

CS/EE 320 Computer Organization and Assembly Language Spring 2024

Shahid Masud Lecture 9

Topics

- Logic and Arithmetic Instructions in Assembly Language
- Binary Numbers, Logic Design and Boolean Algebra
- Basic digital logic functions Invert, AND, OR, NAND, NOR
- Half-Adder (without Carry-In)
- 1-bit Full Adder (with Carry-In)
- Making 2's Complement Subtractor from Full-Adder
- Design of 1-bit ALU containing AND gate, OR gate, Full Adder
- Quiz 2

MIPS Arithmetic Instructions



Category	Instruction	Example		Meaning	Comments
	add	add	\$s1,\$s2,\$s3	\$s1 = \$s2 + \$s3	Three operands; overflow detected
	subtract	sub	\$s1,\$s2,\$s3	\$s1 = \$s2 - \$s3	Three operands; overflow detected
	add immediate	addi	\$s1,\$s2,100	\$s1 = \$s2 + 100	+ constant; overflow detected
	add unsigned	addu	\$s1,\$s2,\$s3	\$s1 = \$s2 + \$s3	Three operands; overflow undetected
	subtract unsigned	subu	\$s1,\$s2,\$s3	\$s1 = \$s2 - \$s3	Three operands; overflow undetected
	add immediate unsigned	addiu	\$s1,\$s2,100	\$s1 = \$s2 + 100	+ constant; overflow undetected
	move from coprocessor register	mfc0	\$s1,\$epc	\$s1 = \$epc	Copy Exception PC + special regs
Arithmetic	multiply	mult	\$s2,\$s3	Hi, Lo = \$s2 × \$s3	64-bit signed product in Hi, Lo
	multiply unsigned	multu	\$ s2, \$ s3	Hi, Lo = $$s2 \times $s3$	64-bit unsigned product in Hi, Lo
	divide	div	\$ s2, \$ s3	Lo = \$s2 / \$s3, Hi = \$s2 mod \$s3	Lo = quotient, Hi = remainder
	divide unsigned	divu	\$s2,\$s3	Lo = \$s2 / \$s3, Hi = \$s2 mod \$s3	Unsigned quotient and remainder
	move from Hi	mfhi	\$ s1	\$s1 = Hi	Used to get copy of Hi
	move from Lo	mflo	\$ s1	\$s1 = Lo	Used to get copy of Lo

MIPS Logical Instructions



Logical	AND	AND	\$s1,\$s2,\$s3	\$s1 = \$s2 & \$s3	Three reg. operands; bit-by-bit AND
	OR	OR	\$s1,\$s2,\$s3	\$s1 = \$s2 \$s3	Three reg. operands; bit-by-bit OR
	NOR	NOR	\$s1,\$s2,\$s3	\$s1 = ~ (\$s2 \$s3)	Three reg. operands; bit-by-bit NOR
	AND immediate	ANDi	\$s1,\$s2,100	\$s1 = \$s2 & 100	Bit-by-bit AND with constant
	OR immediate	ORi	\$s1,\$s2,100	\$s1 = \$s2 100	Bit-by-bit OR with constant
	shift left logical	s11	\$s1,\$s2,10	\$s1 = \$s2 << 10	Shift left by constant
	shift right logical	srl	\$s1,\$s2,10	\$s1 = \$s2 >> 10	Shift right by constant

Arithmetic for Computers

- Operations on integers
 - Addition and subtraction
 - Multiplication and division
 - Dealing with overflow
- Floating-point real numbers (later)
 - Representation and operations

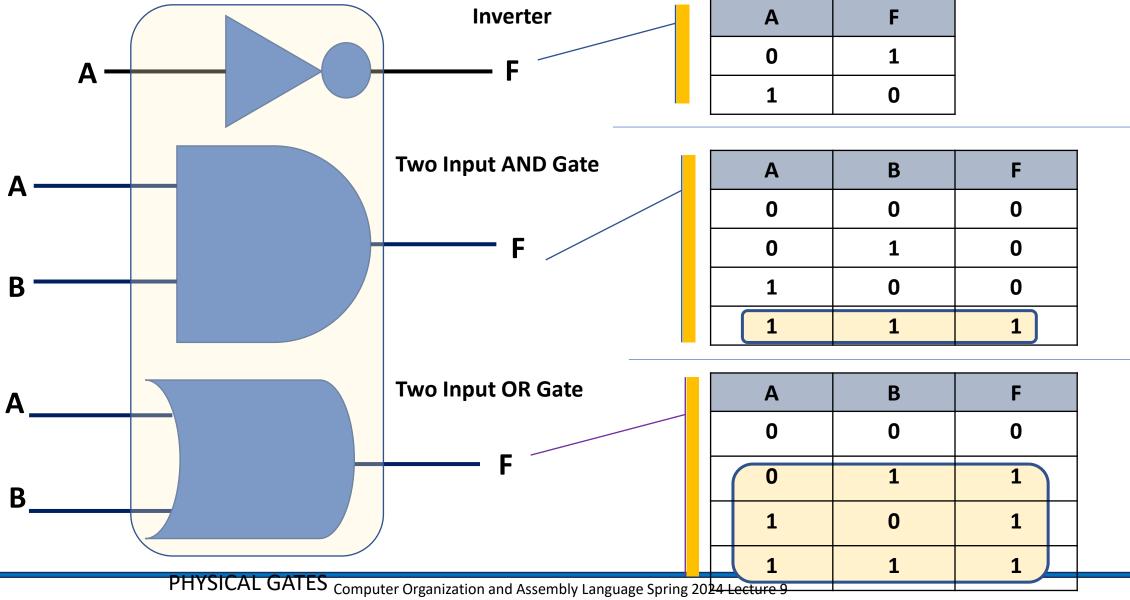
Basic Logic Operations in CPU



- Functions Through Logic Gates
 - NOT
 - AND
 - OR
 - NOR
 - NAND
 - XOR

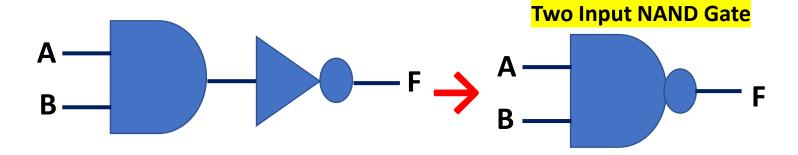
Gates (Primary Gates)





Gates (Compound Gates)





		•
0	0	1
0	1	1
1	0	1
1	1	0

AND Gate followed by Inverter

A	Two Input NOR Gate
В	F
D	OR Gate followed by Inverter
	on date followed by lifetier

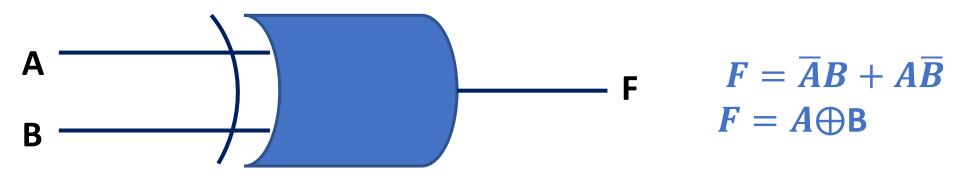
Α	В	F
0	0	1
0	1	0
1	0	0
1	1	0

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Complex Gates – the XOR and XNOR



Two Input Exclusive OR Gate



Α	В	F
0	0	0
0	1	1
1	0	1
1	1	0

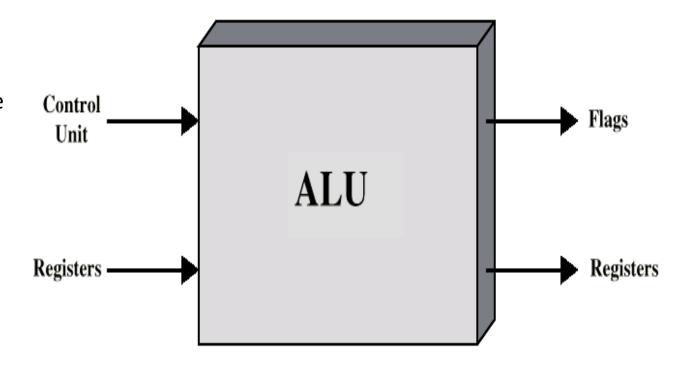
Arithmetic & Logic Unit

- Does the calculations
- Everything else in the computer is there to service this unit
- Handles integers
- May handle floating point (real) numbers
- May be separate FPU (maths coprocessor)
- May be on chip separate FPU (Eg. 486DX)

ALU – Arithmetic and Logic Unit

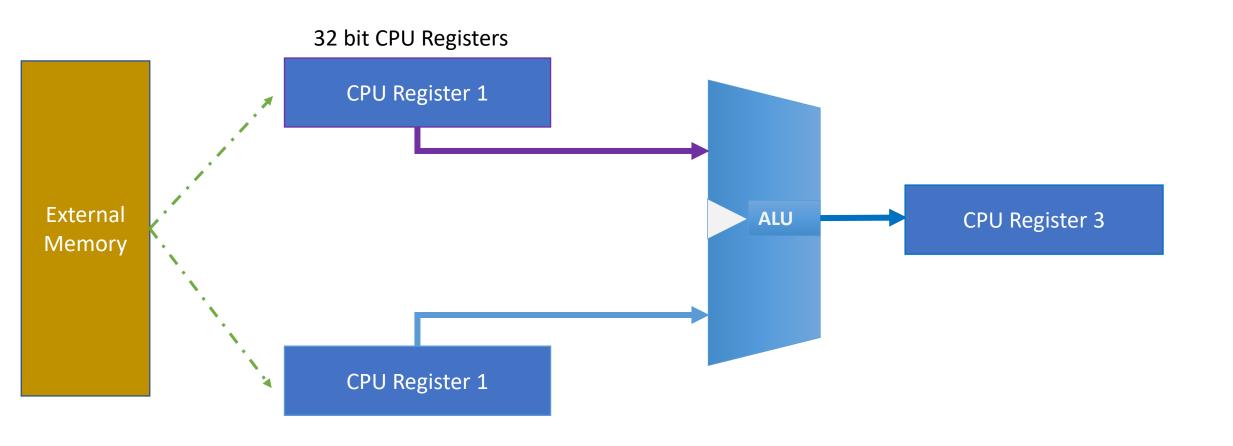


- Does the calculations
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ALU in Load/Store Architecture





Number Systems



- ALU does calculations with binary numbers
- Decimal number system
 - Uses 10 digits (0,1,2,3,4,5,6,7,8,9)
 - In decimal system, a number 84, e.g., means 84 = (8x10) + 4
 - 4728 = (4x1000) + (7x100) + (2x10) + 8
 - Base or radix of 10: each digit in the number is multiplied by 10 raised to a power corresponding to that digit's position
 - E.g. $83 = (8x10^1) + (3x10^0)$
 - $4728 = (4x10^3) + (7x10^2) + (2x10^1) + (8x10^0)$

Binary Number System



- Uses only two digits, 0 and 1
- It is base or radix of 2
- Each digit has a **value** depending on its **position**:
 - $10_2 = (1 \times 2^1) + (0 \times 2^0) = 2_{10}$
 - $11_2 = (1x2^1) + (1x2^0) = 3_{10}$
 - $100_2 = (1x2^2) + (0x2^1) + (0x2^0) = 4_{10}$
 - $1001.101_2 = (1x2^3) + (0x2^2) + (0x2^1) + (1x2^0) + (1x2^{-1}) + (0x2^{-2}) + (1x2^{-3}) = 9.625_{10}$

Decimal to Binary conversion



- Integer and fractional parts are handled separately,
 - Integer part is handled by repeating division by 2
 - Factional part is handled by repeating multiplication by 2
- E.g. convert decimal 11.81 to binary
 - Integer part 11
 - Factional part .81

Decimal number to Binary



- Fractional values, e.g.
 - $472.83 = (4x10^2) + (7x10^1) + (2x10^0) + (8x10^{-1}) + (3x10^{-2})$

In general, for the decimal representation of

$$X = \{... x_2 x_1 x_0. x\}$$

Whole Number Part

Fractional Part

$$X = \sum_{i} x_{i} 10^{i}$$

Decimal to Binary conversion example



- e.g. 11.81 to 1011.11001 (approx)
 - 11/2 = 5 remainder 1
 - 5/2 = 2 remainder 1
 - 2/2 = 1 remainder 0
 - 1/2 = 0 remainder 1
 - Binary number 1011
 - .81x2 = 1.62 integral part 1
 - .62x2 = 1.24 integral part 1
 - .24x2 = 0.48 integral part 0
 - .48x2 = 0.96 integral part 0
 - .96x2 = 1.92 integral part 1
 - Binary number .11001 (approximate)

Hexadecimal Notation



- Compact representation of bus information
- Use 16 digits, (0,1,3,...9,A,B,C,D,E,F)
- $1A_{16} = (1_{16} \times 16^1) + (A_{16} \times 16^0)$ = $(1_{10} \times 16^1) + (10_{10} \times 16^0) = 26_{10}$
- Convert group of four binary digits to/from one hexadecimal digit,
 - 0000=0; 0001=1; 0010=2; 0011=3; 0100=4; 0101=5; 0110=6; 0111=7; 1000=8; 1001=9; 1010=A; 1011=B; 1100=C; 1101=D; 1110=E; 1111=F;
- e.g.
 - 1101 1110 0001. 1110 1101 = DE1.DE

Integer Representation +/- numbers



- Only have 0 & 1 to represent everything
- Positive numbers stored in binary
 - e.g. 41=00101001
- No minus sign
- No period
- How to represent negative number?
 - Sign-Magnitude
 - Two's compliment

Sign-Magnitude Representation



- Add one extra bit on MSB to represent sign
- Left most bit is sign bit
- 0 means positive
- 1 means negative
- +18 = 00010010
- -18 = **1**0010010
- Problems
 - Need to consider both sign and magnitude in arithmetic
 - Two representations of zero (+0 and -0)

Two's Compliment Representation



- +3 = 00000011
- +2 = 00000010
- +1 = 00000001
- +0 = 00000000
- -1 = 11111111
- -2 = 11111110
- -3 = 11111101

2's Compliment Number System



- One representation of zero
- Arithmetic works easily, use same hardware as positive numbers
- Negating is fairly easy (2's compliment operation)
 - 3 = 00000011
 - Boolean **complement** gives 11111100
 - Add +1 to LSB 11111101

Range of 2's Compliment Numbers



- 8 bit 2's compliment
 - $+127 = 011111111 = 2^7 -1$
 - $-128 = 10000000 = -2^7$
- 16 bit 2's compliment

Different word lengths in 2's Compliment

- Positive number pack with leading zeros
- +18 = 00010010 (in 8 bits)
- +18 = 00000000 00010010 (in 16 bits)
- Negative numbers pack with leading ones
- -18 = 10010010 (in 8 bits)
- -18 = 11111111 10010010 (in 16 bits, notice sign bit replication)
- i.e. pack higher bits with MSB (sign bit)

Negation Special Case 1



- 0 = 00000000
- Bitwise not 11111111
- Add 1 to LSB +1
- Result <u>1</u> 0000000
- Overflow is ignored, so:
- -0 = 0 OK!

Negation Special Case 2



- -128 = 10000000
- bitwise not 01111111
- Add 1 to LSB +1
- Result 10000000
- So:
- -(-128) = -128 **Problem here!**
- Monitor MSB (sign bit)
- It should change during negation
- >> There is no representation of +128 in this case. (no $+2^n$)

Addition and Subtraction

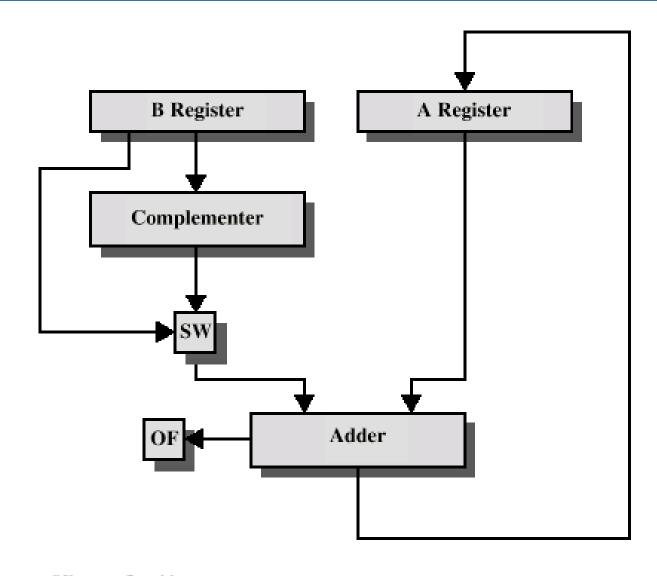


Normal binary addition

- 0011 0101 1100
- +0100 +0100 +1111
- -----
- 0111 $\underline{1}001 = \text{overflow} \ \underline{1}1011$
- Monitor sign bit for overflow (sign bit change as adding two positive numbers or two negative numbers.)
- Subtraction: Take twos compliment of subtrahend then add to minuend
 - i.e. a b = a + (-b)
- So we only need addition and complement circuits

Hardware for Addition and Subtraction



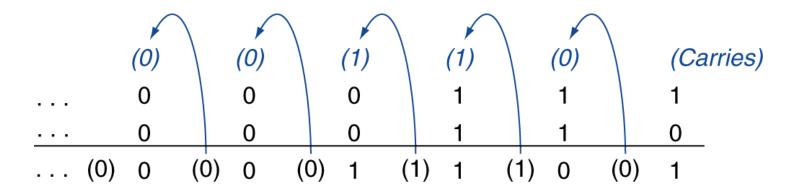


OF = overflow bit SW = Switch (select addition or subtraction)

Integer Addition - Overflow



• Example: 7 + 6



- Overflow if result is out of range
 - Adding +ve and –ve operands, no overflow
 - Adding two +ve operands
 - Overflow if result sign is 1
 - Adding two –ve operands
 - Overflow if result sign is 0

Integer Subtraction - Overflow



- Add negation of second operand
- Example: 7 6 = 7 + (-6)
 - +7: 0000 0000 ... 0000 0111
 -6: 1111 1111 ... 1111 1010
 +1: 0000 0000 ... 0000 0001
- Overflow if result is out of range
 - Subtracting two +ve or two –ve operands, no overflow
 - Subtracting +ve from –ve operand
 - Overflow if result sign is 0
 - Subtracting –ve from +ve operand
 - Overflow if result sign is 1

Overflow in Addition / Subtraction



OVERFLOW RULE: If two numbers are added, and they are both positive or both negative, then overflow occurs if and only if the result has the opposite sign.

1001 = -7 +0101 = 5 1110 = -2 (a) (-7) + (+5)	1100 = -4 +0100 = 4 10000 = 0 (b) (-4) + (+4)
0011 = 3 + 0100 = 4 0111 = 7 (c) (+3) + (+4)	1100 = -4 + 1111 = -1 11011 = -5 (d) (-4) + (-1)
0101 = 5 + 0100 = 4 1001 = Overflow (e) (+5) + (+4)	1001 = -7 + 1010 = -6 10011 = Overflow (f)(-7) + (-6)

Figure 10.3 Addition of Numbers in Twos Complement Representation

Figure 10.4 Subtraction of Numbers in Twos Complement

CPU Dealing with Overflow



- Some languages (e.g., C) ignore overflow
 - Use MIPS addu, addui, subu instructions
- Other languages (e.g., Ada, Fortran) require raising an exception
 - Use MIPS add, addi, sub instructions
 - On overflow, invoke exception handler
 - Save PC in exception program counter (EPC) register
 - Jump to predefined handler address
 - mfc0 (move from coprocessor reg) instruction can retrieve EPC value, to return after corrective action

MIPS Instructions and Overflow



The computer designer must therefore provide a way to ignore overflow in some cases and to recognize it in others. The MIPS solution is to have two kinds of arithmetic instructions to recognize the two choices:

- Add (add), add immediate (addi), and subtract (Sub) cause exceptions on overflow.
- Add unsigned (addu), add immediate unsigned (addiu), and subtract unsigned (Subu) do *not* cause exceptions on overflow.

Because C ignores overflows, the MIPS C compilers will always generate the unsigned versions of the arithmetic instructions addu, addiu, and subu, no matter what the type of the variables. The MIPS Fortran compilers, however, pick the appropriate arithmetic instructions, depending on the type of the operands.

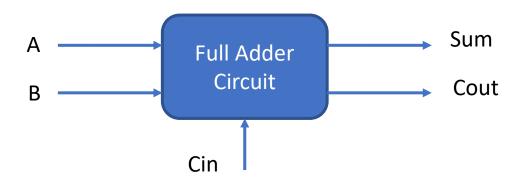


Logic Circuit of Adder / Subtractor

1 Bit Full Adder Circuit



Truth table represents behaviour of inputs and outputs



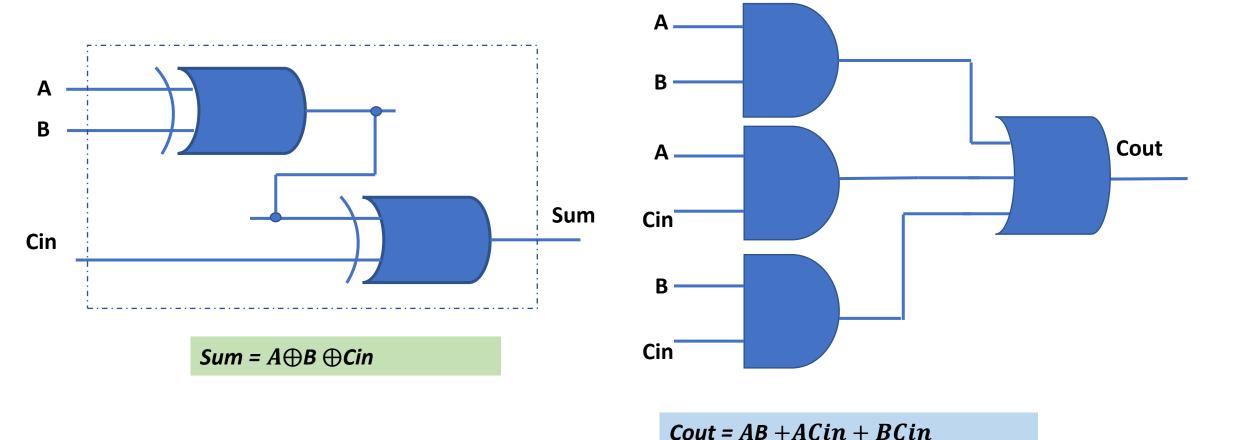
Α	В	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Boolean Expression:

$$Ci+1 = Xi.Yi + Ci(Xi \oplus Yi)$$



Full Adder Implementation using Logic Gates

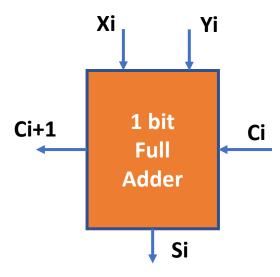




Array of Adders?

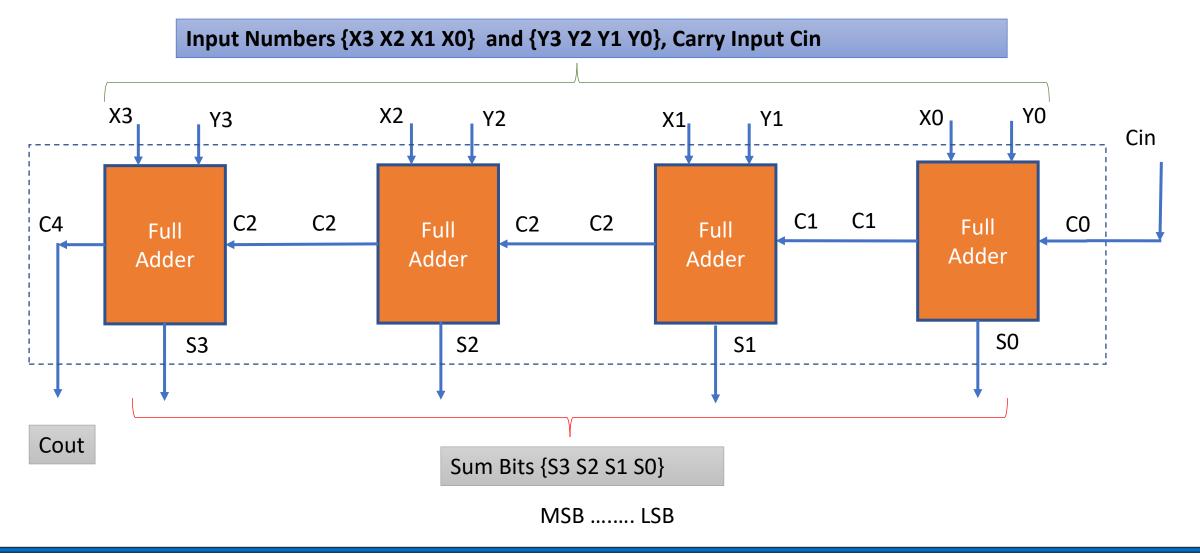
Usually, data types defined in any program are 8 bits, 16 bits or 32 bits

We need array of arithmetic and logic circuits to perform ALU operation on CPU registers





Multiple Bits – Ripple Carry Adder





Readings

- P&H Textbook Chapter 3
- Search Appendix C P&H Textbook online; this contains useful background material on digital logic, ALU and related stuff

Topics

- Computer Performance in light of
 - Power Wall
 - Complexity Wall
 - Moore's Law
 - Dennard Scaling
- Computing ideas in post-PC era
- What is meant by the Hardware / Software Interface
- Calculate Power dissipation
- Notion of Computer Performance



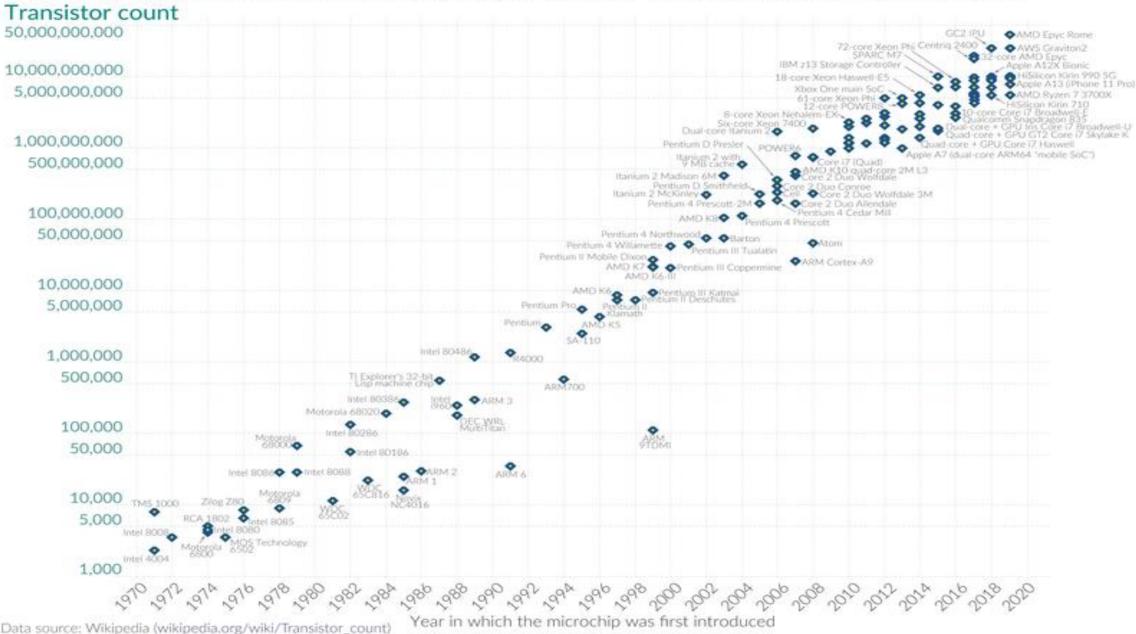
Power Wall, and Complexity Wall

Moore's Law: The number of transistors on microchips doubles every two years Our World



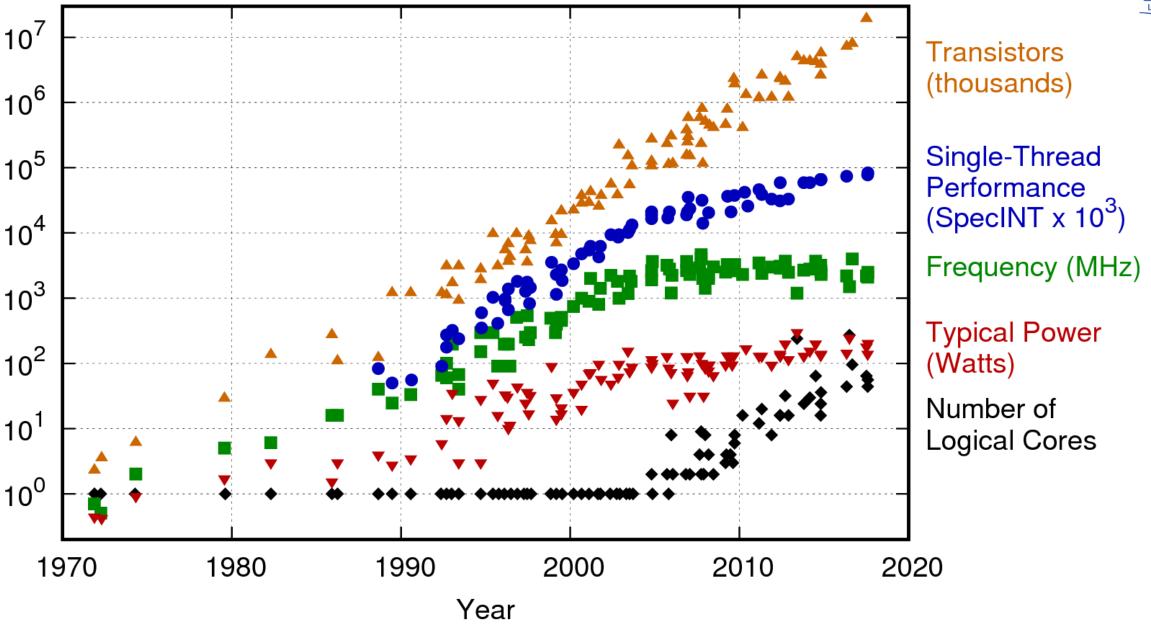
Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years.

This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

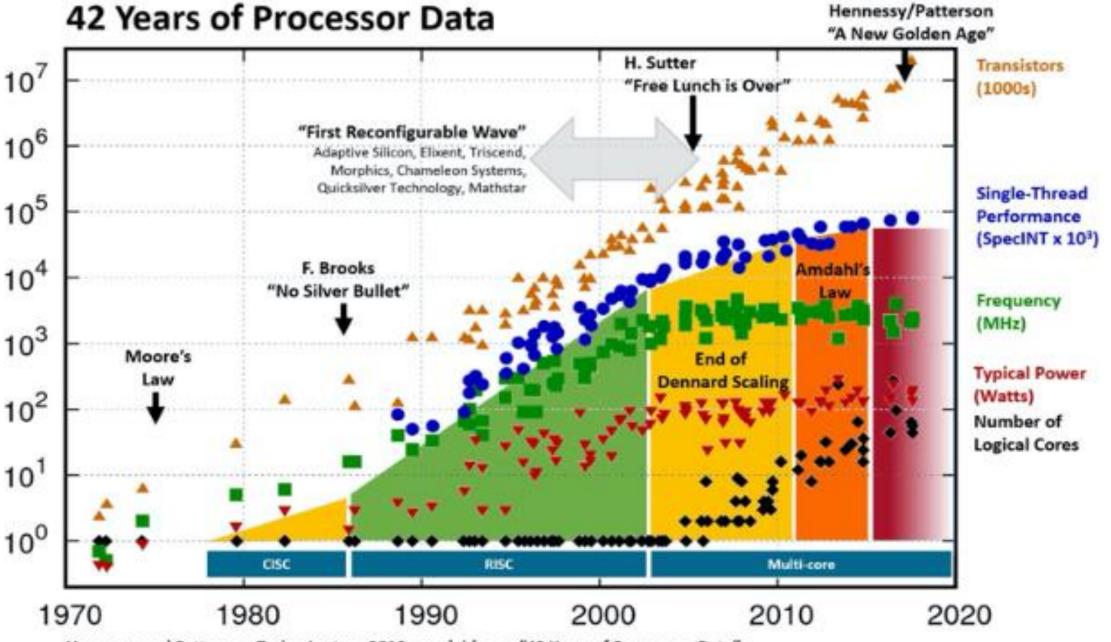


42 Years of Microprocessor Trend Data





Original data up to the year 2010 collected and plotted by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten New plot and data collected for 2010-20d house. Original data collected for 2010-20d house.



Hennessy and Patterson, Turing Lecture 2018, overlaid over "42 Years of Processors Data"

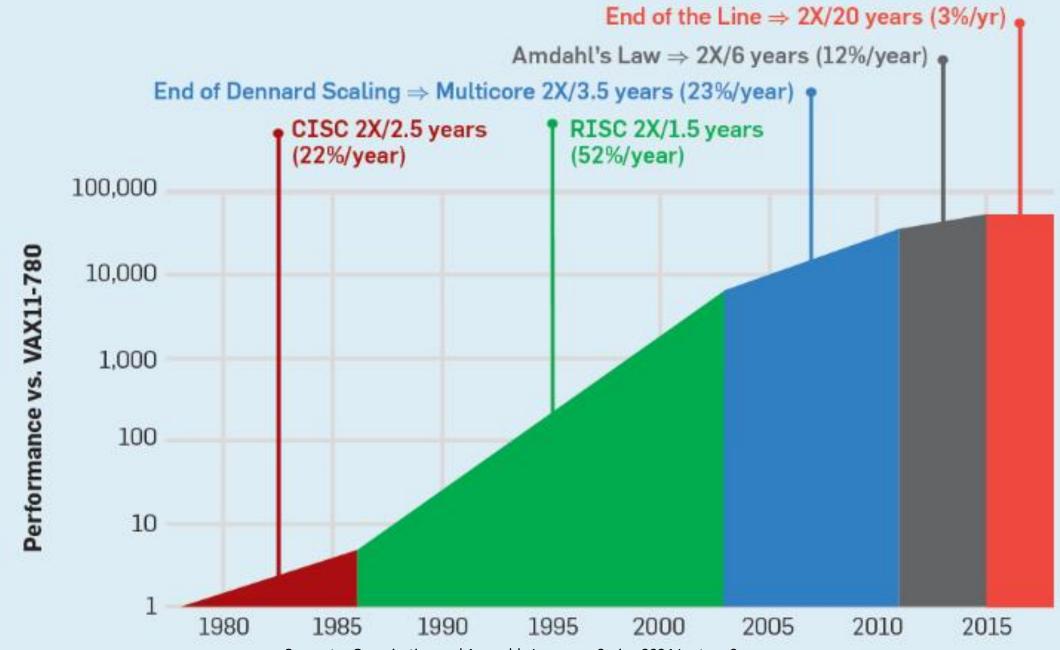
https://www.karlrupp.net/2018/02/42-years-of-microprocessor-trend-data/; "First Wave" added by Les Wilson, Frank Schirrmeister

Original data up to the year 2010 collected and plotted by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten

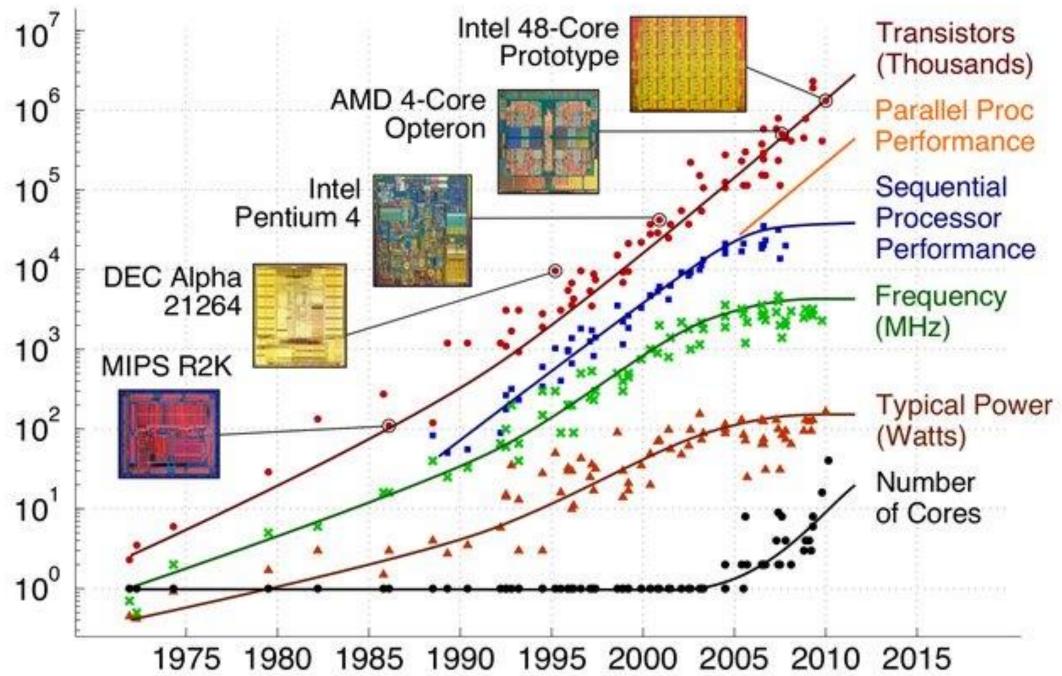
New plot and data collected for 201 Computer Organization and Assembly Language Spring 2024 Lecture 9

LUMS









CPU Architecture Today Heat becoming un unmanageable problem



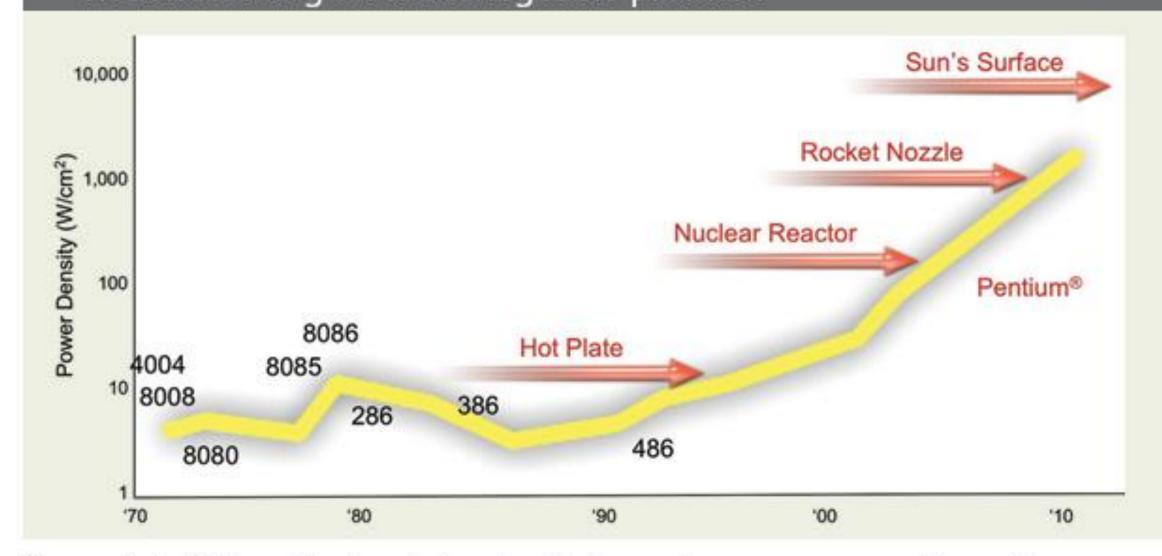


Figure 1. In CPU architecture today, heat is becoming an unmanageable problem. (Courtesy of Pat Gelsinger, Intel Developer Forum, Spring 2004)

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CPU: From GHz to multi-core



Moore's Law:

- the number of transistors on an IC doubles every two years.
 - Less space, more complexity.
 - Shorter gates, higher clock rate.

Strategy of the 80s and 90's:

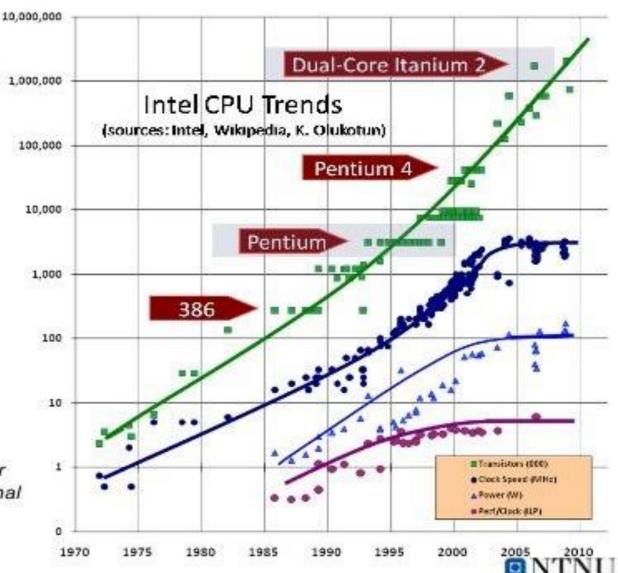
- Add more complexity!
- Increase the clock rate!

Pollack's Rule:

 The performance increase is ~ square root of the increased complexity. [Borkar 2007]

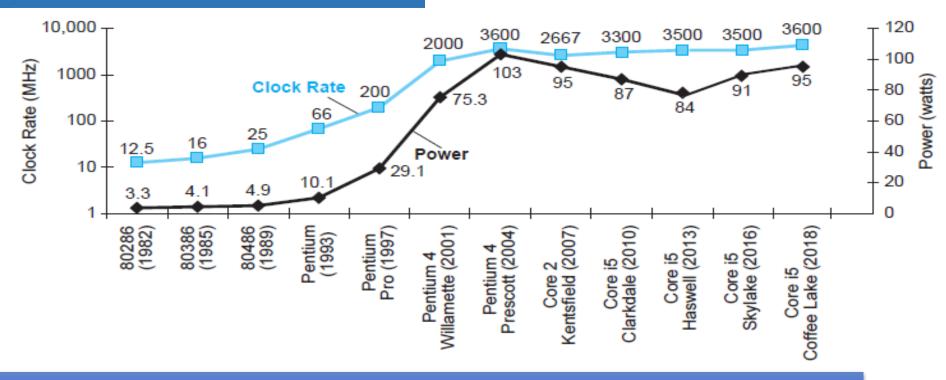
The Power Wall:

 Increasing clock rate and transistor current leakage lead to excess power consumption, while RC delays in signal transmission grow as feature sizes shrink. [Borkar et al. 2005]

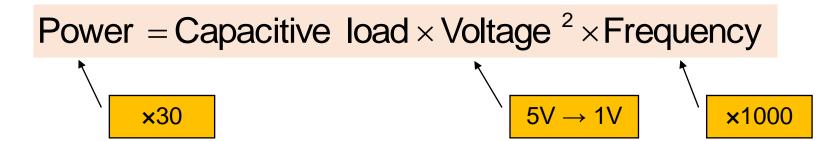


CPU Power Trends



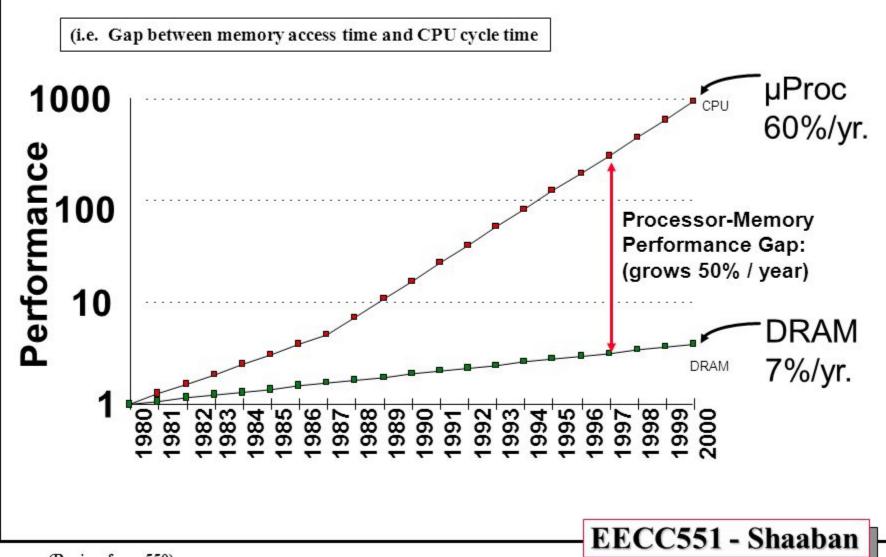


In CMOS IC technology











Reducing Power

- Suppose a new CPU has
 - 85% of capacitive load of old CPU
 - 15% voltage and 15% frequency reduction

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{C_{\text{old}} \times 0.85 \times (V_{\text{old}} \times 0.85)^2 \times F_{\text{old}} \times 0.85}{C_{\text{old}} \times V_{\text{old}}^2 \times F_{\text{old}}} = 0.85^4 = 0.52$$

- The power wall
 - We can't reduce voltage further
 - We can't remove more heat
- How else can we improve performance?

Computing Ideas in Post-PC Era



Current High-Performance Computing Architectures Post-Moore's-Law Era

Model Based Abstract Software Development for Complex Systems

Parallel Processing through Multicores

Instruction Level Parallelism through Pipelining

Speculation and Prediction in RISC Architecture

Performance Evaluation to make the 'Common Case Fast'

Use variety of memory hierarchies to improve performance

Cloud based Computing and Storage

Role of Network in Computer Performance



Upcoming Computing Architectures Post-PC Era

- Mobile and Embedded Computing
- Domain Specific Architectures E.g. Google Tensor Processor
- Domain Specific Languages and Compilers E.g. Tensorflow
- Open Instruction Set Architectures RISC V
- Agile Computing through Reconfigurable Hardware and Customized Instructions
- Security in CPU Hardware



The Scale in Computing

Decimal term	Abbreviation	Value	Binary term	Abbreviation	Value	% Larger
kilobyte	KB	10 ³	kibibyte	KiB	210	2%
megabyte	MB	10 ⁶	mebibyte	MiB	220	5%
gigabyte	GB	10 ⁹	gibibyte	GiB	230	7%
terabyte	TB	1012	tebibyte	TiB	240	10%
petabyte	PB	1015	pebibyte	PiB	250	13%
exabyte	EB	1018	exbibyte	EiB	2 ⁶⁰	15%
zettabyte	ZB	1021	zebibyte	ZiB	270	18%
yottabyte	YB	1024	yobibyte	YiB	280	21%

FIGURE 1.1 The 2^x vs. 10^y bytes ambiguity was resolved by adding a binary notation for all the common size terms. In the last column we note how much larger the binary term is than its corresponding decimal term, which is compounded as we head down the chart. These prefixes work for bits as well as bytes, so *gigabit* (Gb) is 10⁹ bits while *gibibits* (Gib) is 2³⁰ bits.



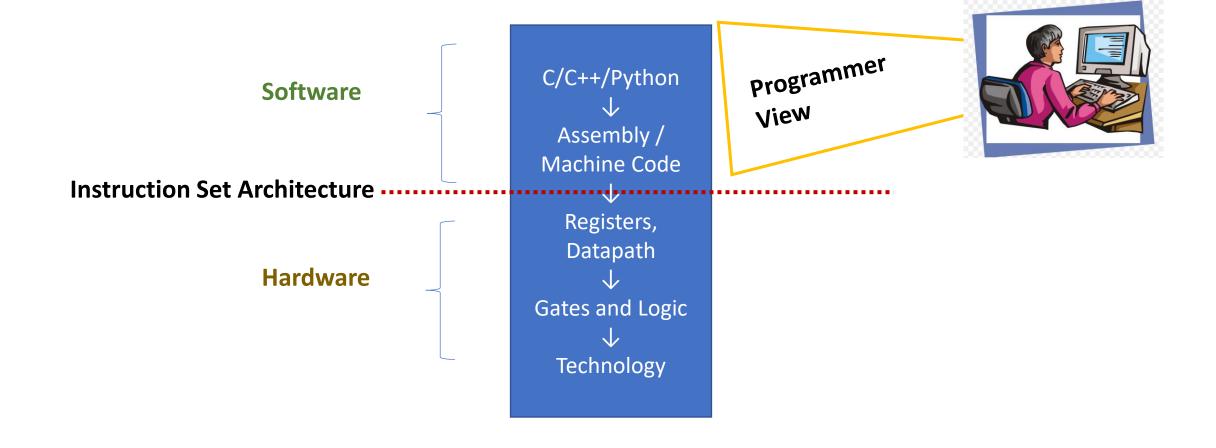
The Hardware / Software Interface

Interface of Hardware and Software

Hardware or software component	How this component affects performance		
Algorithm	Determines both the number of source-level statements and the number of I/O operations executed		
Programming language, compiler, and architecture	Determines the number of computer instructions for each source-level statement		
Processor and memory system	Determines how fast instructions can be executed		
I/O system (hardware and operating system)	Determines how fast I/O operations may be executed		



Hardware / Software Interface



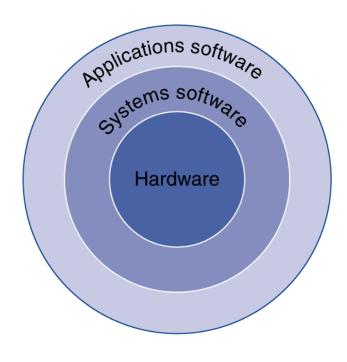


Why Abstraction?

Delving into the depths reveals more information

 An abstraction omits unneeded detail, helps us cope with complexity

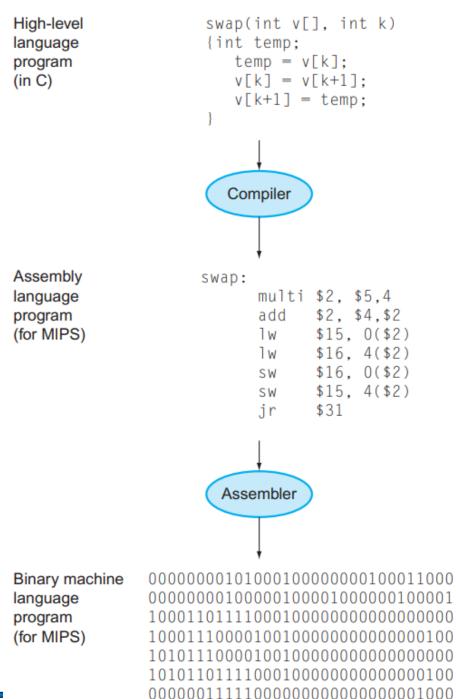
Below a Computer Program



- Application software
 - Written in high-level language
- System software
 - Compiler: translates HLL code to machine code
 - Operating System: service code
 - Handling input/output
 - Managing memory and storage
 - Scheduling tasks & sharing resources
- Hardware
 - Processor, memory, I/O controllers

Compiling a Program

Example of Software Abstraction





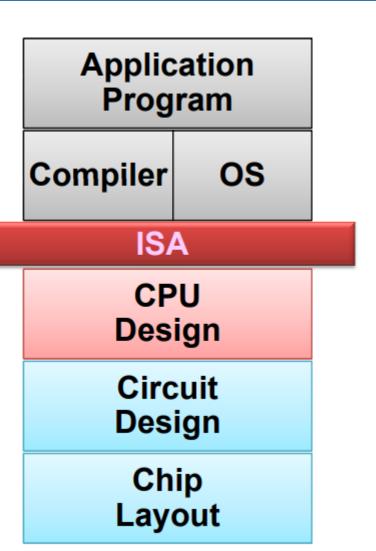
What is an Instruction Set Architecture ISA?

- Lowest level of Computer Architecture that is visible to a programmer
- ISA comprises instructions in Assembly Language and Corresponding Binary Machine Code
- It is the language of the CPU machine
- Primitive syntax compared to a high-level language
- It is easily interpreted and understood by the CPU
- Designed to maximize performance
- Designed to minimize cost and design time



Role of ISA within a Computer

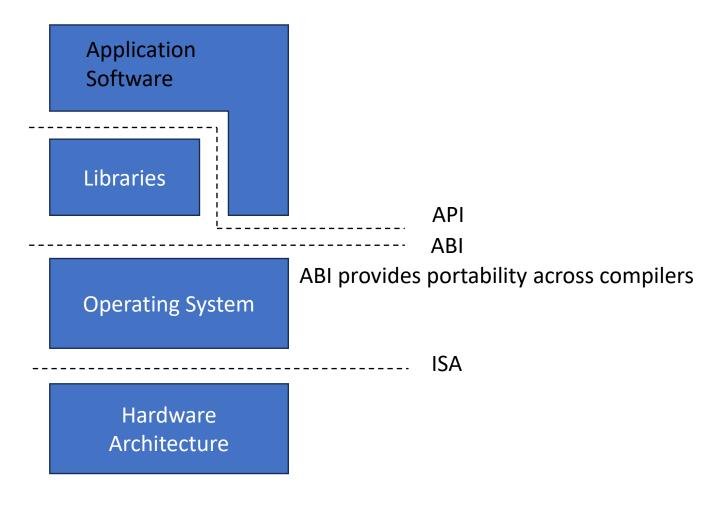
- Assembly Language View
 - Processor state (RF, mem)
 - Instruction set and encoding
- Layer of Abstraction
 - Above: how to program machine HLL, OS
 - Below: what needs to be built
 - tricks to make it run fast

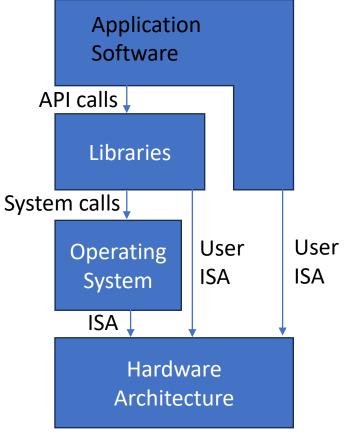


Key:

RF = Register File
Mem = Memory
HLL = High Level Language
OS = Operating System

Operating System Interface Software / Hardware



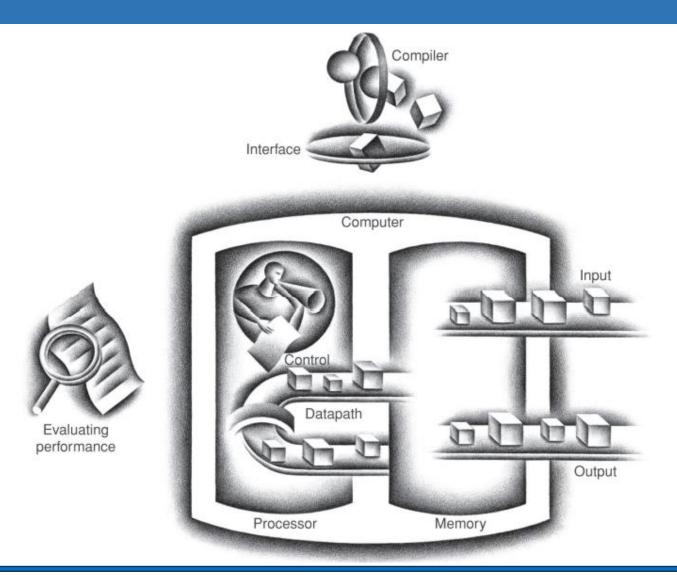




Exploring Computer Organization



Typical Computer Organization



Computer

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Main Components

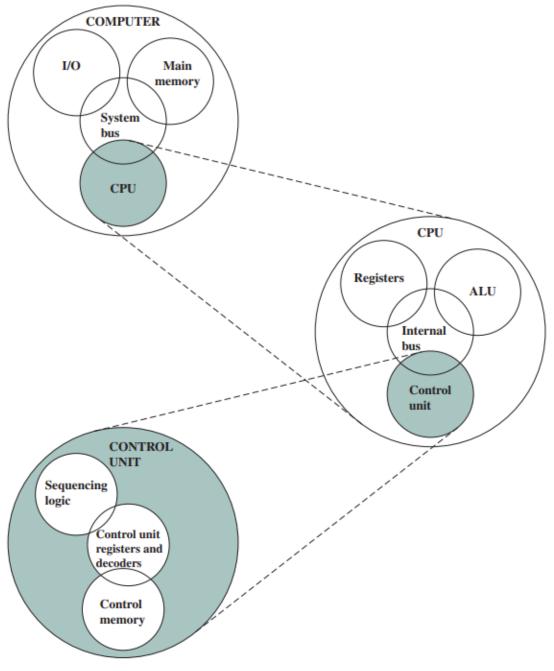


Figure 1.1 The Computer: Top-Level Structure



A Multi-core Computer

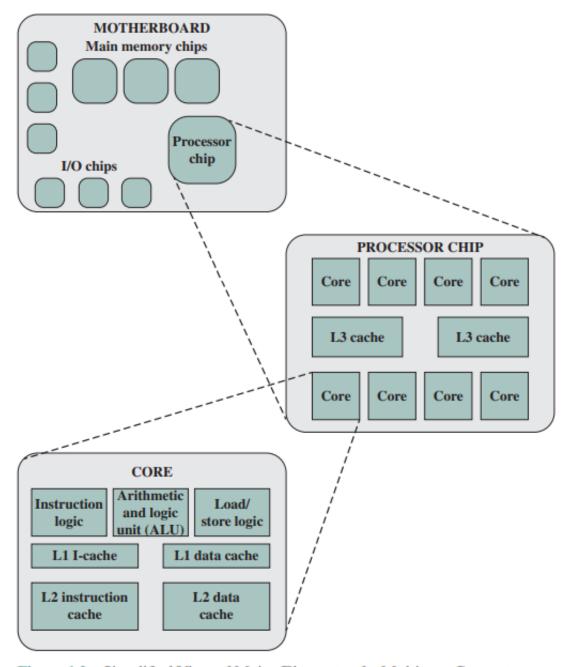
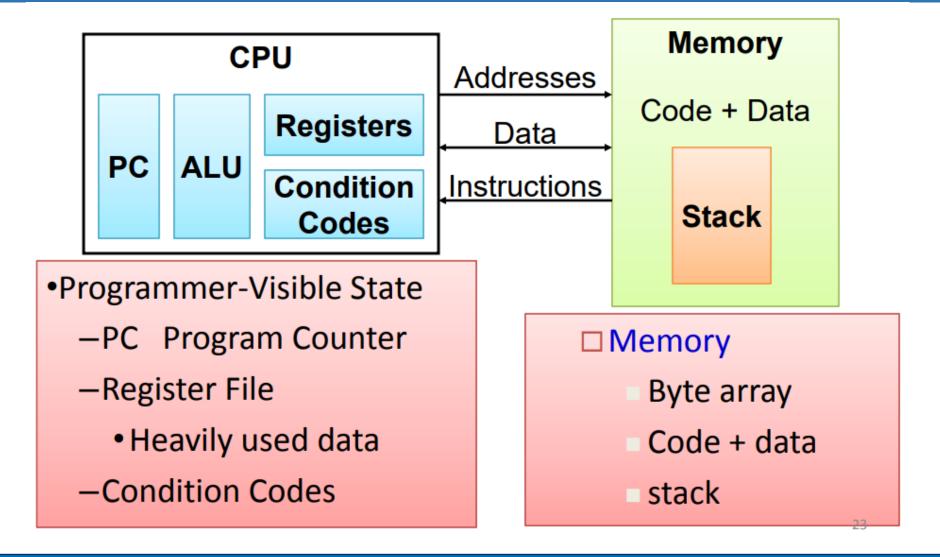


Figure 1.2 Simplified View of Major Elements of a Multicore Computer



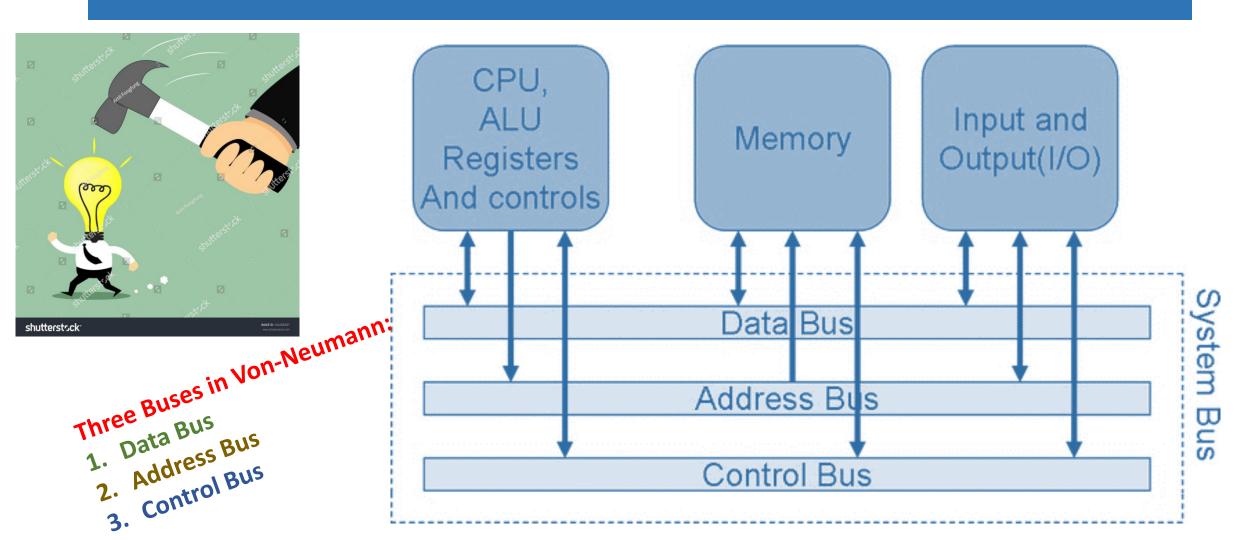


Computer Abstraction



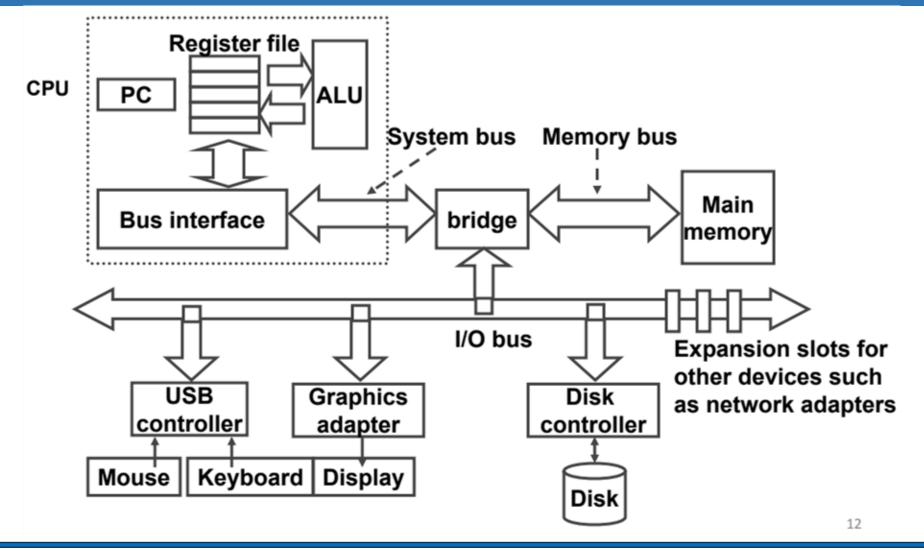


The Von-Neumann Architecture





Hardware Abstraction Example







Simple Computer Performance

- Time to execute a program in seconds
- Clock Speed of Computer MHz or GHz
- In terms of the width of Data Bus (16 bit, 32 bit, 64 bit)
- In terms of the size of different memory (8GB / 256GB)
 - Cache size
 - RAM
 - Hard disk
 - SSD, etc.
- In terms of multiple cores and multiple I/O available
- Software
 - Algorithm
 - Language / Compiler
 - Optimization Parallel, Vector, etc.



What is MIPS?

- MIPS is a leading example of RISC Architecture
- Used in many household products such as Nintendo, Sony, Mobile phones, Routers, etc.
- Simple and Small Instruction Set

MIPS = Microprocessor without Interlock Pipeline Stages ()



MIPS = **M**illion **I**nstructions **P**er **S**econd

What is MFLOPS

FLOPS = Floating Point Linear Operations Per Second

```
= \frac{Number\ of\ executed\ floating\ point\ operations\ in\ a\ program}{Execution\ time\ \times 10^6}
```

Modern notion of computer performance

user



Software Performance

Programming Languages
Operating System
Compilers
Algorithms
Parallelism / Threads

Software optimization

Hardware Performance
Processor Architecture
Vector Processing

Memory Hierarchy
Multicores

Network support

Hardware optimization

Make Common Case FAST