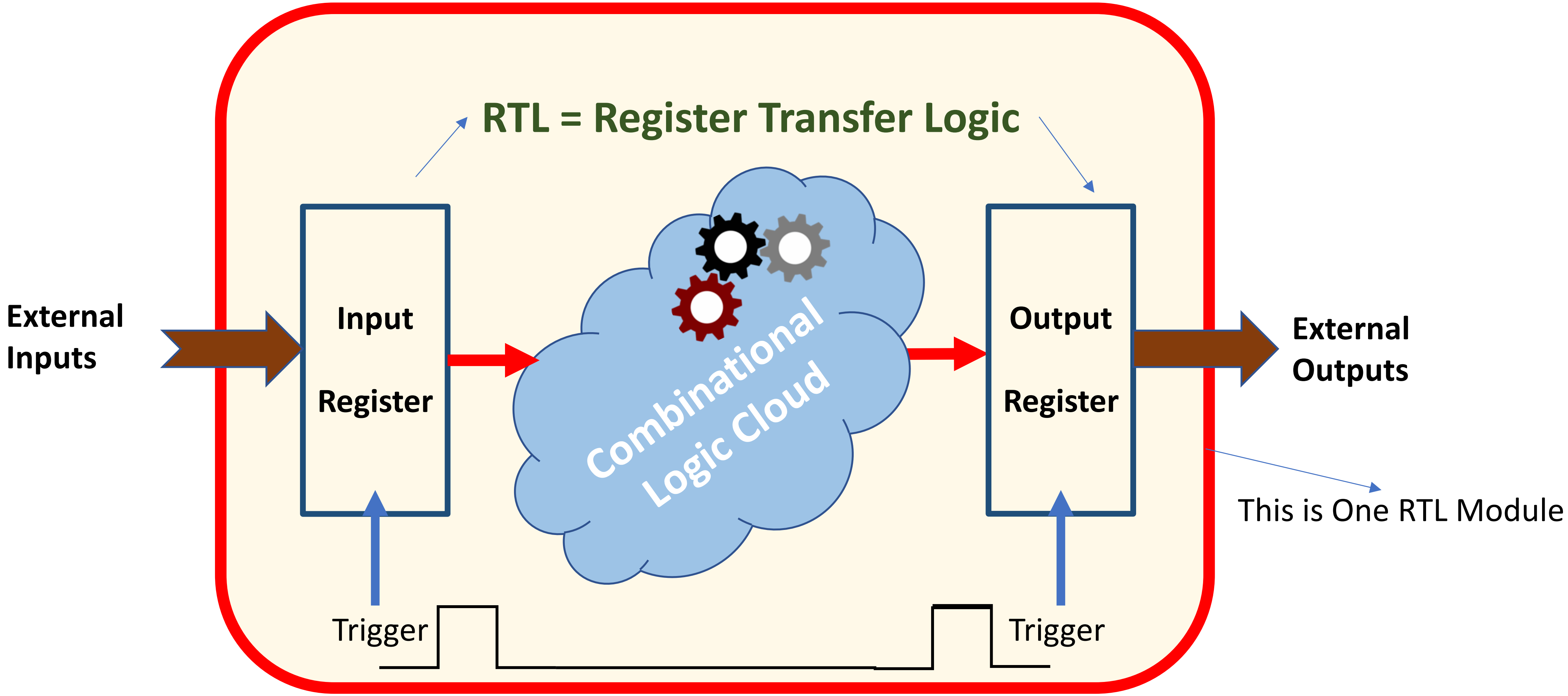
Answer = $(17A)_{16}$

In Binary form:

$(17A)_{16} = (0001\ \ 0111\ \ 1010)_2$

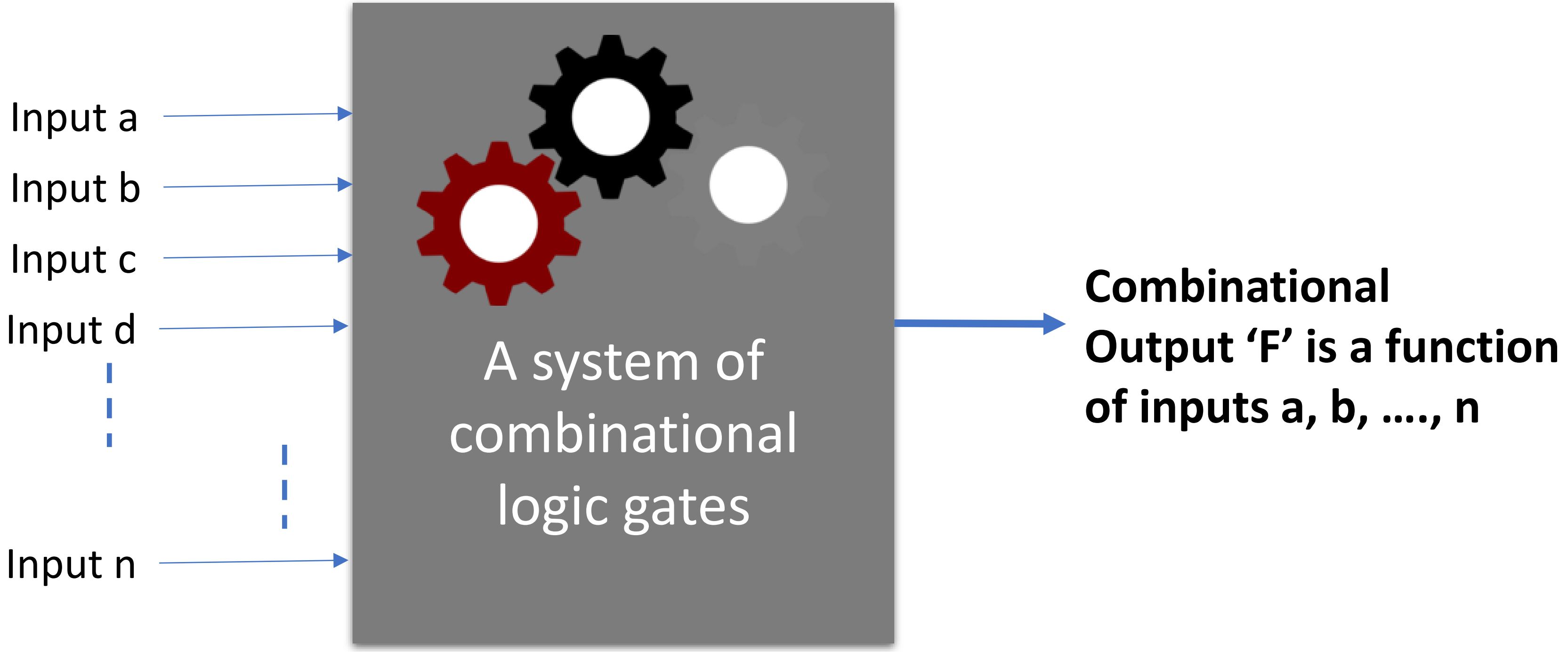
RTL View of Digital System Design

Engineering System has Inputs and Outputs



A Complex Digital System has many RTL Modules

Combinational Logic Circuits



The Language of Digital Systems

- Minterms and Maxterms
- Truth Table
- Boolean Algebra
- K-Maps
- State Tables
- State Machines and Diagrams
- Schematic Diagram
- HDL Code

Combinational Logic

Sequential Logic

All types of complex Logic Functions

linking inputs and outputs at different Abstractions and Hierarchy levels

Truth table, minterms, SOP

P	Q	R	S output	Expression	Minterms
0	0	0	1	$\bar{P}\bar{Q}\bar{R}$	m0
0	0	1	1	$\bar{P}\bar{Q}$R	m1
0	1	0	1	$\bar{P}Q\bar{R}$	m2
0	1	1	1	$\bar{P}QR$	m3
1	0	0	0	0	
1	0	1	0	0	
1	1	0	0	0	
1	1	1	1	*PQR*	m7

$S = \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}R + \bar{P}Q\bar{R} + \bar{P}QR + PQR$ **Sum of Products Expression**

$S = \sum(m0, m1, m1, m3, m7)$ **Minterms Expression**

Truth table, Maxterms, POS

P	Q	R	F output (POS)	Expression	Maxterms
0	0	0	1		
0	0	1	1		
0	1	0	1		
0	1	1	1		
1	0	0	0	$\bar{P} + Q + R$	M4
1	0	1	0	$\bar{P} + Q + \bar{R}$	M5
1	1	0	0	$\bar{P} + \bar{Q} + R$	M6
1	1	1	1		

$$F_{POS} = (\bar{P} + Q + R).(\bar{P} + Q + \bar{R}).(\bar{P} + \bar{Q} + R)$$

Product of Sum Expression

$$F_{POS} = \prod(M4.M5.M6)$$

Maxterms Expression

Output is Zero at F_{POS}

Boolean Algebra theorems

$$x + y = y + x$$

$$x.y = y.x$$

$$x + (y + z) = (x + y) + z = x + y + z$$

$$x(yz) = (xy)z = xyz$$

$$x(y + z) = xy + xz$$

$$(w + x)(y + z) = wy + xy + wz + xz$$

$$x + xy = x$$

$$\left.\begin{aligned} x + \bar{x}y &= x + y \\ \bar{x} + xy &= \bar{x} + y \end{aligned}\right\}$$

Prove through truth table

$$\left.\begin{aligned} \overline{(x + y)} &= \bar{x} . \bar{y} \\ \overline{(x.y)} &= \bar{x} + \bar{y} \end{aligned}\right\}$$

DeMorgan's Laws

Logic Minimization – using Boolean Algebra

From truth table example: $S = \bar{P}\bar{Q}\bar{R} + \bar{P}\bar{Q}R + \bar{P}Q\bar{R} + \bar{P}QR + PQR$

$$S = \bar{P}\bar{Q}(\bar{R} + R) + \bar{P}Q(\bar{R} + R) + PQR$$

Identify and Eliminate Common terms

$$S = \bar{P}\bar{Q} + \bar{P}Q + PQR$$

$$S = \bar{P}(\bar{Q} + Q) + PQR$$

$$S = \bar{P} + PQR$$

Use theorem

$$S = \bar{P} + QR$$

Final simplified expression

Logic Simplification – using Karnaugh K-Maps

2 Variable K-maps

A input	B input	X output
0	0	1
0	1	0
1	0	0
1	1	1

$$X = \overline{A}\,\overline{B} + AB$$

A, B	$\overline{B}$	B
$\overline{A}$	1	0
A	0	1

B, A	$\overline{A}$	A
$\overline{B}$	1	0
B	0	1

Three Variable K-Maps

A input	B input	C input	X output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

AB,C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	1
$\overline{A}B$	1	0
AB	1	0
$A\overline{B}$	0	0

A,BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	1	1	0	1
A	0	0	0	1

$$X = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + AB\overline{C}$$

4 Variable K-Maps

A input	B input	C input	D input	X output
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + ABCD$$

AB, CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	0	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

Different orientation - 4 Variable K-Maps

A input	B input	C input	D input	X output
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

$$X = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + ABCD$$

CD, AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	1	1	1	0
CD	0	0	1	0
$C\bar{D}$	0	0	0	0

K-Maps with Don't Care Literals

CD, AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	1	1	1	0
CD	0	X	1	0
$C\bar{D}$	0	0	0	0

A'C'D

BD

Logic Minimization – using Karnaugh K-Maps

Procedure:

Find biggest groups of adjacent ‘1’s

Edges can be folded

The literals that remain fixed in the group are written down

The literals that change, eg A to A’ are removed

All 1’s to be covered at least once

2 Variable K-maps

Example 1

A, B	$\bar{B}$	B
$\bar{A}$	1	0
A	1	0

A and A’ are literals that change, so removed

B’ remains common in the group

The minimized expression is **B’**

Example 2

B, A	$\bar{A}$	A
$\bar{B}$	0	0
B	1	1

A and A’ are literals that change, hence dropped

B remains common in the group, retained

The minimized expression is **B**

LUMS

Logic Minimization – 3 variable K-Maps

Example 3-1

AB,C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	1
$\overline{A}B$	1	0
AB	1	0
$A\overline{B}$	0	0

The 1 at A'BC' is covered in two groups, only one is sufficient
So minimization will be on two non-redundant groups
Minimized Expression is A'B' (where C and C' are removed)
And BC' (where A and A' are removed).
Hence Answer is **A'B' + BC'** (in the form of Minterms)

A,BC	$\overline{B}\overline{C}$	$\overline{B}C$	BC	$B\overline{C}$
$\overline{A}$	1	1	0	1
A	0	0	0	1

Example 3-2

23

Three groups containing two 1's each are identified
One group A'B'C' and A'BC' is redundant and can be
Left out
C and C' can be removed to give A'B' as one expression
And A and A' can be removed to give BC' as the other
Final Minterms expression is **A'B' + BC'**

More Examples – 3 variable K-Maps

Example 3-3

AB,C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	1
$\overline{A}B$	1	1
AB	0	0
$A\overline{B}$	0	0

Group of 4 1's
C' and B' change and can be removed
The answer is only **A'**

Example 3-4

AB,C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	0
$\overline{A}B$	1	0
AB	1	0
$A\overline{B}$	1	0

Another group of 4 1's
A and A' can be removed
B and B' can be removed
The answer is only **C'**

Example 3-5

AB,C	$\overline{C}$	C
$\overline{A}\overline{B}$	1	1
$\overline{A}B$	0	0
AB	0	0
$A\overline{B}$	1	1

Another group of 4 1's with folded edges
A and A' change so can be removed
C and C' change so can be removed
The answer is only **B'**

Logic Minimization – 4 Variable K-maps

Example 4-1

AB, CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
AB	0	1	1	0
$A\bar{B}$	0	0	0	0

Group of 4   1's
A' changes along vertical axis
C' changes along horizontal axis
Answer is remaining terms = **BD**

Example 4-2

AB, CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	0	0	1

Group of 4 1's with folded edges
C and C' change along horizontal axis
A and A' change along vertical axis
Answer is terms that do not change = **B'D'**

Logic Minimization – 4 Variable K-maps

Example 4-3

AB, CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

Group of 8 1's
A and A' change in vertical axis
All variables change in horizontal axis
Answer = **B**

Example 4-4

AB, CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	1	0	0	1

Group of 8 1's
Folded along horizontal edges for biggest group
All variables change along vertical axis
C and C' change along horizontal axis
Answer = **D'**

K-Maps for 5 variables ??

Starts to look complicated for manual working
For any function X=f{A,B,C,D,E}
Visualize two 4 input truth tables for {B,C,D,E},
First truth table is when A=0, second is when A=1

K-map for {B,C,D,E}

When A=0

A can be eliminated if is common for any
Literal in both K-maps

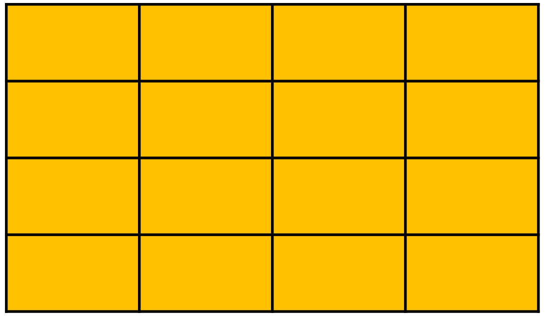
K-map for {B,C,D,E}

When A=1

K-Maps for 6 or more variables

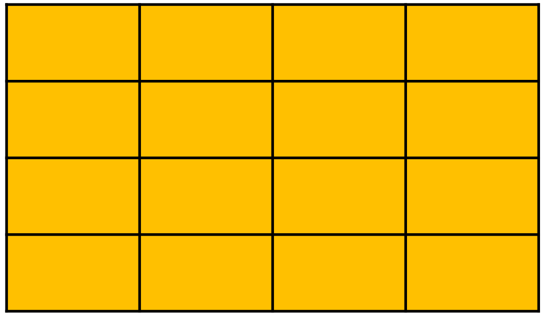
Generally computer programs are used to model multivariable K-Maps through multidimensional arrays
Conceptually, we can continue to extend 4-variable K-Maps
Eg for X=f{A,B,C,D,E,F}:
Consider there are four K-Maps of 4 variables {C,D,E,F}
There is one dedicated K-Map each for AB=00, AB=01, AB=10 and AB=11
After simplification in K-Maps, further simplification is done through Boolean Algebra

K-map for {C,D,E,F}



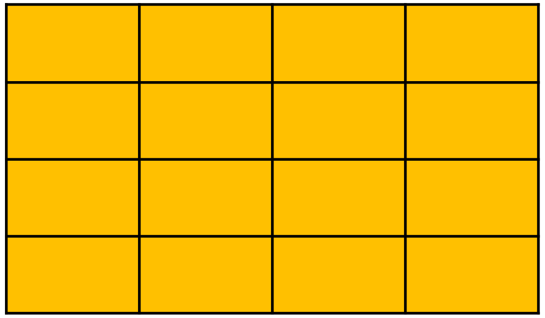
When AB=00

K-map for {C,D,E,F}



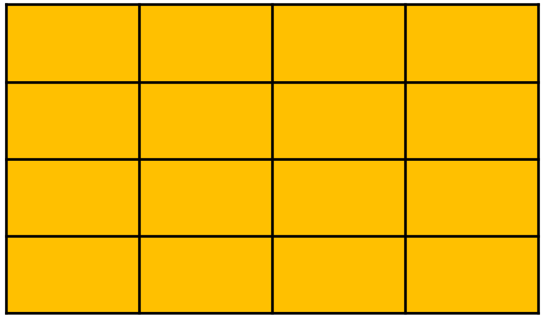
When AB=01

K-map for {C,D,E,F}
K-map for {C,D,E,F}



When AB=10

K-map for {C,D,E,F}



When AB=11

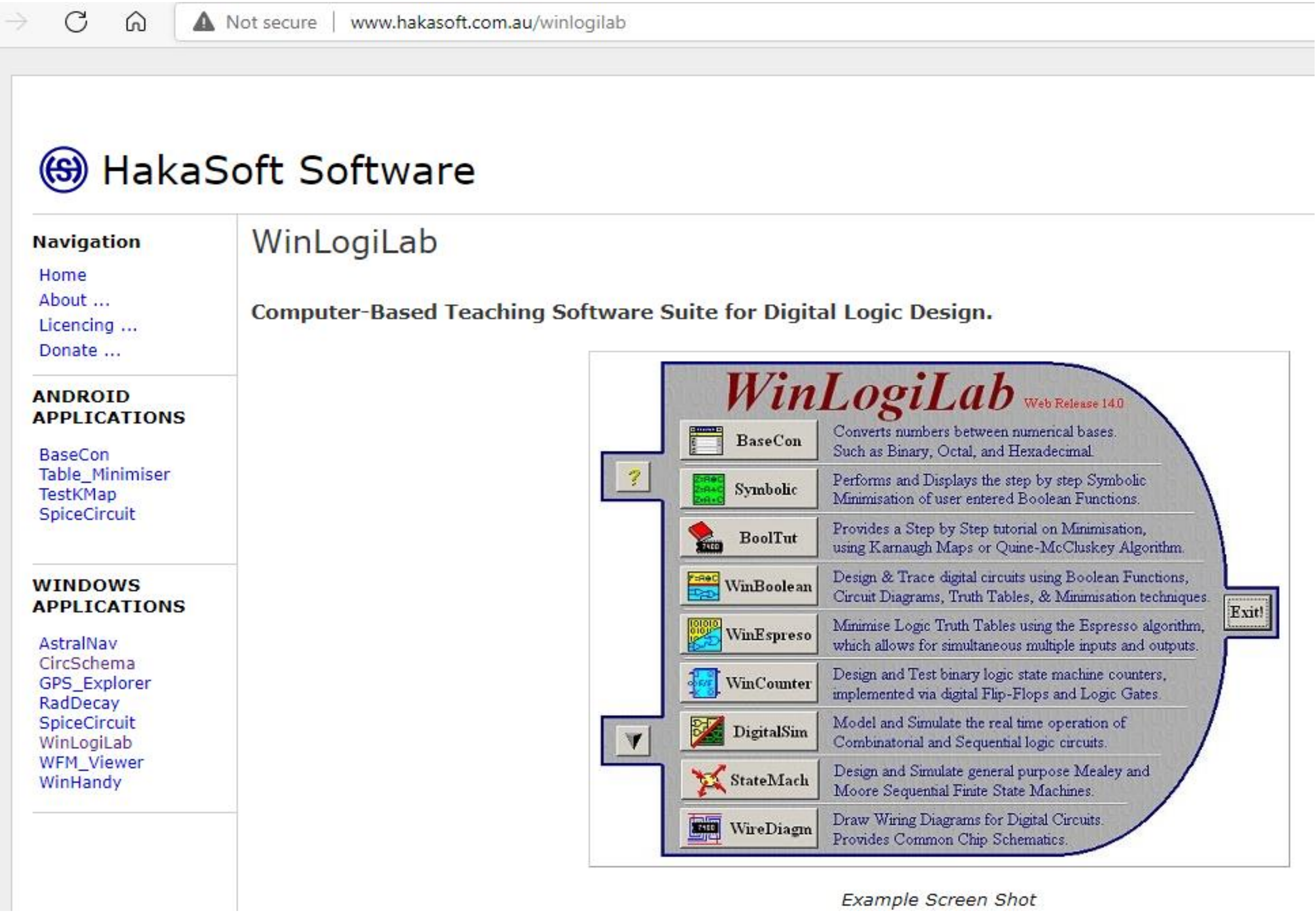
Higher level Logic Minimization

Quine McCluskey Tabular Method is a famous algorithm that is used in many computer programs
Logic Minimizer https://www.logicminimizer.com/ (up to 8 variables, tabular method)
QMC Logic Minimizer https://sourceforge.net/projects/qmclm/ (up to 16 variables, based on QM)
Logic Friday https://logic-friday.software.informer.com/ (up to 16 variables, based on Espresso)
Mini Log https://softwiki.net/Logic_Minimizer.html (up to 40 variables, based on Espresso)
Espresso uses Graph theory and Cubes
https://ptolemy.berkeley.edu/projects/embedded/pubs/downloads/espresso/index.htm
An overview document by digilent https://learn.digilentinc.com/Documents/412
WinLogiLab program can minimize using both QM and Espresso algorithms

All this leads to:

LOGIC SYNTHESIS Synopsys, Cadence (discuss further in labs)
This is built into modern design automation tools for digital design and implementation on FPGA and ASIC

UC Berkeley is considered as the source of many simulation and synthesis tools, Eg SPICE, ESPRESSO and others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SpiceCircuit
WinLogiLab
WFM_Viewer
WinHandy
WinLogiLab
Computer-Based Teaching Software Suite for Digital Logic Design.
WinLogiLab
Web Release 14.0
BaseCon
Converts numbers between numerical bases. Such as Binary, Octal, and Hexadecimal.
Symbolic
Performs and Displays the step by step Symbolic Minimisation of user entered Boolean Functions.
BoolTut
Provides a Step by Step tutorial on Minimisation, using Karnaugh Maps or Quine-McCluskey Algorithm.
WinBoolean
Design & Trace digital circuits using Boolean Functions, Circuit Diagrams, Truth Tables, & Minimisation techniques.
WinEspreso
Minimise Logic Truth Tables using the Espresso algorithm, which allows for simultaneous multiple inputs and outputs.
WinCounter
Design and Test binary logic state machine counters, implemented via digital Flip-Flops and Logic Gates.
DigitalSim
Model and Simulate the real time operation of Combinatorial and Sequential logic circuits.
StateMach
Design and Simulate general purpose Mealey and Moore Sequential Finite State Machines.
WireDiagm
Draw Wiring Diagrams for Digital Circuits. Provides Common Chip Schematics.
Exit!
Example Screen Shot