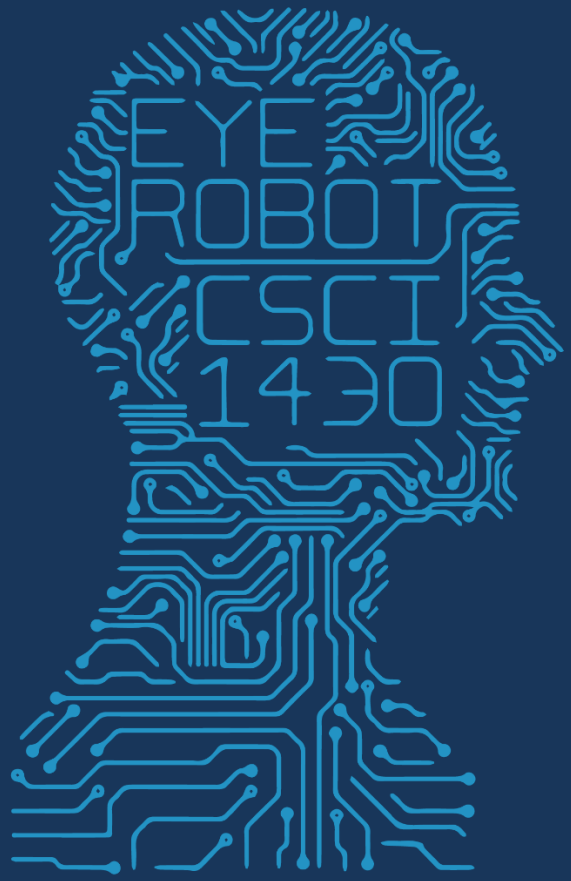
1950

Future Vision



2017 MWF 1pm

Computer Vision







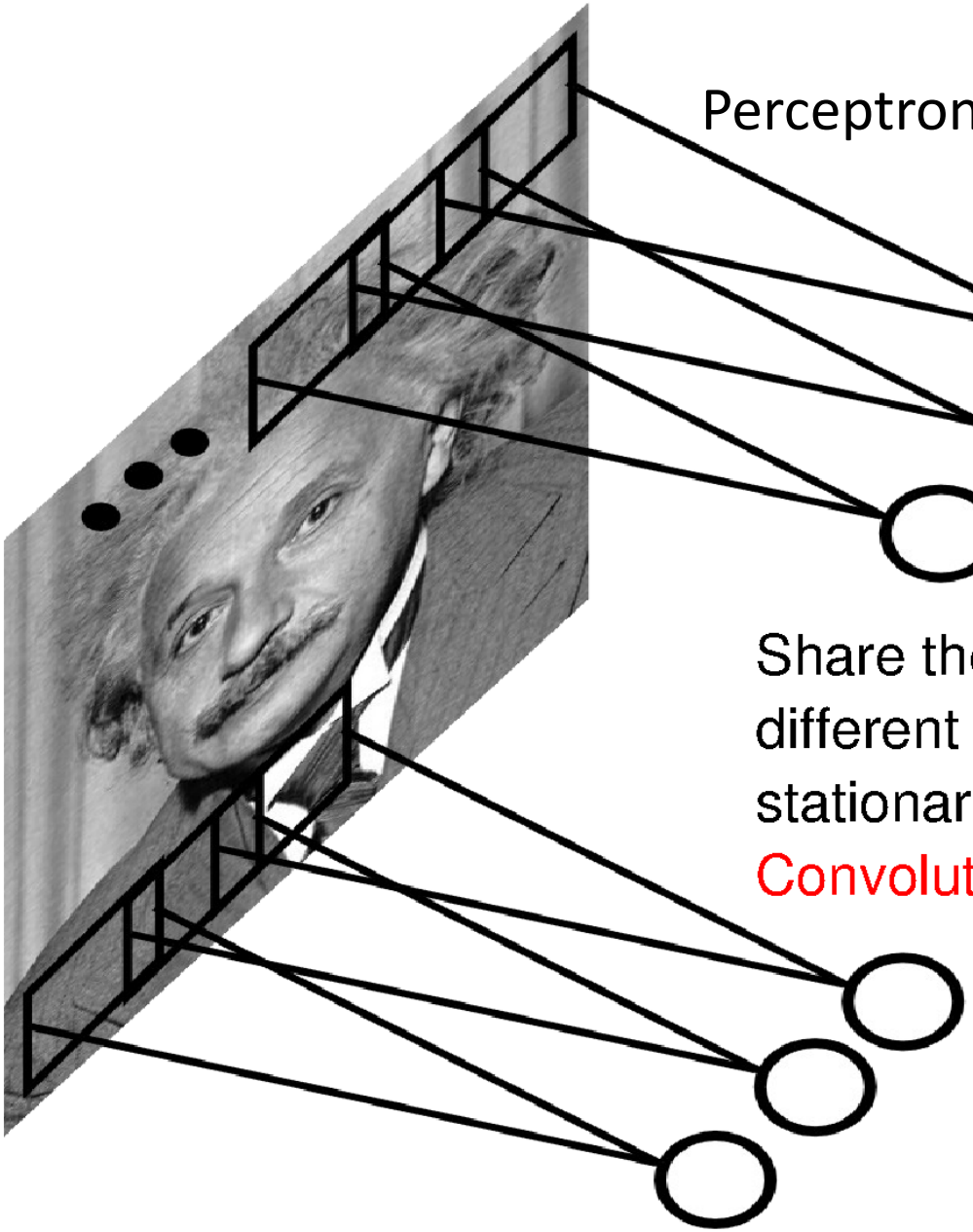









Convolutional Layer



Perceptron: $\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$

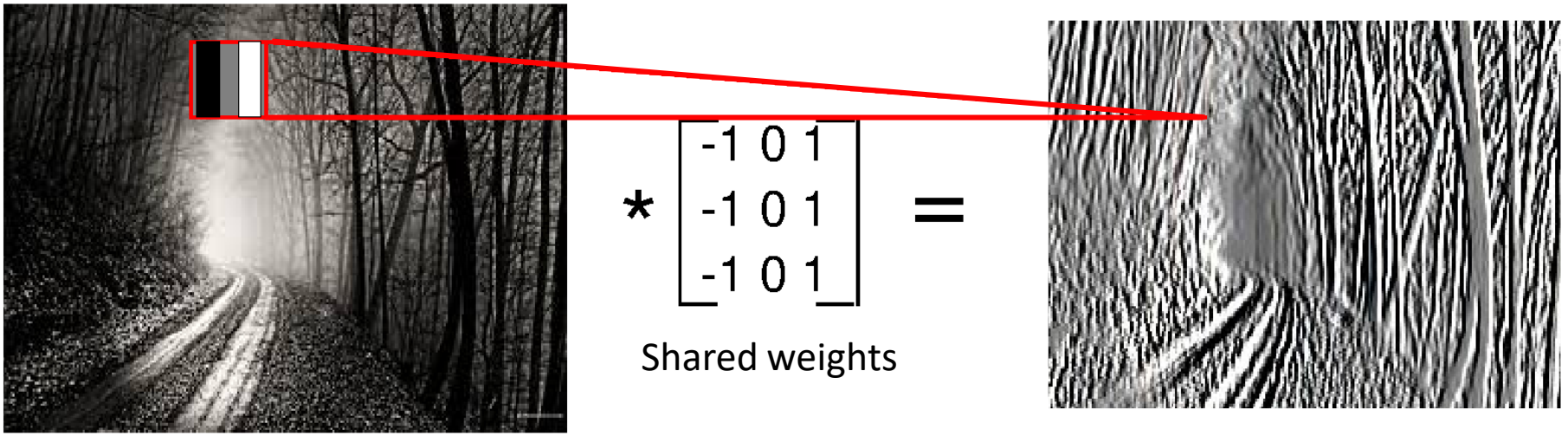
$w \cdot x \equiv \sum_j w_j x_j$

This is convolution!

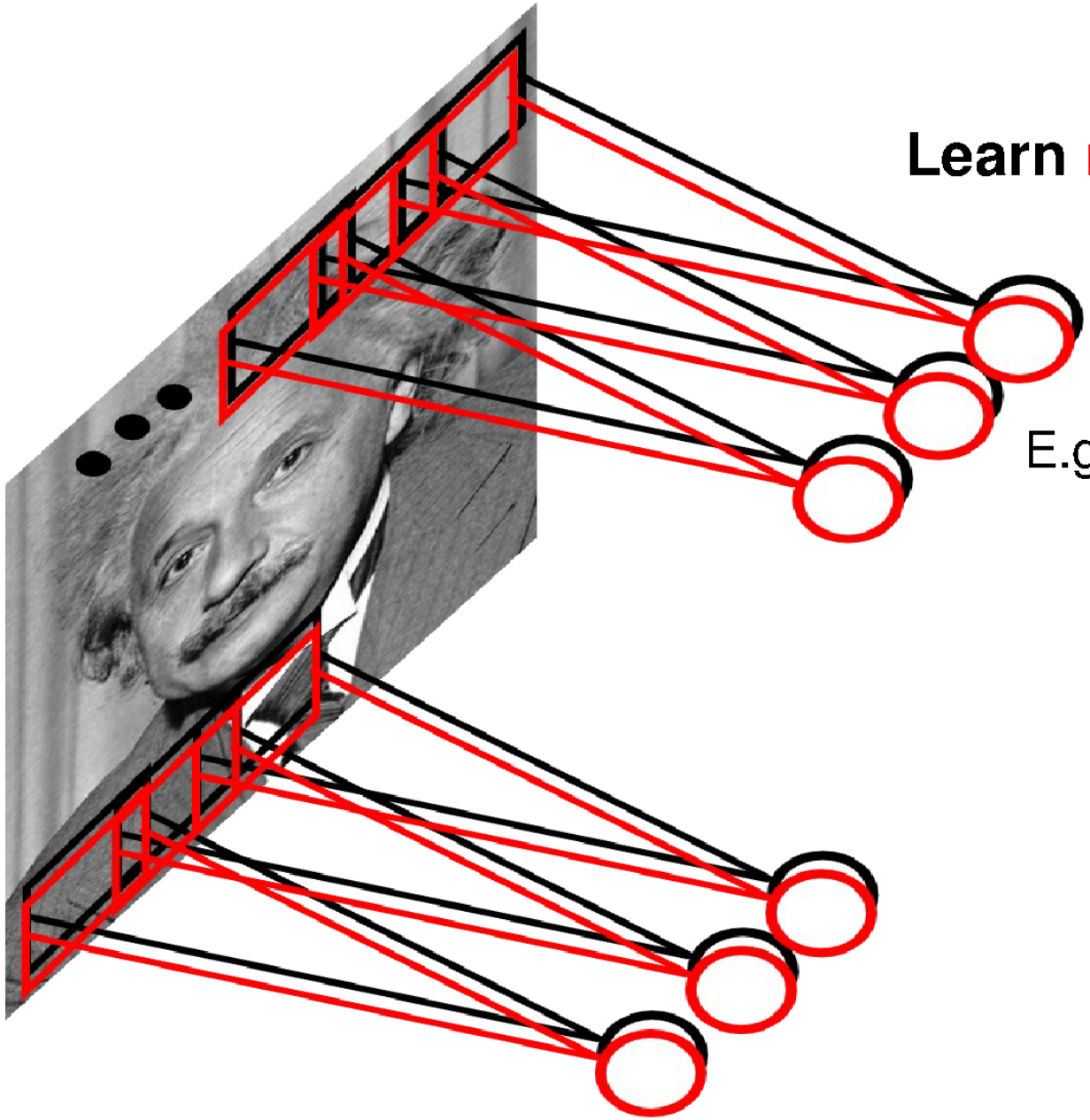
Share the same parameters across different locations (assuming input is stationary):

Convolutions with learned kernels

Convolutional Layer



Convolutional Layer

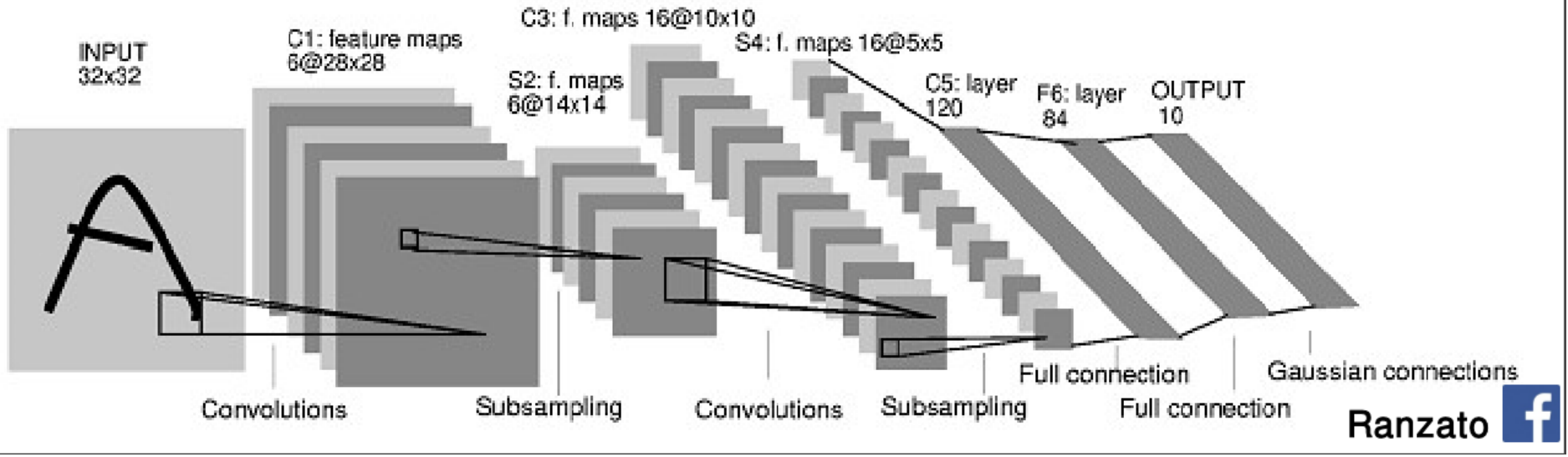


Learn multiple filters.

Filter = 'local' perceptron.
Also called *kernel.*

E.g.: 200x200 image
100 Filters
Filter size: 10x10
10K parameters

Yann LeCun's MNIST CNN architecture



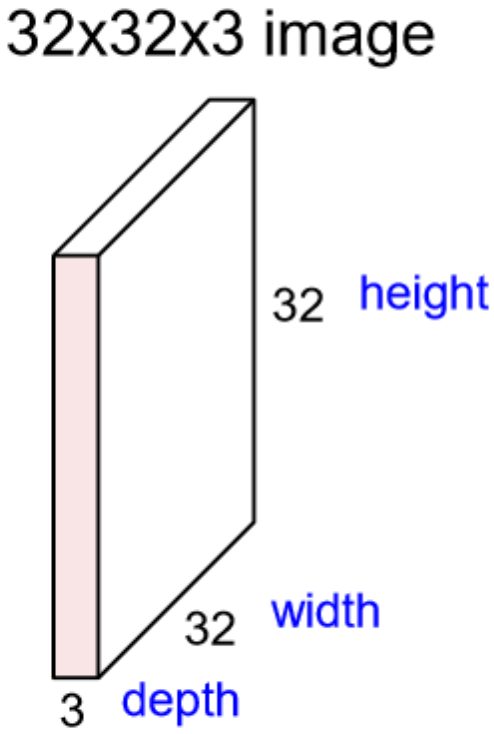
DEMO

http://scs.ryerson.ca/~aharley/vis/conv/

Thanks to Adam Harley for making this.

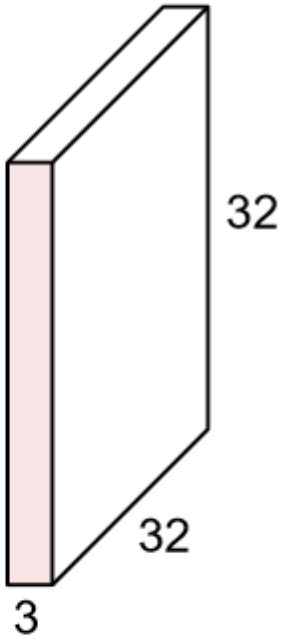
More here: http://scs.ryerson.ca/~aharley/vis

Convolutions: More detail



Convolutions: More detail

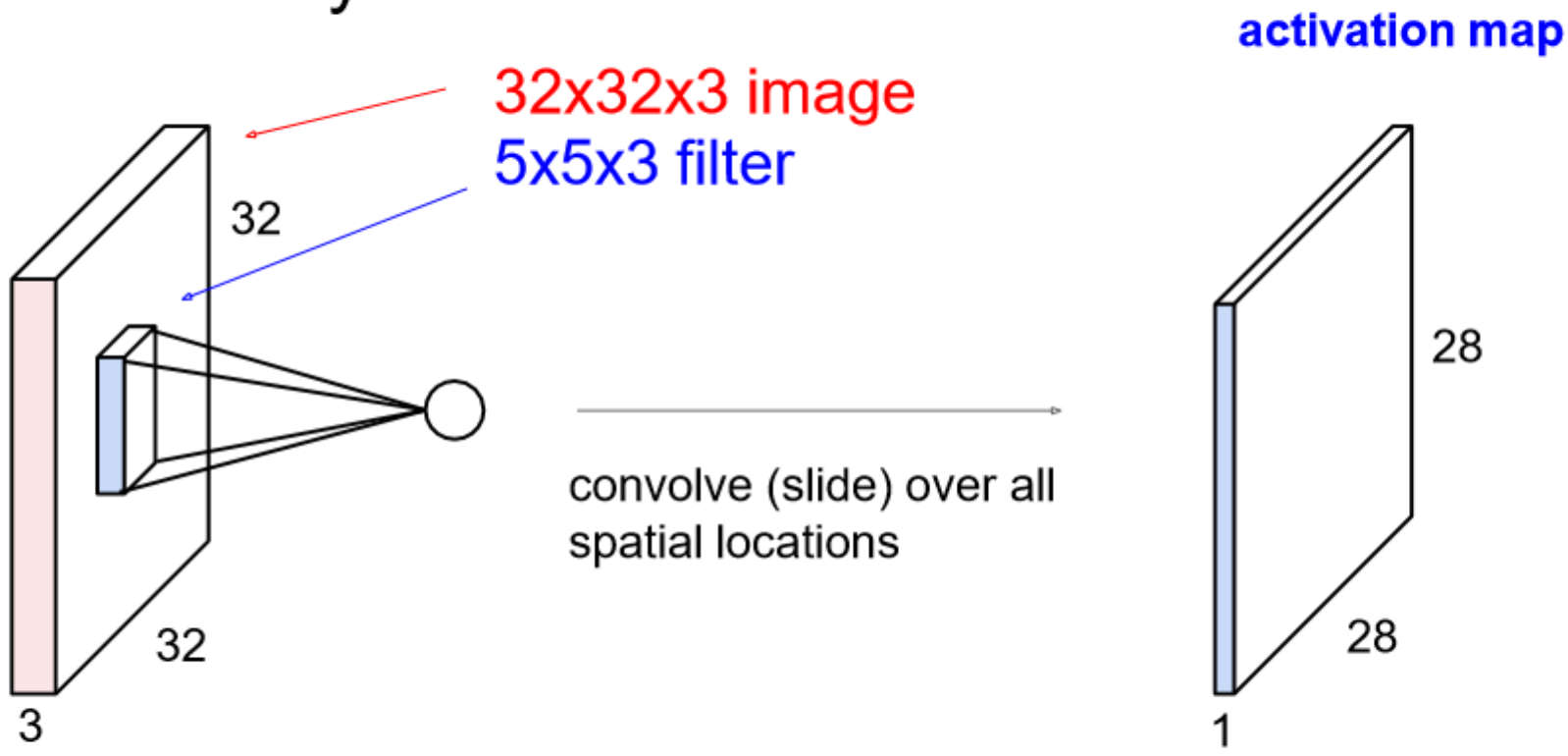
32x32x3 image



5x5x3 filter

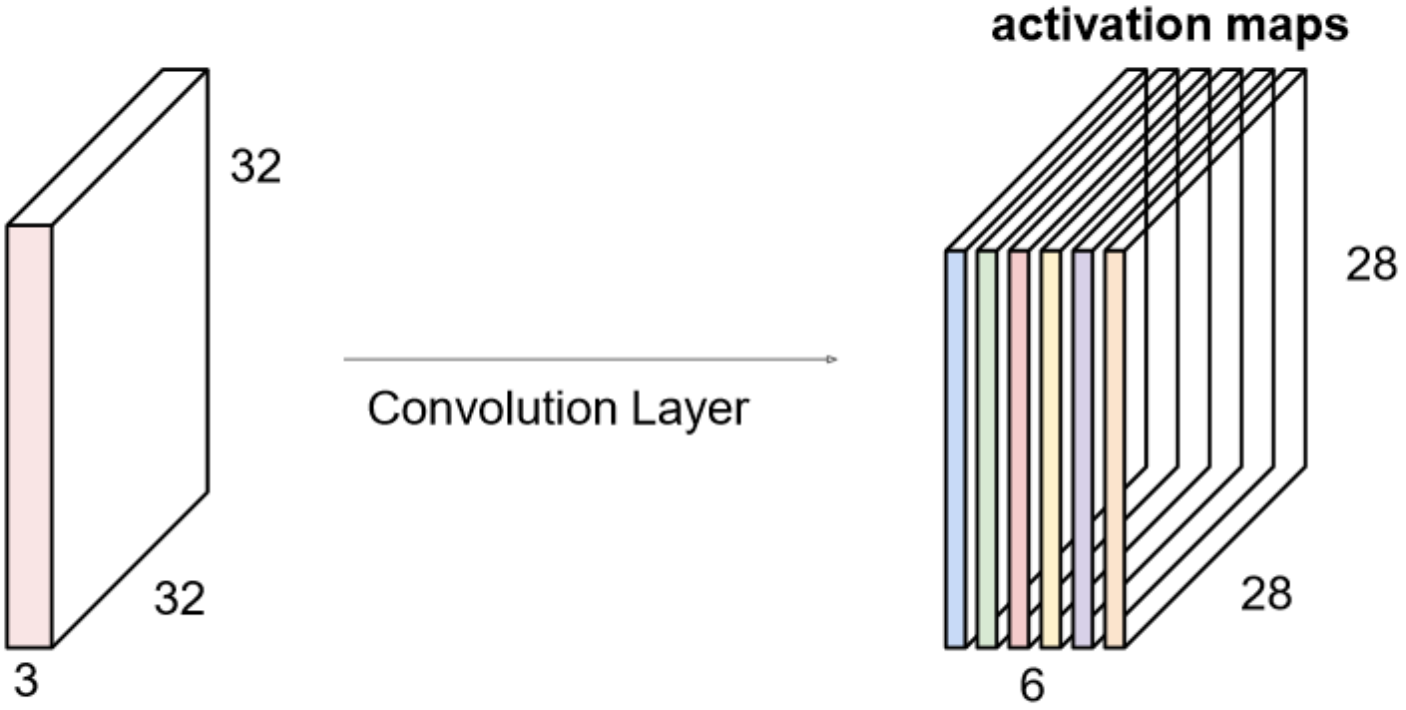
Convolutions: More detail

Convolution Layer



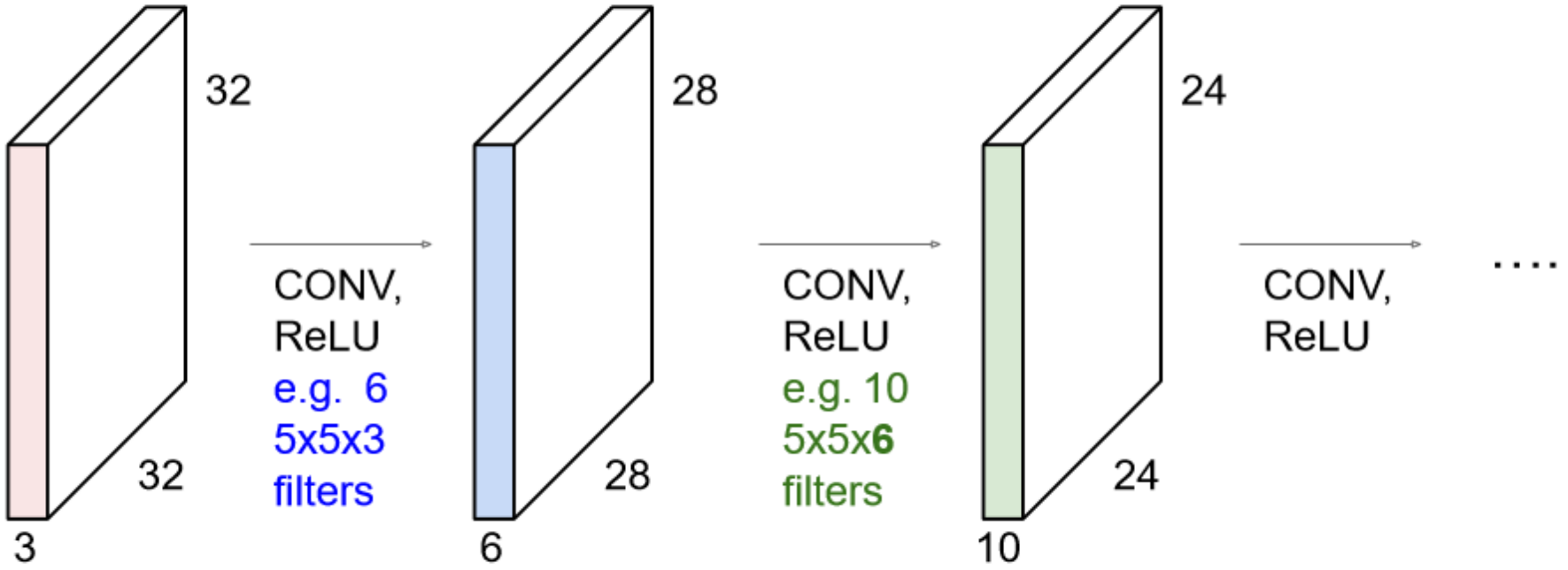
Convolutions: More detail

For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Convolutions: More detail



Think-Pair-Share

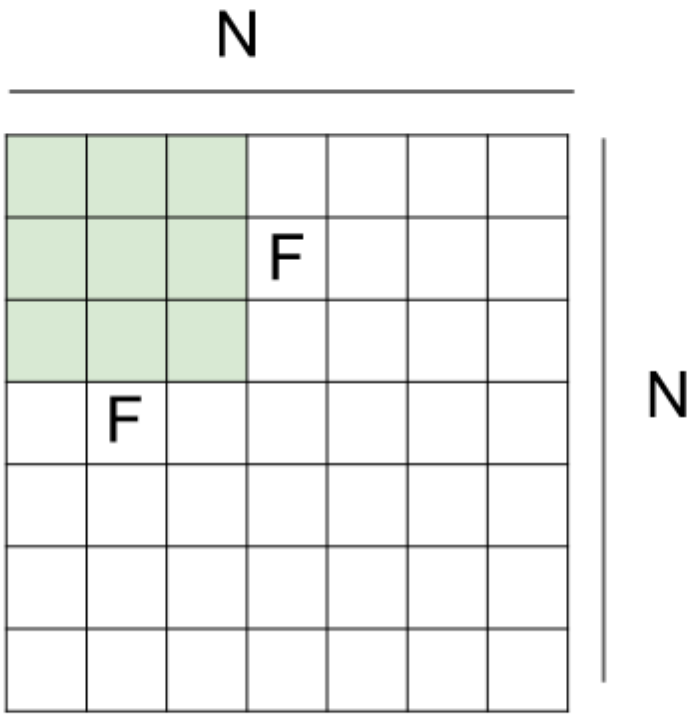
Input size: 96 x 96 x 3
Kernel size: 5 x 5 x 3
Stride: 1
Max pooling layer: 4 x 4

Output feature map size?
a) 5 x 5
b) 22 x 22
c) 23 x 23
d) 24 x 24
e) 25 x 25

Input size: 96 x 96 x 3
Kernel size: 3 x 3 x 3
Stride: 3
Max pooling layer: 8 x 8

Output feature map size?
a) 2 x 2
b) 3 x 3
c) 4 x 4
d) 5 x 5
e) 12 x 12

Convolutions: More detail

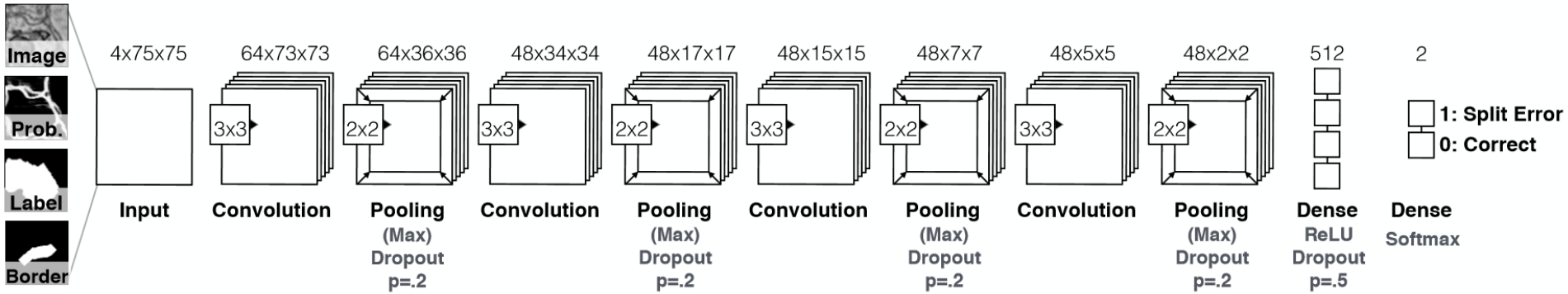


Output size:
(N - F) / stride + 1

Our connectomics diagram

Auto-generated from network declaration by *nolearn* (for Lasagne / Theano)

Input
75x75x4

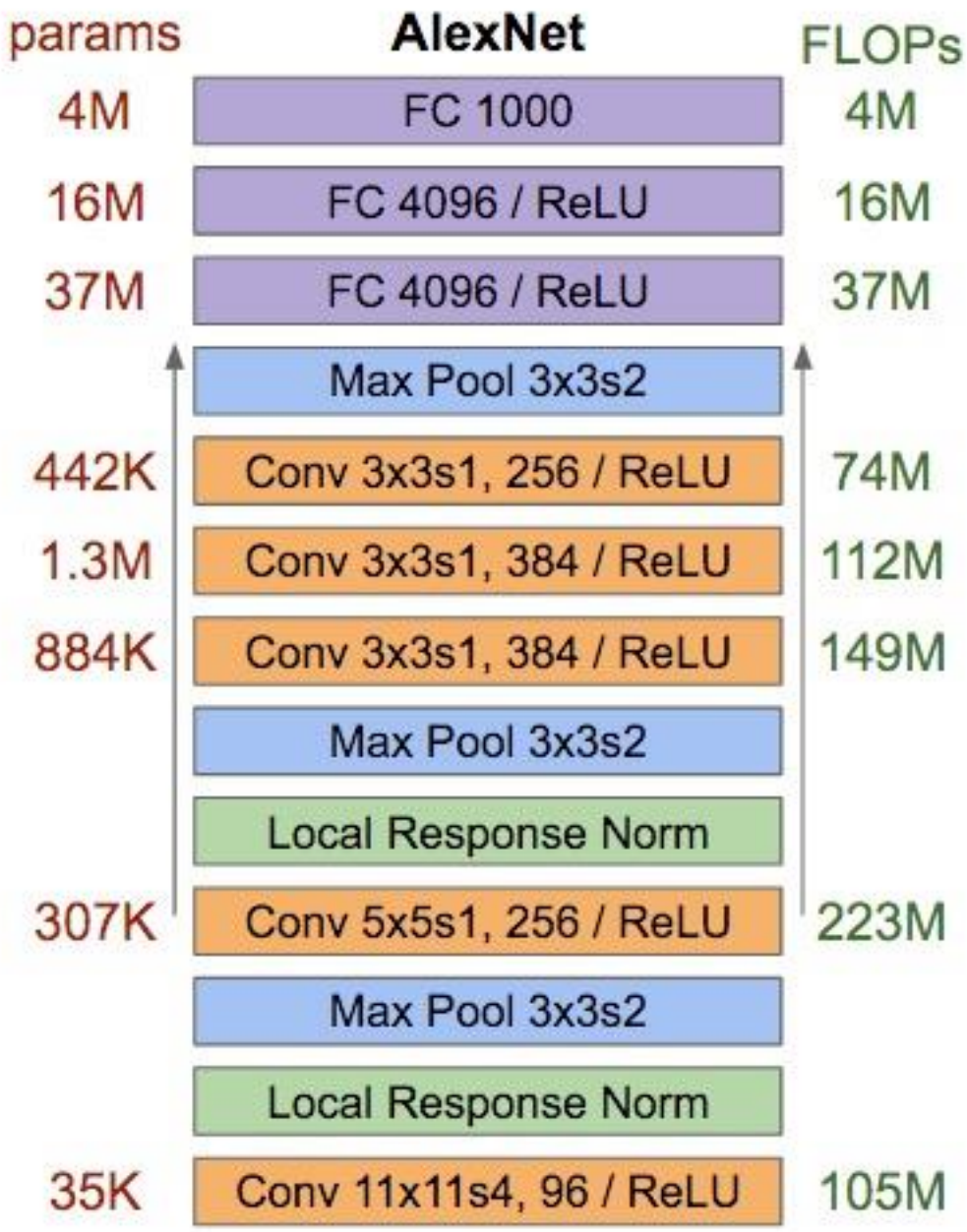


Conv 1	Conv 2	Conv 3	Conv 4
3x3x4	3x3x64	3x3x48	3x3x48
64 filters	48 filters	48 filters	48 filters

Max pooling	Max pooling	Max pooling	Max pooling
2x2 per filter	2x2 per filter	2x2 per filter	2x2 per filter

Reading architecture diagrams

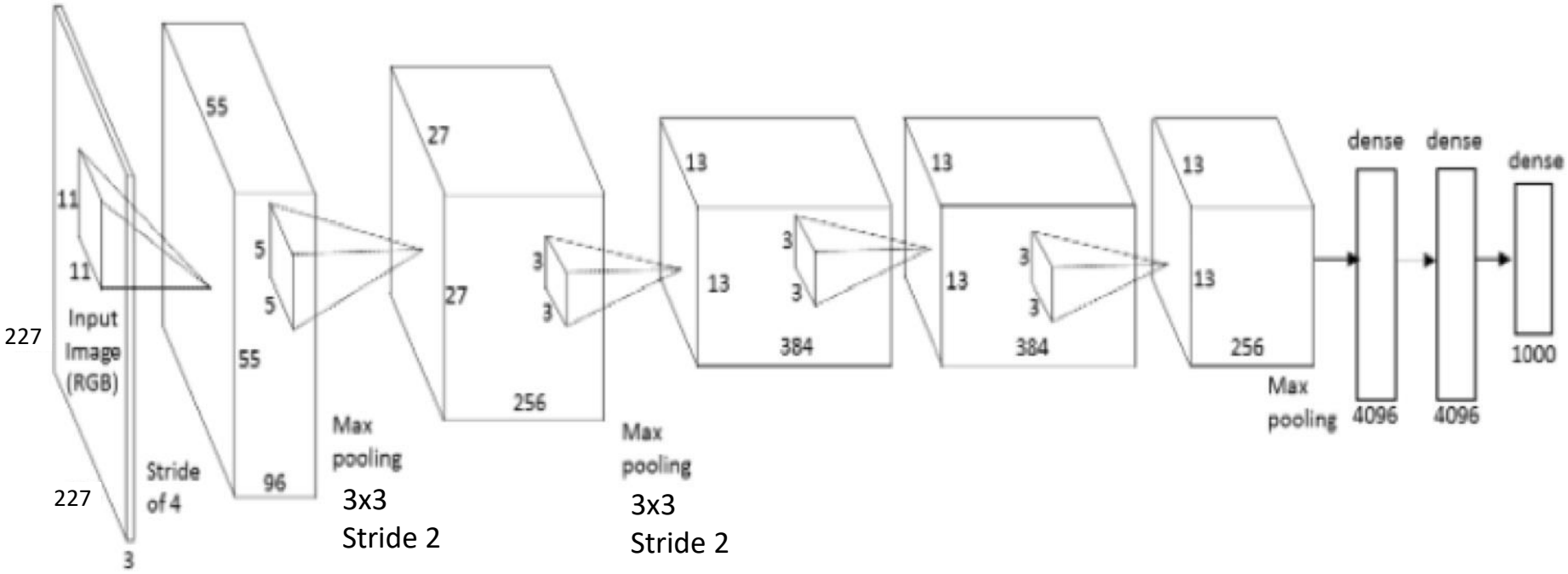
Layers
- Kernel sizes
- Strides
- # channels
- # kernels
- Max pooling



[Krizhevsky et al. 2012]

AlexNet diagram (simplified)

Input size
227 x 227 x 3



Conv 1
11 x 11 x 3
Stride 4
96 filters

Conv 2
5 x 5 x 96
Stride 1
256 filters

Conv 3
3 x 3 x 256
Stride 1
384 filters

Conv 4
3 x 3 x 192
Stride 1
384 filters

Conv 4
3 x 3 x 192
Stride 1
256 filters

Wait, why isn't it called a correlation neural network?

It could be.

Deep learning libraries actually implement correlation.

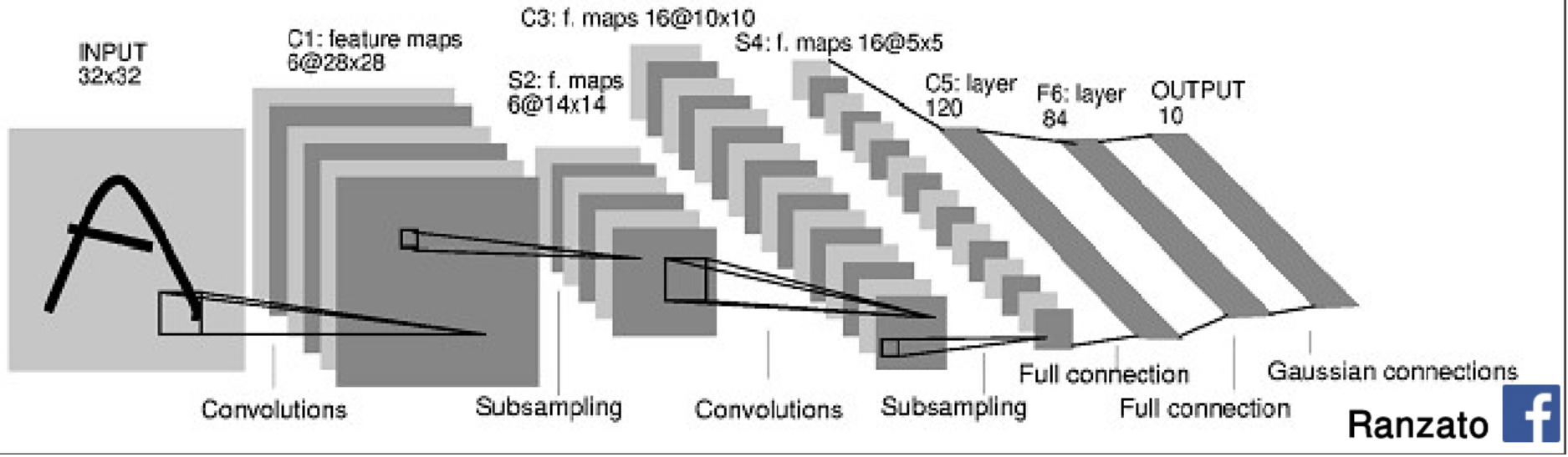
Correlation relates to convolution via a 180deg rotation of the kernel. When we *learn* kernels, we could easily learn them flipped.

Associative property of convolution ends up not being important to our application, so we just ignore it.

[p.323, Goodfellow]

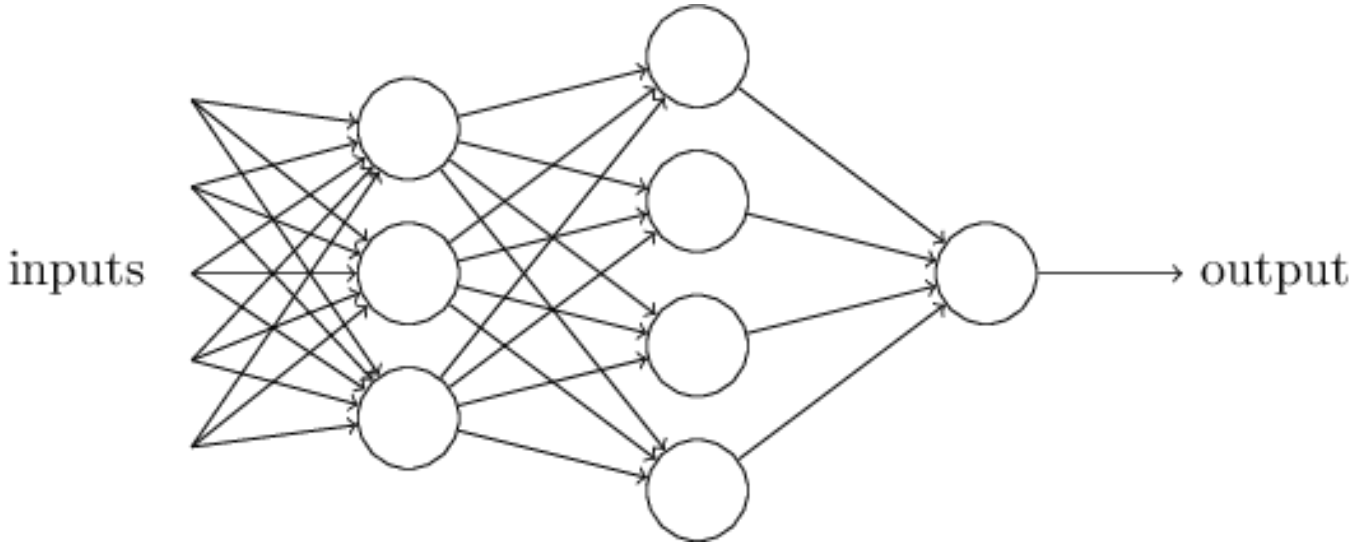
What does it mean to convolve over greater-than-first-layer hidden units?

Yann LeCun's MNIST CNN architecture



Multi-layer perceptron (MLP)

…is a '*fully connected*' neural network with non-linear activation functions.

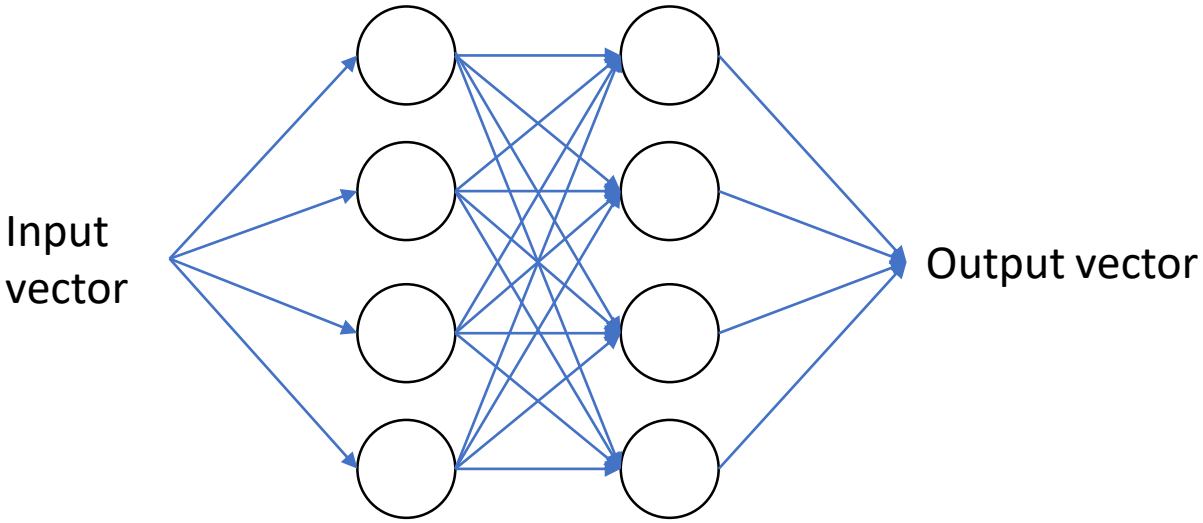


'*Feed-forward*' neural network

Nielson

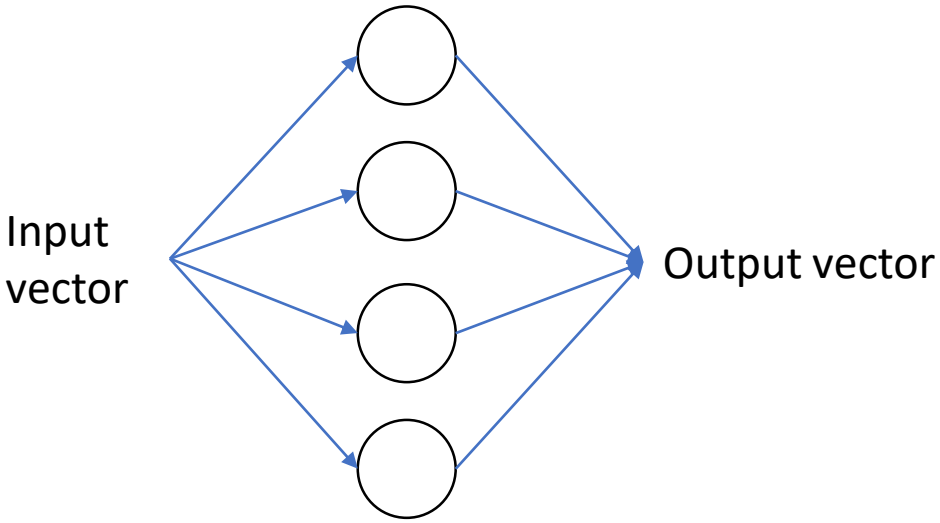
Does anyone pass along the weight without an activation function?

No – this is linear chaining.



Does anyone pass along the weight without an activation function?

No – this is linear chaining.



Are there other activation functions?

Yes, many.

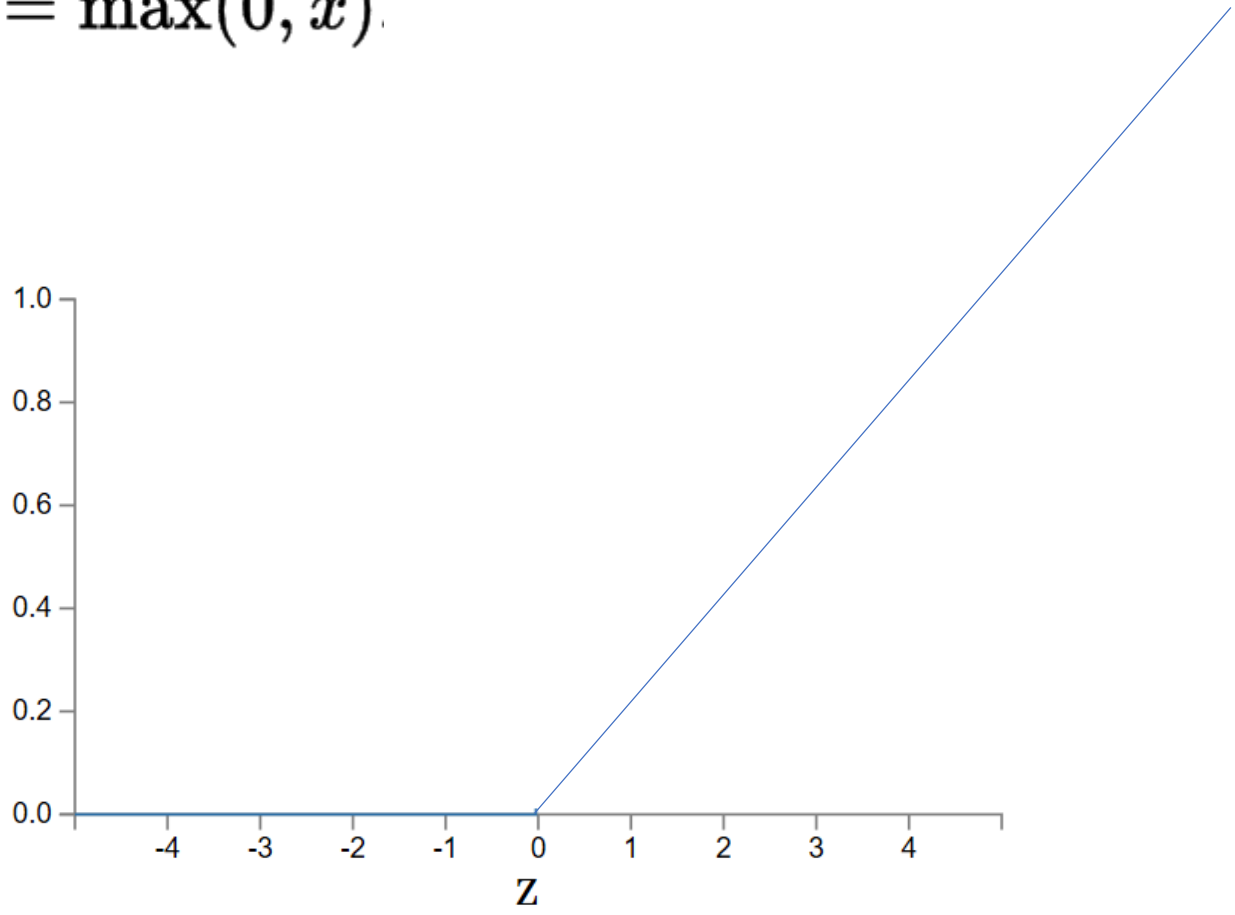
As long as:

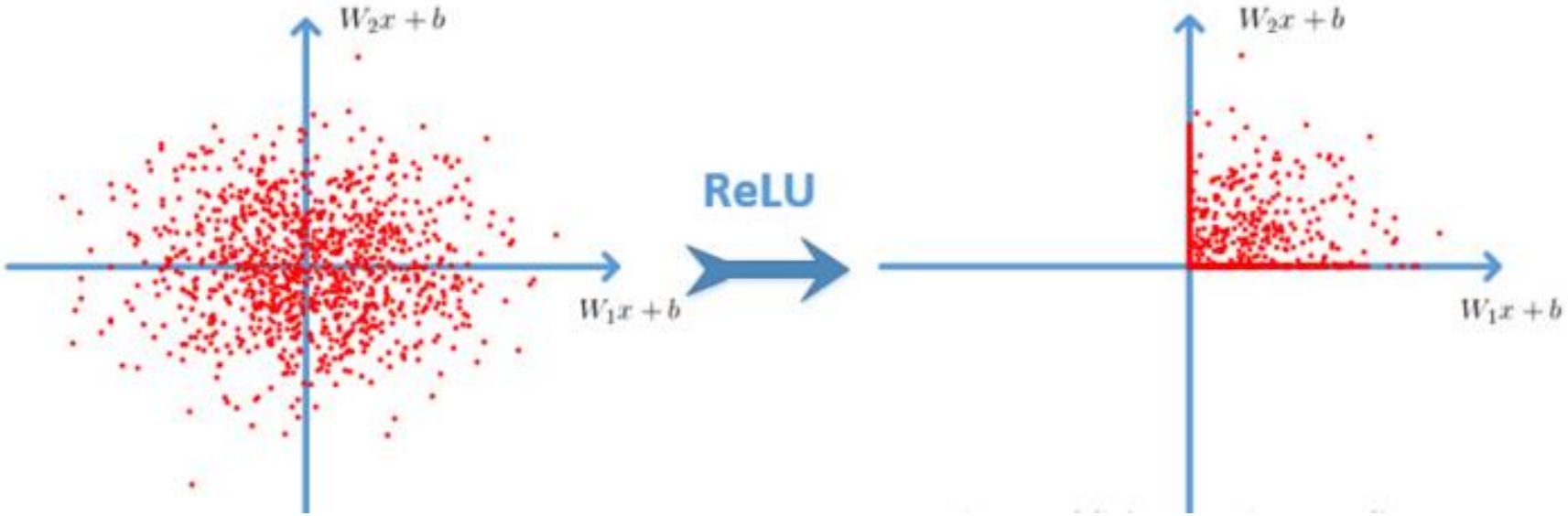
- Activation function s(z) is well-defined as z -> -∞ and z -> ∞
- These limits are different

Then w*e can make a step!* [Think visual proof]
It can be shown that it is universal for function approximation.

Activation functions: Rectified Linear Unit

- ReLU $f(x) = \max(0, x)$

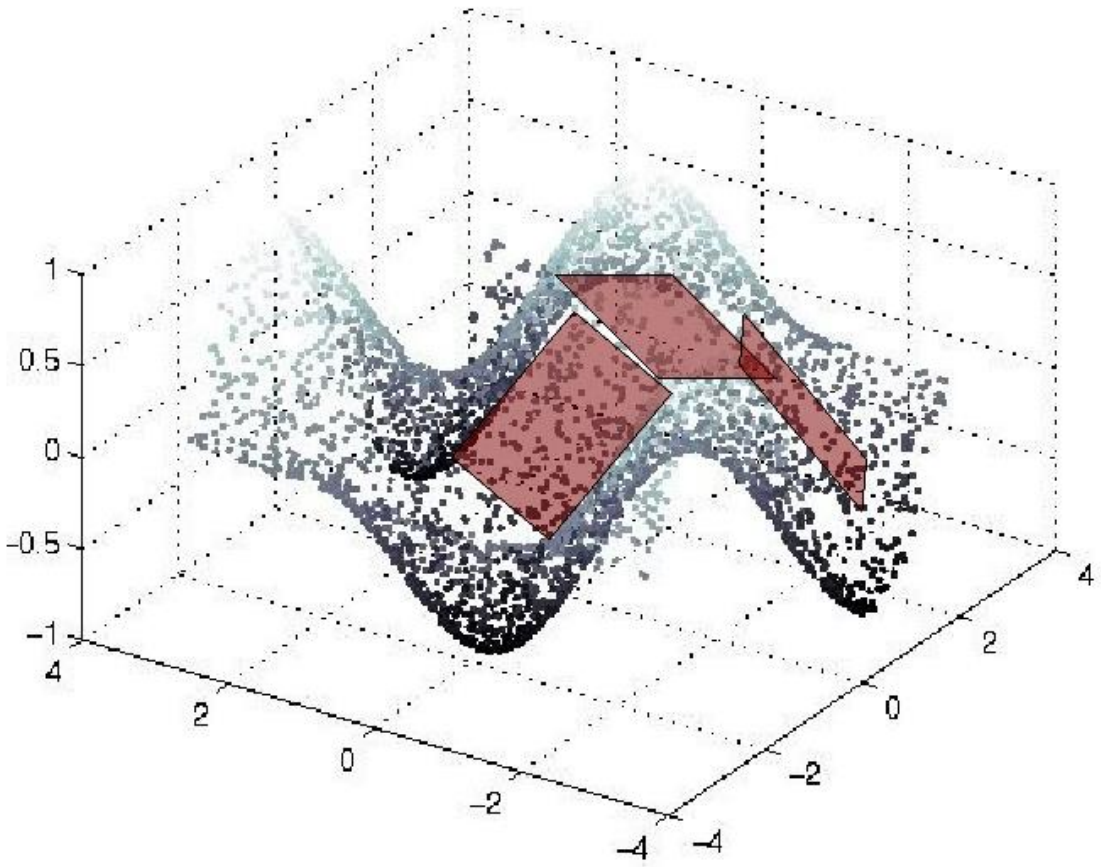



$W_2x + b$
$W_1x + b$
ReLU
$W_2x + b$
$W_1x + b$

Rectified Linear Unit

Question: What do ReLU layers accomplish?

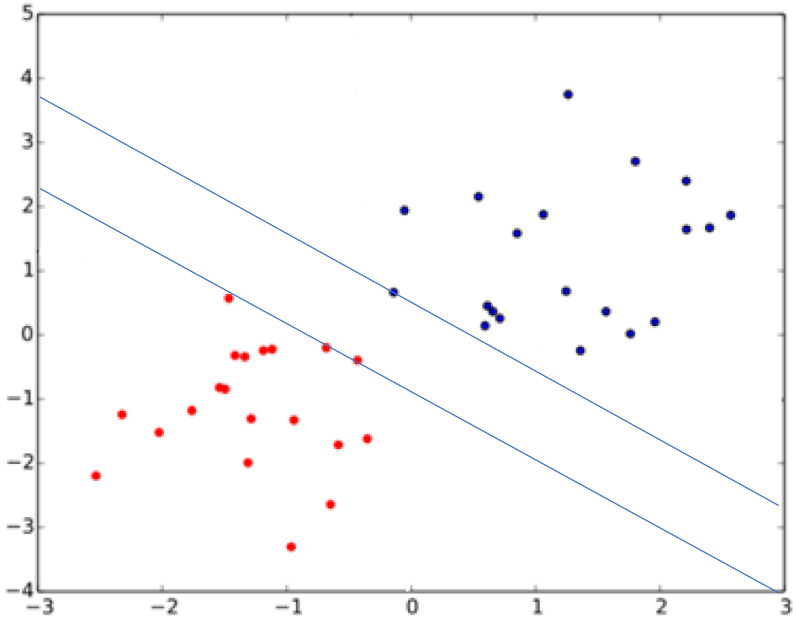
Answer: Piece-wise linear tiling: mapping is locally linear.



Ranzato

What is the relationship between SVMs and perceptrons?

SVMs attempt to learn the support vectors which maximize the margin between classes.

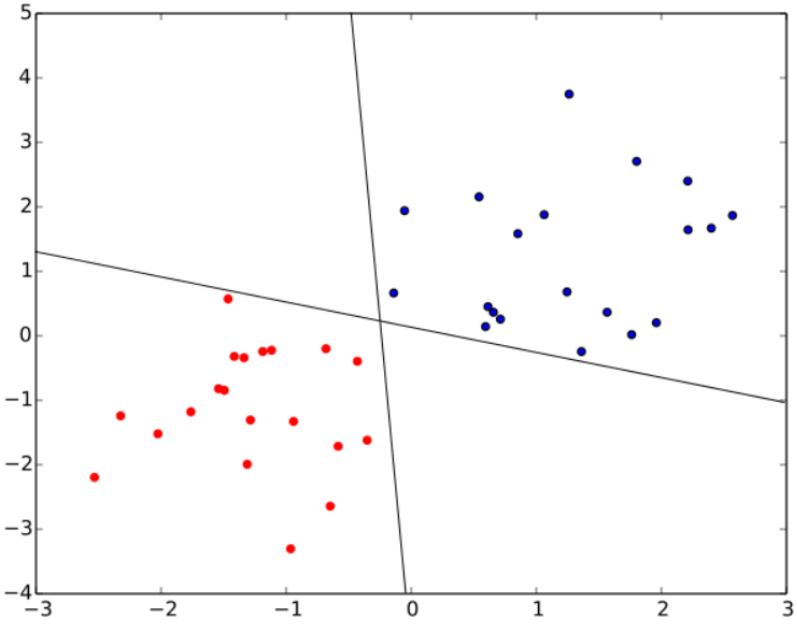


What is the relationship between SVMs and perceptrons?

SVMs attempt to learn the support vectors which maximize the margin between classes.

A perceptron does not.
Both of these perceptron classifiers are equivalent.

‘Perceptron of optimal stability’ is used in SVM:



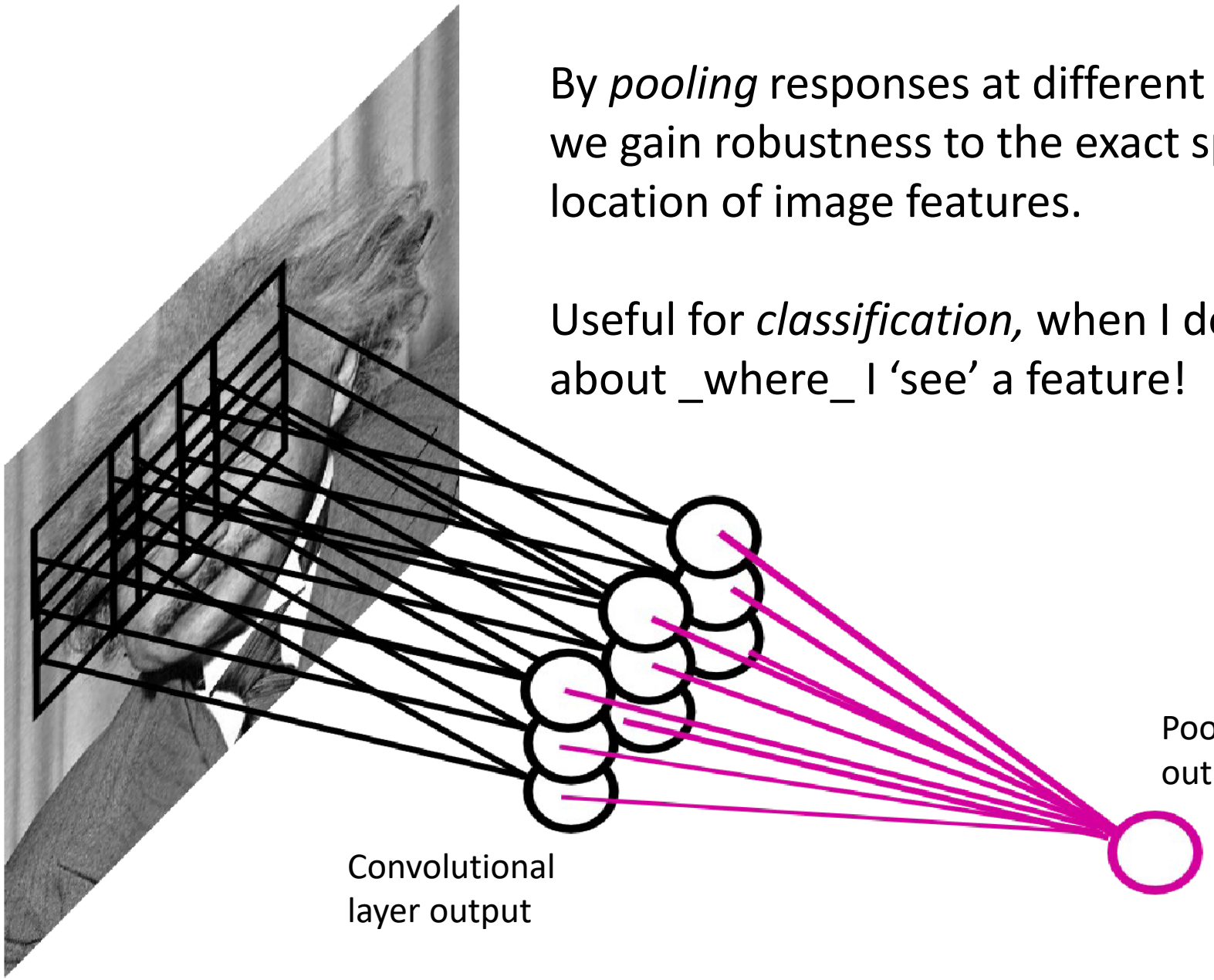
Perceptron
+ optimal stability
+ kernel trick
= foundations of SVM

Why is pooling useful again?
What kinds of pooling operations might we consider?

Pooling Layer

By *pooling* responses at different locations, we gain robustness to the exact spatial location of image features.

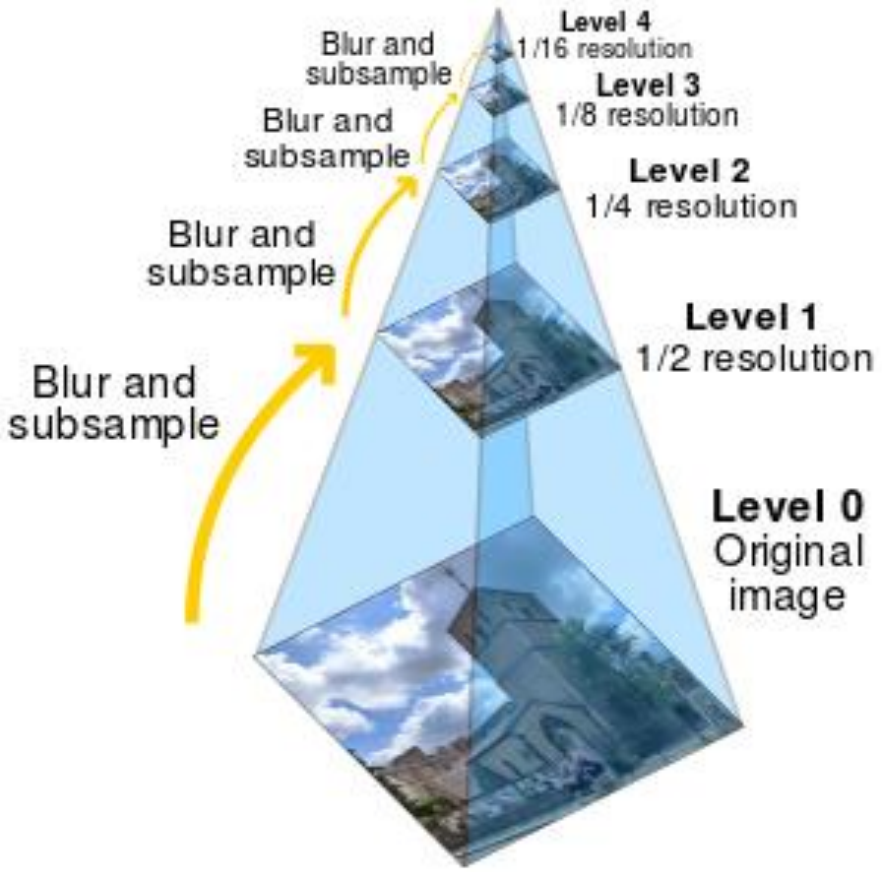
Useful for *classification,* when I don't care about _where_ I 'see' a feature!



Pooling is similar to downsampling

…but on feature maps,
not the input!

…except sometimes we
don't want to blur,
as other functions might
be better for classification.



Pooling Layer: Examples

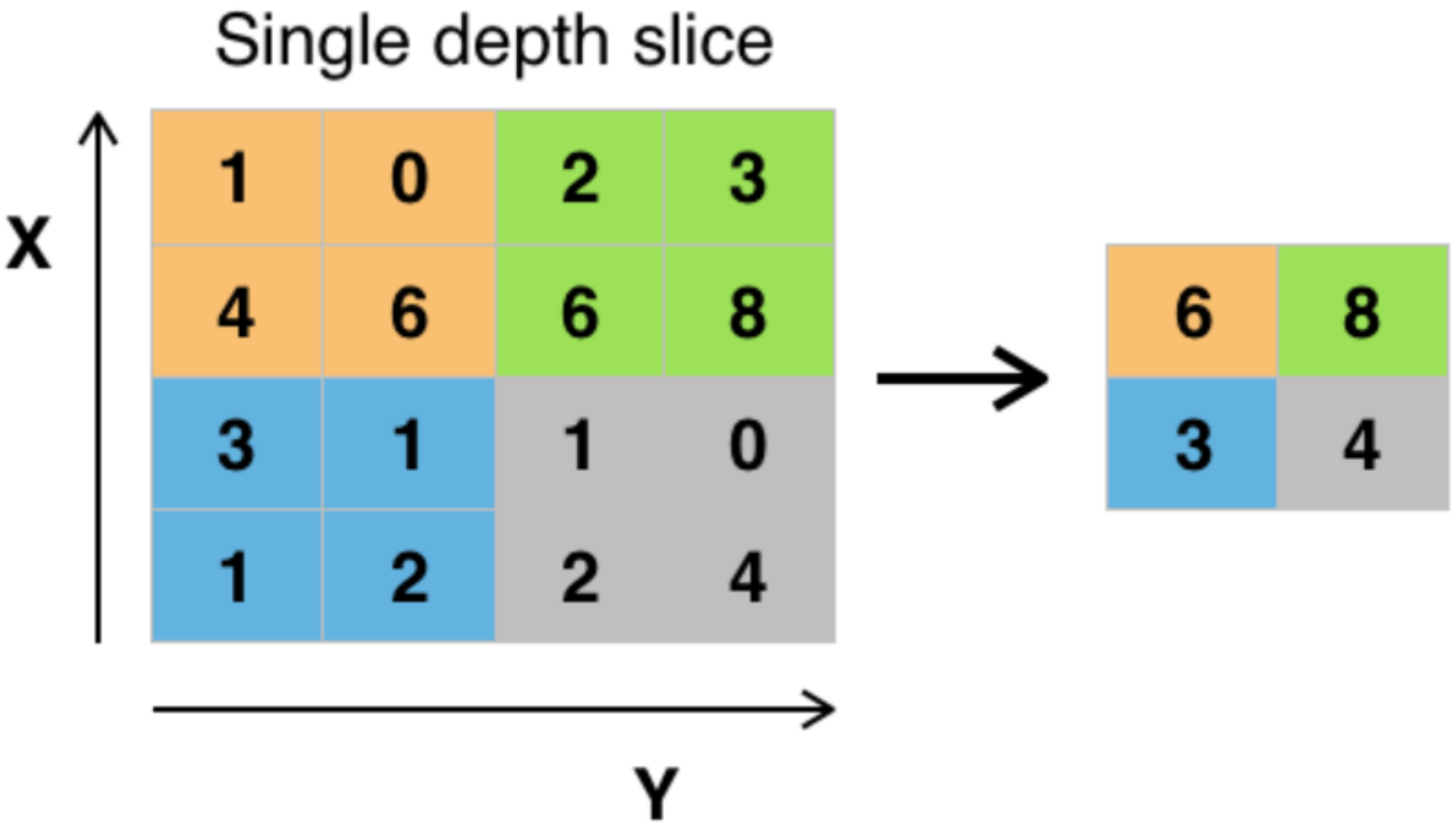
Max-pooling:

$$h_j^n(x, y) = max_{\bar{x} \in N(x),\, \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Average-pooling:

$$h_j^n(x, y) = 1/K \sum_{\bar{x} \in N(x),\, \bar{y} \in N(y)} h_j^{n-1}(\bar{x}, \bar{y})$$

Max pooling



Wikipedia

OK, so does pooling give us rotation invariance?

What about scale invariance?

Convolution is translation invariant ('shift-invariant') – we could shift the image and the kernel would give us a corresponding shift in the feature map.

But if we rotated or scaled the input, the same kernel would give a very different response.

Pooling lets us aggregate (avg) or pick from (max) responses, but the kernels themselves must be trained on and so learn to activate on scaled or rotated instances of the object.

If we max pooled over depth (# kernels)…

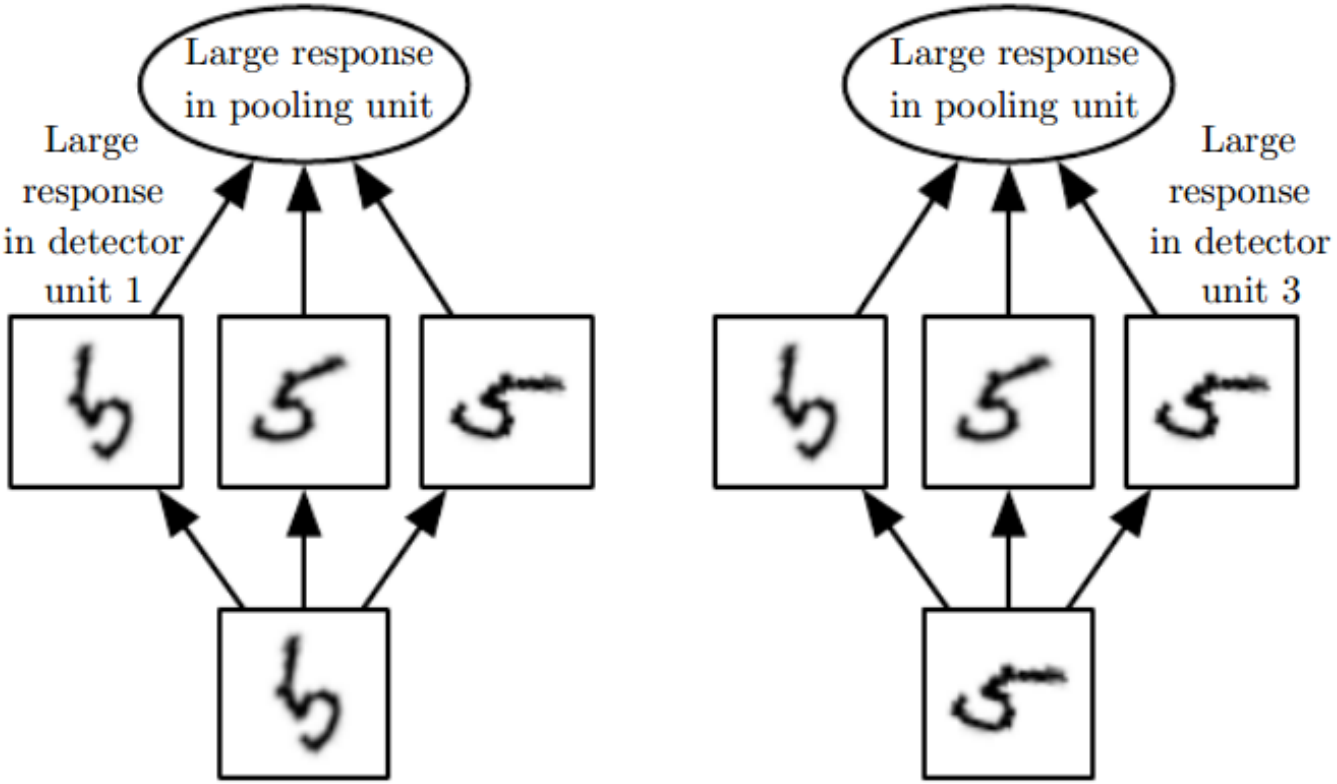
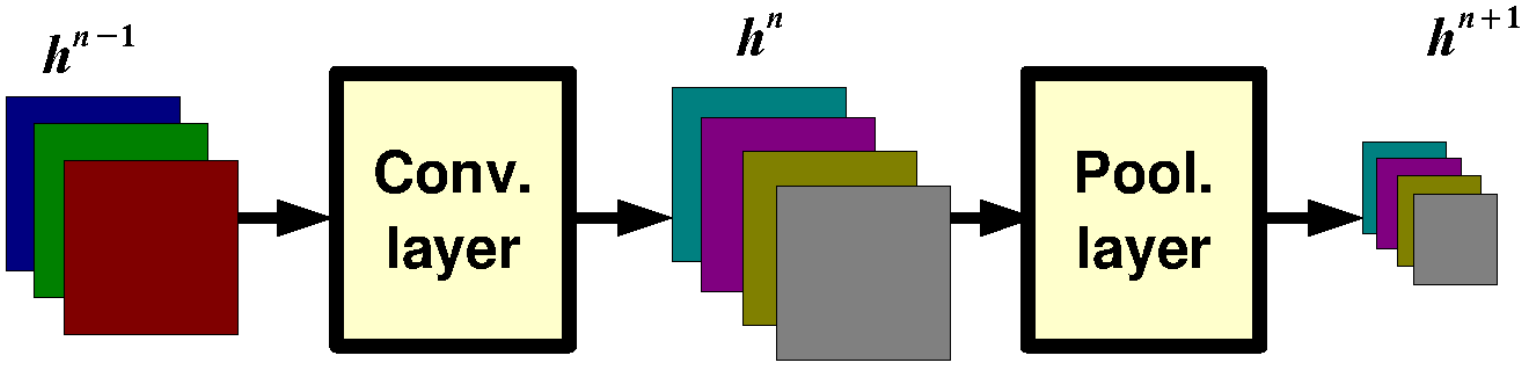
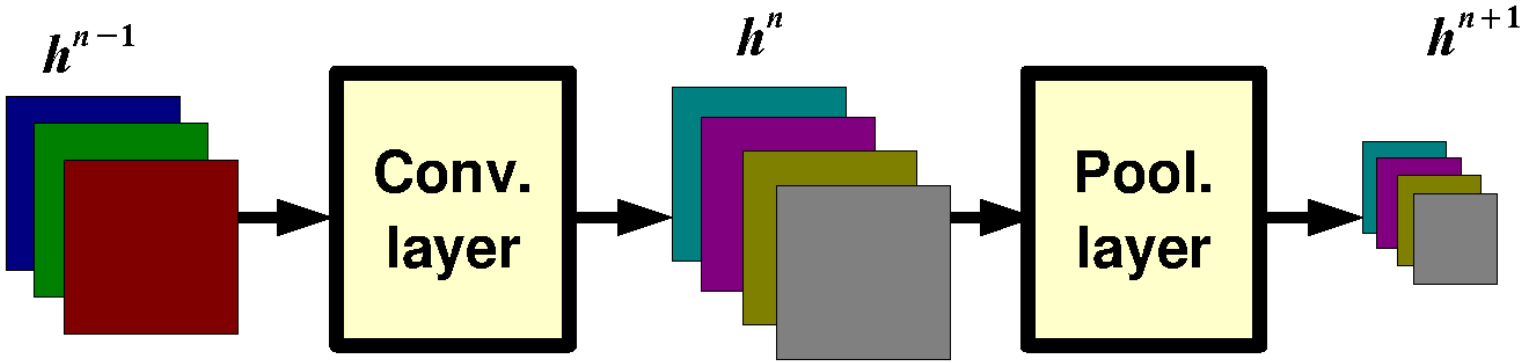


Fig 9.9, Goodfellow et al. [the book]

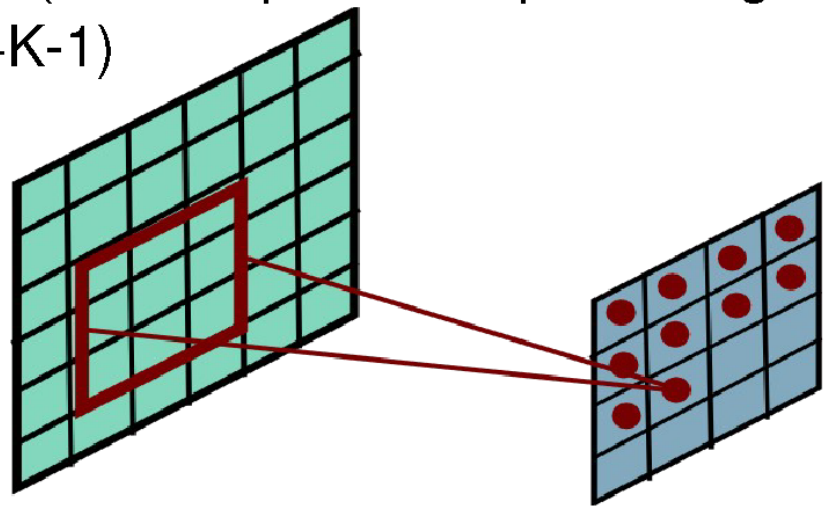
Pooling Layer: Receptive Field Size



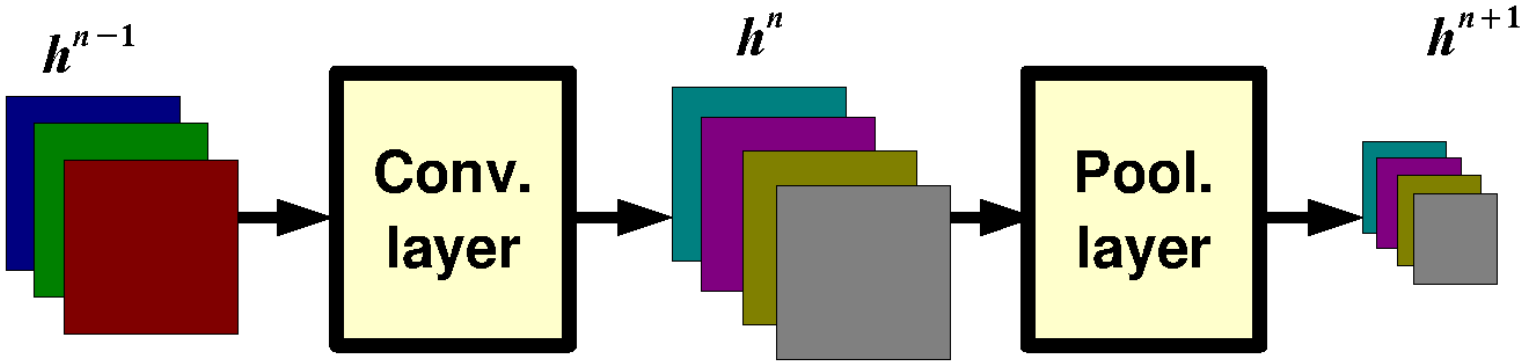
Pooling Layer: Receptive Field Size



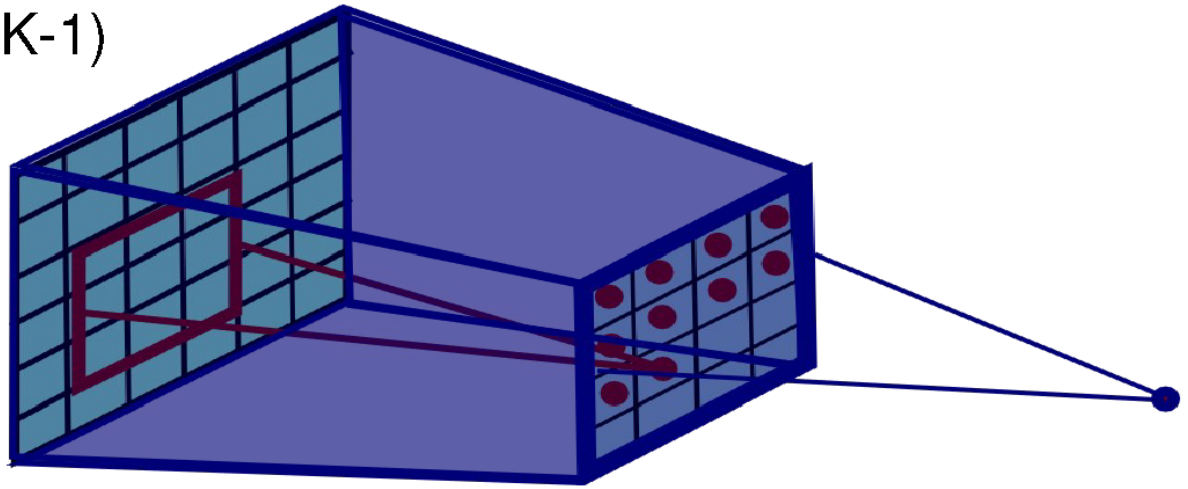
If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)



Pooling Layer: Receptive Field Size



If convolutional filters have size KxK and stride 1, and pooling layer has pools of size PxP, then each unit in the pooling layer depends upon a patch (at the input of the preceding conv. layer) of size: (P+K-1)x(P+K-1)



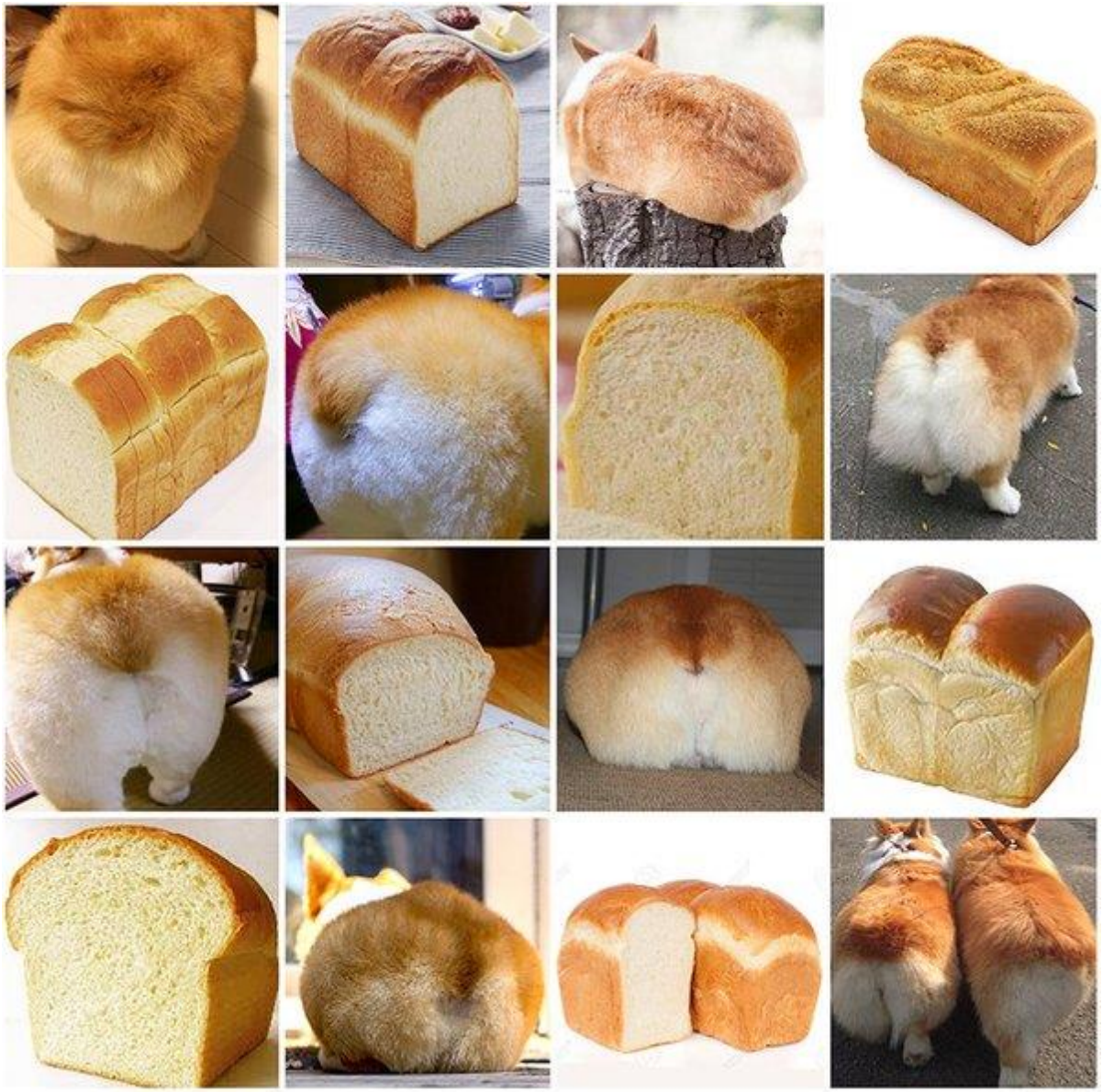
I've heard about many more terms of jargon!

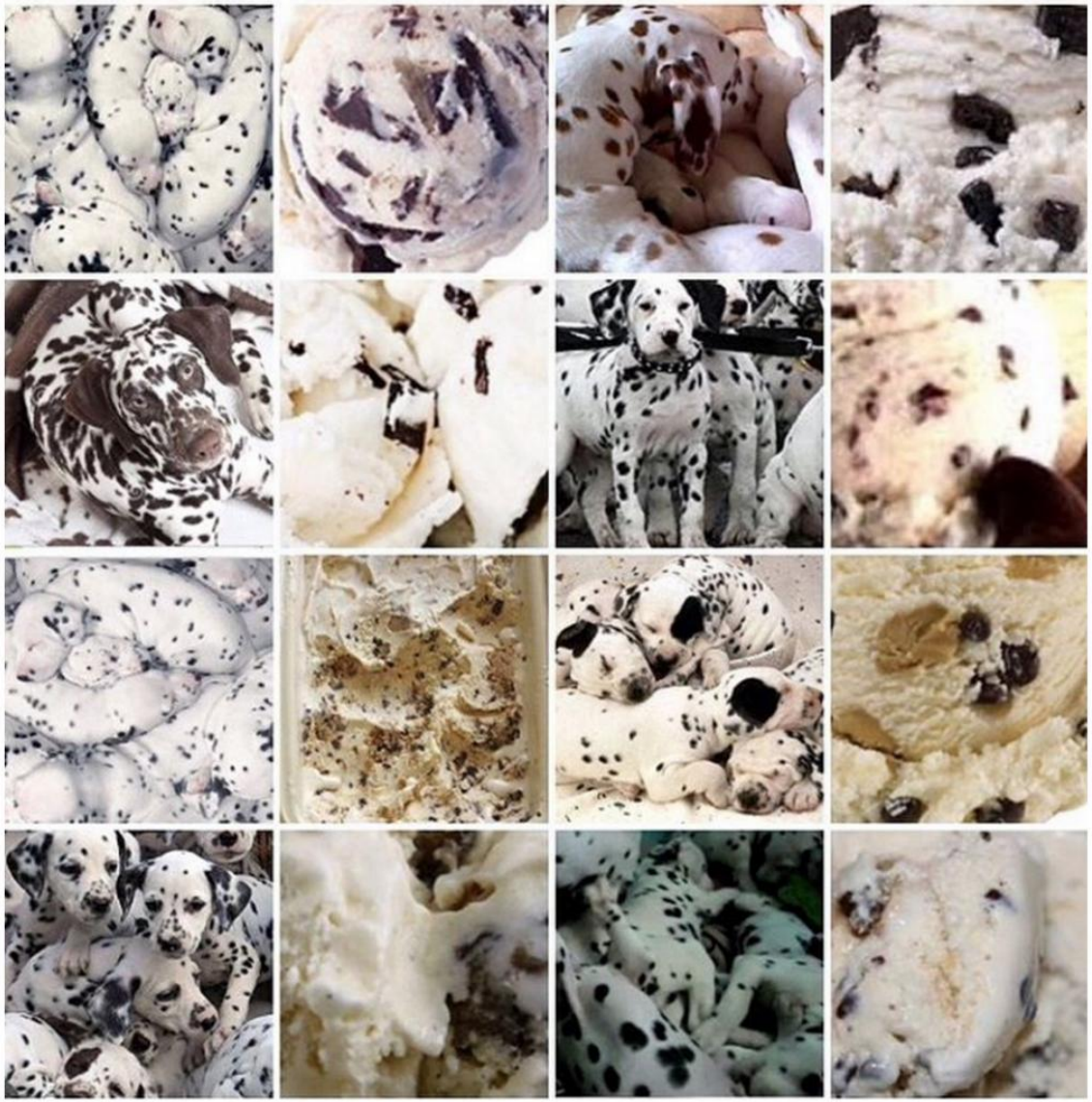
Skip connections

Residual connections

Batch normalization

…we'll get to these in a little while.

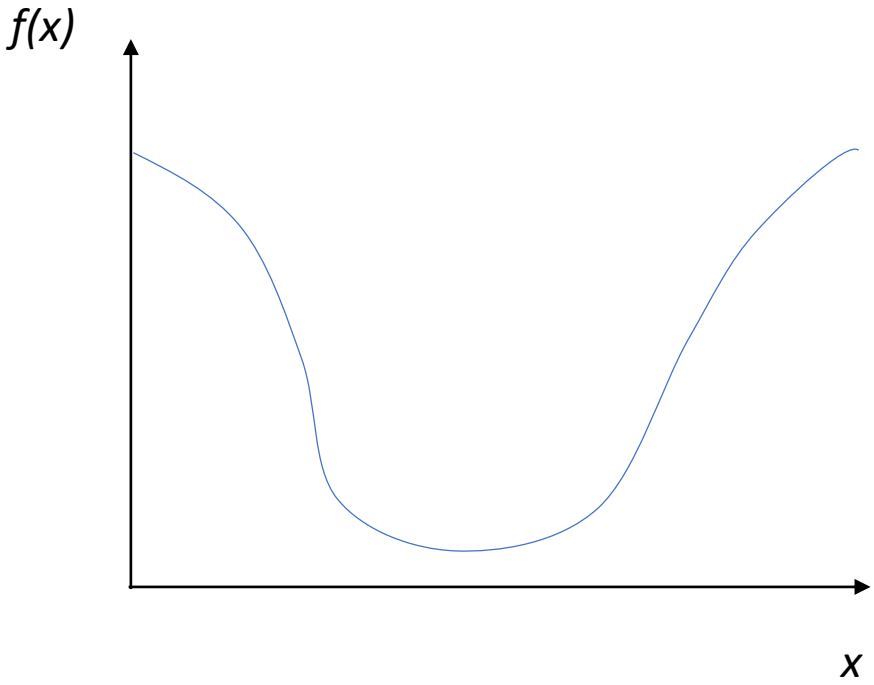




Training Neural Networks

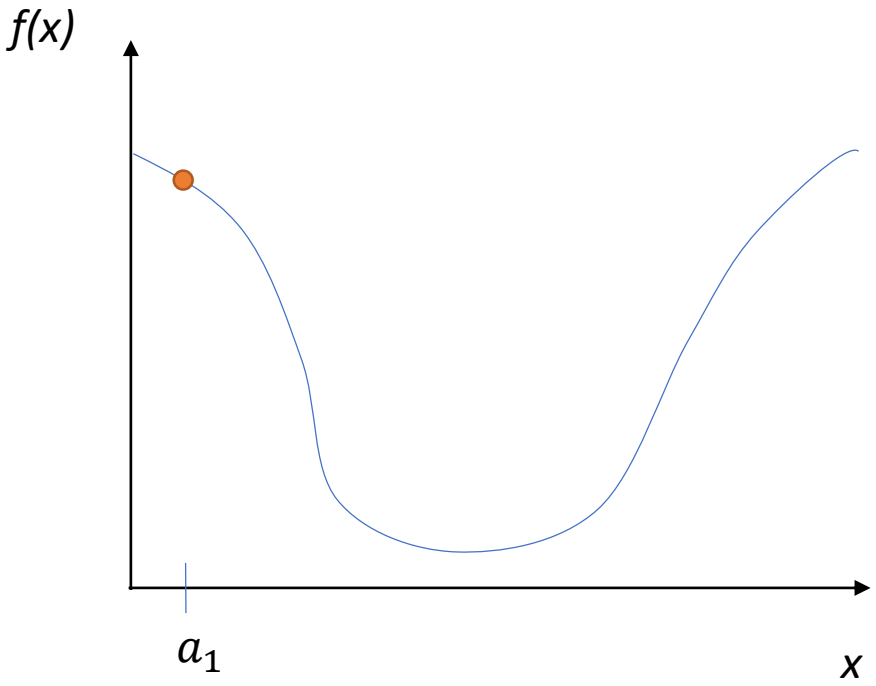
Learning the weight matrices ***W***

Gradient descent



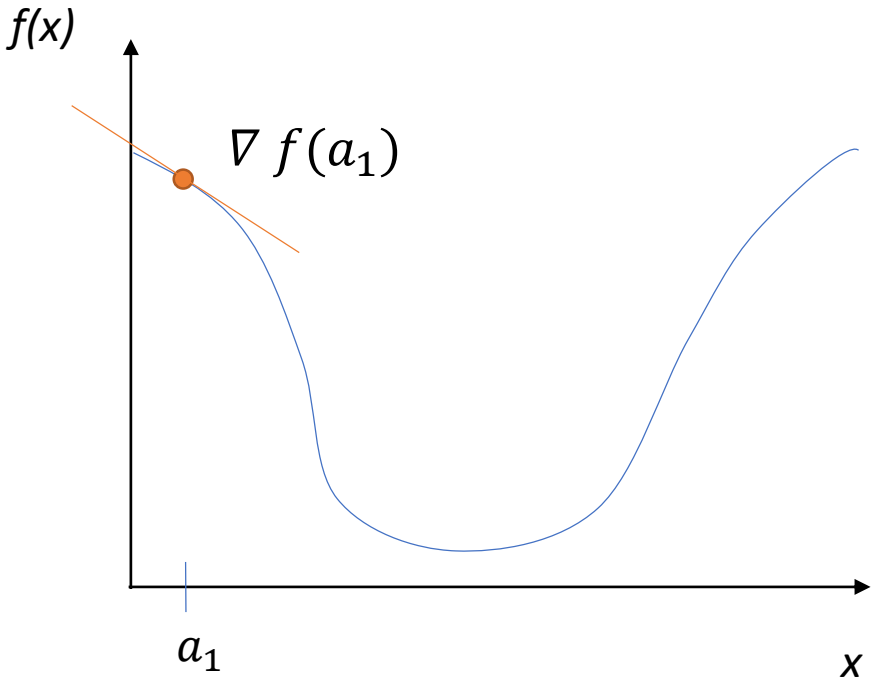
General approach

Pick random starting point.



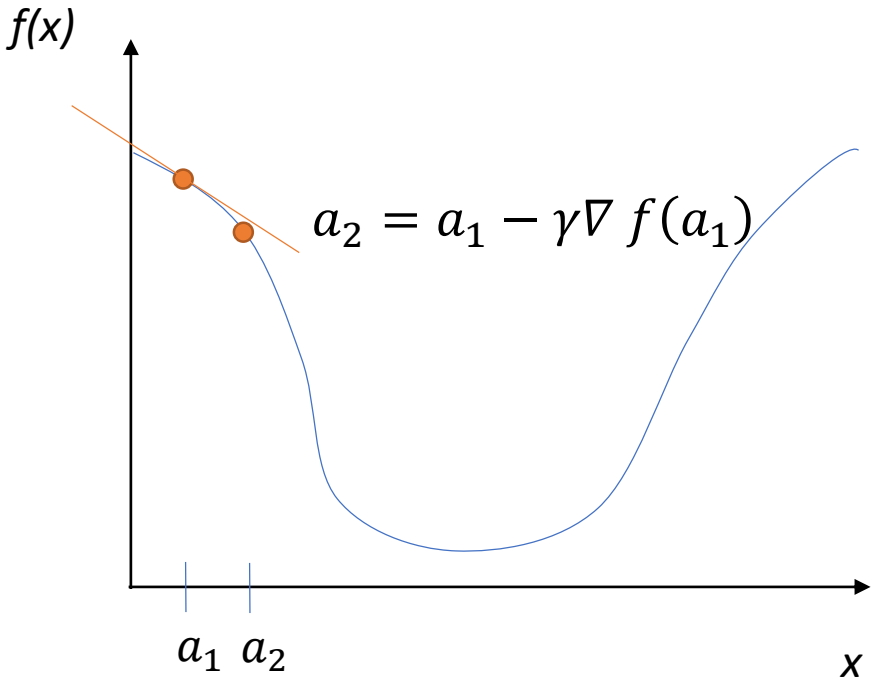
General approach

Compute gradient at point (analytically or by finite differences)



General approach

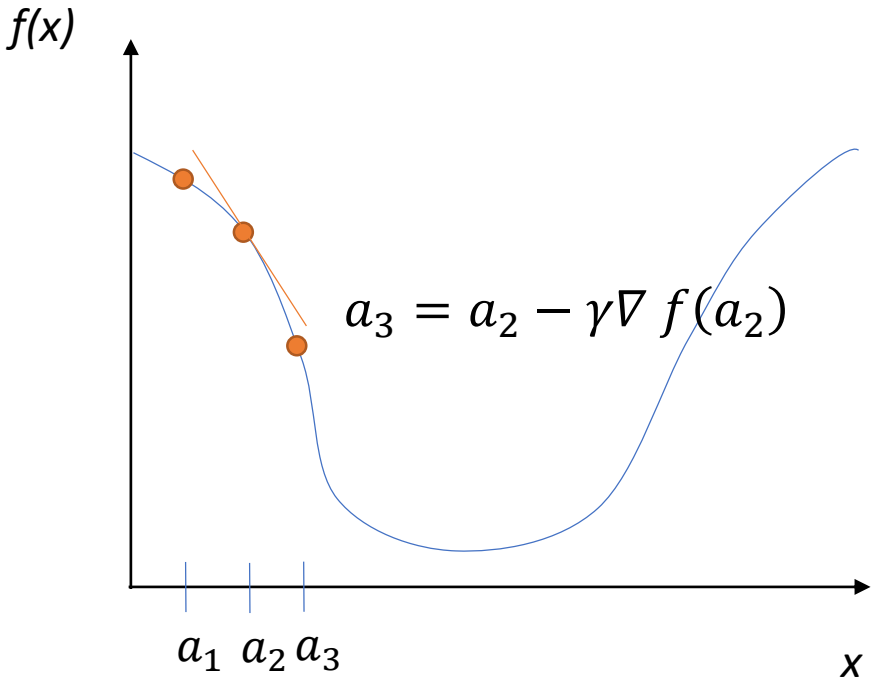
Move along parameter space in direction of negative gradient



γ = amount to move
= *learning rate*

General approach

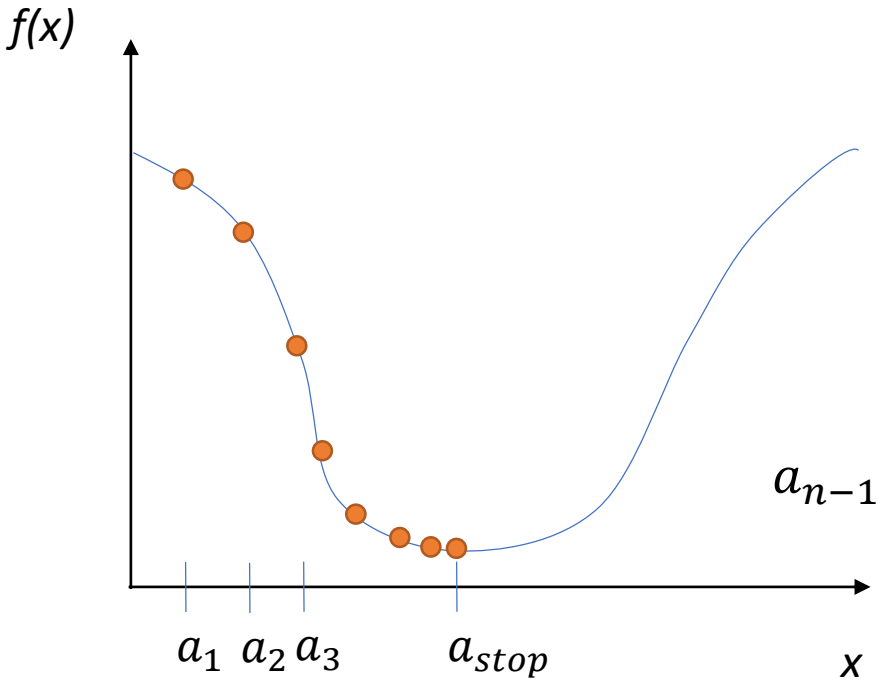
Move along parameter space in direction of negative gradient.



γ = amount to move
= *learning rate*

General approach

Stop when we don't move any more.

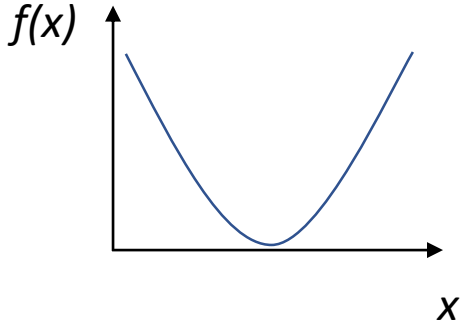


a_{stop}:

$$a_{n-1} - \gamma \nabla f(a_{n-1}) = 0$$

Gradient descent

- Optimizer for functions.
- Guaranteed to find optimum for convex functions.
  - Non-convex = find *local* optimum.
  - Most vision problems aren't convex.



- Works for multi-variate functions.
  - Need to compute matrix of *partial derivatives ("Jacobian")*

Why would I use this over Least Squares?

If my function is convex,
why can't I just use linear least squares?

$$A\mathbf{x} - \mathbf{b} = 0$$

$$F(\mathbf{x}) = \|A\mathbf{x} - \mathbf{b}\|^2.$$

$$\nabla F(\mathbf{x}) = 2A^T(A\mathbf{x} - \mathbf{b}).$$

Analytic solution = normal equations $x = (A^T A)^{-1} A^T b$

You can, yes.

Why would I use this over Least Squares?

But now imagine that I have 1,000,000 data points.

Matrices are _huge_.

Even for convex functions, gradient descent allows me to iteratively solve the solution without requiring very large matrices.

We'll see how.

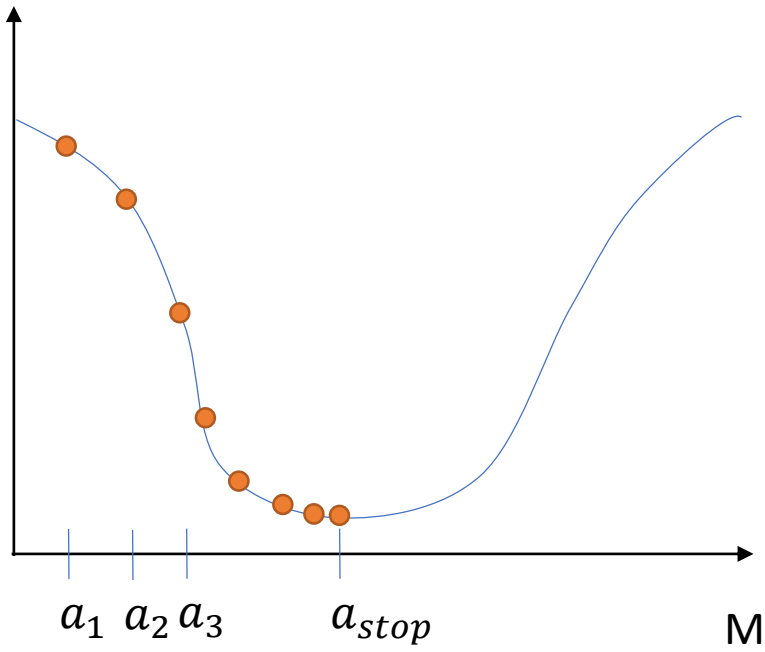
Train NN with Gradient Descent

- x^i, y^i = *n* training examples
- $f(\boldsymbol{x})$ = feed forward neural network
- L(**x**, y; **θ**) = some *loss function*

- *Loss function* measures how 'good' our network is at classifying the training examples wrt. the parameters of the model (the perceptron weights).

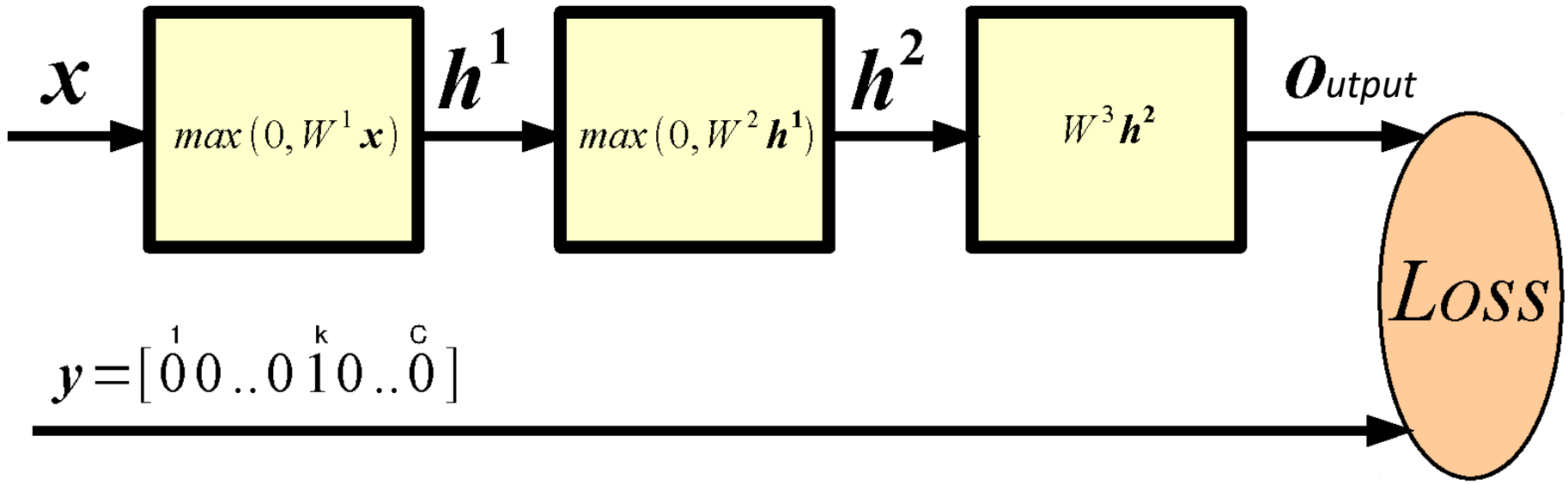
Train NN with Gradient Descent

Loss function
(Evaluate NN
on training data)



Model parameters
(perceptron weights)

How Good is a Network?



What is an appropriate loss?

- Define some output threshold on detection
- Classification: compare training class to output class
- Zero-one loss L (per class)

$y = true\ label$
$\hat{y} = predicted\ label$

$$L(\hat{y}, y) = I(\hat{y} \neq y),$$

- Is it good?
  - Nope – it's a step function.
  - I need to compute the *gradient* of the loss.
  - This loss is not differentiable, and 'flips' easily.

Classification as probability

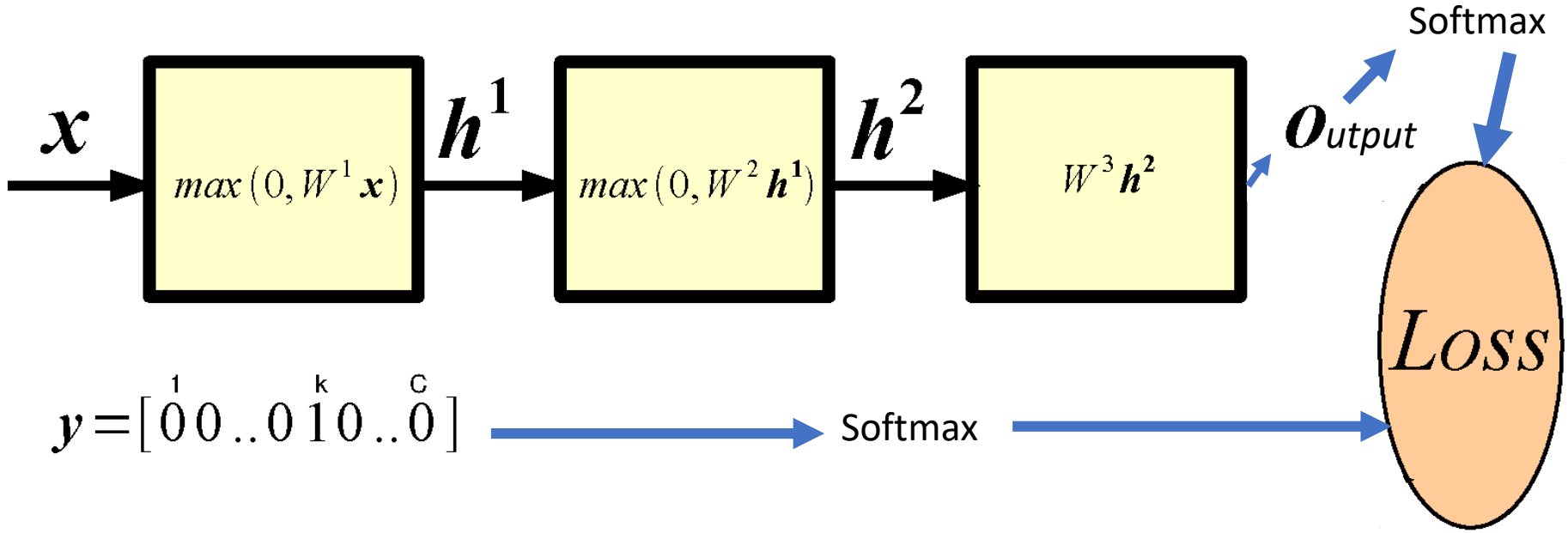
Special function on last layer - 'S*oftmax*':

"Squashes" a *C*-dimensional vector ***O*** of arbitrary real values to a *C*-dimensional vector σ(***O***) of real values in the range (0, 1) that add up to 1.

Turns the output into a probability distribution on classes.

$$p(c_k = 1 | \boldsymbol{x}) = \frac{e^{o_k}}{\sum_{j=1}^{C} e^{o_j}}$$

How Good is a Network?



Probability of class k given input (softmax):

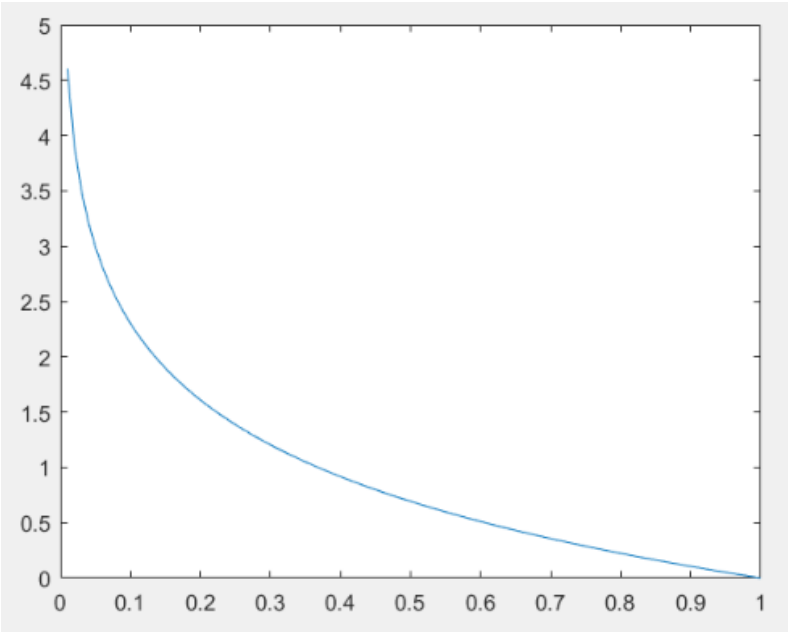
$$p(c_k = 1 | \boldsymbol{x}) = \frac{e^{o_k}}{\sum_{j=1}^{C} e^{o_j}}$$

Cross-entropy loss function

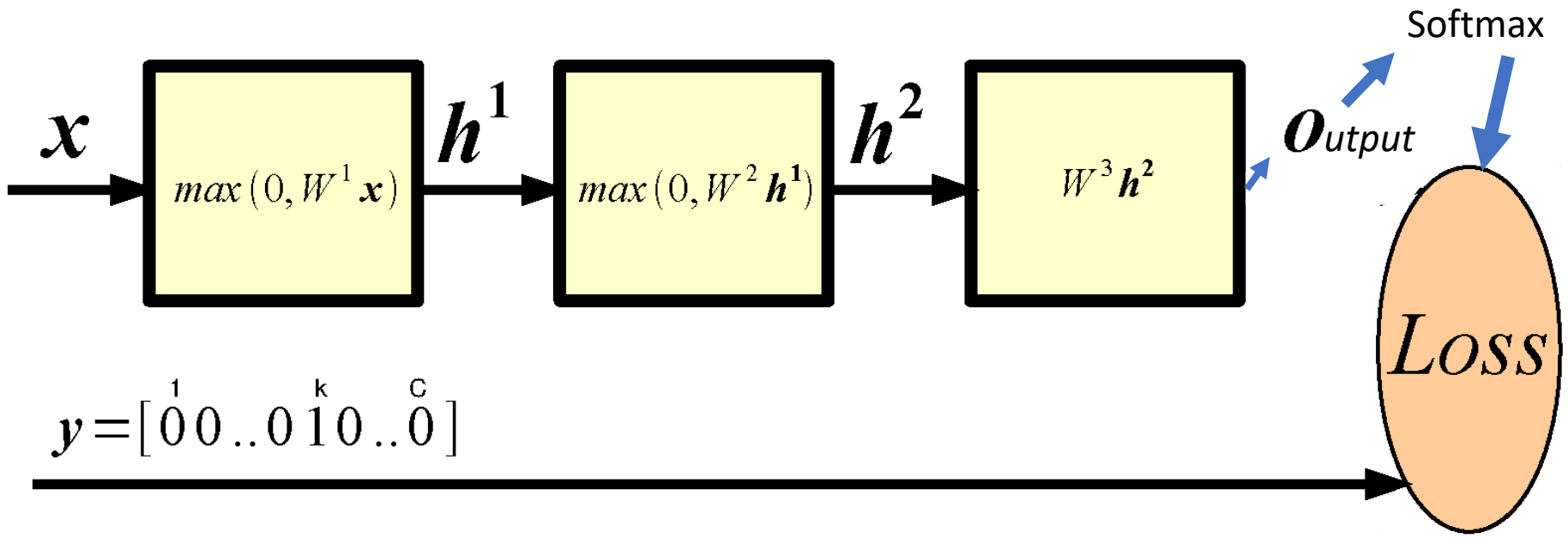
- Negative log-likelihood

$$L(\boldsymbol{x}, y; \boldsymbol{\theta}) = -\sum_j y_j \log p(c_j|\boldsymbol{x})$$

- Minimizing this is equivalent to minimizing KL-divergence on predicted and target probability distributions
- Is it a good loss?
  - Differentiable
  - Cost decreases as probability increases



How Good is a Network?



Probability of class k given input (softmax):

$$p(c_k = 1|\boldsymbol{x}) = \frac{e^{o_k}}{\sum_{j=1}^{C} e^{o_j}}$$

(Per-sample) **Loss**; e.g., negative log-likelihood (good for classification of small number of classes):

$$L(\boldsymbol{x}, y; \boldsymbol{\theta}) = -\sum_j y_j \log p(c_j|\boldsymbol{x})$$

Training

Learning consists of minimizing the loss (plus some regularization term) w.r.t. parameters over the whole training set.

$$\boldsymbol{\theta}^* = arg\, min_{\boldsymbol{\theta}} \sum_{n=1}^{P} L\left(\boldsymbol{x}^n, y^n; \boldsymbol{\theta}\right)$$

Training

Learning consists of minimizing the loss (plus some regularization term) w.r.t. parameters over the whole training set.

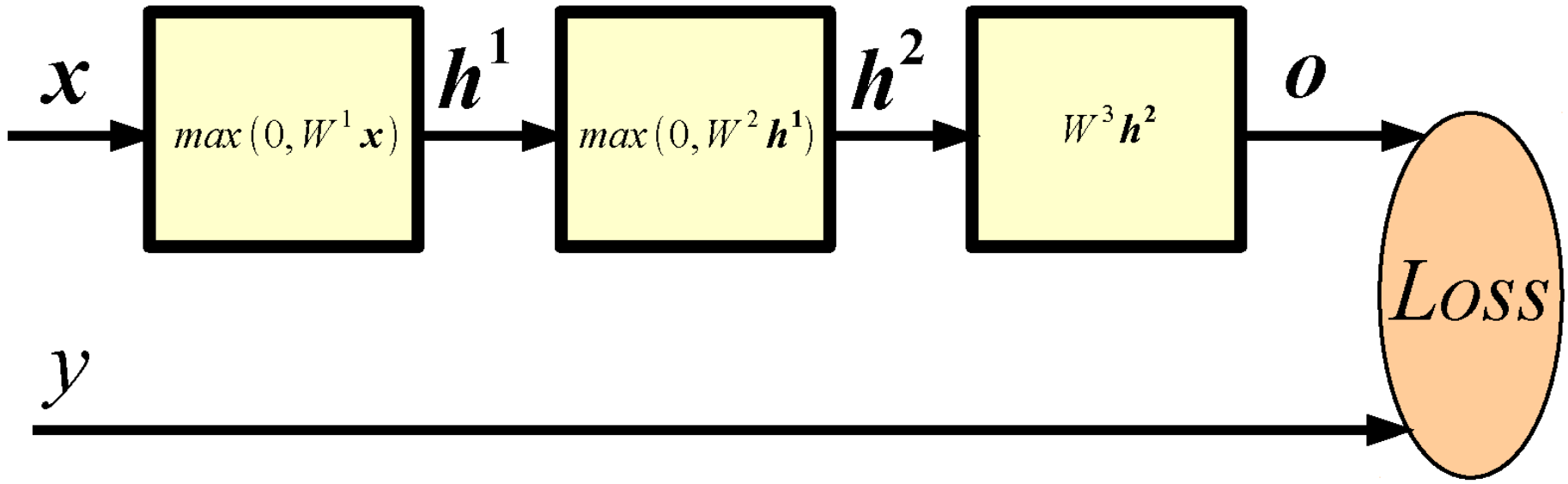
$$\boldsymbol{\theta}^* = arg\,min_{\boldsymbol{\theta}} \sum_{n=1}^{P} L(\boldsymbol{x}^n, y^n; \boldsymbol{\theta})$$

Question: How to minimize a complicated function of the parameters?

Answer: Chain rule, a.k.a. **Backpropagation**! That is the procedure to compute gradients of the loss w.r.t. parameters in a multi-layer neural network.

Rumelhart et al. "Learning internal representations by back-propagating.." Nature 1986

Key Idea: Wiggle To Decrease Loss

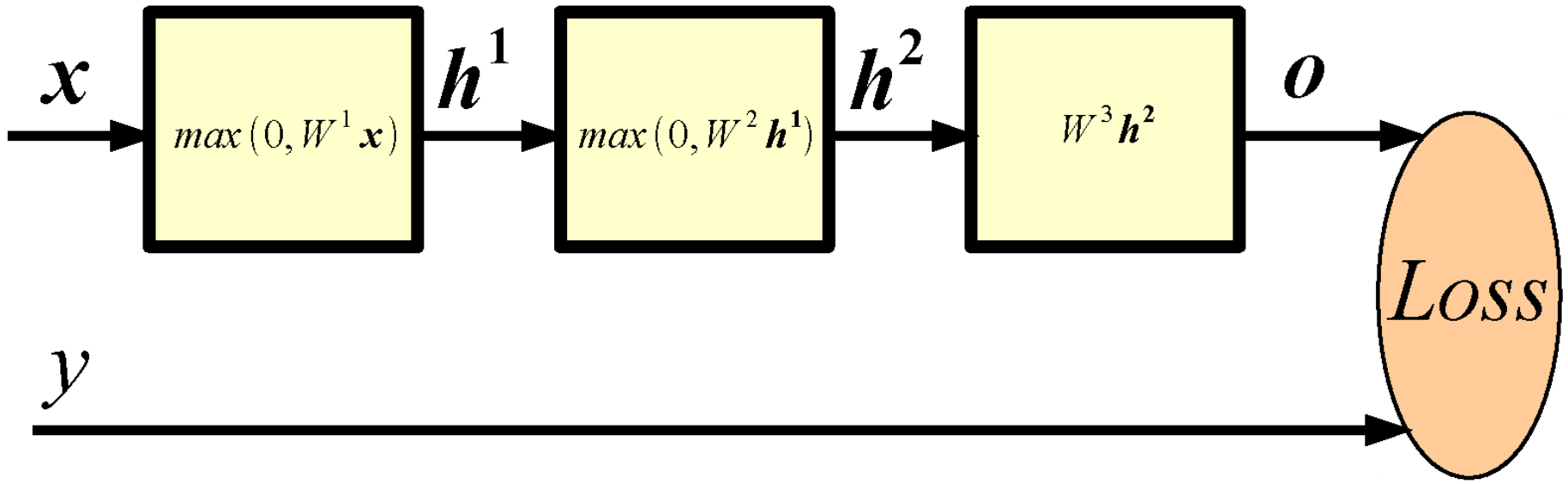


Let's say we want to decrease the loss by adjusting $W^1_{i,j}$.
We could consider a very small $\epsilon = 1\text{e-}6$ and compute:

$$L(\boldsymbol{x}, y; \boldsymbol{\theta})$$

$$L(\boldsymbol{x}, y; \boldsymbol{\theta} \setminus W^1_{i,j}, W^1_{i,j} + \epsilon)$$

Key Idea: Wiggle To Decrease Loss



Let's say we want to decrease the loss by adjusting $W^1_{i,j}$.
We could consider a very small $\epsilon = 1\text{e-}6$ and compute:

$$L(\boldsymbol{x}, y; \boldsymbol{\theta})$$

$$L(\boldsymbol{x}, y; \boldsymbol{\theta} \setminus W^1_{i,j}, W^1_{i,j} + \epsilon)$$

Then, update:

$$W^1_{i,j} \leftarrow W^1_{i,j} + \epsilon \, sgn\left(L(\boldsymbol{x}, y; \boldsymbol{\theta}) - L(\boldsymbol{x}, y; \boldsymbol{\theta} \setminus W^1_{i,j}, W^1_{i,j} + \epsilon) \right)$$

Derivative w.r.t. Input of Softmax

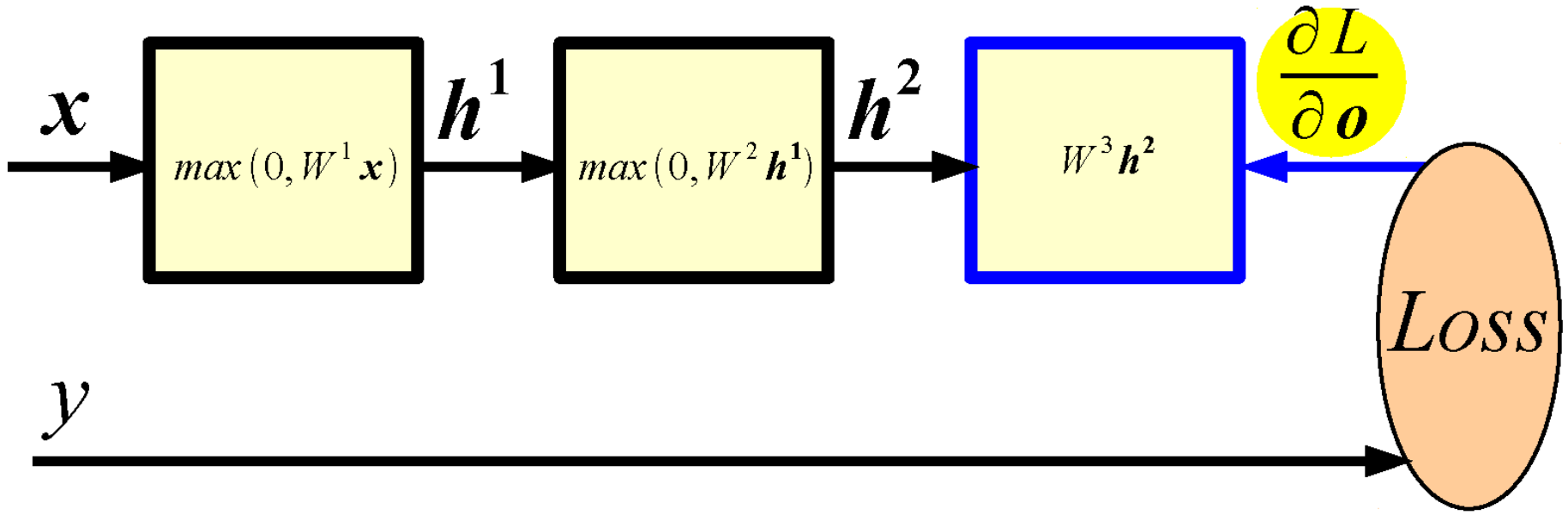
$$p(c_k=1|\boldsymbol{x})=\frac{e^{o_k}}{\sum_j e^{o_j}}$$

$$L(\boldsymbol{x}, y; \boldsymbol{\theta})=-\sum_j y_j \log p(c_j|\boldsymbol{x}) \qquad \boldsymbol{y}=[\overset{1}{0}0..0\overset{k}{1}0..\overset{C}{0}]$$

By substituting the fist formula in the second, and taking the derivative w.r.t. $\boldsymbol{o}$ we get:

$$\frac{\partial L}{\partial o} = p(c|\boldsymbol{x}) - \boldsymbol{y}$$

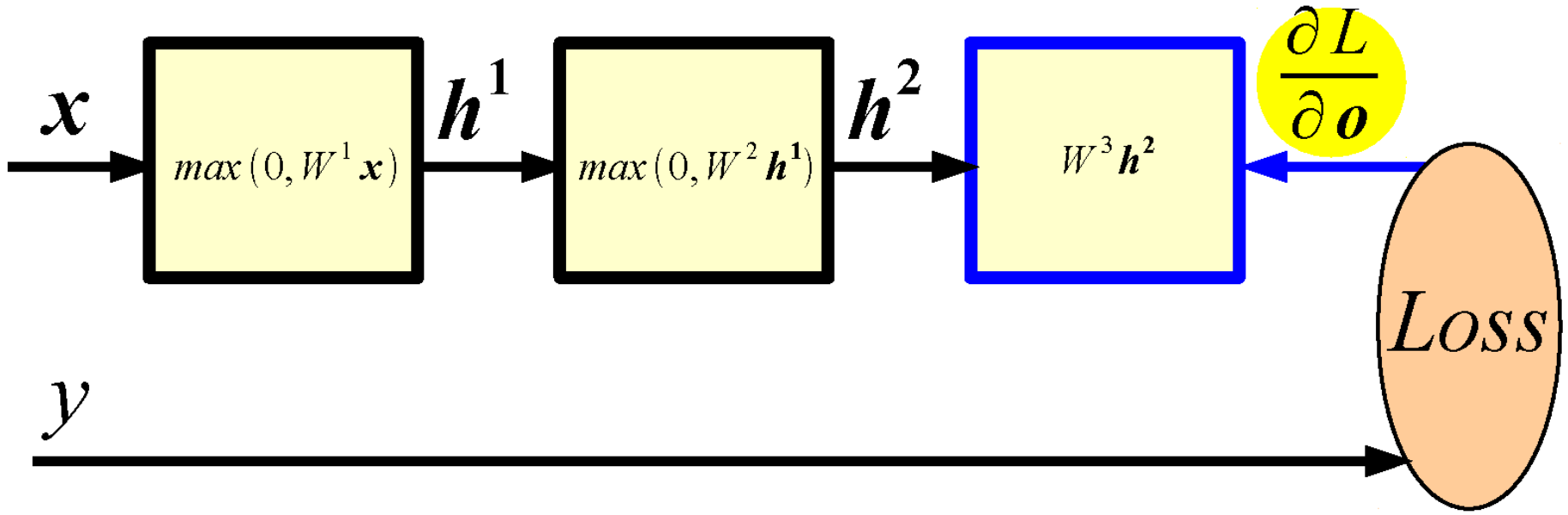
Backward Propagation



Given $\partial L / \partial \boldsymbol{o}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial W^3}$$

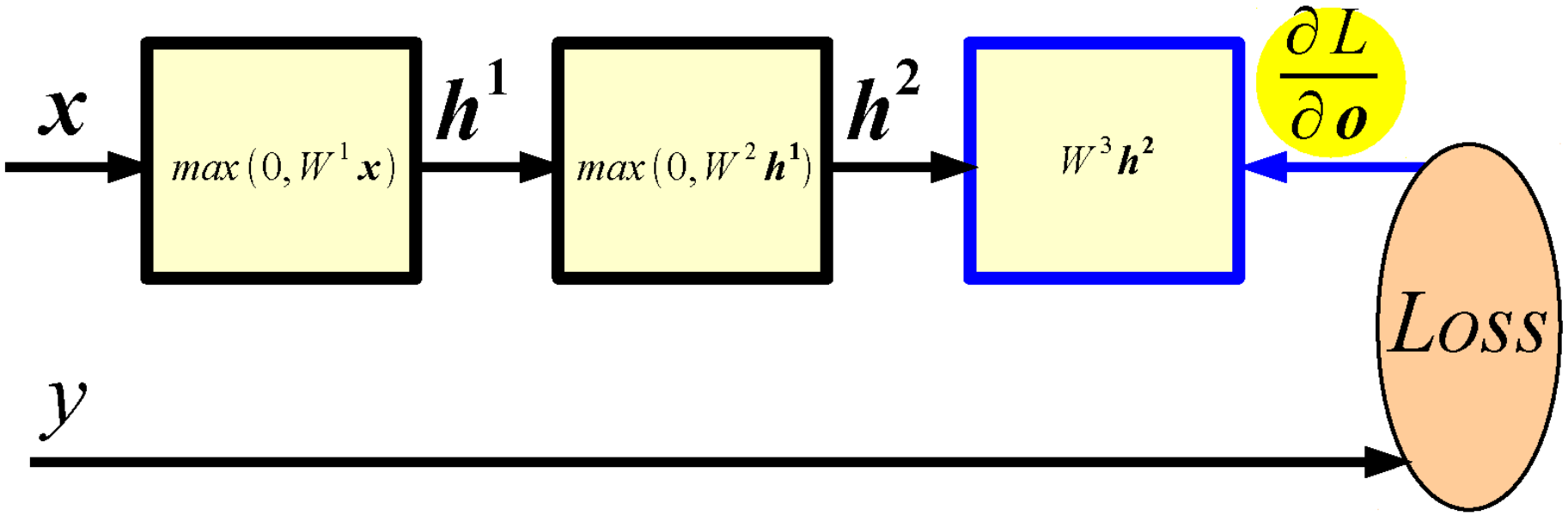
Backward Propagation



Given $\partial L / \partial \boldsymbol{o}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial W^3} \qquad \frac{\partial L}{\partial \boldsymbol{h}^2} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{h}^2}$$

Backward Propagation

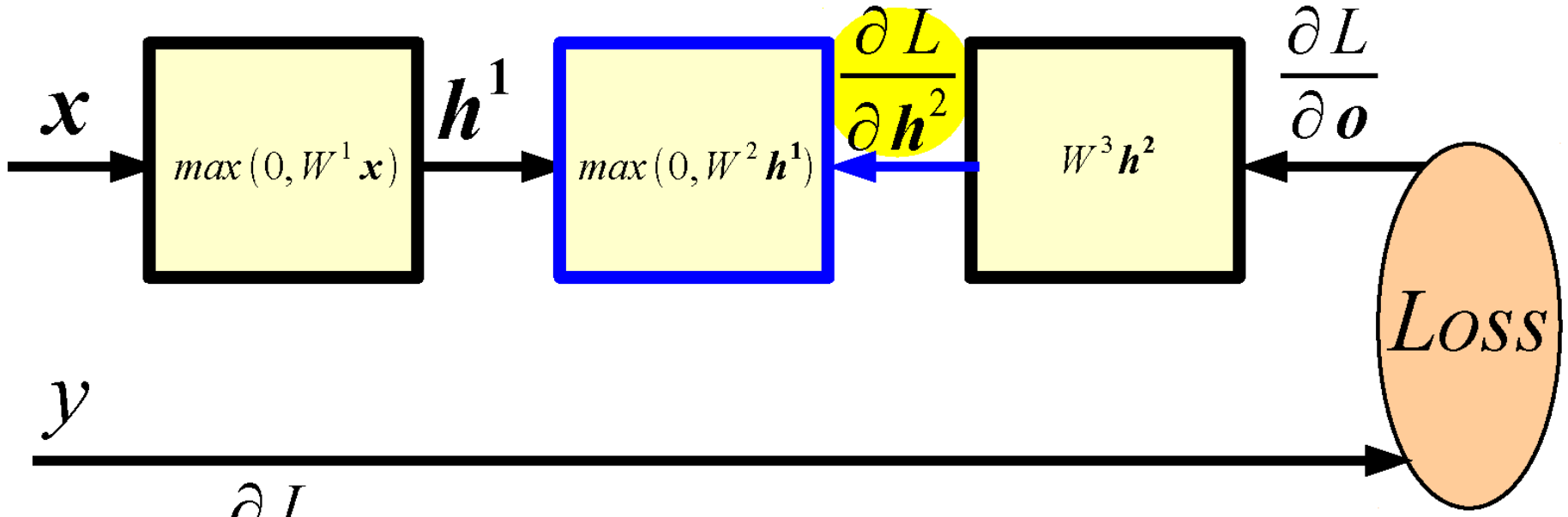


Given $\partial L / \partial \boldsymbol{o}$ and assuming we can easily compute the Jacobian of each module, we have:

$$\frac{\partial L}{\partial W^3} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial W^3} \qquad \frac{\partial L}{\partial \boldsymbol{h}^2} = \frac{\partial L}{\partial \boldsymbol{o}} \frac{\partial \boldsymbol{o}}{\partial \boldsymbol{h}^2}$$

$$\frac{\partial L}{\partial W^3} = (p(c|\boldsymbol{x}) - \boldsymbol{y})\ \boldsymbol{h}^{2\,T} \qquad \frac{\partial L}{\partial \boldsymbol{h}^2} = W^{3\,T}(p(c|\boldsymbol{x}) - \boldsymbol{y})$$

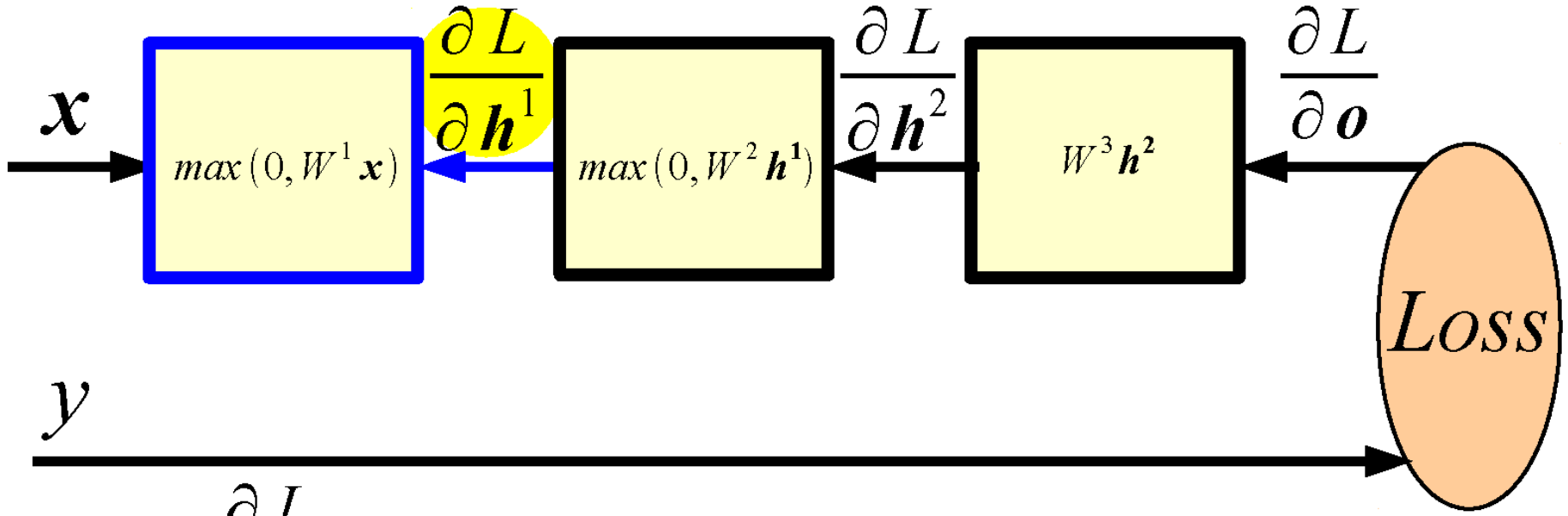
Backward Propagation



Given $\frac{\partial L}{\partial \boldsymbol{h}^2}$ we can compute now:

$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial \boldsymbol{h}^2}\frac{\partial \boldsymbol{h}^2}{\partial W^2} \qquad \frac{\partial L}{\partial \boldsymbol{h}^1} = \frac{\partial L}{\partial \boldsymbol{h}^2}\frac{\partial \boldsymbol{h}^2}{\partial \boldsymbol{h}^1}$$

Backward Propagation



Given $\frac{\partial L}{\partial h^1}$ we can compute now:

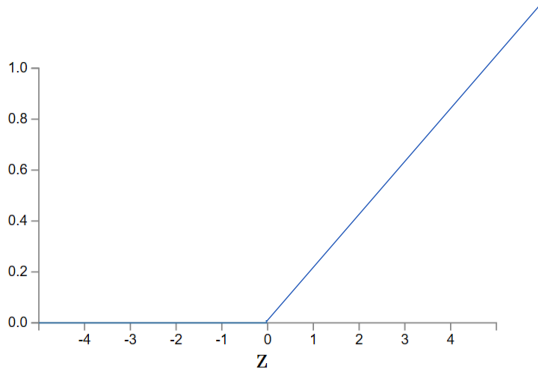
$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial h^1} \frac{\partial h^1}{\partial W^1}$$

Backward Propagation

Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

But the ReLU is not differentiable at 0!



Right. Fudge!

- ‘0’ is the best place for this to occur, because we don’t care about the result (it is no activation).
- ‘Dead’ perceptrons
- ReLU has unbounded positive response:
  - Potential faster convergence / overstep

Backward Propagation

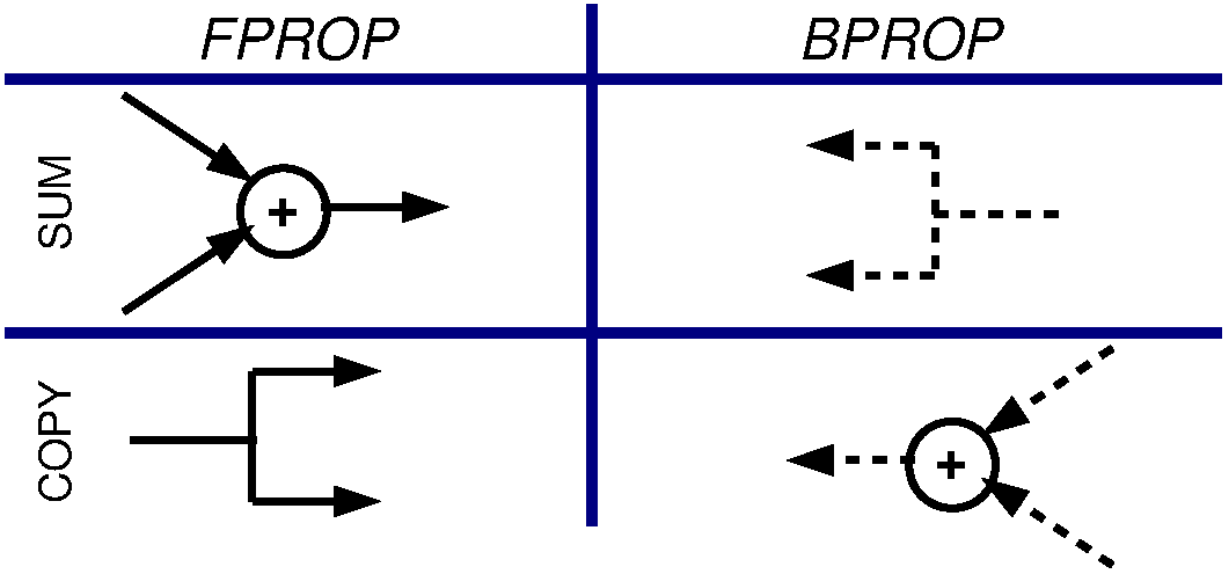
Question: Does BPROP work with ReLU layers only?

Answer: Nope, any a.e. differentiable transformation works.

Question: What's the computational cost of BPROP?

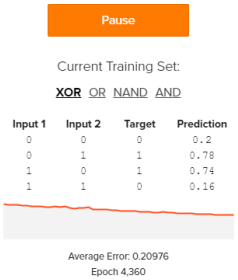
Answer: About twice FPROP (need to compute gradients w.r.t. input and parameters at every layer).

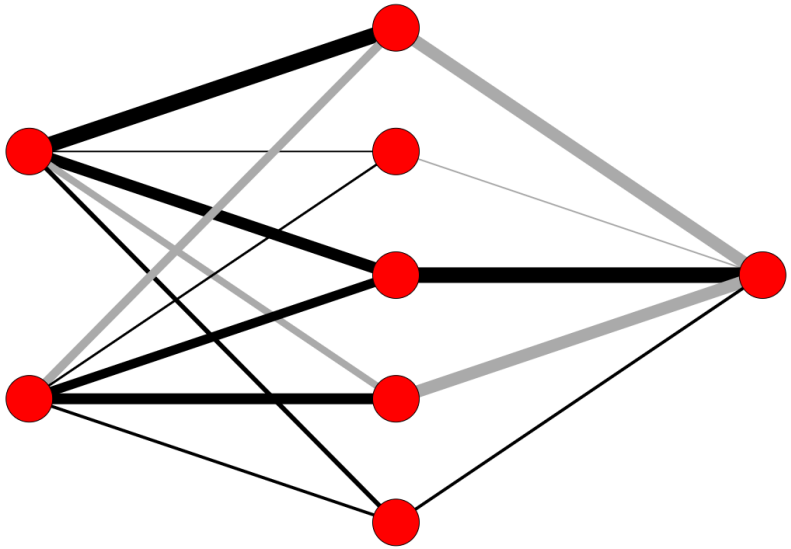
Note: FPROP and BPROP are dual of each other. E.g.,:



Optimization demo

- http://www.emergentmind.com/neural-network
- Thank you Matt Mazur





Toy Code (Matlab): Neural Net Trainer

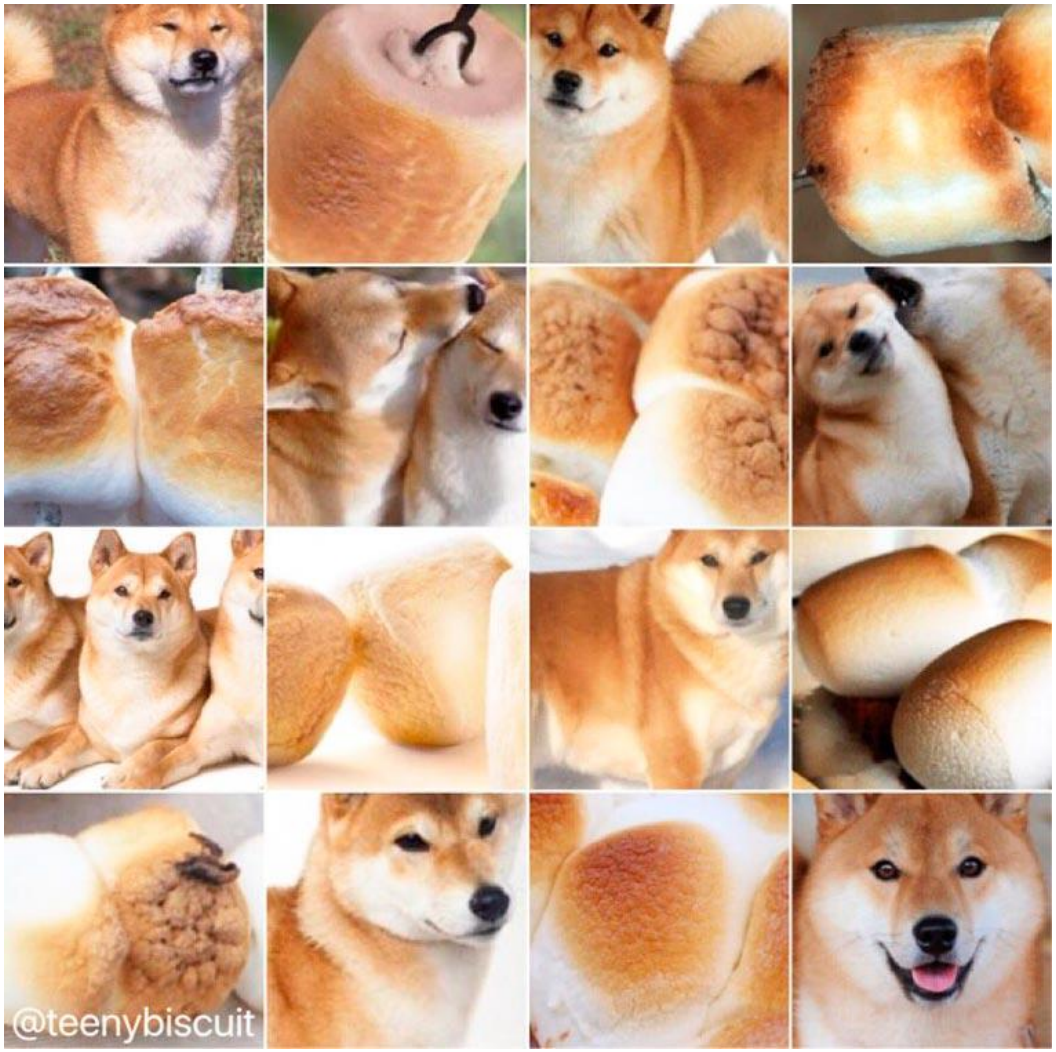
```
% F-PROP
for i = 1 : nr_layers - 1
  [h{i}  jac{i}]  =  nonlinearity(W{i} * h{i-1} +  b{i});
end
h{nr_layers-1}  =  W{nr_layers-1} * h{nr_layers-2}  +  b{nr_layers-1};
prediction  =  softmax(h{l-1});


% CROSS ENTROPY LOSS
loss  =  -  sum(sum(log(prediction)  .*  target)) / batch_size;


% B-PROP
dh{l-1}  =  prediction  -  target;
for i = nr_layers - 1 : -1 : 1
  Wgrad{i}  =  dh{i} * h{i-1}';
  bgrad{i}  =  sum(dh{i}, 2);
  dh{i-1}  =  (W{i}' * dh{i})  .*  jac{i-1};
end


% UPDATE
for i = 1 : nr_layers - 1
  W{i}  =  W{i}  -  (lr / batch_size)  *  Wgrad{i};
  b{i}  =  b{i}  -  (lr / batch_size)  *  bgrad{i};
end
```

Wow



false positives

what class

no good filtr

so misclassified

cool kernel

Stochastic Gradient Descent

- Dataset can be too large to strictly apply gradient descent.
- Instead, randomly sample a data point, perform gradient descent per point, and iterate.
  - True gradient is approximated only
  - Picking a subset of points: "*mini-batch*"

Pick starting W and learning rate γ

While not at minimum:

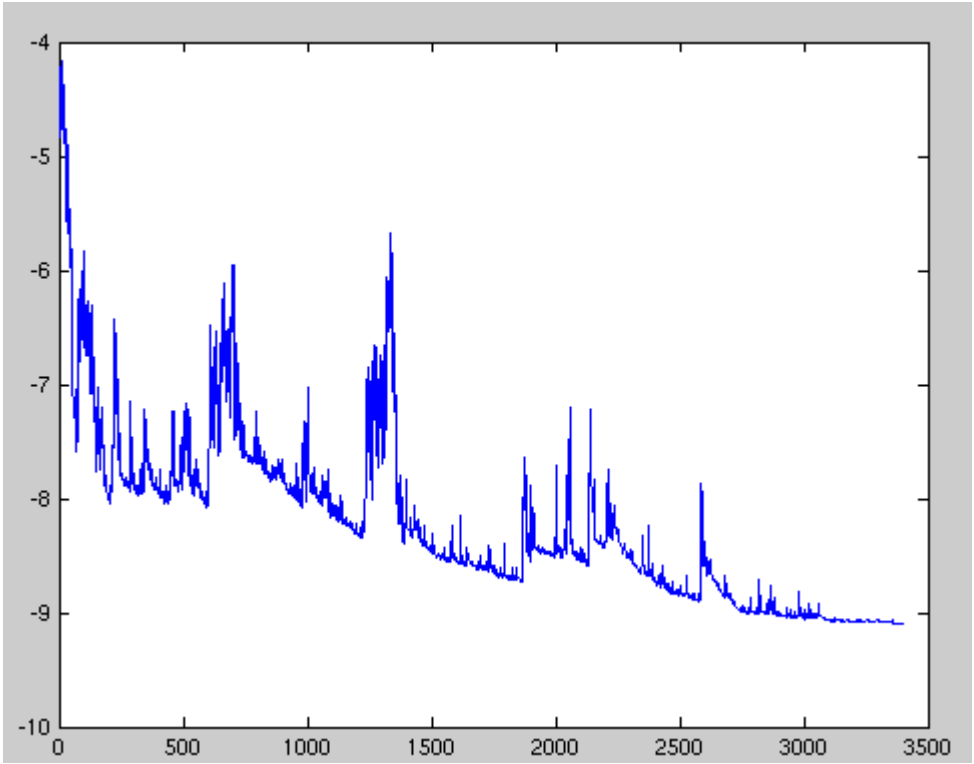
- Shuffle training set
- For each data point *i=1…n (maybe as mini-batch)*
  - *Gradient descent*

"Epoch"

Stochastic Gradient Descent

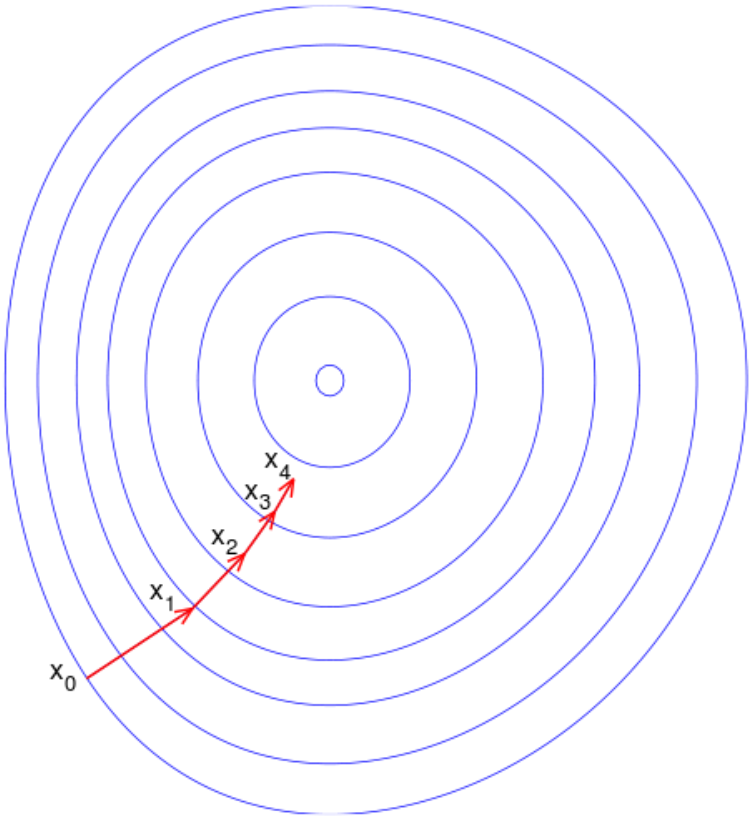
Loss will not always decrease (locally) as training data point is random.

Still converges over time.



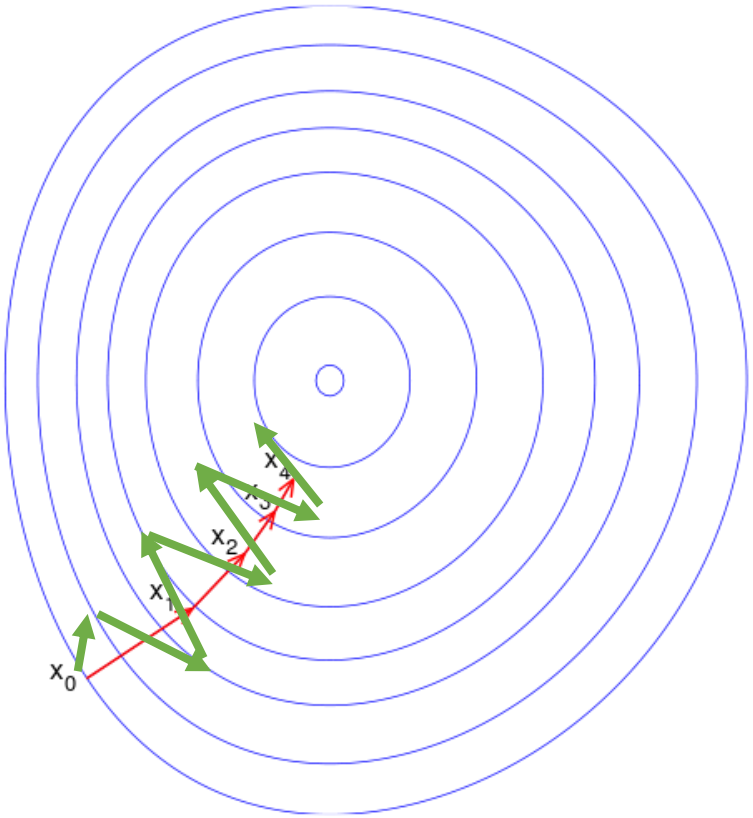
Wikipedia

Gradient descent oscillations



Wikipedia

Gradient descent oscillations



Slow to converge to the (local) optimum

Wikipedia

Momentum

- Adjust the gradient by a weighted sum of the previous amount plus the current amount.

- Without momentum: $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \gamma \frac{\partial L}{\partial \boldsymbol{\theta}}$

- With momentum (new α parameter):

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t - \gamma \left(\alpha \left[\frac{\partial L}{\partial \boldsymbol{\theta}} \right]_{t-1} + \left[\frac{\partial L}{\partial \boldsymbol{\theta}} \right]_t \right)$$

But James…

…I thought we were going to treat machine learning like a black box? I like black boxes.

Deep learning is:
- a black box

But James…

…I thought we were going to treat machine learning like a black box? I like black boxes.

Deep learning is:
- a black box
- *also a black art*.

http://www.isrtv.com/

But James…

…I thought we were going to treat machine learning like a black box? I like black boxes.

Many approaches and hyperparameters:

- Activation functions, learning rate, mini-batch size, momentum…

Often these need tweaking, and you need to know what they do to change them intelligently.

Nailing hyperparameters + trade-offs

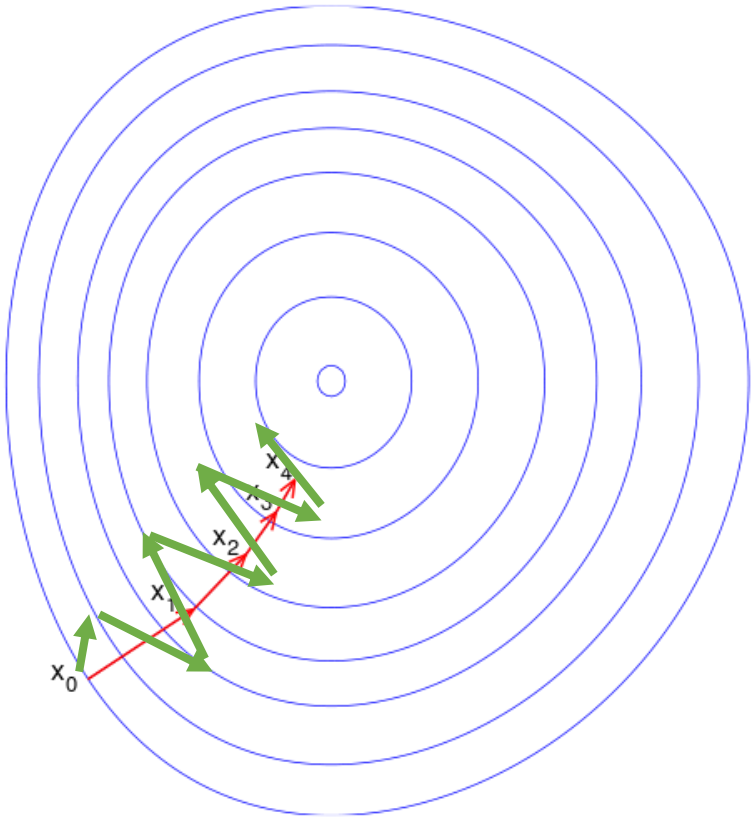
agokasla 6:56 PM
uploaded and commented on this image: **image.png**

> WOOT! Nailed the hyperparameters. 4 generator updates per discriminator update. Wait extra long before you initiate the switch.

jamestompkin 6:57 PM
Well done - I wonder if we can turn hyperparameter nailing into the next e-Sport?

agokasla 4:30 AM
I am starting to think that the numeric instability of the model is starting to become a real issue. Lowering the learning rate could make it more stable, but it would require lowering it by two orders of magnitude which would make it take 100x longer to train right? 😕

Lowering the learning rate = smaller steps in SGD

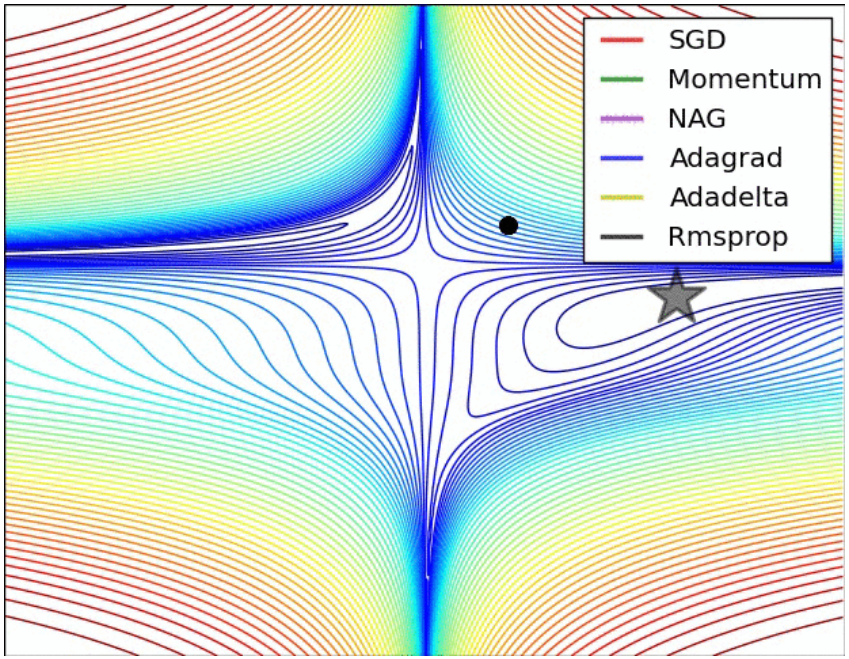


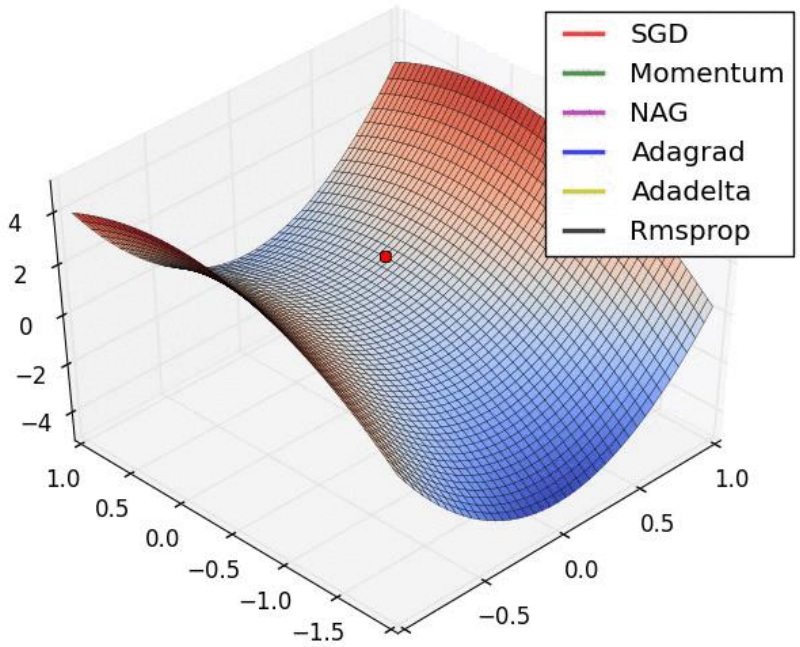
-Less ‘ping pong’

-Takes longer to get to the optimum

Wikipedia

Flat regions in energy landscape






@teenybiscuit


@teenybiscuit

Problem of fitting

- Too many parameters = overfitting
- Not enough parameters = underfitting

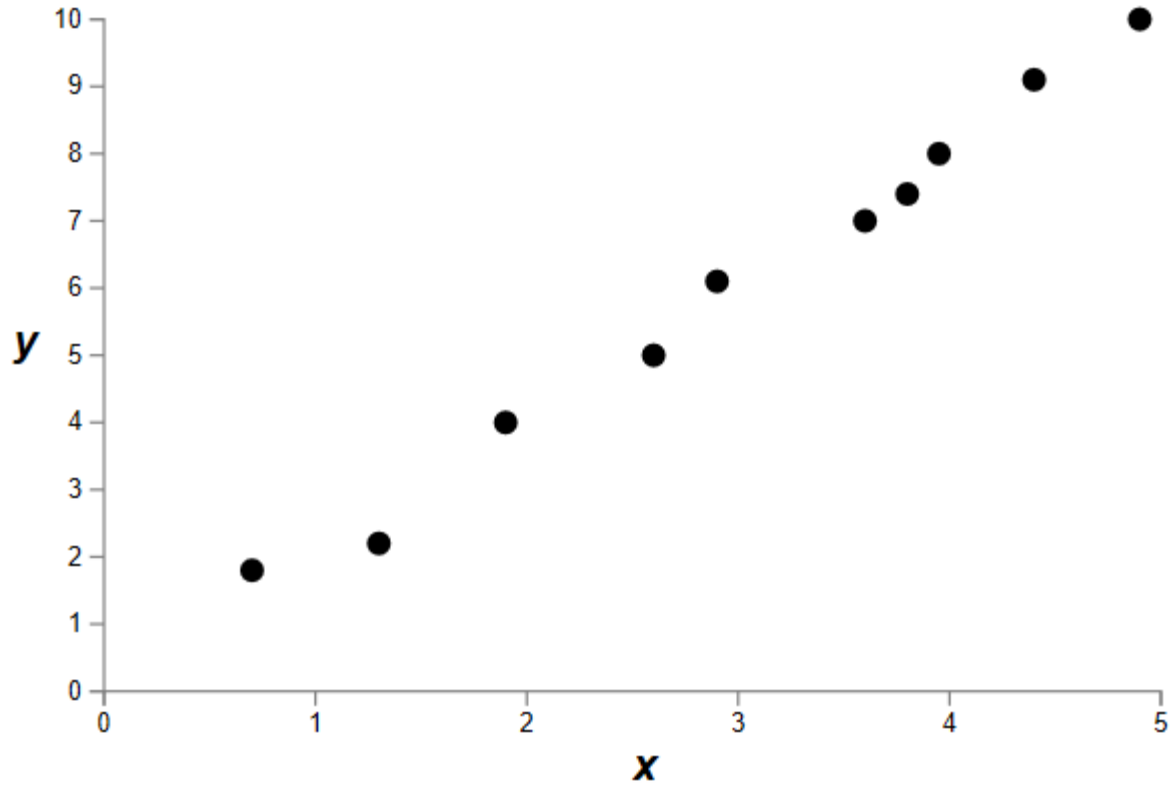
- More data = less chance to overfit

- How do we know what is required?

Regularization

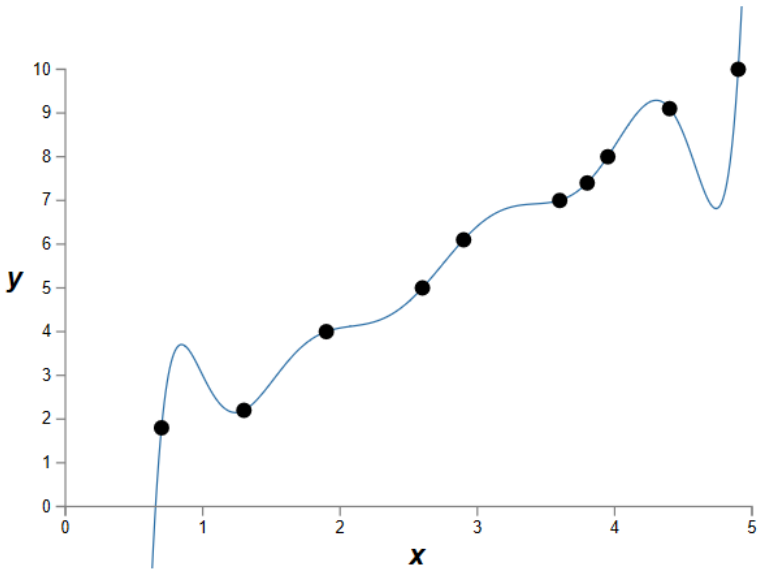
- Attempt to guide solution to *not overfit*
- But still give freedom with many parameters

Data fitting problem

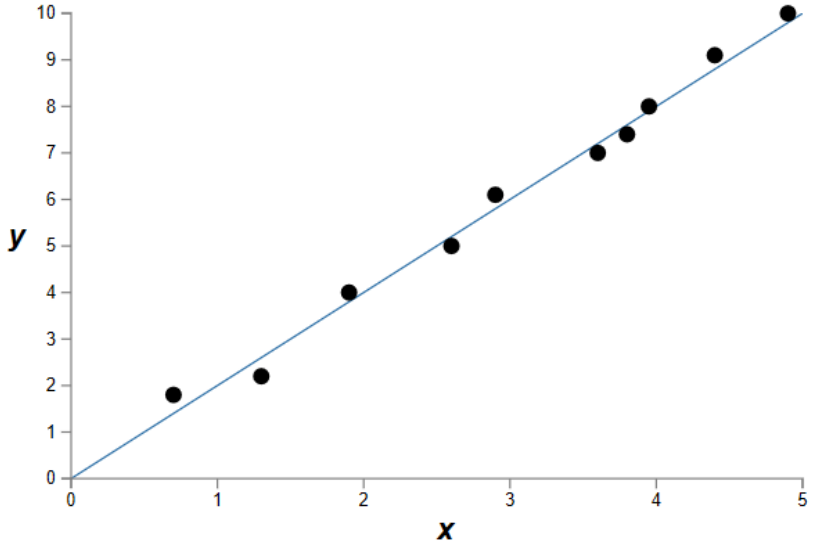


[Nielson]

Which is better?
Which is better *a priori*?



9th order polynomial



1st order polynomial

[Nielson]

Regularization

- Attempt to guide solution to *not overfit*
- But still give freedom with many parameters

- Idea:
  Penalize the use of parameters to prefer small weights.

Regularization:

- Idea: add a cost to having high weights
- λ = regularization parameter

$$C = C_0 + \lambda \sum_w w^2,$$

[Nielson]

Both can describe the data…

- …but one is simpler.

- Occam's razor:

  "Among competing hypotheses, the one with the fewest assumptions should be selected"

For us:

Large weights cause large changes in behaviour in response to small changes in the input.

Simpler models (or smaller changes) are more robust to noise.

Regularization

- Idea: add a cost to having high weights
- λ = regularization parameter

$$C = C_0 + \lambda \sum_w w^2,$$

$$C = -\frac{1}{n} \sum_{xj} \left[y_j \ln a_j^L + (1 - y_j) \ln(1 - a_j^L) \right] + \lambda \sum_w w^2.$$

Normal cross-entropy loss (binary classes)

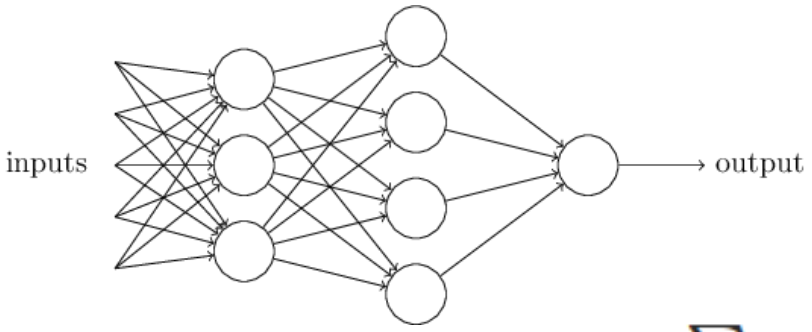
Regularization term

[Nielson]

Regularization: Dropout

- Our networks typically start with random weights.
- Every time we train = slightly different outcome.

- Why random weights?
- If weights are all equal, response across filters will be equivalent.
  - Network doesn't train.



$$w \cdot x \equiv \sum_j w_j x_j$$

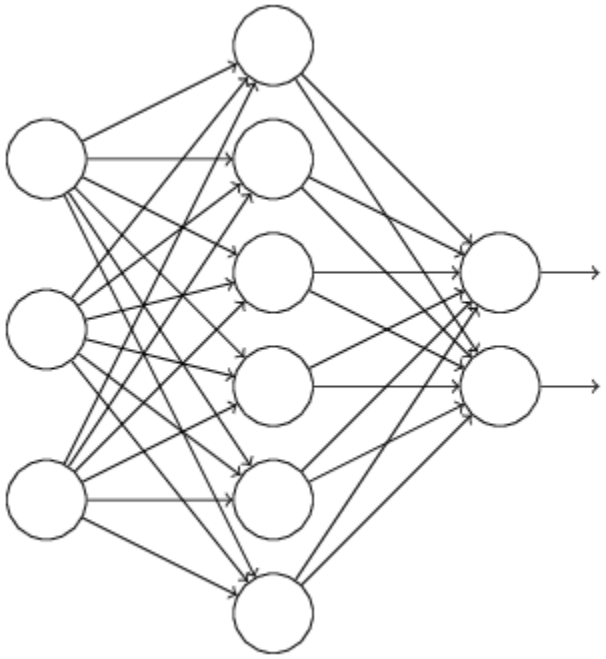
[Nielson]

Regularization

- Our networks typically start with random weights.
- Every time we train = slightly different outcome.

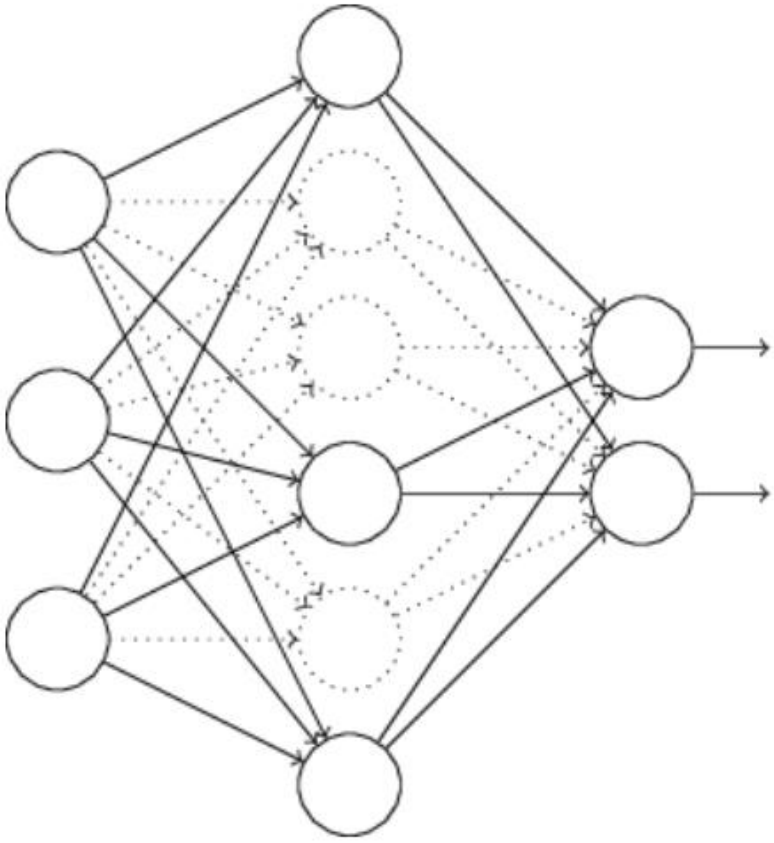
- Why not train 5 different networks with random starts and vote on their outcome?
  - Works fine!
  - Helps generalization because error is averaged.

Regularization: Dropout



[Nielson]

Regularization: Dropout



At each mini-batch:

- Randomly select a subset of neurons.
- Ignore them.

On test: half weights outgoing to compensate for training on half neurons.

Effect:

- Neurons become less dependent on output of connected neurons.
- Forces network to learn more robust features that are useful to more subsets of neurons.
- Like averaging over many different trained networks with different random initializations.
- Except cheaper to train.

[Nielson]

Many forms of ‘regularization’

- Adding more data is a kind of regularization
- Pooling is a kind of regularization
- Data augmentation is a kind of regularization