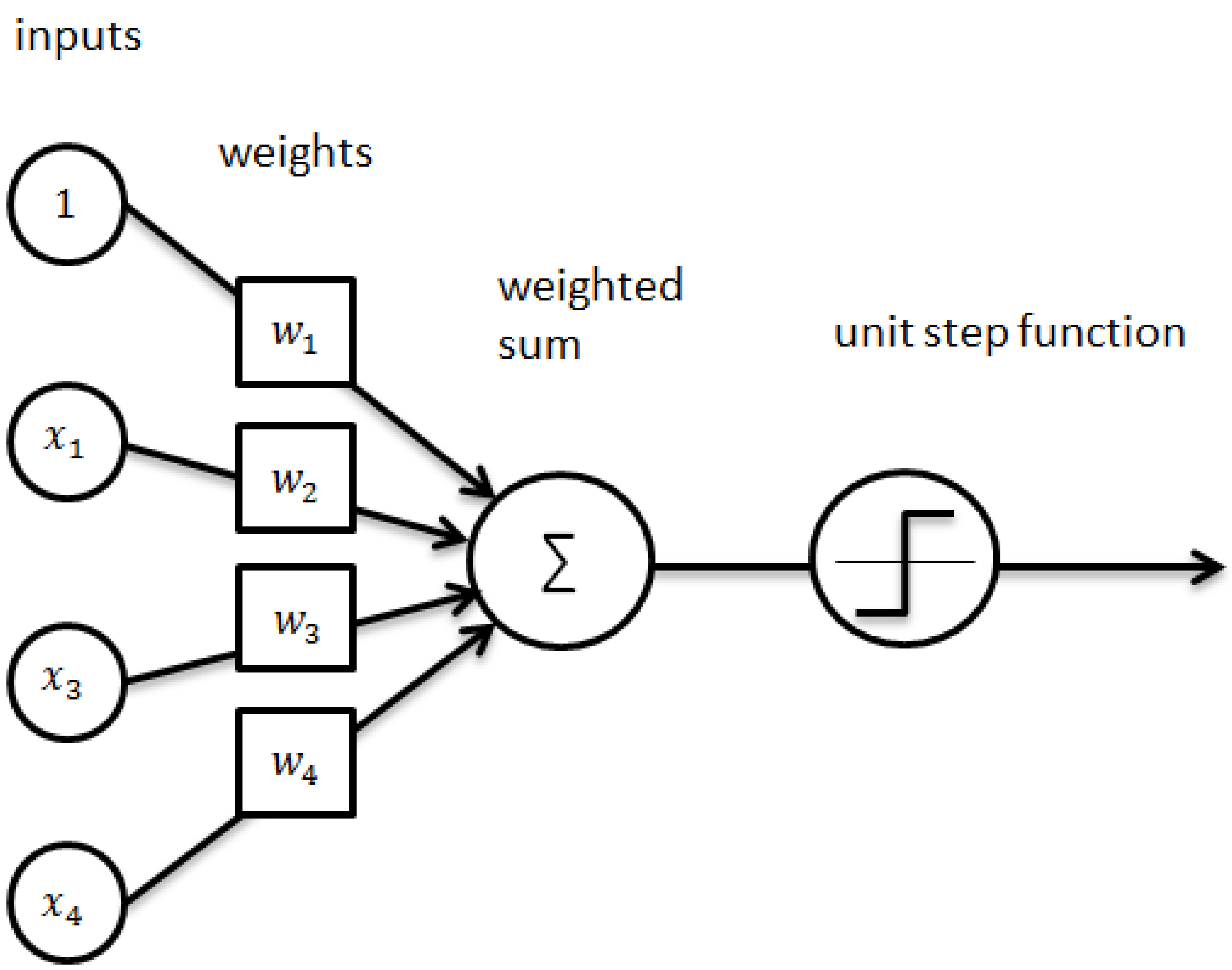
Introduction to neural networks



16-385 Computer Vision
Spring 2019, Lecture 19

http://www.cs.cmu.edu/~16385/

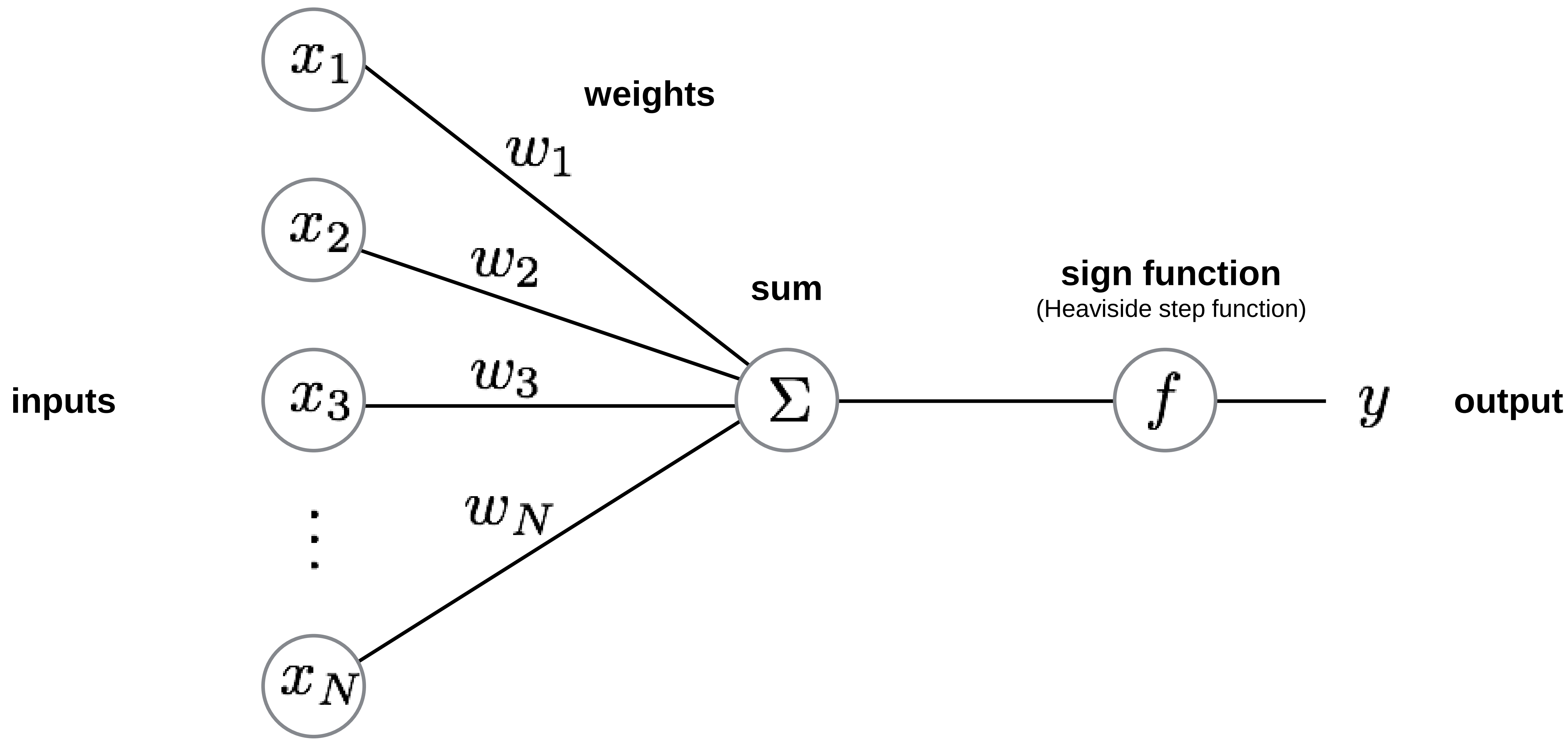
Slide credits

Most of these slides were adapted from:

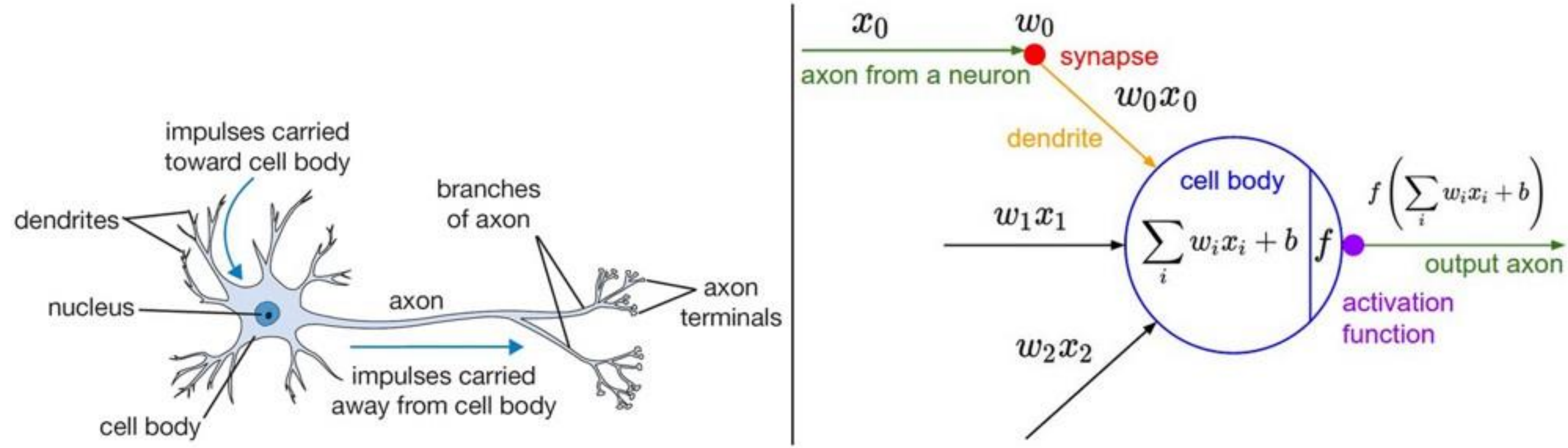
- Kris Kitani (16-385, Spring 2017).
- Noah Snavely (Cornell University).
- Fei-Fei Li (Stanford University).
- Andrej Karpathy (Stanford University).

Perceptron

The Perceptron



Aside: Inspiration from Biology

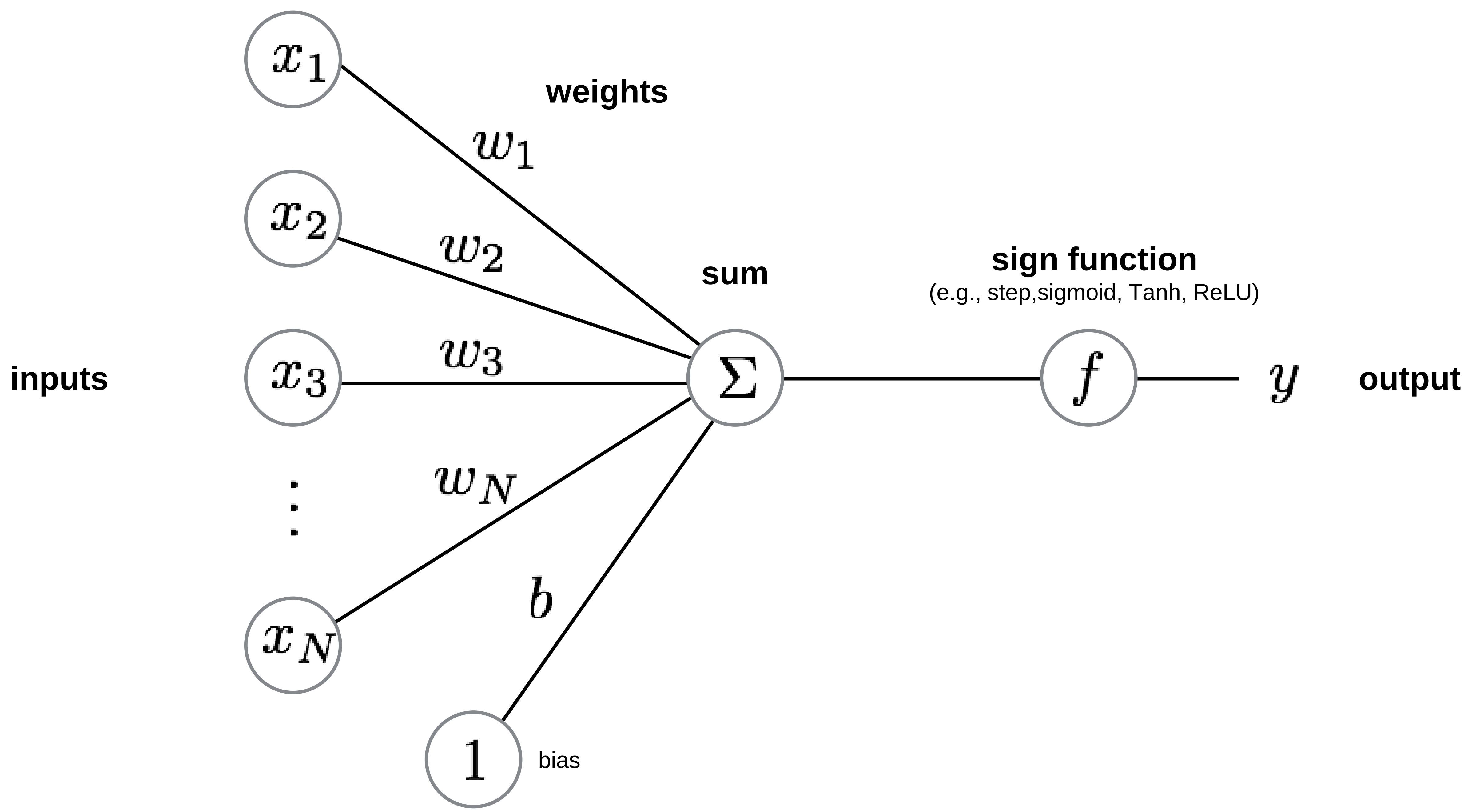


A cartoon drawing of a biological neuron (left) and its mathematical model (right).

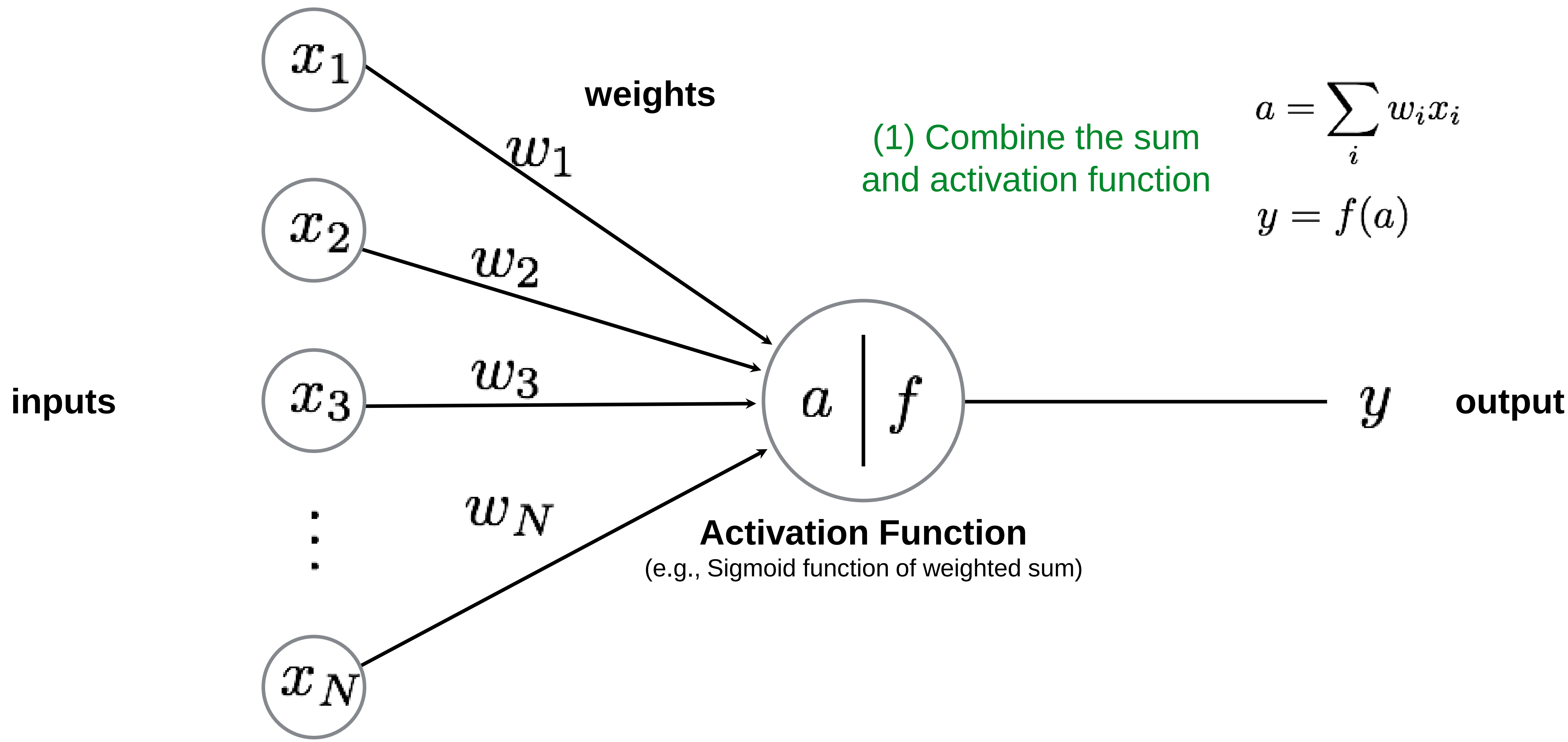
Neural nets/perceptrons are **loosely** inspired by biology.

But they certainly are **not** a model of how the brain works, or even how neurons work.

The Perceptron



Another way to draw it…



(1) Combine the sum and activation function

$$a = \sum_i w_i x_i$$

$$y = f(a)$$

(2) suppress the bias term (less clutter)

$$x_N = 1$$

$$w_N = b$$

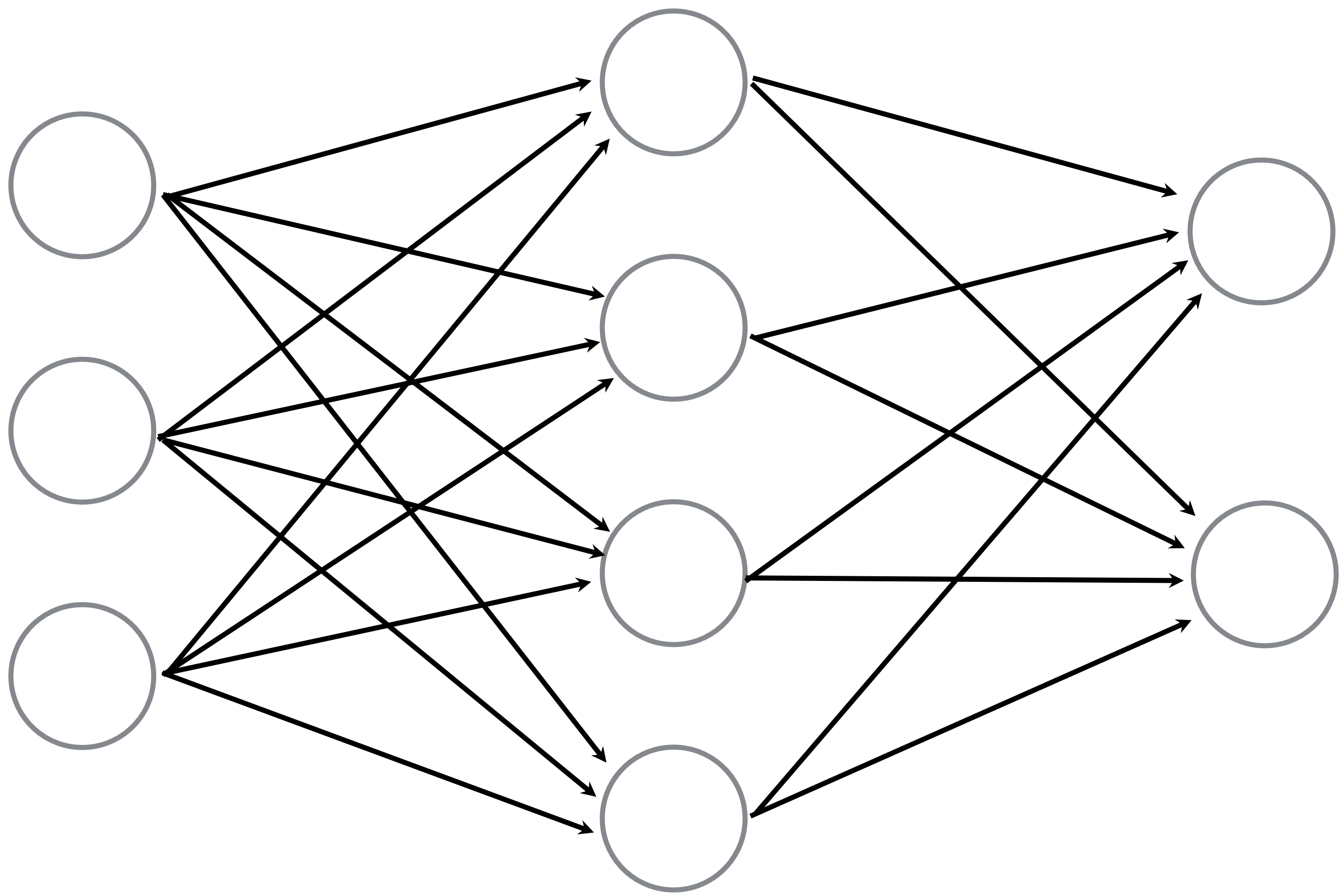
Neural networks

Connect a bunch of perceptrons together …

Connect a bunch of perceptrons together …

Neural Network

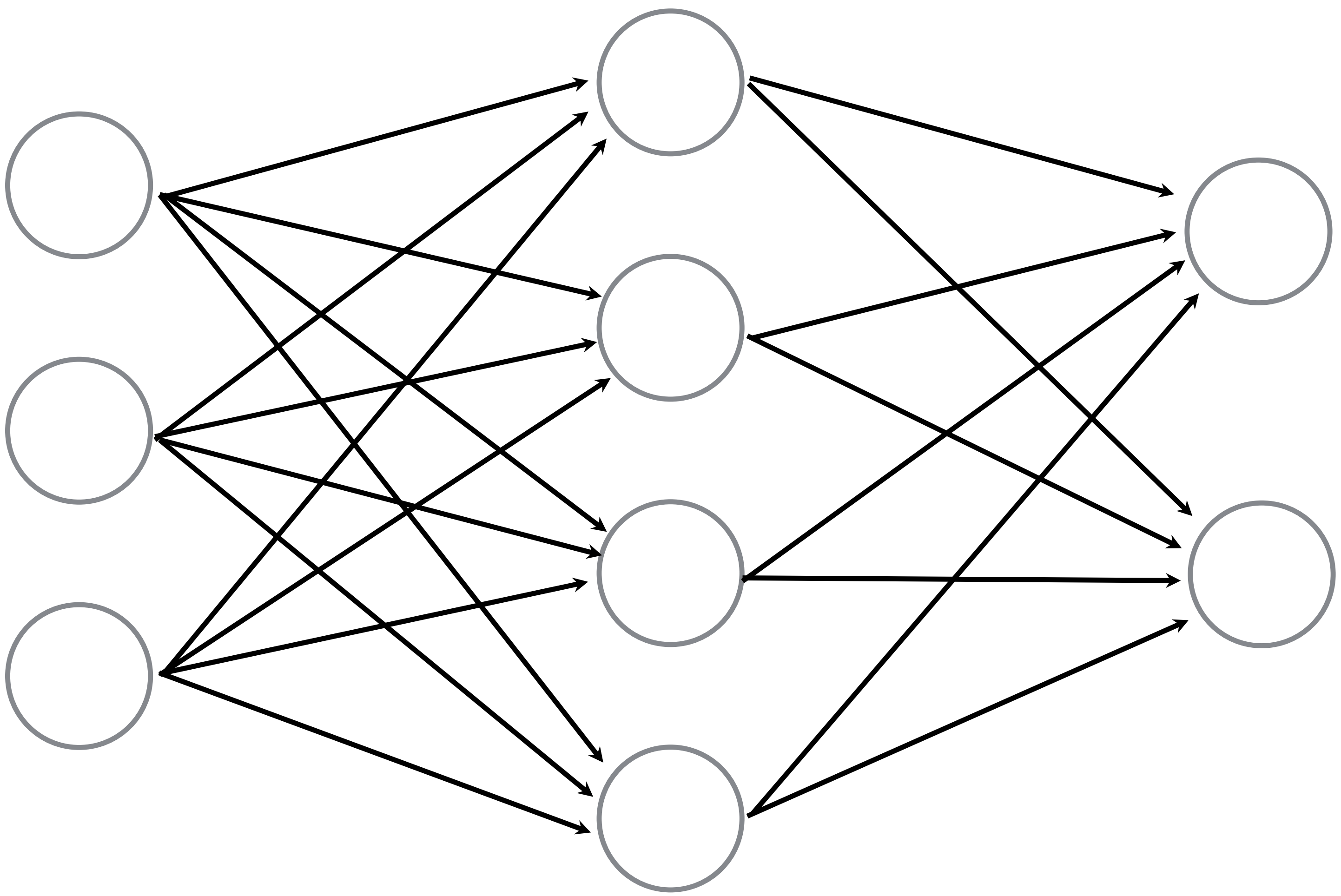
a collection of connected perceptrons



Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons

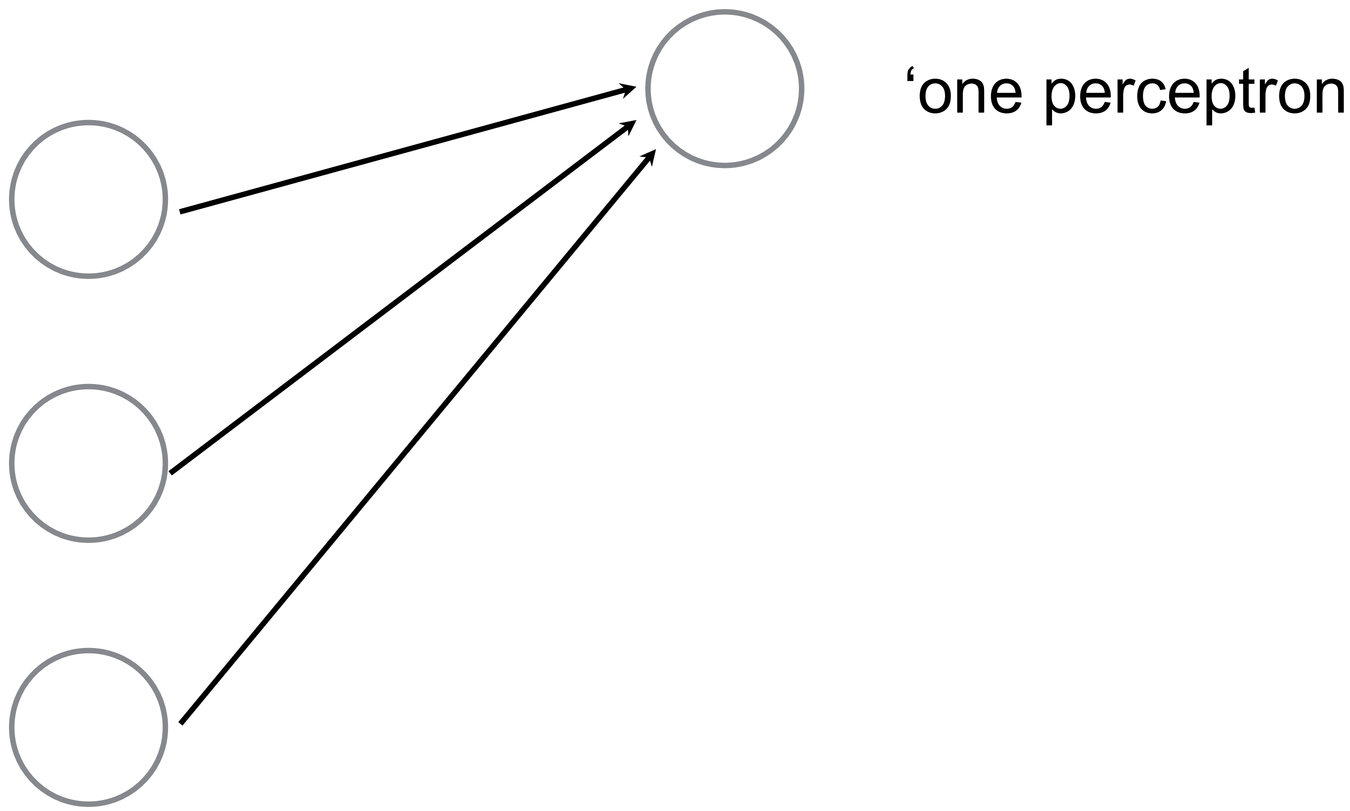


How many perceptrons in this neural network?

Connect a bunch of perceptrons together …

Neural Network

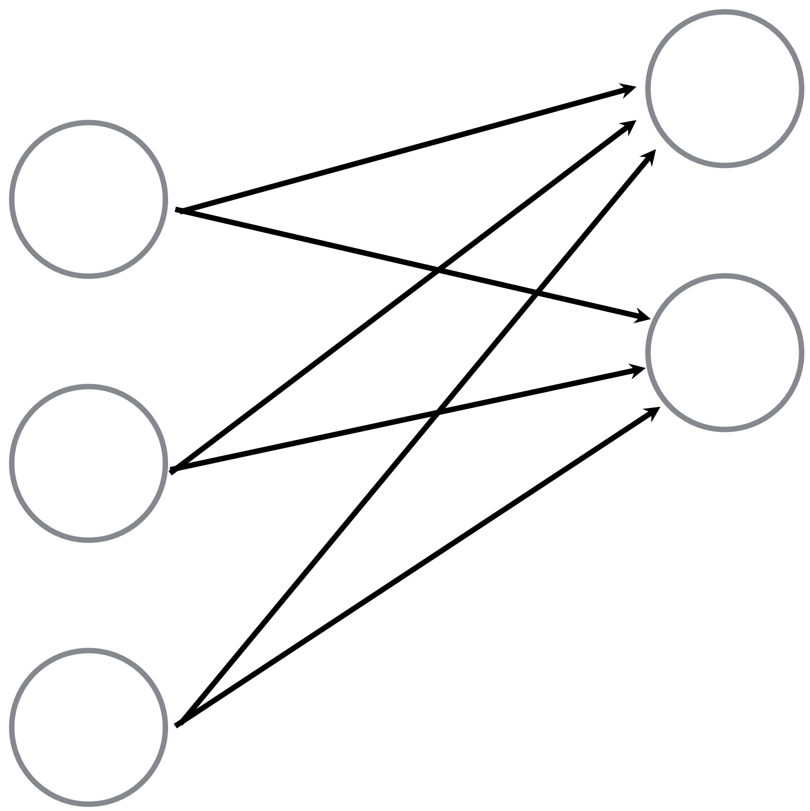
a collection of connected perceptrons



Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons

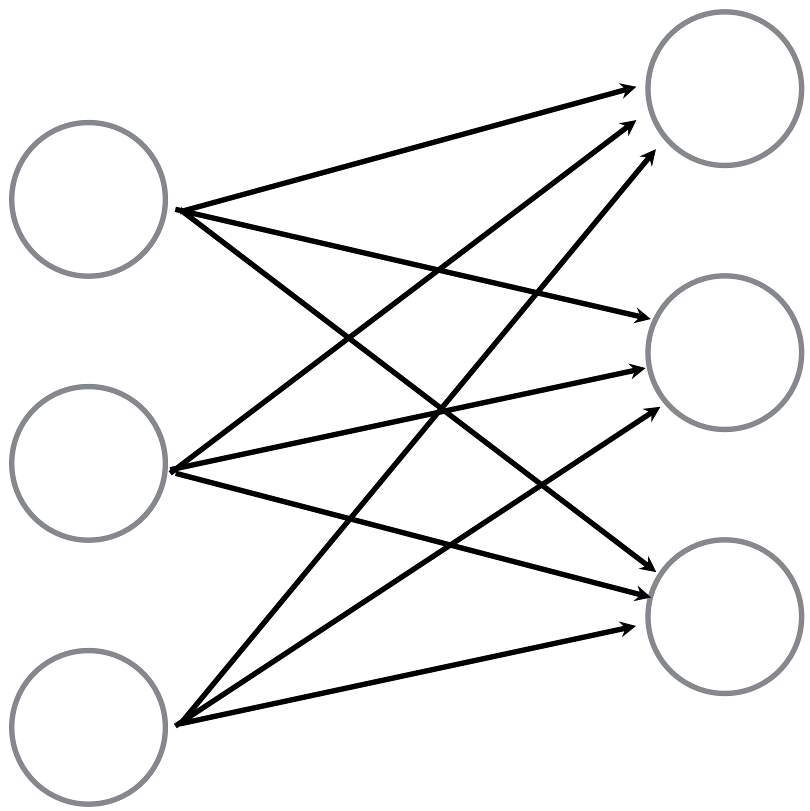


‘two perceptrons’

Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons

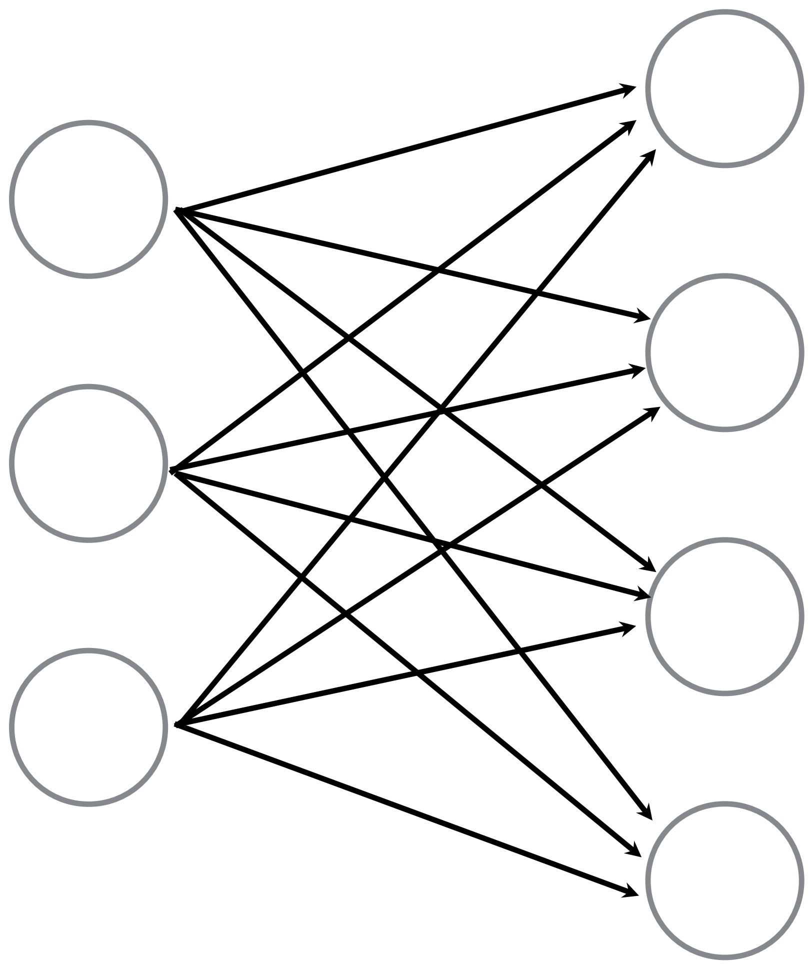


'three perceptrons'

Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons



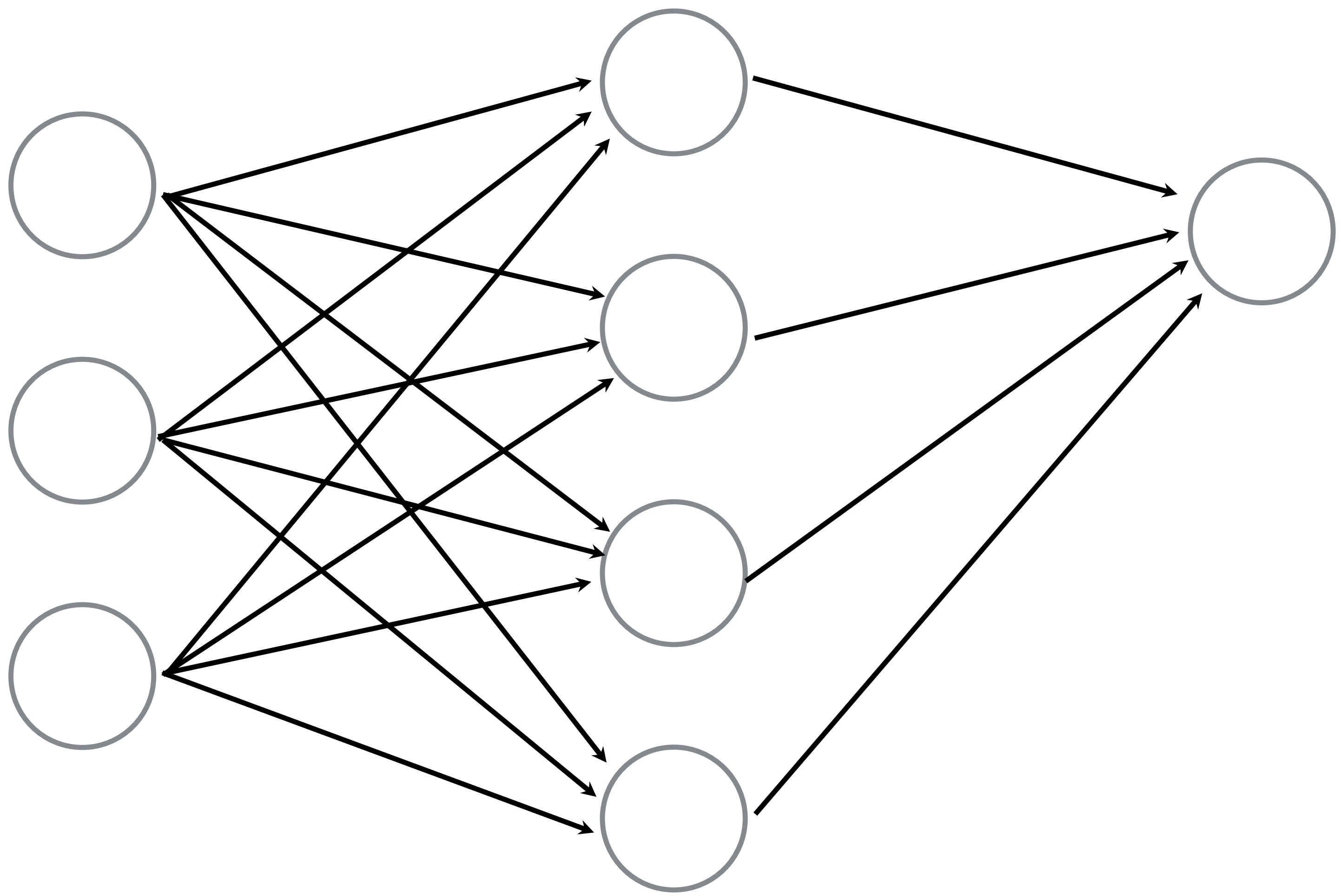
‘four perceptrons’

Connect a bunch of perceptrons together …

Neural Network

a collection of connected perceptrons

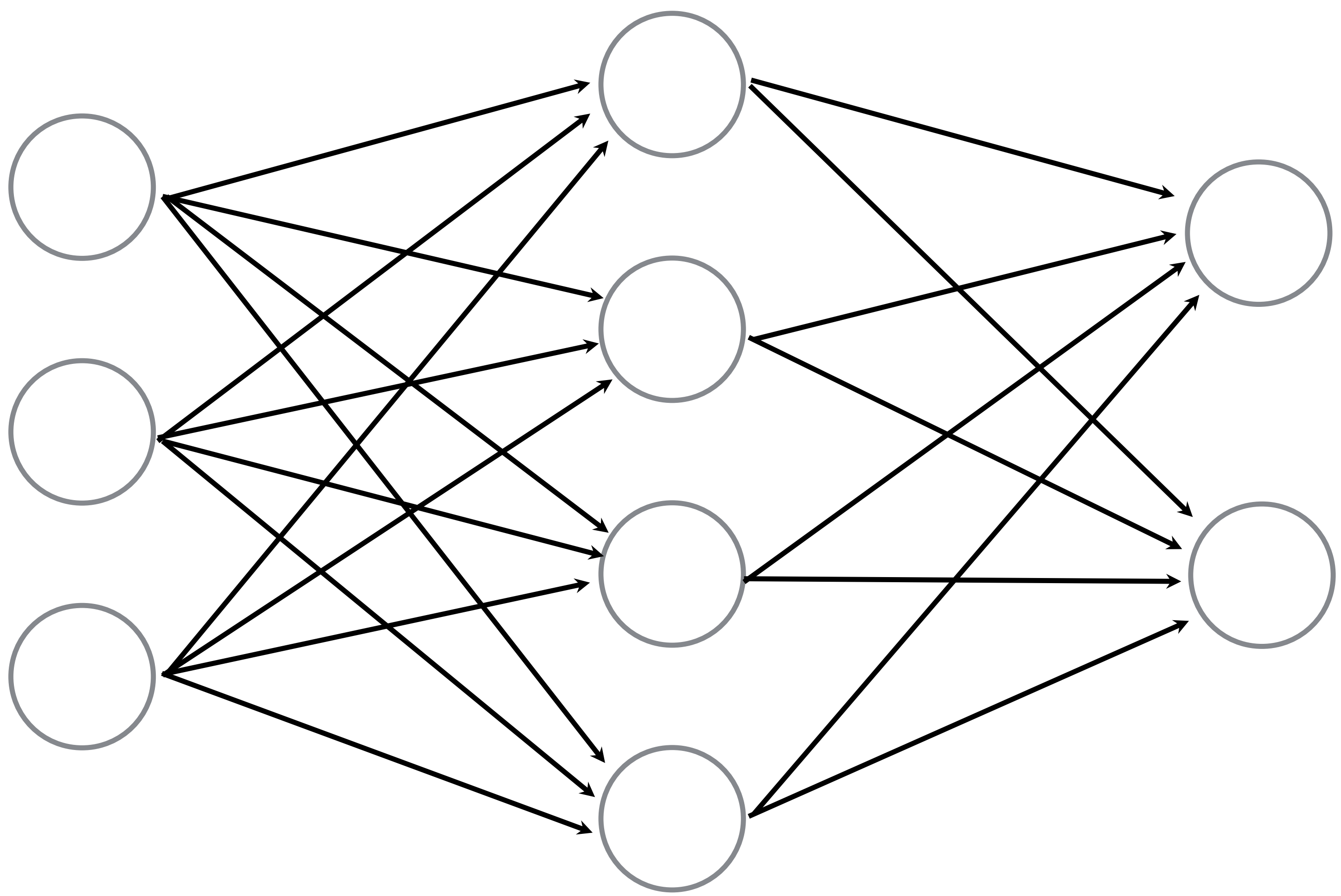
‘five perceptrons’



Connect a bunch of perceptrons together …

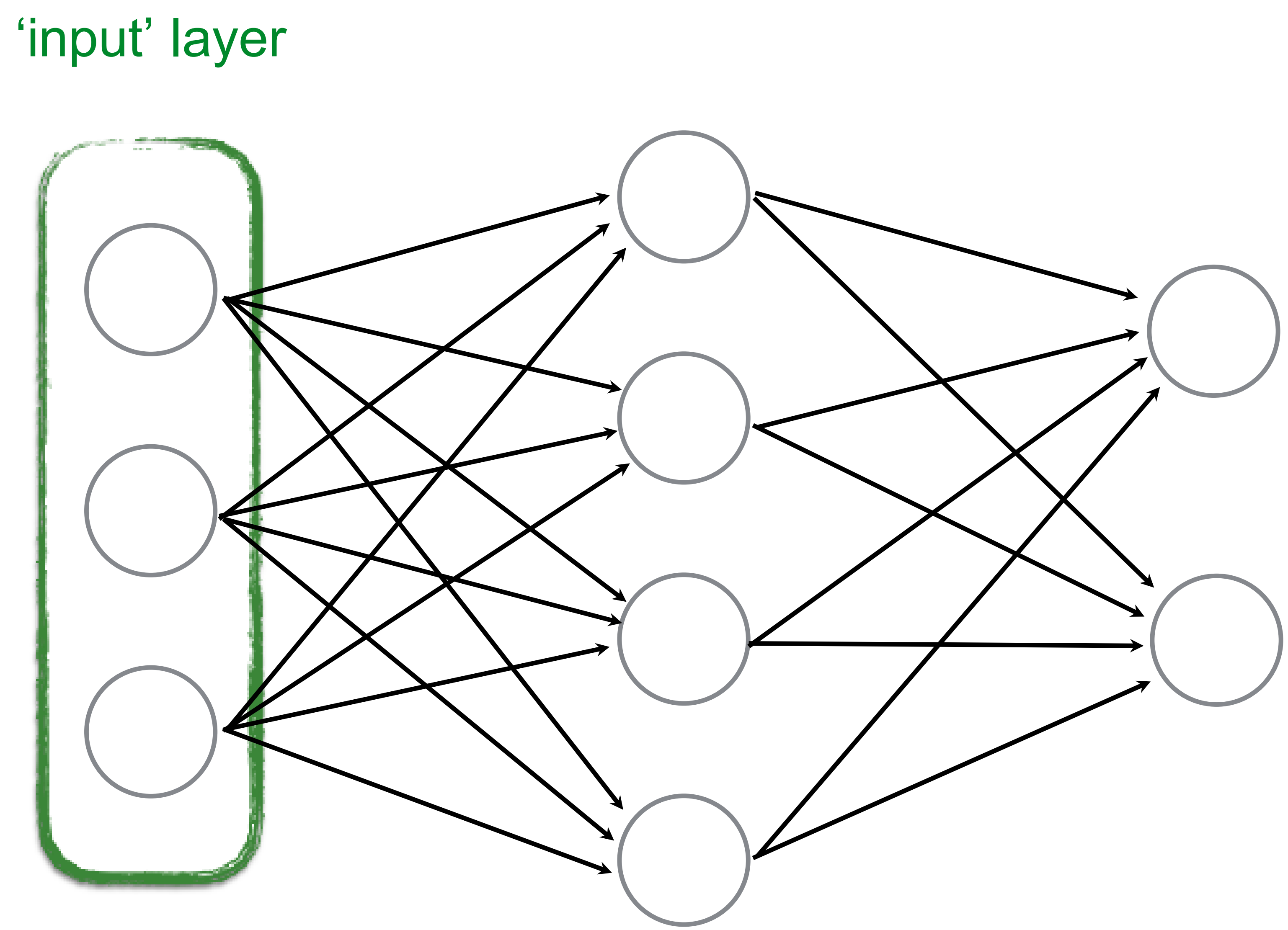
Neural Network

a collection of connected perceptrons



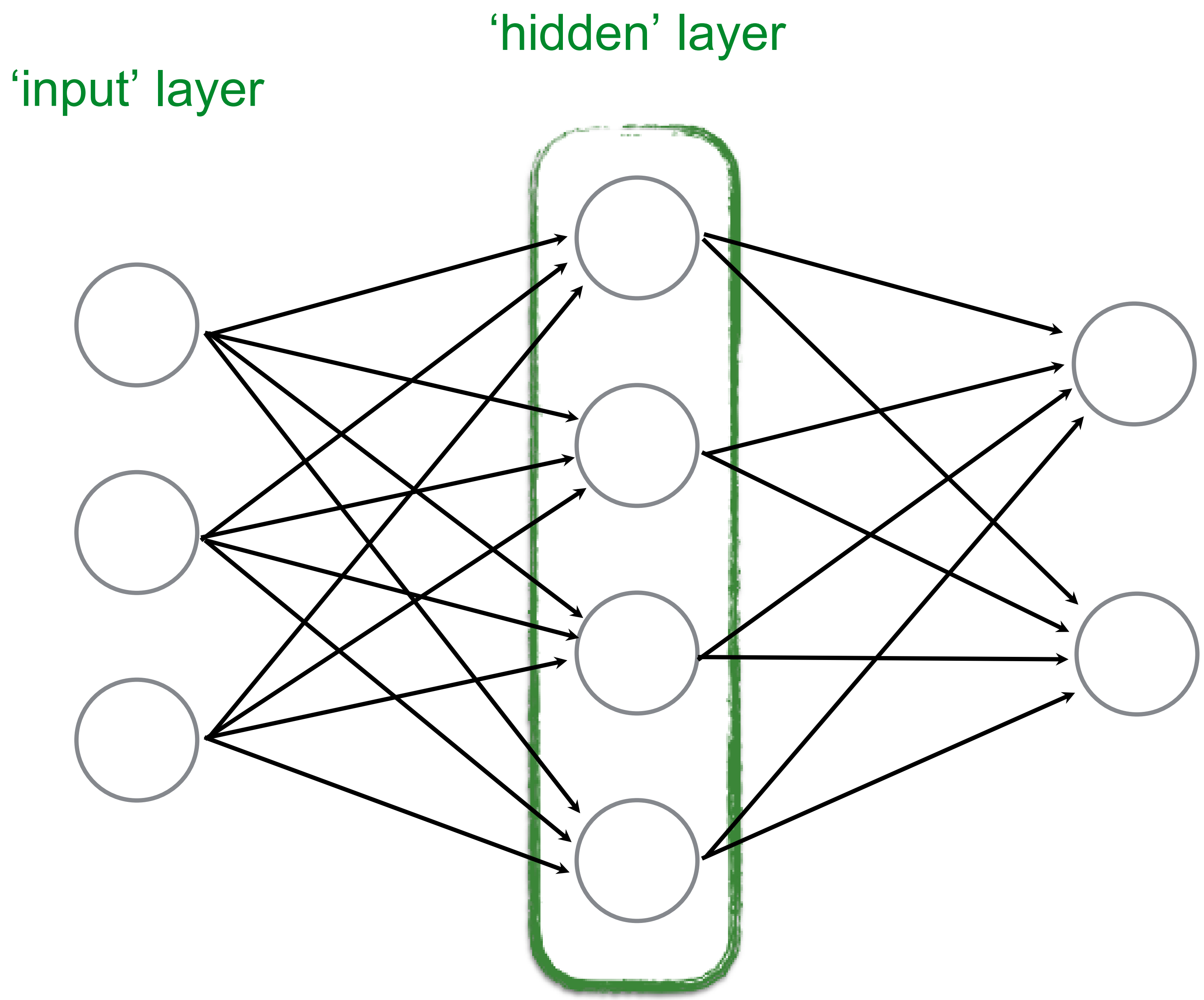
'six perceptrons'

Some terminology…



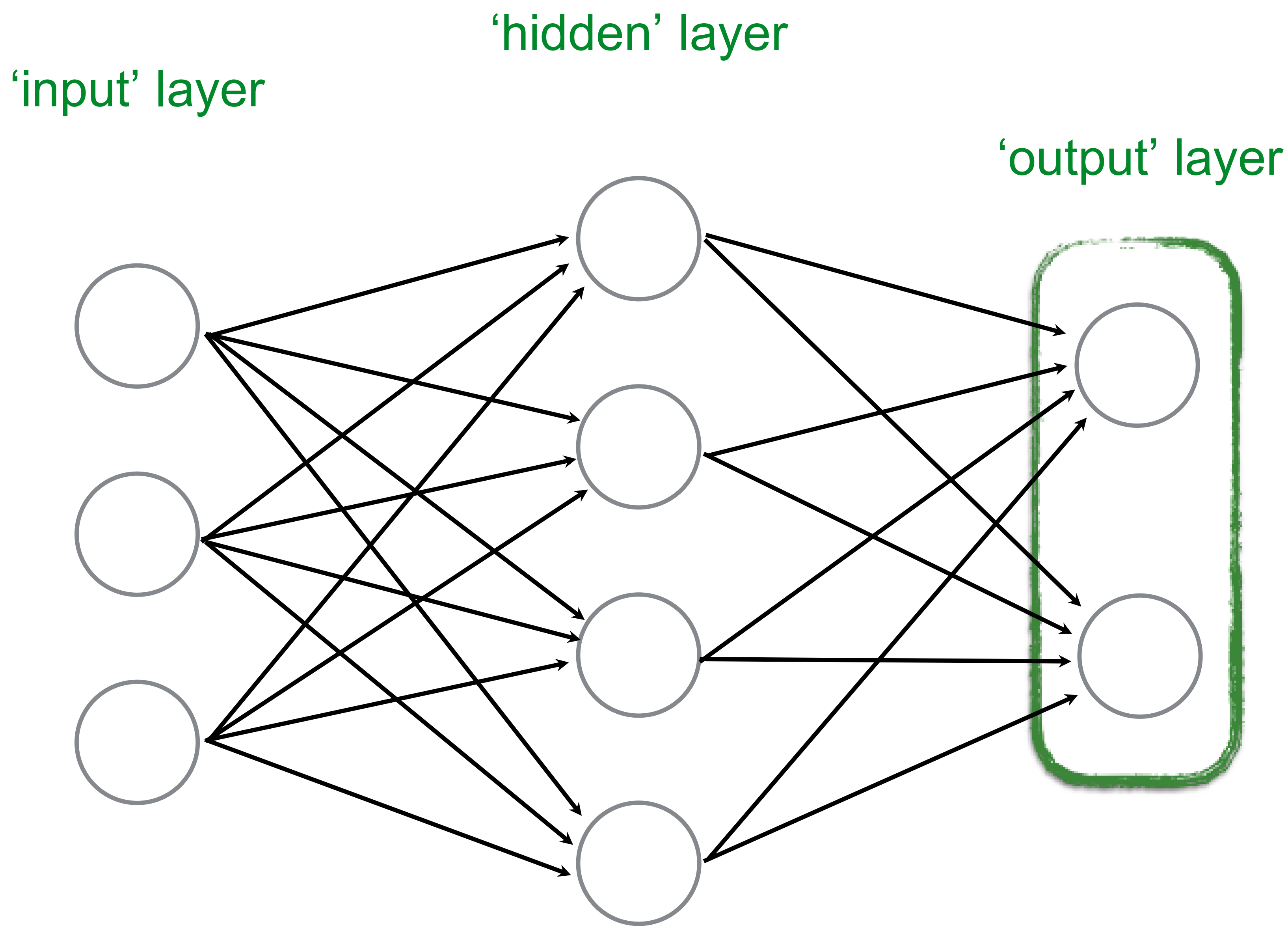
…also called a **Multi-layer Perceptron** (MLP)

Some terminology…

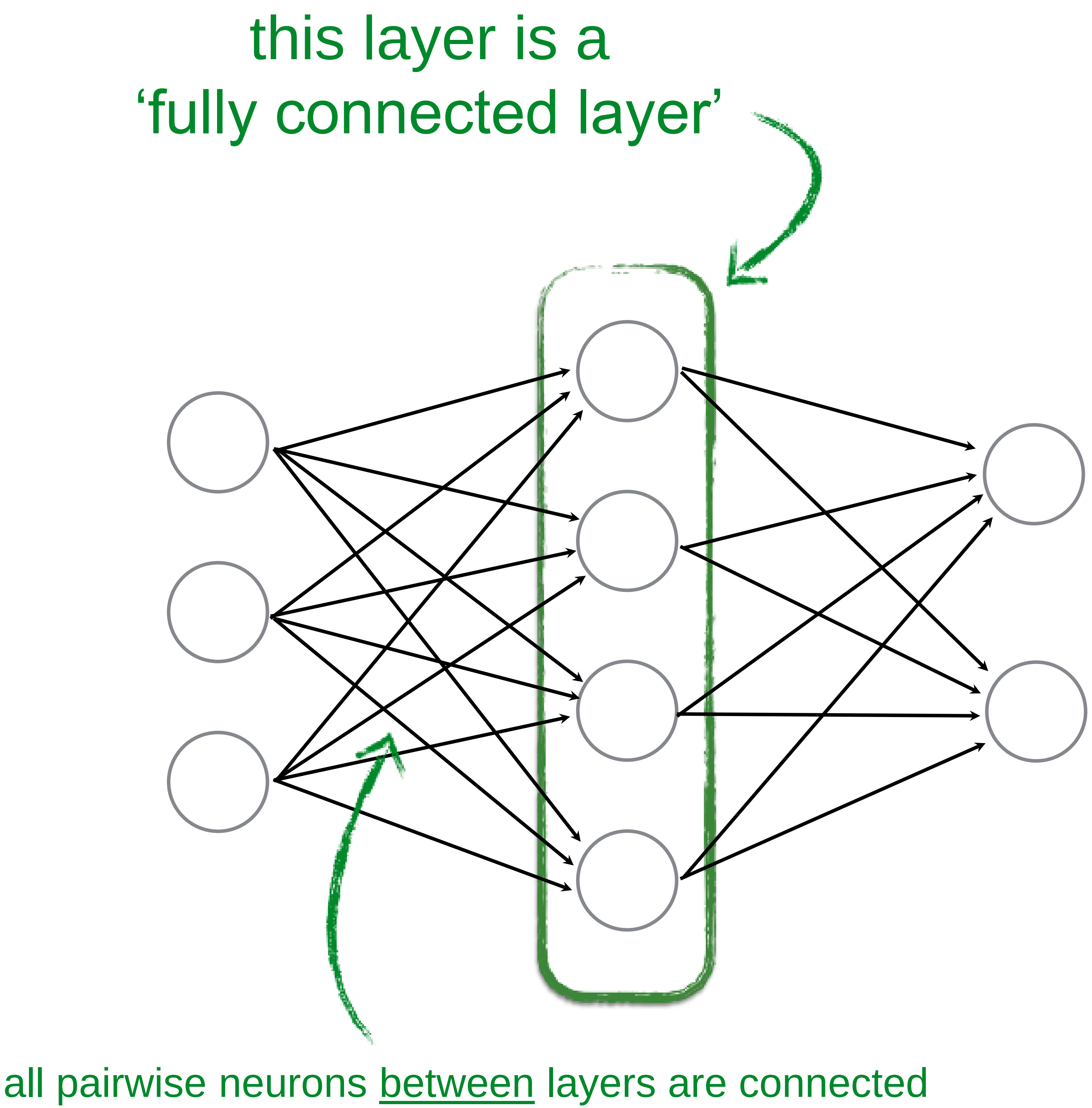


…also called a **Multi-layer Perceptron** (MLP)

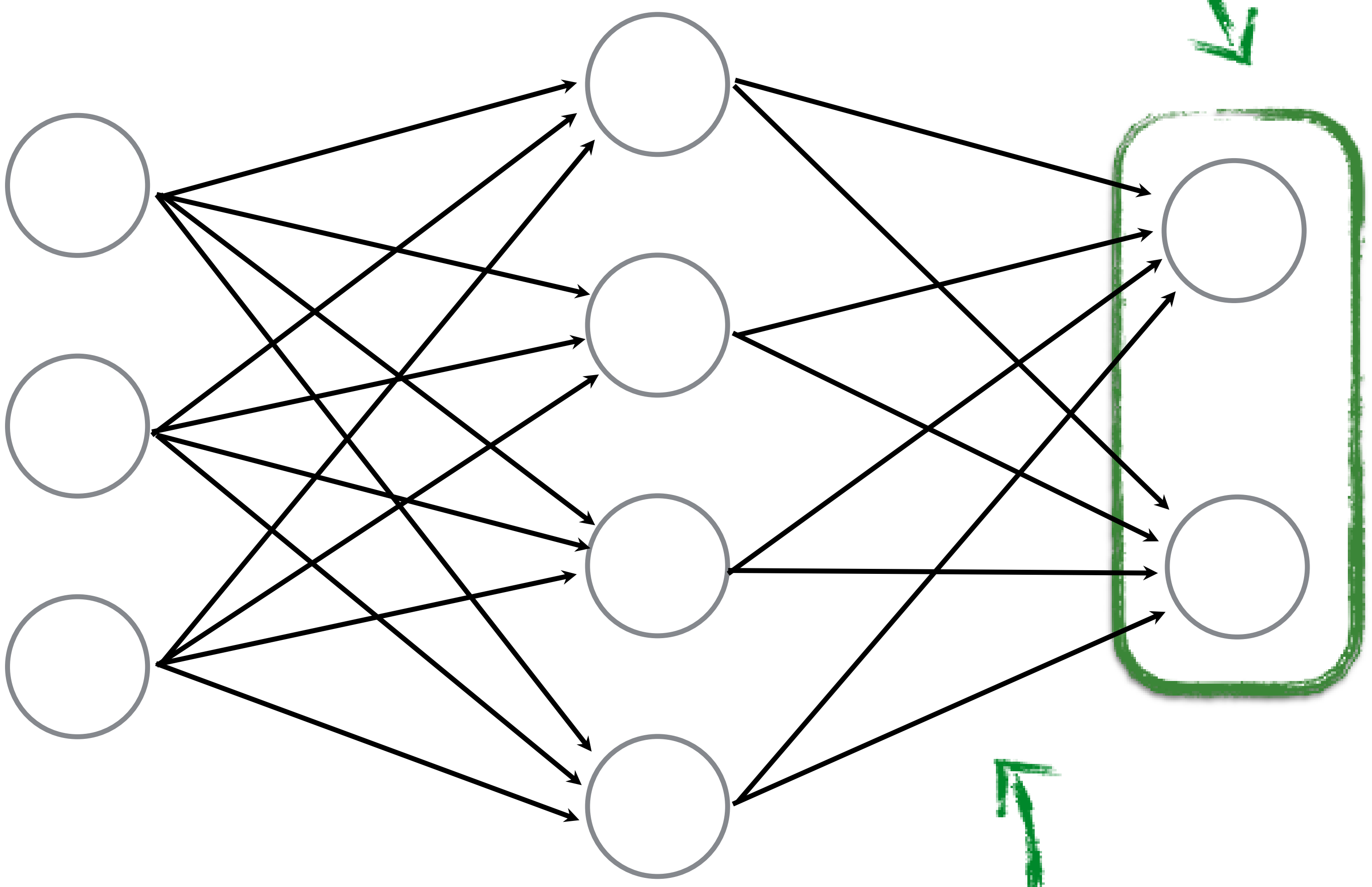
Some terminology…



…also called a **Multi-layer Perceptron** (MLP)


this layer is a
'fully connected layer'
all pairwise neurons between layers are connected

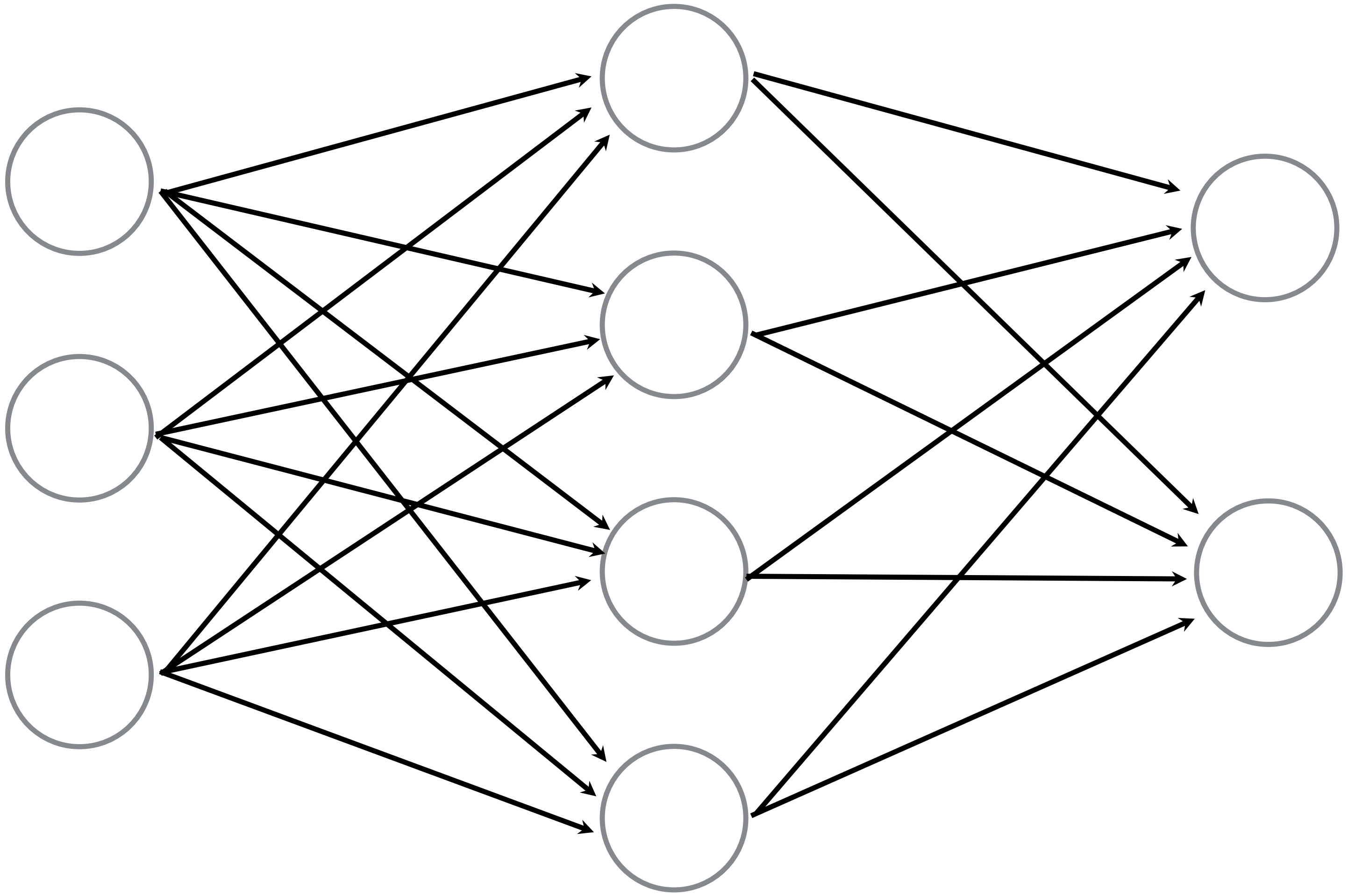
so is this



all pairwise neurons between layers are connected

How many neurons (perceptrons)?

How many weights (edges)?

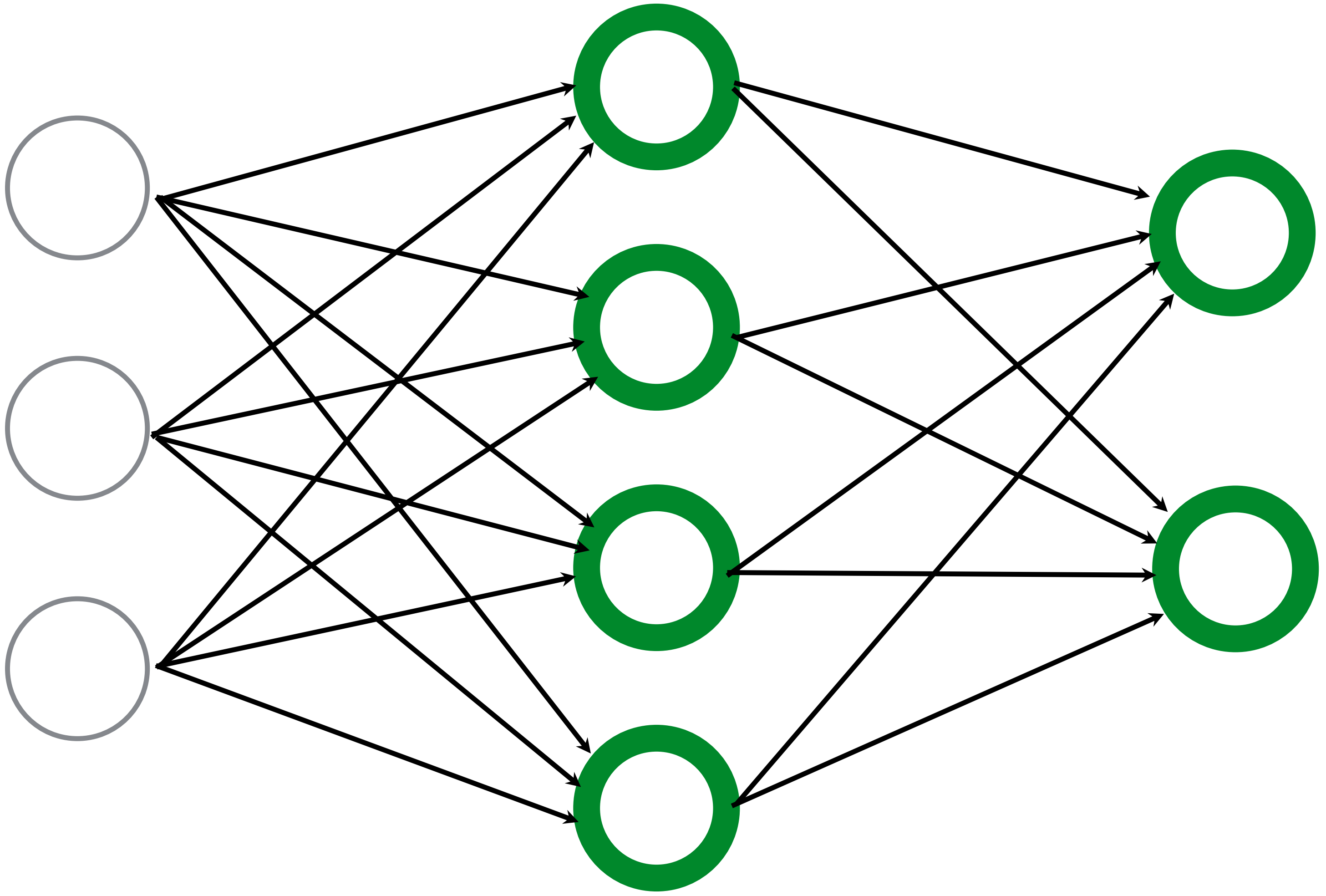


How many learnable parameters total?

How many neurons (perceptrons)?

4 + 2 = 6

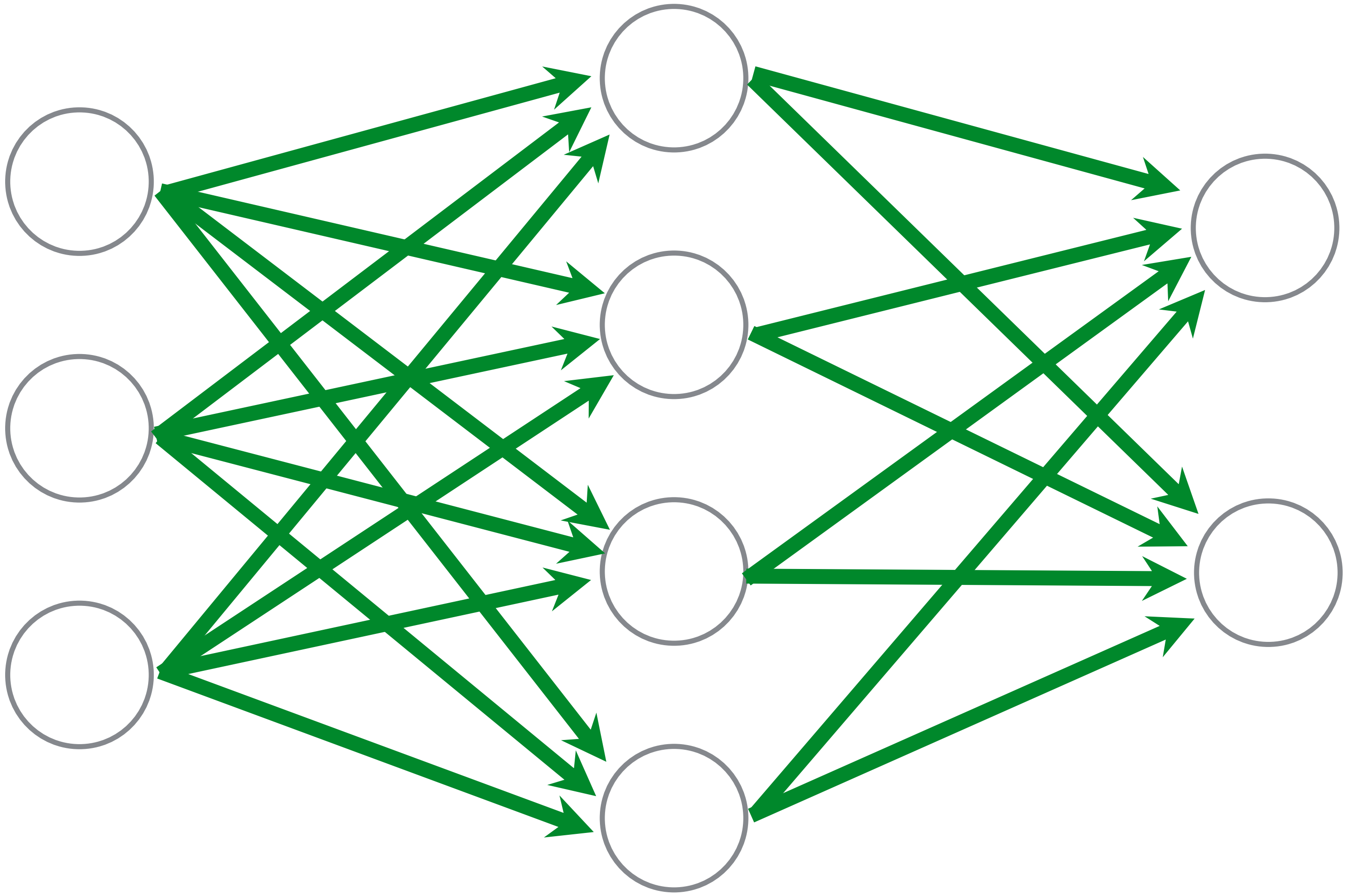
How many weights (edges)?



How many learnable parameters total?

How many neurons (perceptrons)? $4 + 2 = 6$

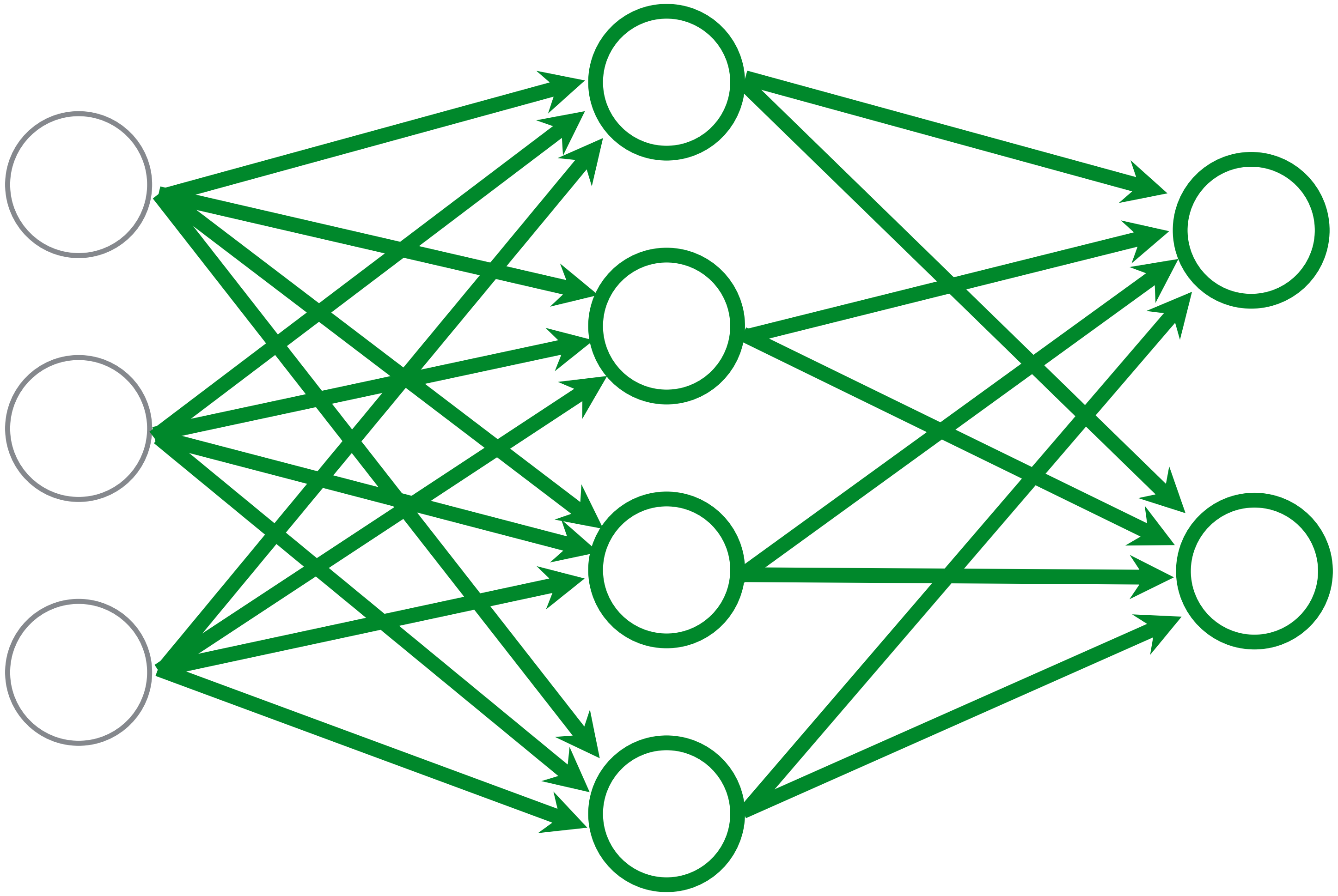
How many weights (edges)? $(3 \times 4) + (4 \times 2) = 20$



How many learnable parameters total?

How many neurons (perceptrons)? 4 + 2 = 6

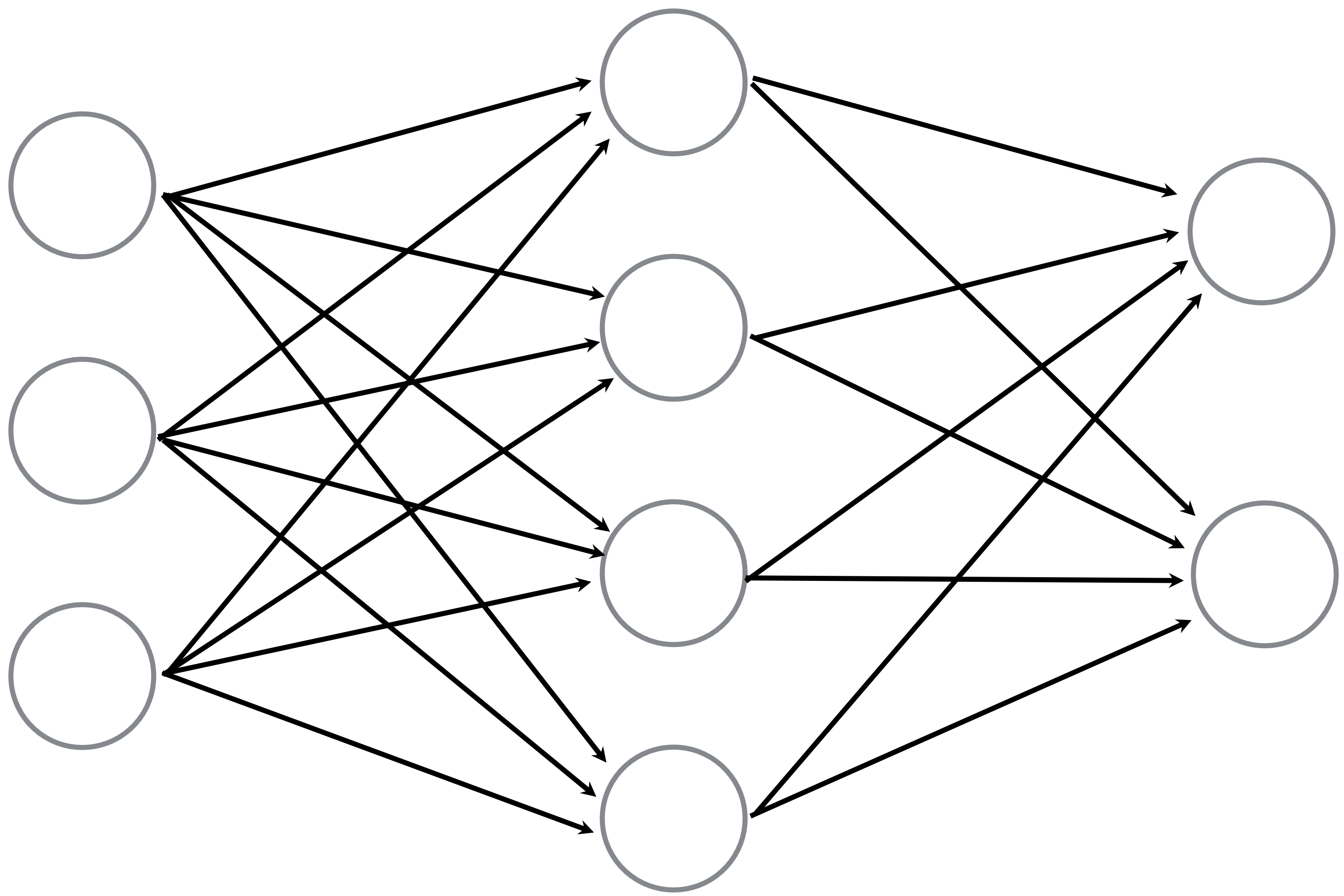
How many weights (edges)? (3 x 4) + (4 x 2) = 20



How many learnable parameters total? 20 + 4 + 2 = 26

bias terms

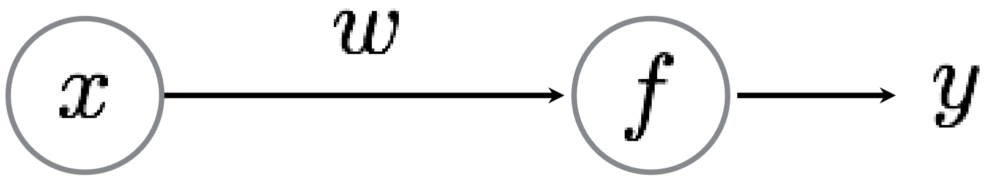
performance usually tops out at 2-3 layers,
deeper networks don’t really improve performance...



...with the exception of **convolutional** networks for images

Training perceptrons

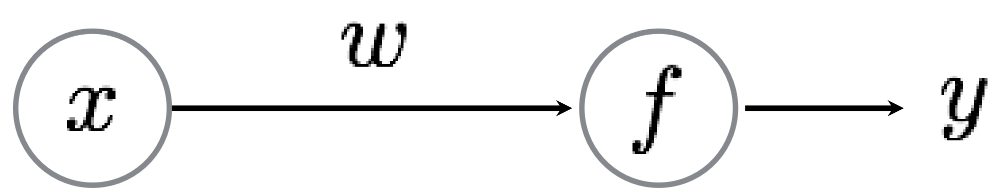
world's smallest perceptron!



$$y = wx$$

What does this look like?

world's smallest perceptron!



$$y = wx$$

(a.k.a. line equation, linear regression)

Learning a Perceptron

Given a set of samples and a Perceptron

$$\{x_i, y_i\}$$

$$y = f_{\text{PER}}(x; w)$$

Estimate the parameters of the Perceptron

$$w$$

Given training data:

x	y
10	10.1
2	1.9
3.5	3.4
1	1.1

What do you think the weight parameter is?

$$y = wx$$

Given training data:

x	y
10	10.1
2	1.9
3.5	3.4
1	1.1

What do you think the weight parameter is?

$$y = wx$$

not so obvious as the network gets more complicated so we use …

An Incremental Learning Strategy

(gradient descent)

Given several examples

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

and a perceptron

$$\hat{y} = wx$$

An Incremental Learning Strategy

(gradient descent)

Given several examples

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

and a perceptron

$$\hat{y} = wx$$

Modify weight w such that $\hat{y}$ gets **'closer'** to y

An Incremental Learning Strategy

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perceptron parameter

perceptron output

true label

An Incremental Learning Strategy

(gradient descent)

Given several examples

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

and a perceptron

$$\hat{y} = wx$$

Modify weight w such that $\hat{y}$ gets **'closer'** to y

perceptron parameter

perceptron output

what does this mean?

true label

Before diving into gradient descent, we need to understand …

Loss Function

defines what is means to be

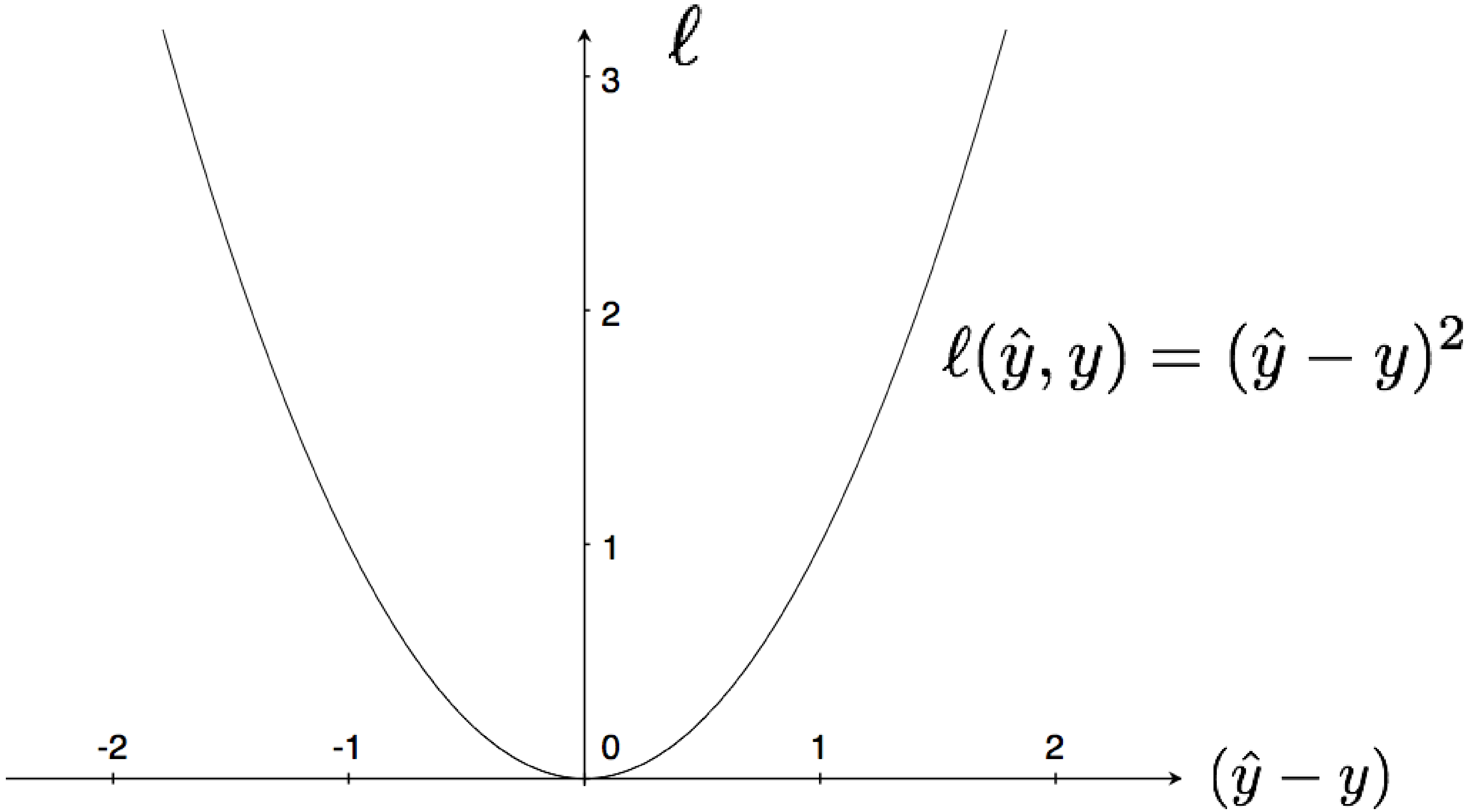
close to the true solution

YOU get to chose the loss function!

(some are better than others depending on what you want to do)

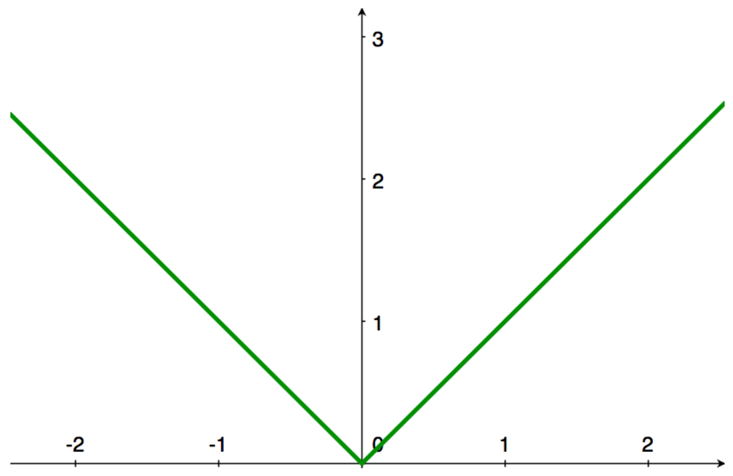
Squared Error (L2)

(a popular loss function) ((why?))



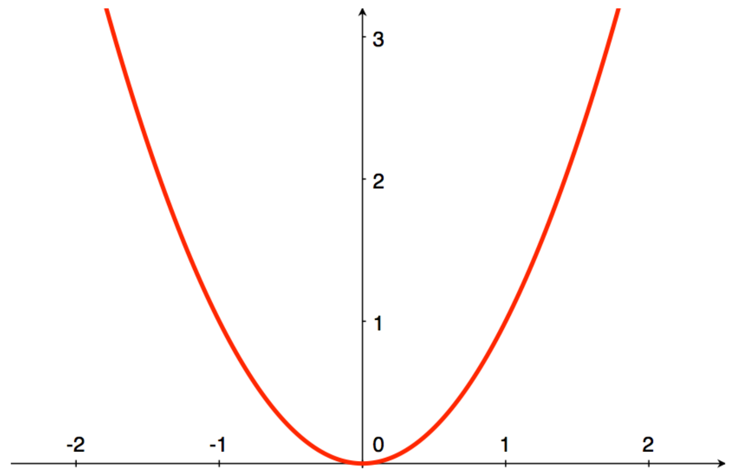
L1 Loss

$$\ell(\hat{y}, y) = |\hat{y} - y|$$



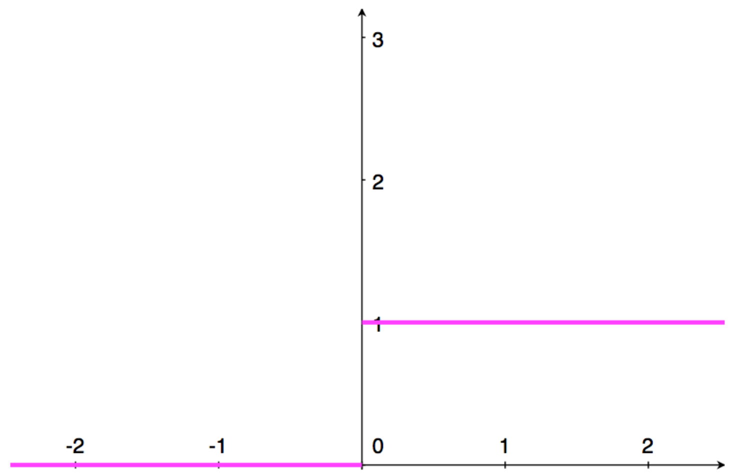
L2 Loss

$$\ell(\hat{y}, y) = (\hat{y} - y)^2$$



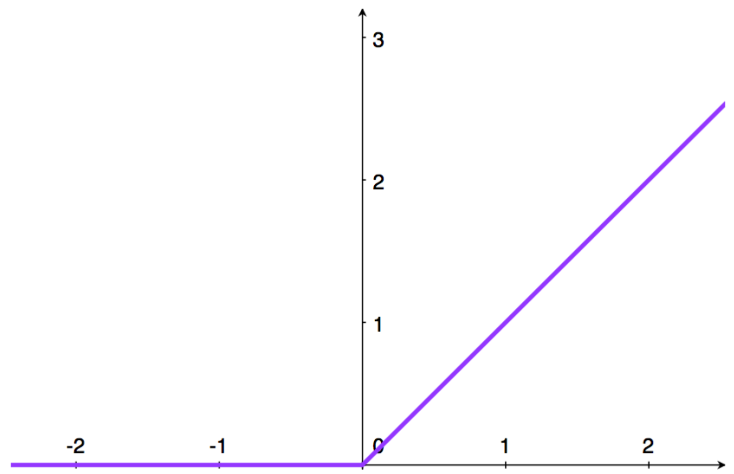
Zero-One Loss

$$\ell(\hat{y}, y) = \mathbf{1}[\hat{y} = y]$$



Hinge Loss

$$\ell(\hat{y}, y) = \max(0, 1 - y \cdot \hat{y})$$

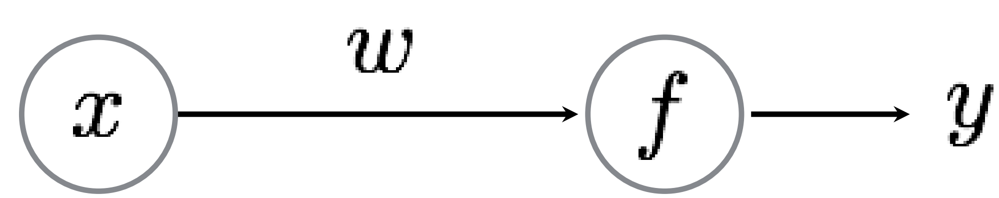


Cross Entropy Loss/Negative Log Likelihood

$$CrossEntropyLoss = -(y_i log(\hat{y}_i) + (1 - y_i) log(1 - \hat{y}_i))$$

back to the…

World’s Smallest Perceptron!



$$y = wx$$

(a.k.a. line equation, linear regression)

function of **ONE** parameter!

Learning a Perceptron

Given a set of samples and a Perceptron

$$\{x_i, y_i\}$$

$$y = f_{\mathrm{PER}}(x; w)$$

what is this activation function?

Estimate the parameter of the Perceptron

$$w$$

Learning a Perceptron

Given a set of samples and a Perceptron

$$\{x_i, y_i\}$$

$$y = f_{\mathrm{PER}}(x; w)$$

what is this activation function?

linear function! $f(x) = wx$

Estimate the parameter of the Perceptron

$$w$$

Learning Strategy

(gradient descent)

Given several examples

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)\}$$

and a perceptron

$$\hat{y} = wx$$

Modify weight w such that $\hat{y}$ gets **'closer'** to y

perceptron parameter

perceptron output

true label

Let’s demystify this process first…

Code to train your perceptron:

Let's demystify this process first…

Code to train your perceptron:

$$\text{for } n = 1 \ldots N$$
$$w = w + (y_n - \hat{y})x_i;$$

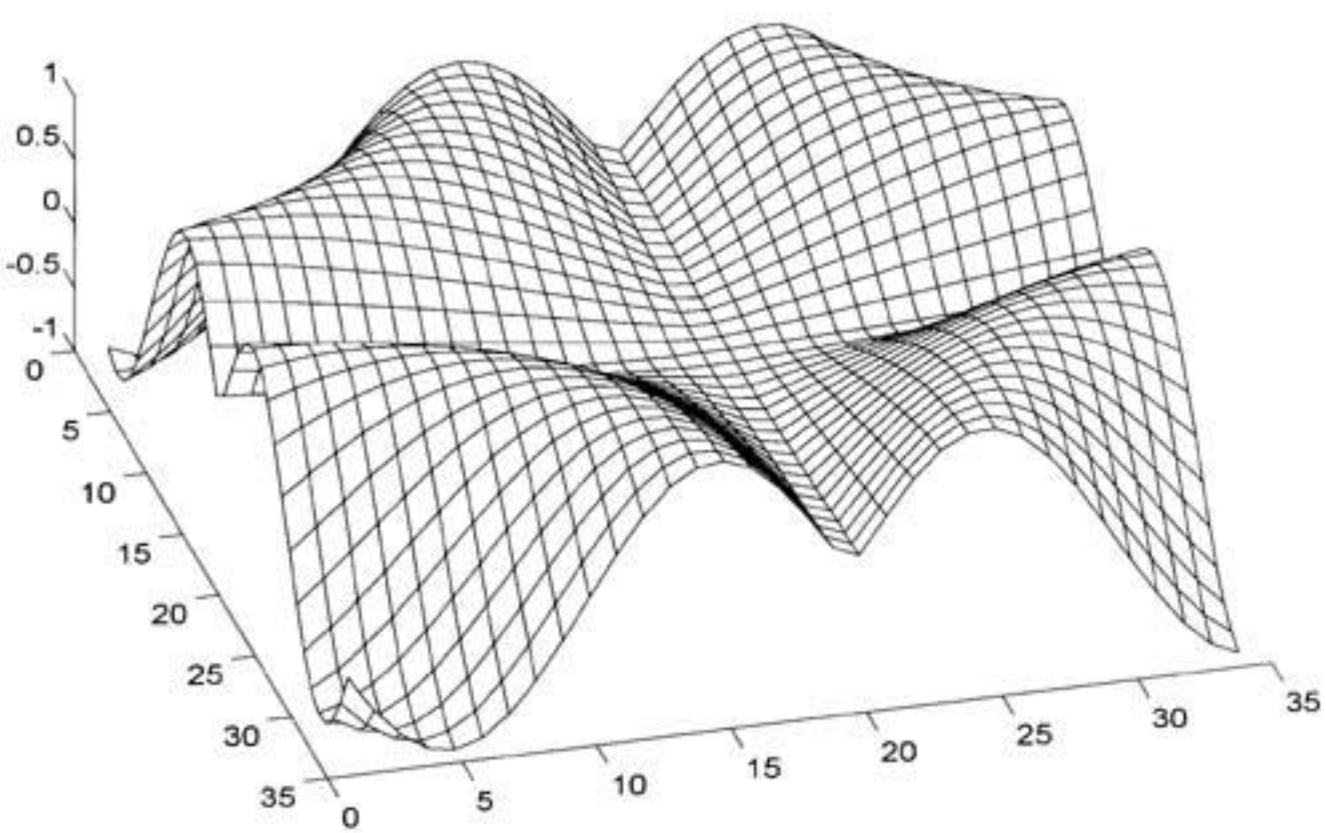
just one line of code!

Now where does this come from?

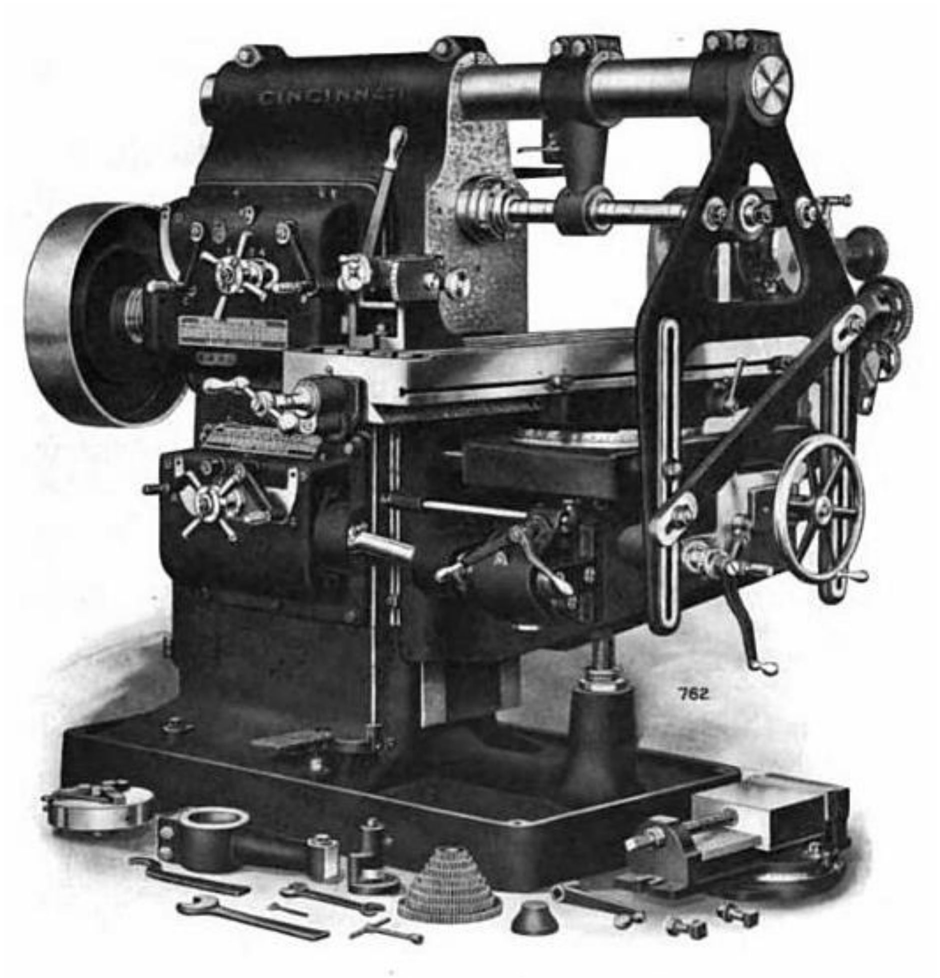
Gradient descent

(partial) derivatives tell us how much one variable affects another

Two ways to think about them:

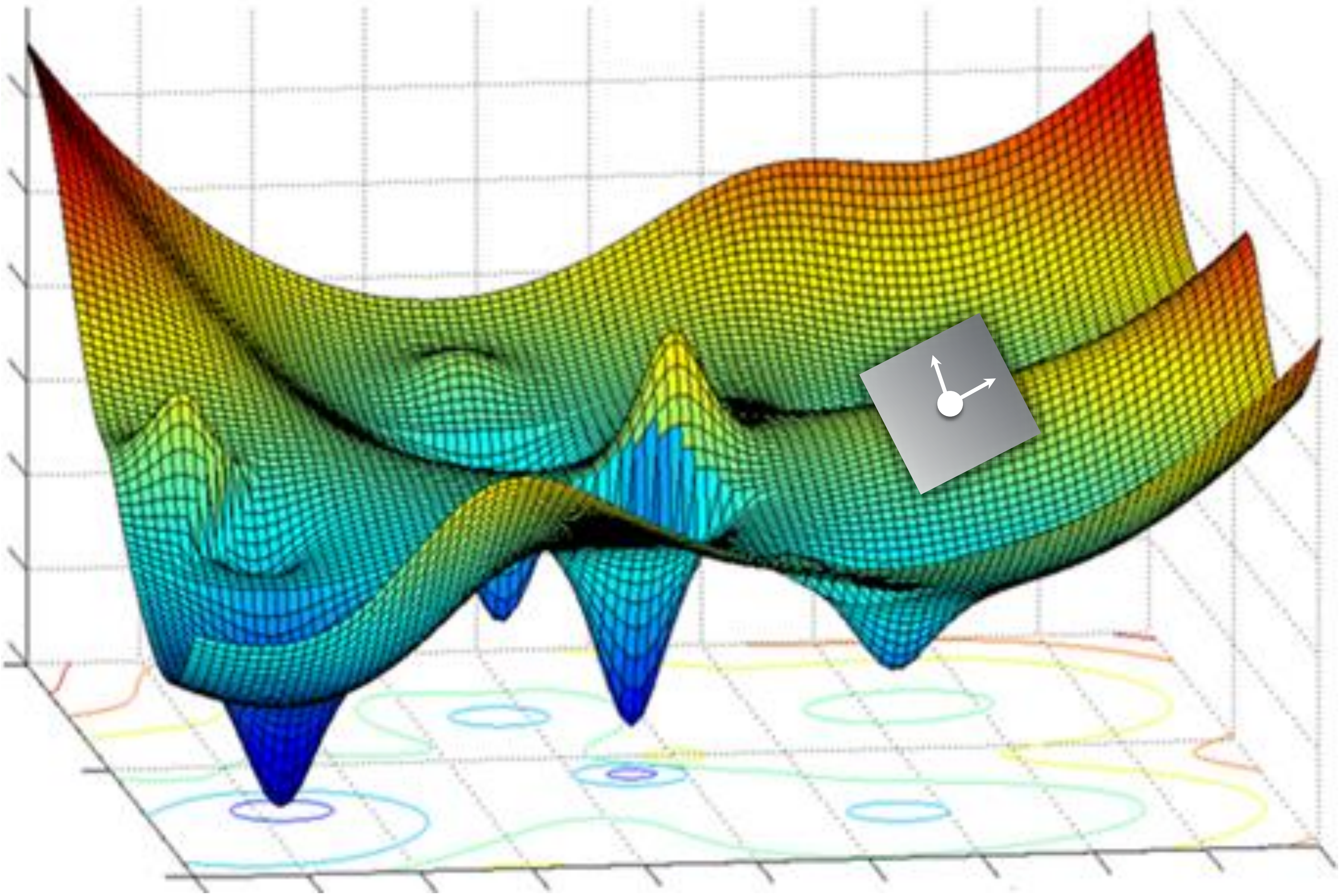


Slope of a function



Knobs on a machine

1. Slope of a function:



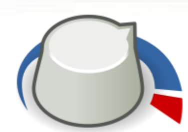
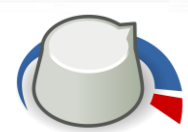
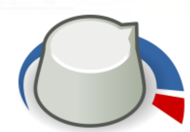
$$\frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} = \left[\frac{\partial f(\boldsymbol{x})}{\partial x}, \frac{\partial f(\boldsymbol{x})}{\partial y}\right]$$

describes the slope around a point

2. Knobs on a machine:

input
$\boldsymbol{x}$

output
$f(\boldsymbol{x};\boldsymbol{w})$

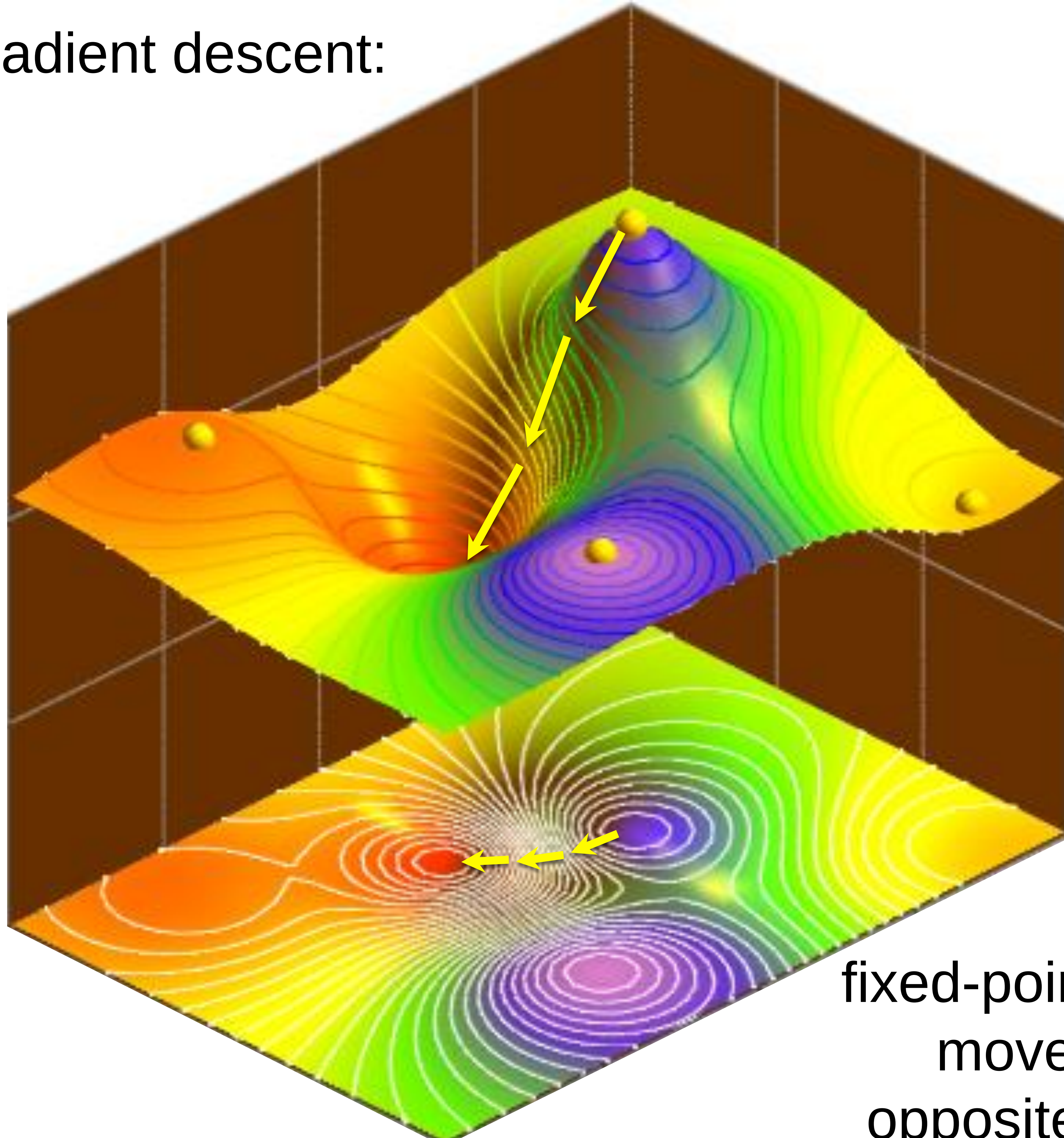




describes how each
'knob' affects the output

$$\frac{\partial f(x)}{\partial w_1} \qquad \frac{\partial f(x)}{\partial w_2} \qquad \frac{\partial f(x)}{\partial w_3}$$

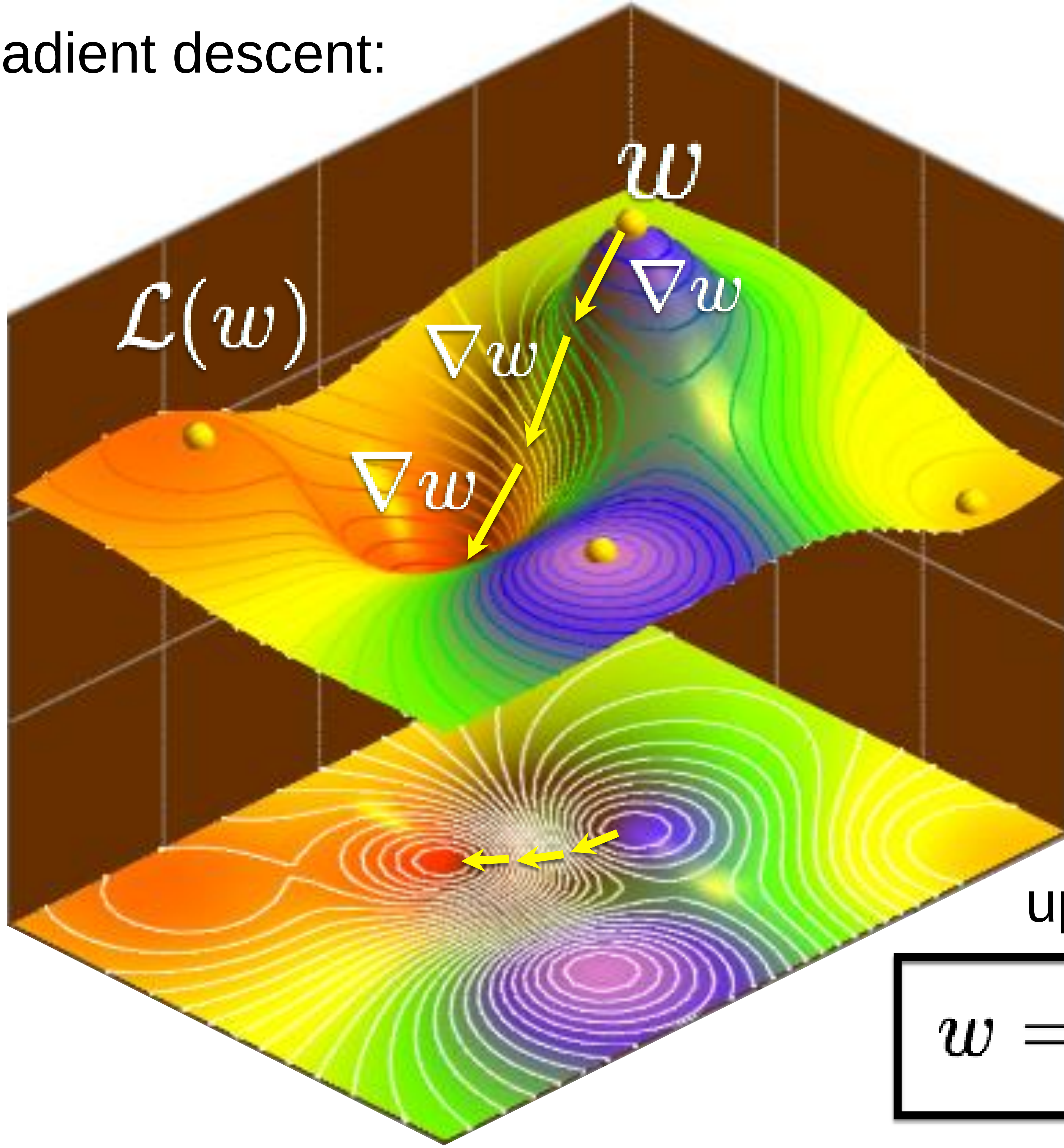
small change in parameter Δw_1 → output will change by $\dfrac{\partial f(x)}{\partial w_1}\,\Delta w_1$

Gradient descent:



Given a
fixed-point on a function,
move in the direction
opposite of the gradient

Gradient descent:



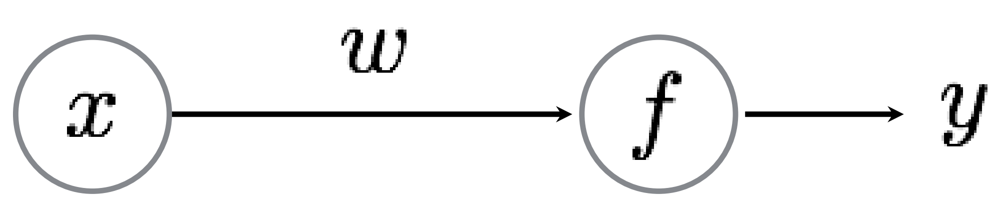
update rule:

$$\boxed{w = w - \nabla w}$$

Backpropagation

back to the…

World’s Smallest Perceptron!



$$y = wx$$

(a.k.a. line equation, linear regression)

function of **ONE** parameter!

Training the world's smallest perceptron

$$\text{for } n = 1 \dots N$$
$$w = w + \underline{(y_n - \hat{y})x_i};$$

This is just gradient descent, that means…

this should be the gradient of the loss function

Now where does this come from?

$$\frac{d\mathcal{L}}{dw}$$ …is the rate at which **this** will change…

$$\mathcal{L} = \frac{1}{2}(y - \hat{y})^2$$

the loss function

… per unit change of **this**

$$y = wx$$

the weight parameter

Let’s compute the derivative…

Compute the derivative

$$\frac{d\mathcal{L}}{dw} = \frac{d}{dw}\left\{\frac{1}{2}(y-\hat{y})^2\right\}$$

$$= -(y-\hat{y})\frac{dwx}{dw}$$

$$= -(y-\hat{y})x = \nabla w$$ just shorthand

That means the weight update for **gradient descent** is:

$$w = w - \nabla w$$ move in direction of negative gradient

$$= w + (y-\hat{y})x$$

Gradient Descent (world's smallest perceptron)

For each sample $\{x_i, y_i\}$

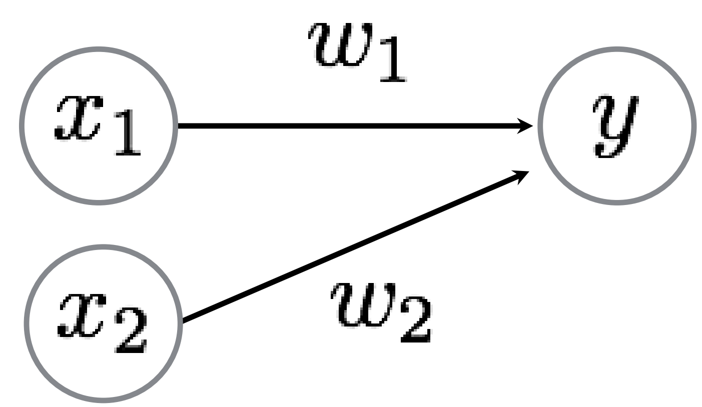
1. Predict
   a. Forward pass $\hat{y} = wx_i$
   b. Compute Loss $\mathcal{L}_i = \frac{1}{2}(y_i - \hat{y})^2$
2. Update
   a. Back Propagation $\frac{d\mathcal{L}_i}{dw} = -(y_i - \hat{y})x_i = \nabla w$
   b. Gradient update $w = w - \nabla w$

Training the world's smallest perceptron

$$\text{for } n = 1 \ldots N$$

$$w = w + (y_n - \hat{y})x_i;$$

world's (second) smallest **perceptron**!



function of **two** parameters!

Gradient Descent

For each sample $\{x_i, y_i\}$

1. Predict
   a. Forward pass
   b. Compute Loss
2. Update
   a. Back Propagation
   b. Gradient update

we just need to compute partial derivatives for this network

Derivative computation

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w_1} &= \frac{\partial}{\partial w_1}\left\{\frac{1}{2}(y-\hat{y})^2\right\} \\
&= -(y-\hat{y})\frac{\partial \hat{y}}{\partial w_1} \\
&= -(y-\hat{y})\frac{\partial \sum_i w_i x_i}{\partial w_1} \\
&= -(y-\hat{y})\frac{\partial w_1 x_1}{\partial w_1} \\
&= -(y-\hat{y})x_1 = \nabla w_1
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w_2} &= \frac{\partial}{\partial w_2}\left\{\frac{1}{2}(y-\hat{y})^2\right\} \\
&= -(y-\hat{y})\frac{\partial \hat{y}}{\partial w_2} \\
&= -(y-\hat{y})\frac{\partial \sum_i w_i x_i}{\partial w_1} \\
&= -(y-\hat{y})\frac{\partial w_2 x_2}{\partial w_2} \\
&= -(y-\hat{y})x_2 = \nabla w_2
\end{aligned}
$$

Derivative computation

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial w_1} &= \frac{\partial}{\partial w_1}\left\{\frac{1}{2}(y-\hat{y})^2\right\} \\ &= -(y-\hat{y})\frac{\partial \hat{y}}{\partial w_1} \\ &= -(y-\hat{y})\frac{\partial \sum_i w_i x_i}{\partial w_1} \\ &= -(y-\hat{y})\frac{\partial w_1 x_1}{\partial w_1} \\ &= -(y-\hat{y})x_1 = \nabla w_1\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial w_2} &= \frac{\partial}{\partial w_2}\left\{\frac{1}{2}(y-\hat{y})^2\right\} \\ &= -(y-\hat{y})\frac{\partial \hat{y}}{\partial w_2} \\ &= -(y-\hat{y})\frac{\partial \sum_i w_i x_i}{\partial w_1} \\ &= -(y-\hat{y})\frac{\partial w_2 x_2}{\partial w_2} \\ &= -(y-\hat{y})x_2 = \nabla w_2\end{aligned}$$

Gradient Update

$$\begin{aligned}w_1 &= w_1 - \eta \nabla w_1 \\ &= w_1 + \eta(y-\hat{y})x_1\end{aligned}$$

$$\begin{aligned}w_2 &= w_2 - \eta \nabla w_2 \\ &= w_2 + \eta(y-\hat{y})x_2\end{aligned}$$

Gradient Descent

For each sample $\{x_i, y_i\}$

1. Predict
   a. Forward pass $\hat{y} = f_{\text{MLP}}(x_i; \theta)$
   b. Compute Loss $\mathcal{L}_i = \frac{1}{2}(y_i - \hat{y})$

(side computation to track loss. not needed for backprop)

2. Update
   a. Back Propagation
   b. Gradient update

two lines now

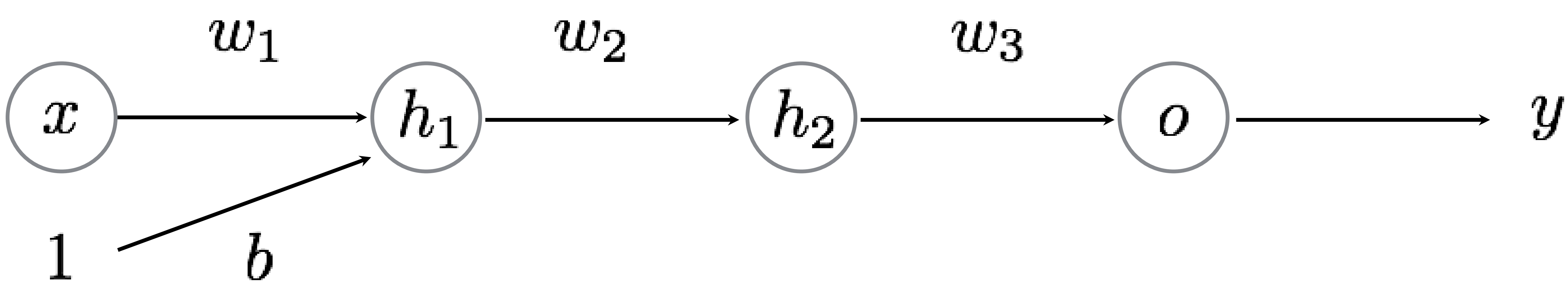
$$\nabla w_{1i} = -(y_i - \hat{y})x_{1i}$$
$$\nabla w_{2i} = -(y_i - \hat{y})x_{2i}$$

$$w_{1i} = w_{1i} + \eta(y - \hat{y})x_{1i}$$
$$w_{2i} = w_{2i} + \eta(y - \hat{y})x_{2i}$$

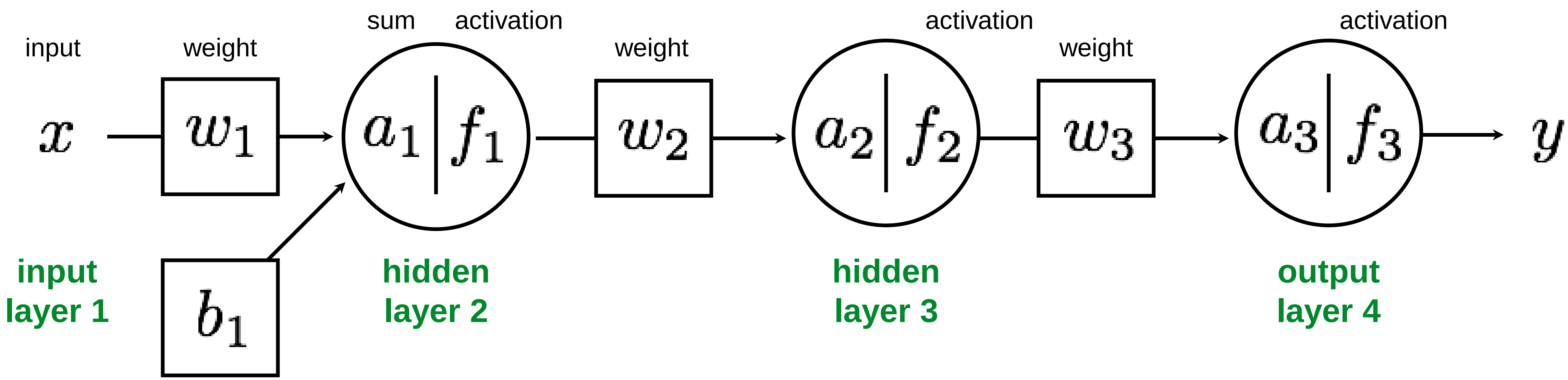
(adjustable step size)

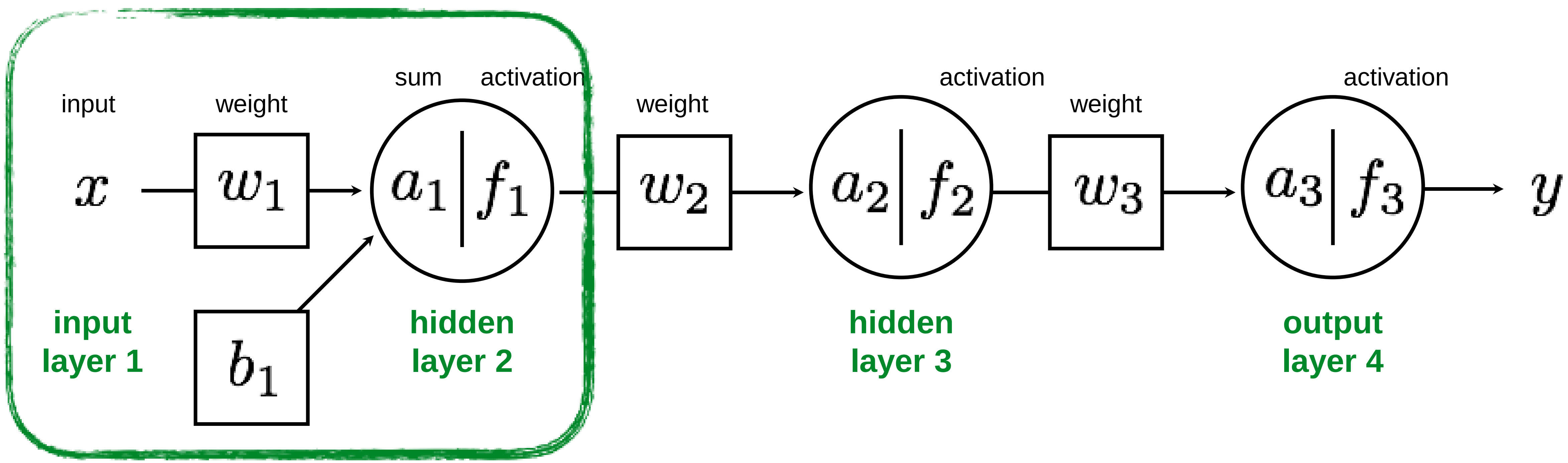
We haven’t seen a lot of ‘propagation’ yet because our perceptrons only had <u>one</u> layer…

multi-layer perceptron

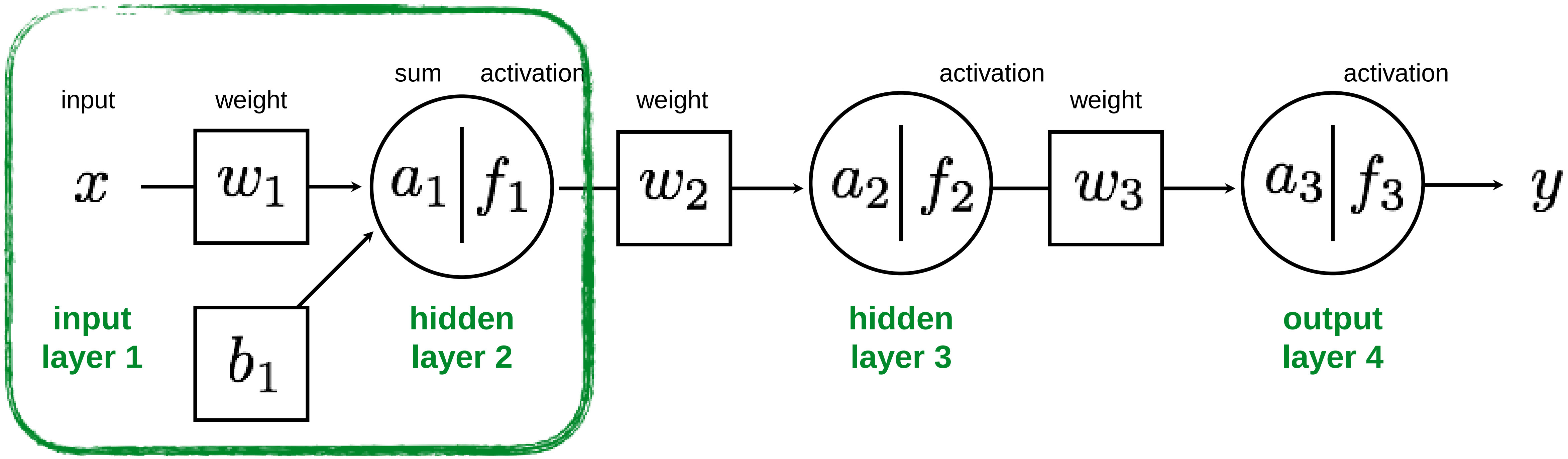


function of **FOUR** parameters and **FOUR** layers!

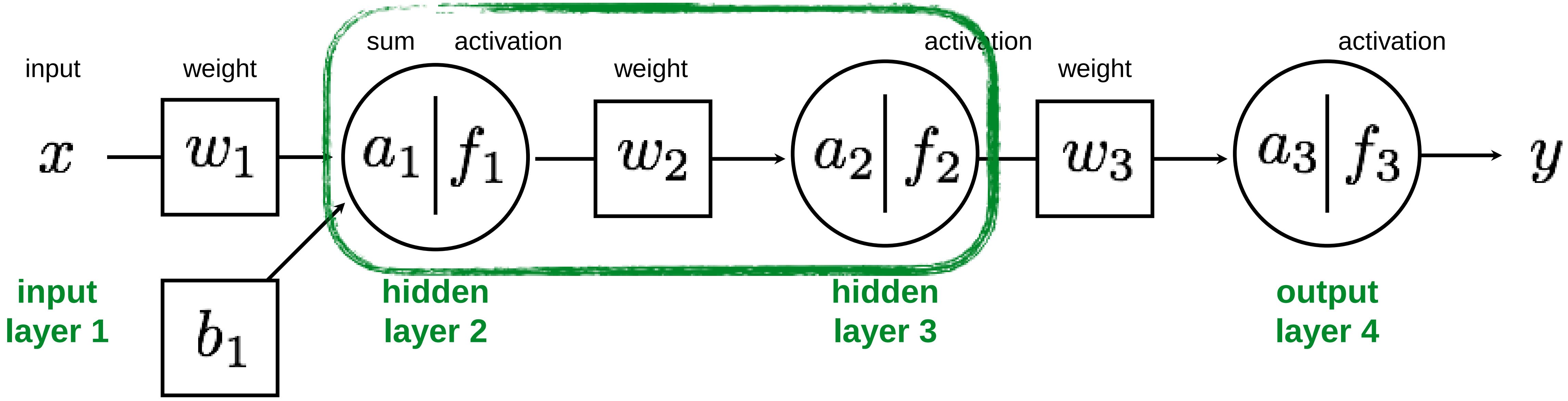

input
weight
sum
activation
weight
activation
weight
activation
x
w_1
a_1 | f_1
w_2
a_2 | f_2
w_3
a_3 | f_3
y
b_1
input
layer 1
hidden
layer 2
hidden
layer 3
output
layer 4



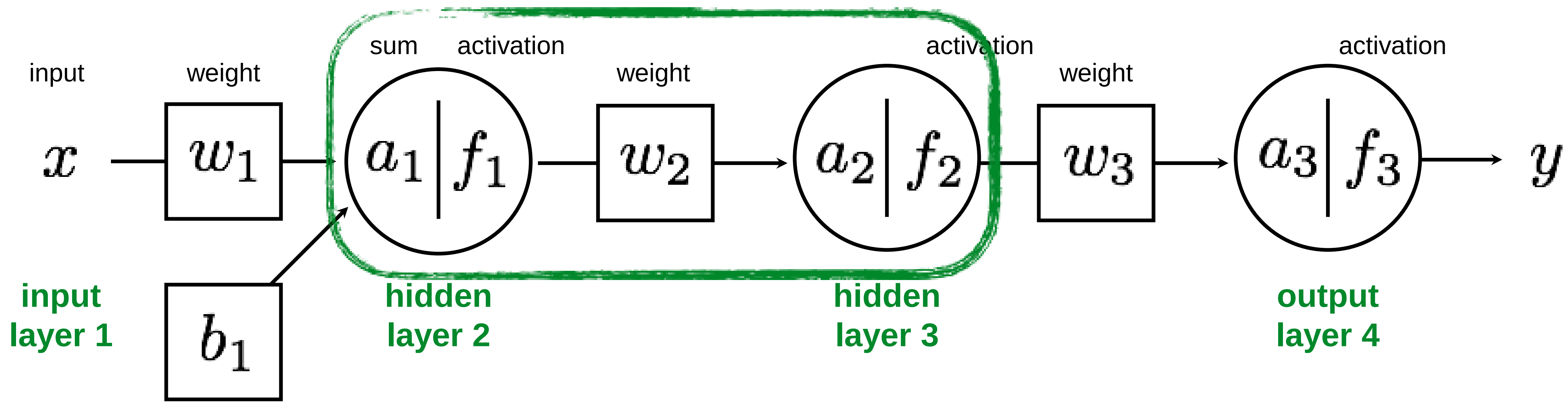
input
weight
sum
activation
weight
activation
weight
activation
x
w_1
a_1 | f_1
w_2
a_2 | f_2
w_3
a_3 | f_3
y
b_1
input layer 1
hidden layer 2
hidden layer 3
output layer 4



$$a_1 = w_1 \cdot x + b_1$$

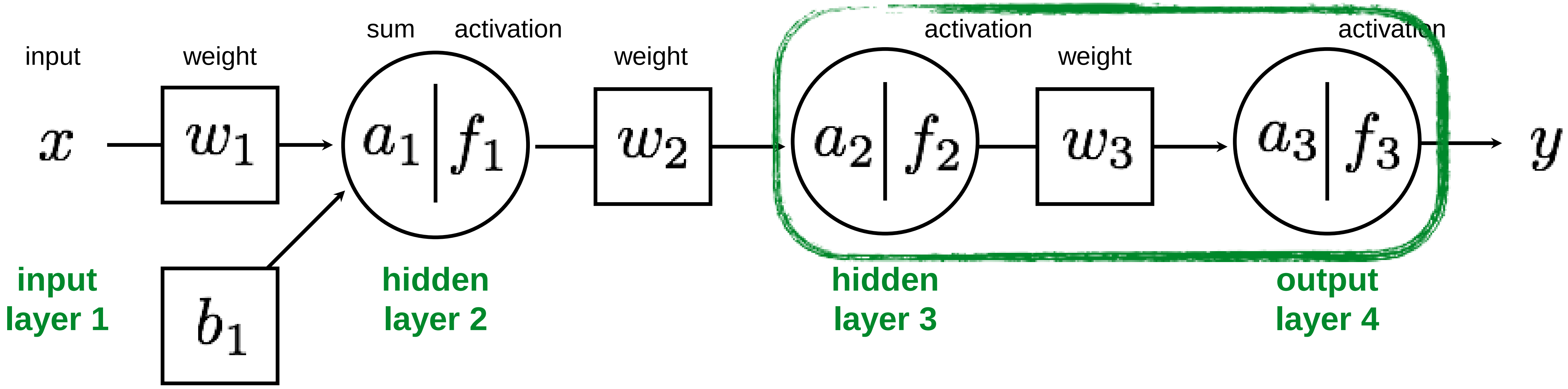


$$a_1 = w_1 \cdot x + b_1$$



$$a_1 = w_1 \cdot x + b_1$$

$$a_2 = w_2 \cdot f_1(w_1 \cdot x + b_1)$$



$$a_1 = w_1 \cdot x + b_1$$

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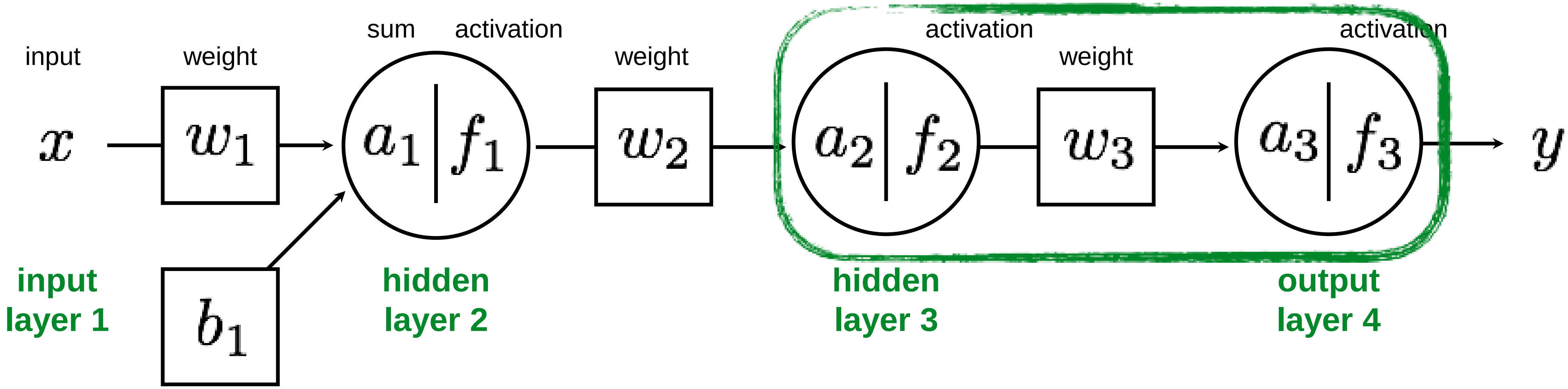
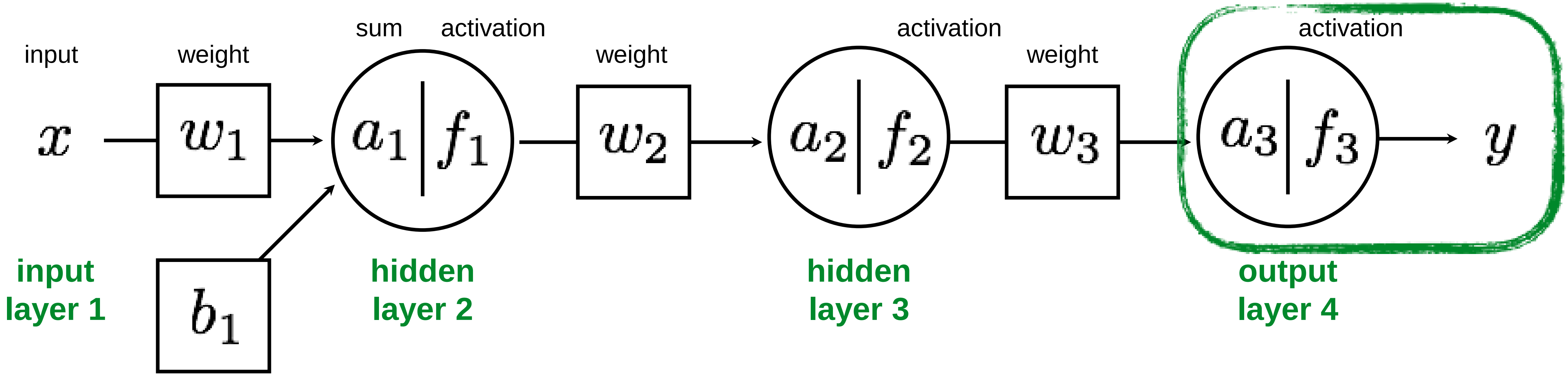


$$a_1 = w_1 \cdot x + b_1$$

$$a_2 = w_2 \cdot f_1(w_1 \cdot x + b_1)$$

$$a_3 = w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1))$$



$$a_1 = w_1 \cdot x + b_1$$

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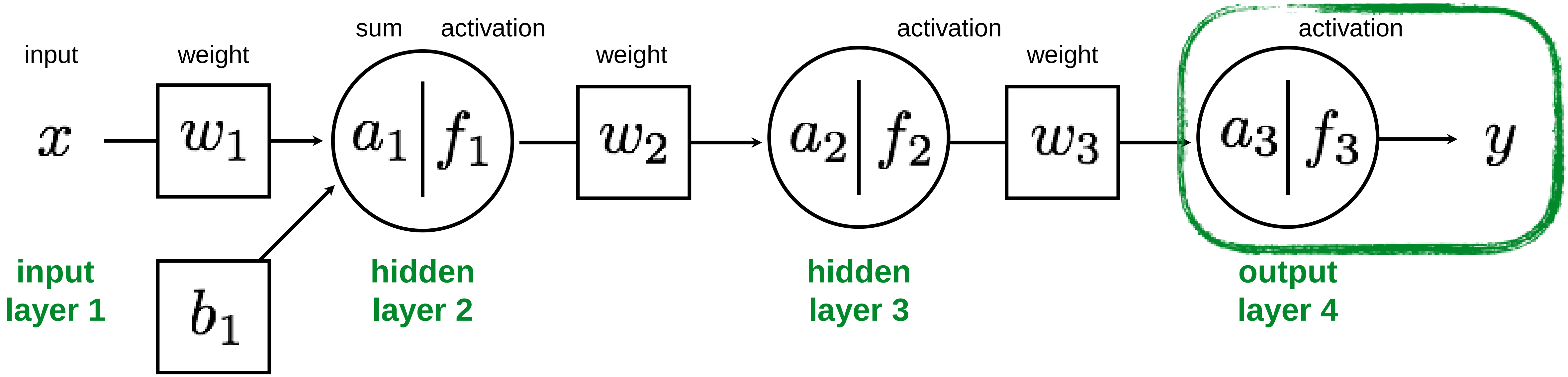


$$a_1 = w_1 \cdot x + b_1$$

$$a_2 = w_2 \cdot f_1(w_1 \cdot x + b_1)$$

$$a_3 = w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1))$$

$$y = f_3(w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1)))$$

Entire network can be written out as one long equation

$$y = f_3(w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1)))$$

We need to train the network:

What is known? What is unknown?

Entire network can be written out as a long equation

$$y = f_3(w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1)))$$

known

We need to train the network:

What is known? What is unknown?

Entire network can be written out as a long equation

$$y = f_3(w_3 \cdot f_2(w_2 \cdot f_1(w_1 \cdot x + b_1)))$$

activation function sometimes has parameters

unknown

We need to train the network:

What is known? What is unknown?

Learning an MLP

Given a set of samples and a MLP

$$\{x_i, y_i\}$$

$$y = f_{\text{MLP}}(x; \theta)$$

Estimate the parameters of the MLP

$$\theta = \{f, w, b\}$$

Gradient Descent

For each **random** sample $\{x_i, y_i\}$

1. Predict

    a. Forward pass $\hat{y} = f_{\text{MLP}}(x_i; \theta)$

    b. Compute Loss

2. Update

    a. Back Propagation $\frac{\partial \mathcal{L}}{\partial \theta}$ vector of parameter partial derivatives

    b. Gradient update $\theta \leftarrow \theta - \eta \nabla \theta$

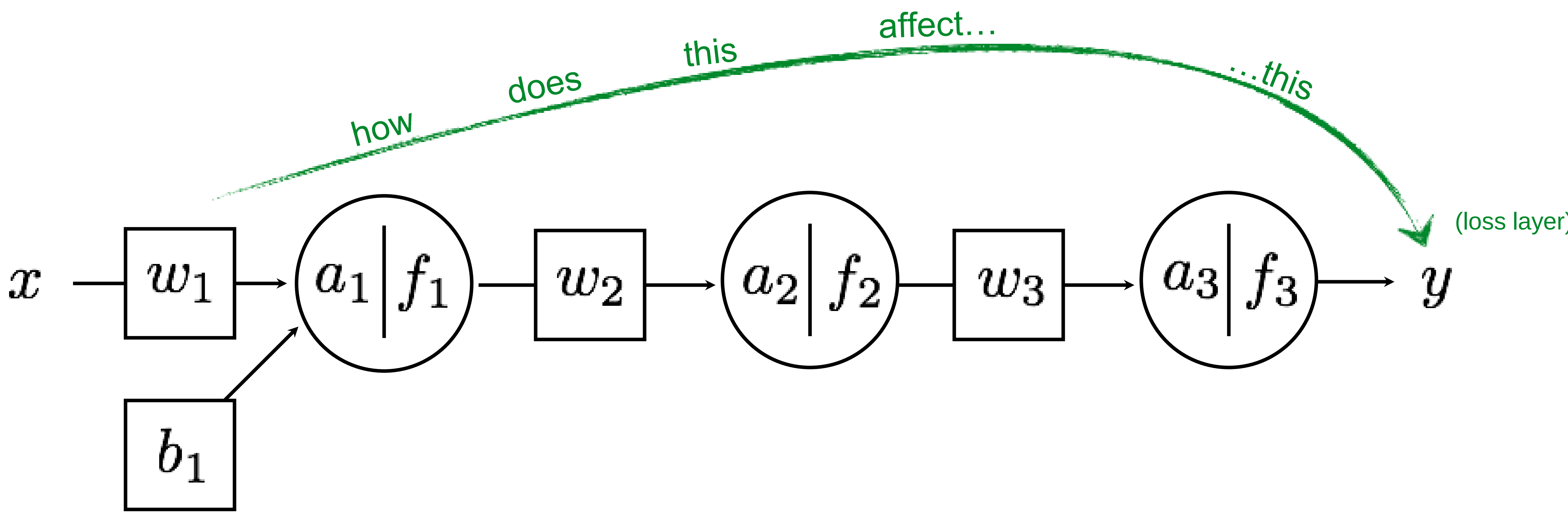
vector of parameter update equations

So we need to compute the partial derivatives

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}} = \left[\frac{\partial \mathcal{L}}{\partial w_3} \frac{\partial \mathcal{L}}{\partial w_2} \frac{\partial \mathcal{L}}{\partial w_1} \frac{\partial \mathcal{L}}{\partial b} \right]$$

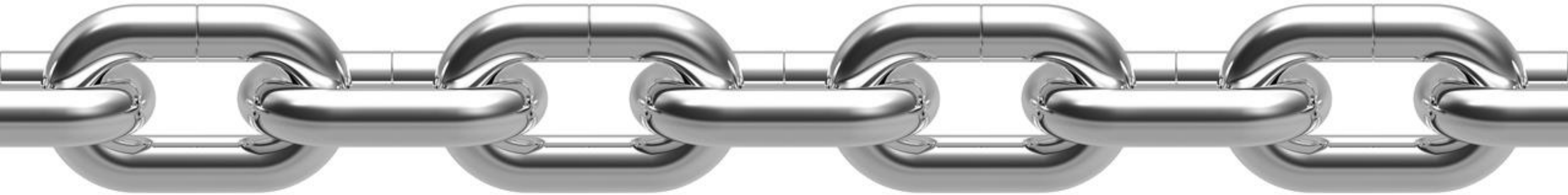
Remember,

Partial derivative $\frac{\partial L}{\partial w_1}$ describes…



So, how do you compute it?

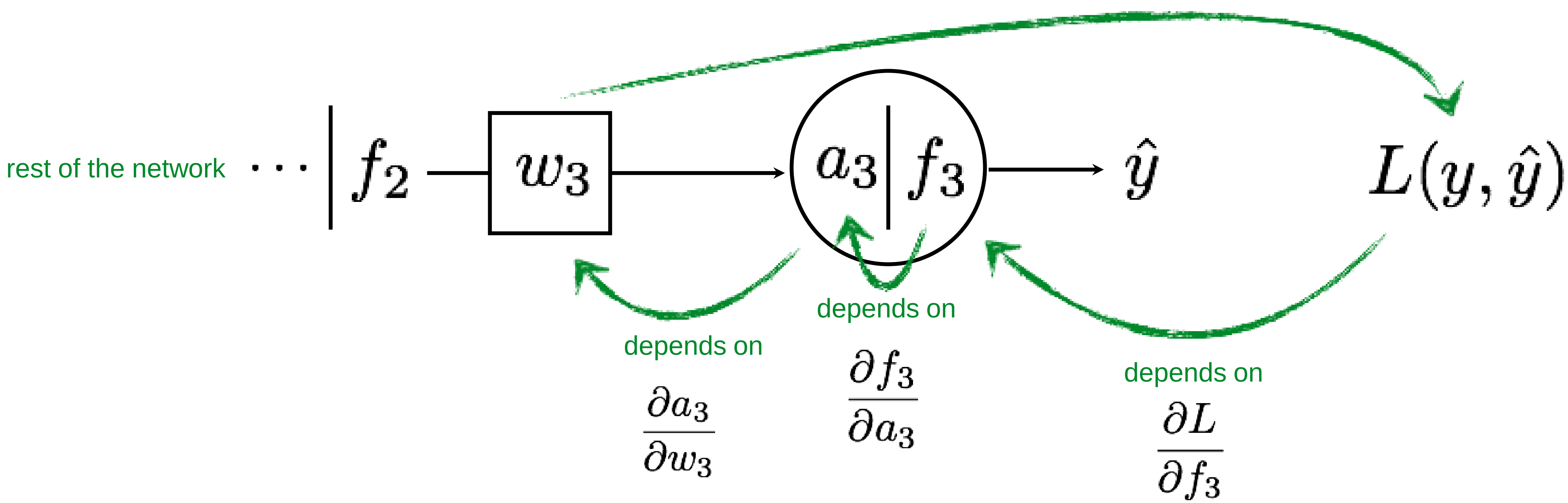
The Chain Rule

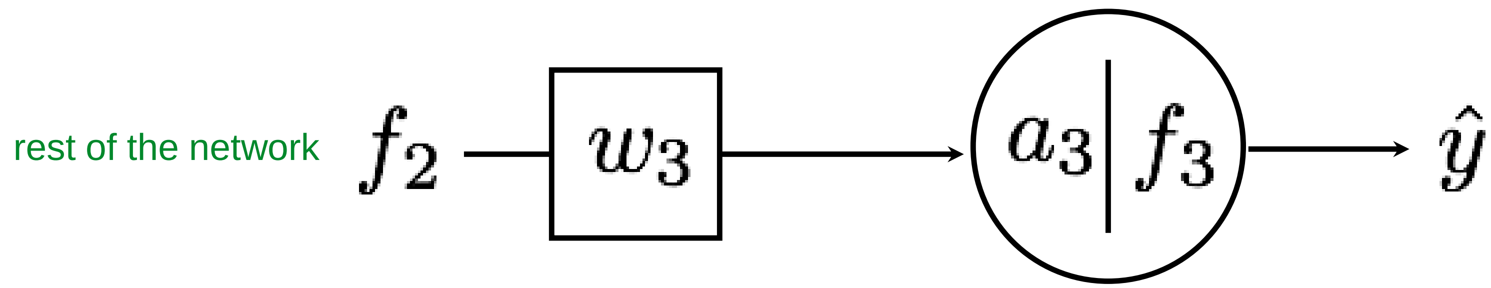


According to the chain rule…

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$

Intuitively, the effect of weight on loss function : $\frac{\partial L}{\partial w_3}$

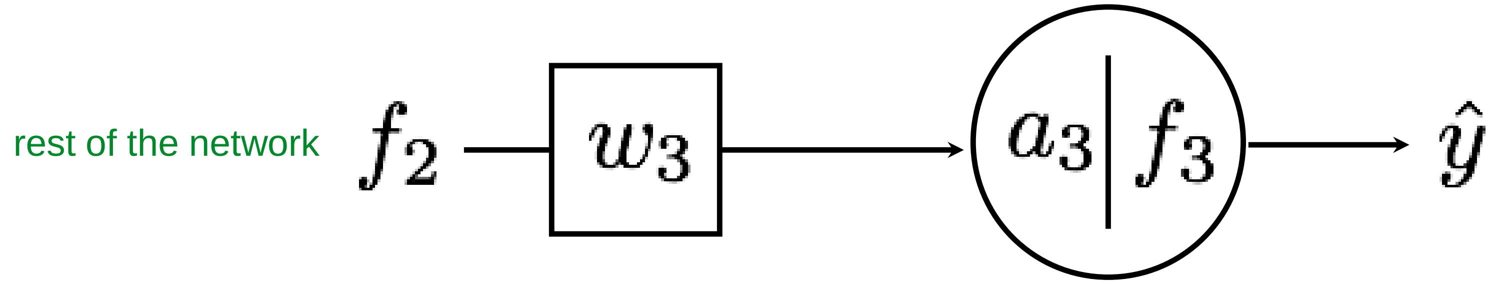




$L(y, \hat{y})$

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial f_3} \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3}$$

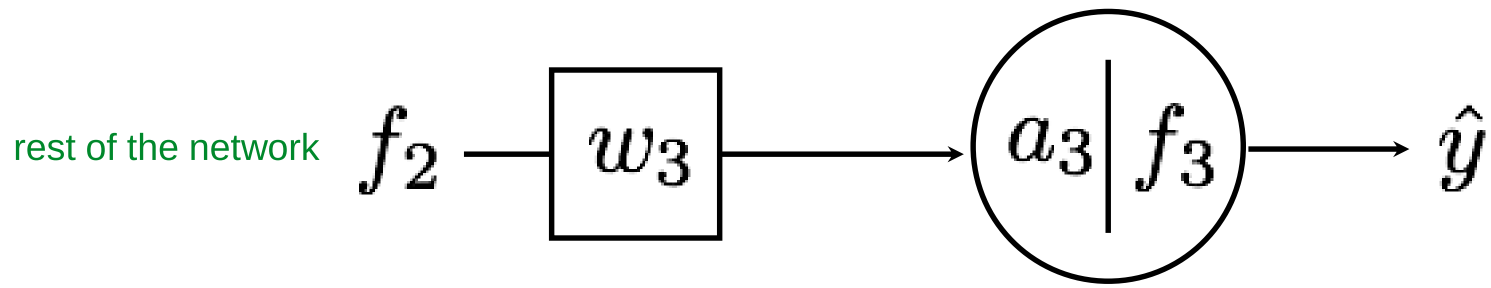
Chain Rule!



$$L(y, \hat{y})$$

$$\begin{aligned}
\frac{\partial L}{\partial w_3} &= \frac{\partial L}{\partial f_3} \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3}
\end{aligned}$$

Just the partial
derivative of L2 loss



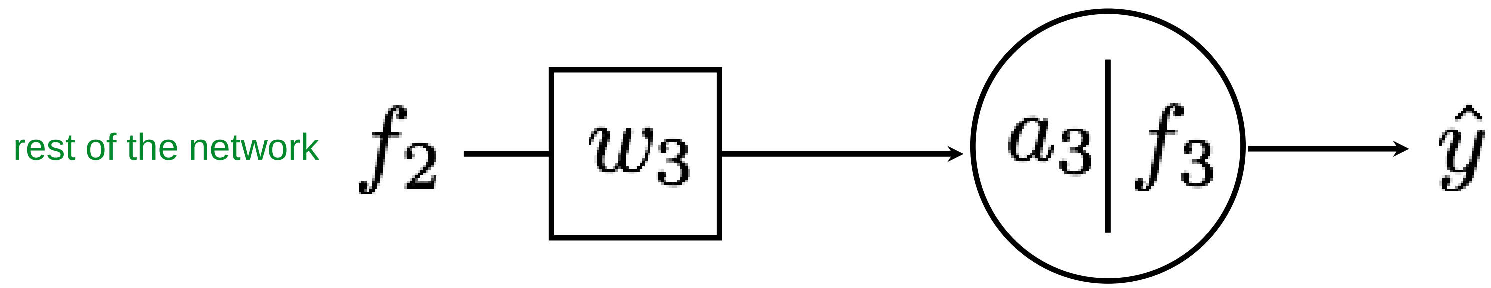
$$L(y, \hat{y})$$

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$

$$= -\eta(y - \hat{y})\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$

Let's use a Sigmoid function

$$\frac{ds(x)}{dx} = s(x)(1 - s(x))$$

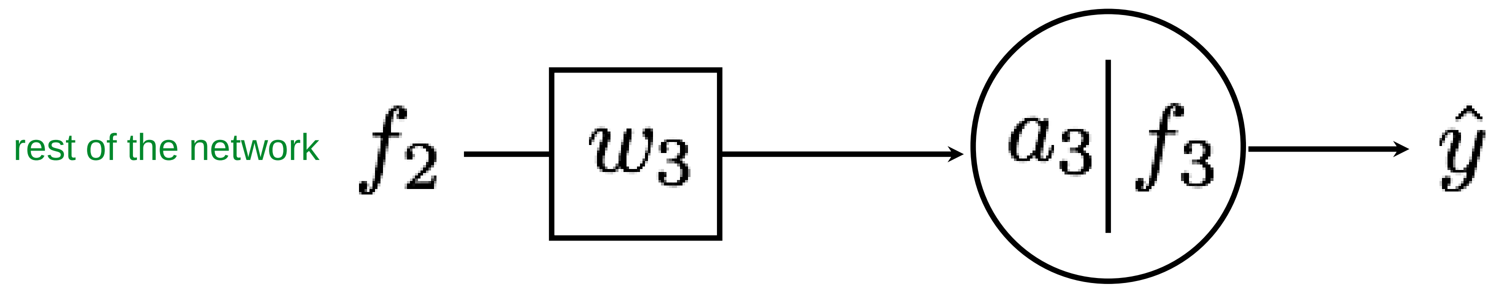


$$L(y, \hat{y})$$

$$\begin{aligned}
\frac{\partial L}{\partial w_3} &= \frac{\partial L}{\partial f_3} \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) f_3(1 - f_3) \frac{\partial a_3}{\partial w_3}
\end{aligned}$$

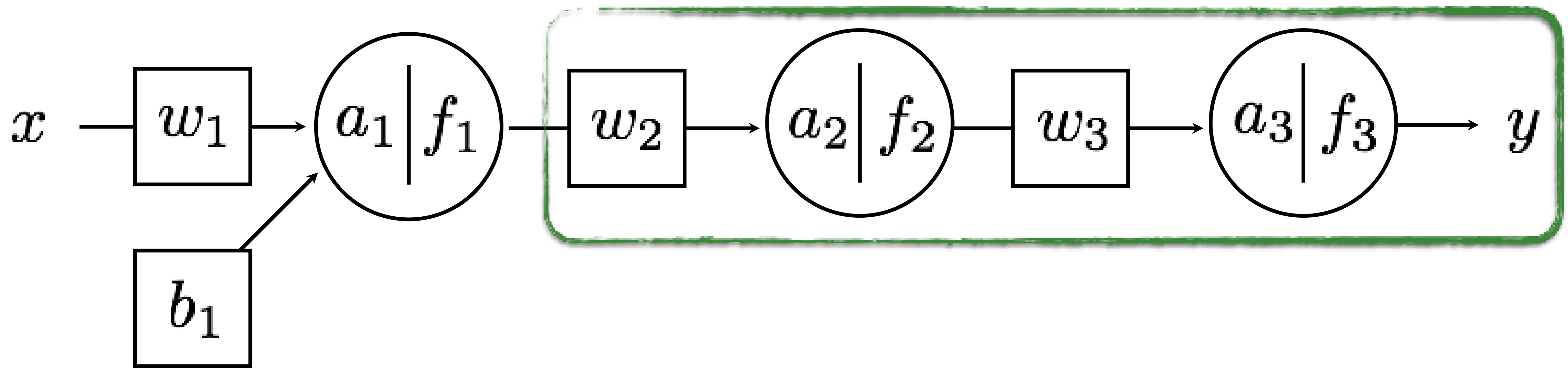
Let’s use a Sigmoid function

$$\frac{ds(x)}{dx} = s(x)(1 - s(x))$$

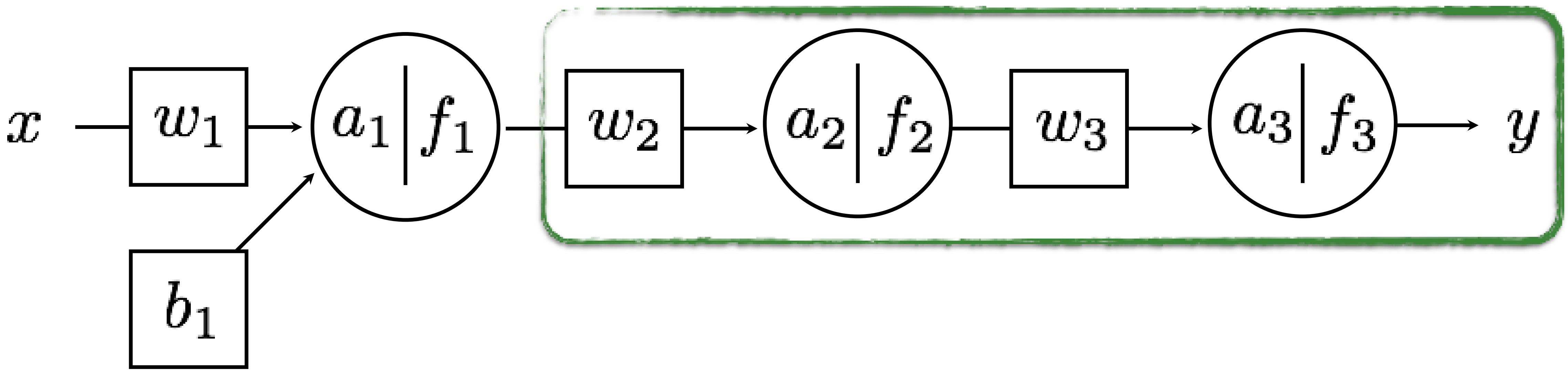


$$L(y, \hat{y})$$

$$\begin{aligned}
\frac{\partial L}{\partial w_3} &= \frac{\partial L}{\partial f_3} \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) \frac{\partial f_3}{\partial a_3} \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) f_3(1 - f_3) \frac{\partial a_3}{\partial w_3} \\
&= -\eta(y - \hat{y}) f_3(1 - f_3) f_2
\end{aligned}$$



$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}$$



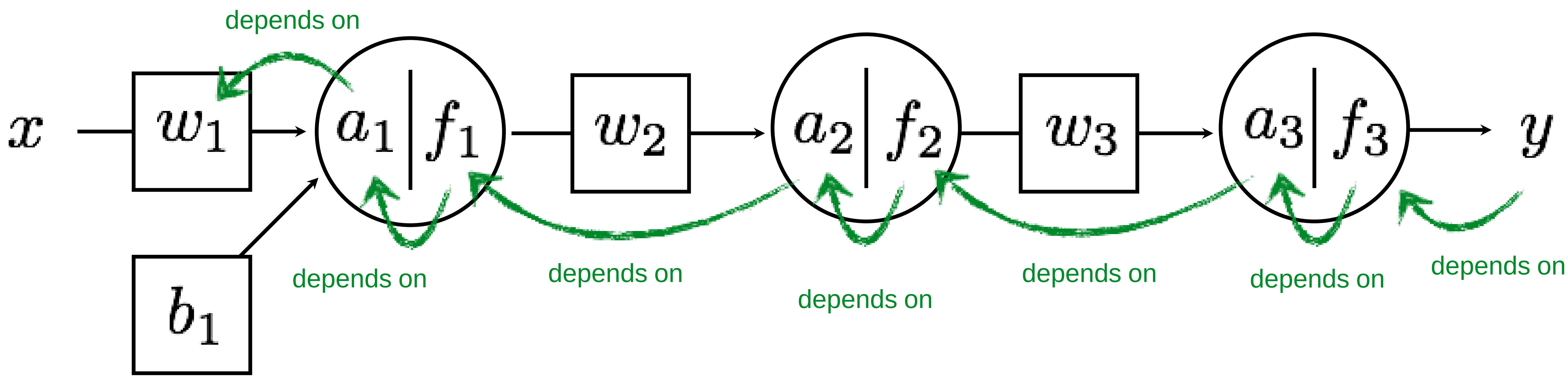
$$\frac{\partial L}{\partial w_2} = \boxed{\frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}$$

already computed.
re-use (propagate)!

The Chain Rule

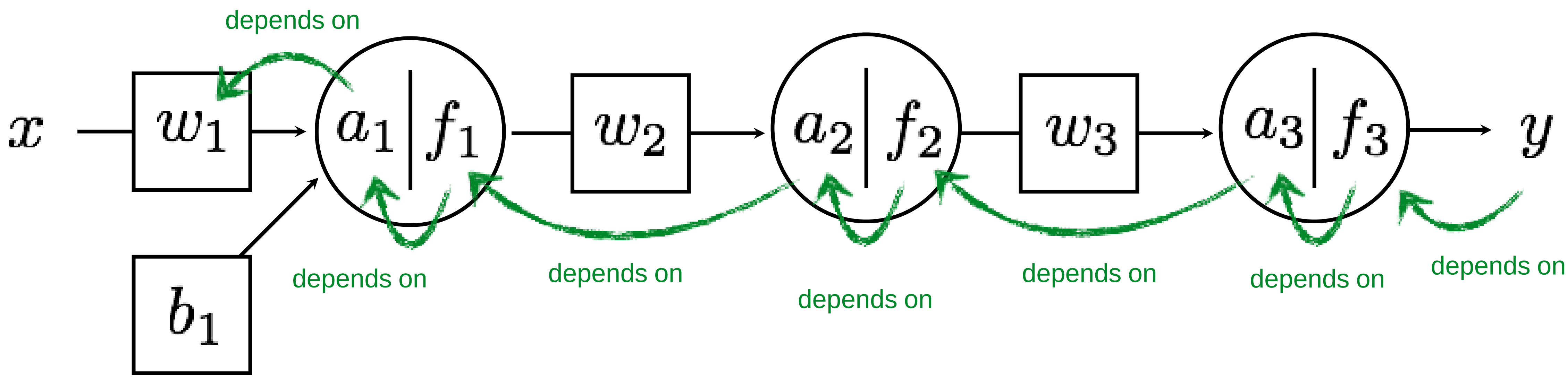
a.k.a. backpropagation

The chain rule says…



$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial w_1}$$

The chain rule says…



$$\frac{\partial L}{\partial w_1} = \boxed{\frac{\partial L}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial w_1}$$

already computed.
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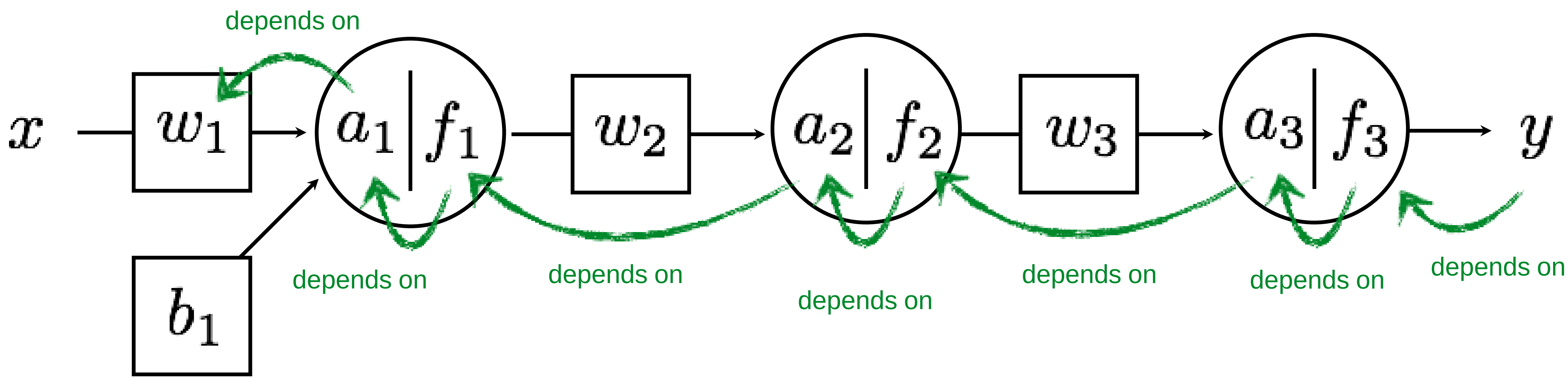
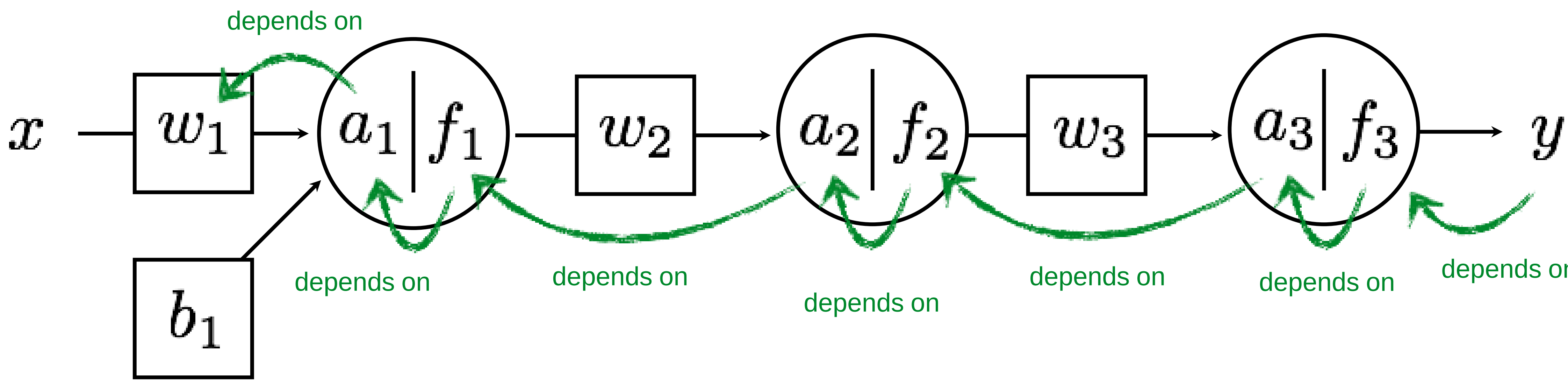


$$\frac{\partial \mathcal{L}}{\partial w_3} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}}\frac{\partial a_3}{\partial w_3}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial w_1}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial b}$$

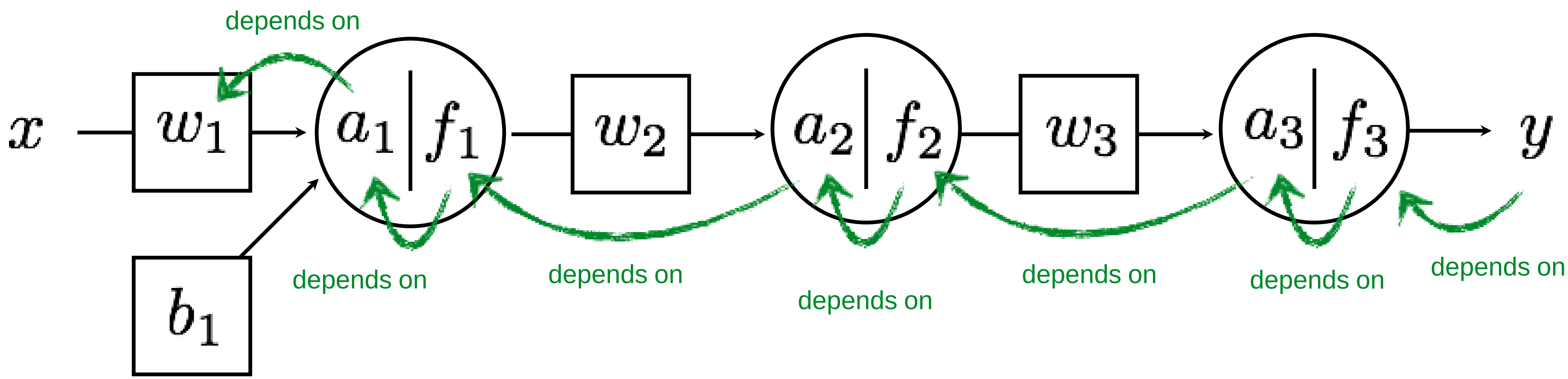


$$\frac{\partial \mathcal{L}}{\partial w_3} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}}\frac{\partial a_2}{\partial w_2}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial w_1}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial b}$$



$$\frac{\partial \mathcal{L}}{\partial w_3} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}$$

$$\frac{\partial \mathcal{L}}{\partial w_1} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}}\frac{\partial a_1}{\partial w_1}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \boxed{\frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}}\frac{\partial a_1}{\partial b}$$

Gradient Descent

For each example sample $\{x_i, y_i\}$

1. Predict

   a. Forward pass $\hat{y} = f_{\text{MLP}}(x_i; \theta)$

   b. Compute Loss $\mathcal{L}_i$

2. Update

   a. Back Propagation

$$\frac{\partial \mathcal{L}}{\partial w_3} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial w_3}$$
$$\frac{\partial \mathcal{L}}{\partial w_2} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial w_2}$$
$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial w_1}$$
$$\frac{\partial \mathcal{L}}{\partial b} = \frac{\partial \mathcal{L}}{\partial f_3}\frac{\partial f_3}{\partial a_3}\frac{\partial a_3}{\partial f_2}\frac{\partial f_2}{\partial a_2}\frac{\partial a_2}{\partial f_1}\frac{\partial f_1}{\partial a_1}\frac{\partial a_1}{\partial b}$$

   b. Gradient update

$$w_3 = w_3 - \eta \nabla w_3$$
$$w_2 = w_2 - \eta \nabla w_2$$
$$w_1 = w_1 - \eta \nabla w_1$$
$$b = b - \eta \nabla b$$

Gradient Descent

For each example sample $\{x_i, y_i\}$

1. Predict

   a. Forward pass $\hat{y} = f_{\text{MLP}}(x_i; \theta)$

   b. Compute Loss $\mathcal{L}_i$

2. Update

   a. Back Propagation $\frac{\partial \mathcal{L}}{\partial \theta}$

   vector of parameter partial derivatives

   b. Gradient update $\theta \leftarrow \theta + \eta \frac{\partial \mathcal{L}}{\partial \theta}$

   vector of parameter update equations

Stochastic gradient descent

What we are truly minimizing:

$$\min_{\theta} \sum_{i=1}^{N} L(y_i, f_{MLP}(x_i))$$

The gradient is:

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$$\min_{\theta} \sum_{i=1}^{N} L(y_i, f_{MLP}(x_i))$$

The gradient is:

$$\sum_{i=1}^{N} \frac{\partial L(y_i, f_{MLP}(x_i))}{\partial \theta}$$

What we use for gradient update is:

What we are truly minimizing:

$$\min_{\theta} \sum_{i=1}^{N} L(y_i, f_{MLP}(x_i))$$

The gradient is:

$$\sum_{i=1}^{N} \frac{\partial L(y_i, f_{MLP}(x_i))}{\partial \theta}$$

What we use for gradient update is:

$$\frac{\partial L(y_i, f_{MLP}(x_i))}{\partial \theta} \quad \text{for some i}$$

Stochastic Gradient Descent

For each example sample $\{x_i, y_i\}$

1. Predict

   a. Forward pass $\hat{y} = f_{\text{MLP}}(x_i; \theta)$

   b. Compute Loss $\mathcal{L}_i$

2. Update

   a. Back Propagation $\frac{\partial \mathcal{L}}{\partial \theta}$

   vector of parameter partial derivatives

   b. Gradient update $\theta \leftarrow \theta + \eta \frac{\partial \mathcal{L}}{\partial \theta}$

   vector of parameter update equations

How do we select which sample?

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- Select randomly!

Do we need to use only one sample?

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- You can use a *minibatch* of size $B < N$.

Why not do gradient descent with all samples?

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- Select randomly!

Do we need to use only one sample?

- You can use a *minibatch* of size B < N.

Why not do gradient descent with all samples?

- It’s very expensive when N is large (big data).

Do I lose anything by using stochastic GD?

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- Select randomly!

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- You can use a *minibatch* of size $B < N$.

Why not do gradient descent with all samples?

- It’s very expensive when N is large (big data).

Do I lose anything by using stochastic GD?

- Same convergence guarantees and complexity!
- Better generalization.

References

Basic reading: No standard textbooks yet! Some good resources:

- https://sites.google.com/site/deeplearningsummerschool/
- http://www.deeplearningbook.org/
- http://www.cs.toronto.edu/~hinton/absps/NatureDeepReview.pdf