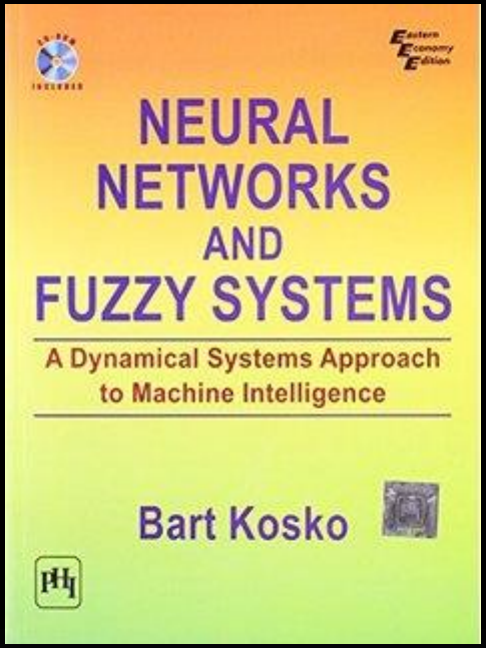
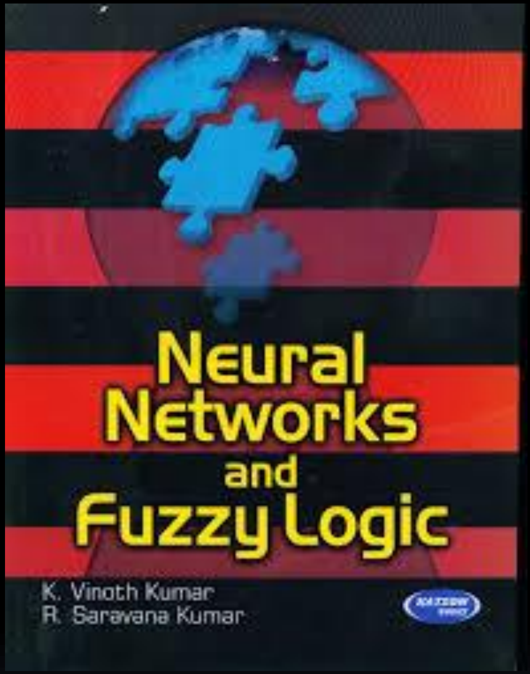
Fuzzy Logic and Neural Network





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Lecture

Fuzzy expert systems: Fuzzy logic

- Introduction, or what is fuzzy thinking?
- Fuzzy sets
- Linguistic variables and hedges
- Operations of fuzzy sets
- Fuzzy rules
- Summary

Introduction, or what is fuzzy thinking?

- Experts rely on **common sense** when they solve problems.
- **How can we represent expert knowledge that uses vague and ambiguous terms in a computer?**
- Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness. Fuzzy logic is the theory of fuzzy sets, sets that calibrate vagueness.
- Fuzzy logic is based on the idea that all things admit of degrees. Temperature, height, speed, distance, beauty – all come on a sliding scale. The motor is running *really hot*. Tom is a *very tall* guy.

- Fuzzy Logic is particularly good at handling uncertainty, vagueness and imprecision.
- This is especially useful where a problem can be described linguistically (using words) or, as with neural networks, where there is data and you are looking for relationships or patterns within that data.
- Fuzzy Logic uses imprecision to provide robust, tractable solutions to problems.
- Fuzzy logic relies on the concept of a ***fuzzy set***.

- Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non-members. For instance, we may say, Tom is tall because his height is 181 cm. If we drew a line at 180 cm, we would find that David, who is 179 cm, is small. Is David really a small man or we have just drawn an arbitrary line in the sand?
- Fuzzy logic reflects how people think. It attempts to model our sense of words, our decision making and our common sense. As a result, it is leading to new, more human, intelligent systems.

- Fuzzy, or multi-valued logic was introduced in the 1930s by **Jan Lukasiewicz** , a Polish philosopher. While classical logic operates with only two values 1 (true) and 0 (false), Lukasiewicz introduced logic that extended the range of truth values to all real numbers in the interval between 0 and 1. He used a number in this interval to represent the *possibility* that a given statement was true or false. For example, the possibility that a man 181 cm tall is really tall might be set to a value of 0.86. It is *likely* that the man is tall. This work led to an inexact reasoning technique often called **possibility theory**.

- In 1965 **Lotfi Zadeh**, published his famous paper "Fuzzy sets". Zadeh extended the work on possibility theory into a formal system of mathematical logic, and introduced a new concept for applying natural language terms. This new logic for representing and manipulating fuzzy terms was called **fuzzy logic**, and Zadeh became the Master of *fuzzy logic*.

- ***Why fuzzy?***

  As Zadeh said, the term is concrete, immediate and descriptive; we all know what it means. However, many people in the West were repelled by the word *fuzzy* , because it is usually used in a negative sense.

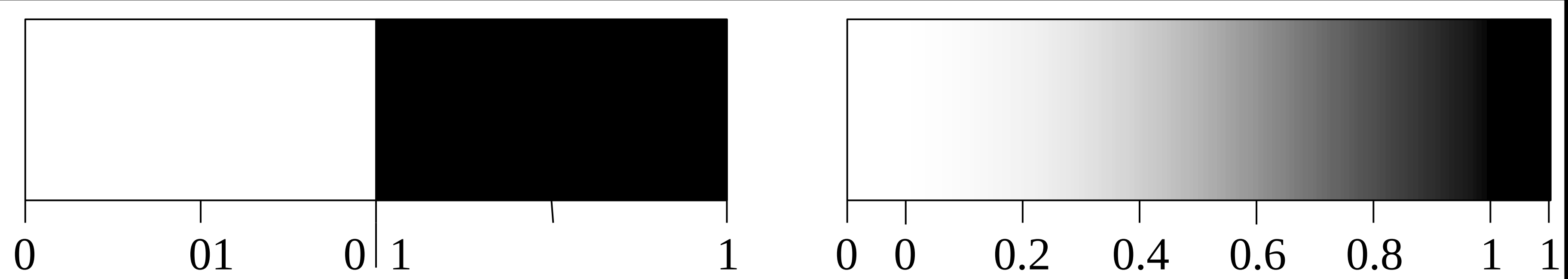
- ***Why logic?***

  Fuzziness rests on fuzzy set theory, and fuzzy logic is just a small part of that theory.

Fuzzy logic is a set of mathematical principles for knowledge representation based on degrees of membership.

Unlike two-valued Boolean logic, fuzzy logic is **multi-valued**. It deals with **degrees of membership** and **degrees of truth**. Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true). Instead of just black and white, it employs the spectrum of colours, accepting that things can be partly true and partly false at the same time.

Range of logical values in Boolean and fuzzy logic



(*a*) Boolean Logic. (*b*) Multi-valued Logic.

What does it offer?

- The first applications of fuzzy theory were primarily industrial, such as process control for cement kilns.
- Since then, the applications of Fuzzy Logic technology have virtually exploded, affecting things we use everyday. Take for example, the *fuzzy washing machine* .
- A load of clothes in it and press start, and the machine begins to turn, automatically choosing the best cycle. The fuzzy microwave, Place chili, potatoes, or etc in a *fuzzy microwave* and push single button, and it cooks for the right time at the proper temperature.
- The *fuzzy car*, maneuvers itself by following simple verbal instructions from its driver. It can even stop itself when there is an obstacle immediately ahead using sensors.

Fuzzy sets

- The concept of a **set** is fundamental to mathematics.
- However, our own language is also the supreme expression of sets. For example, *car* indicates the *set of cars*. When we say *a car* , we mean one out of the set of cars.

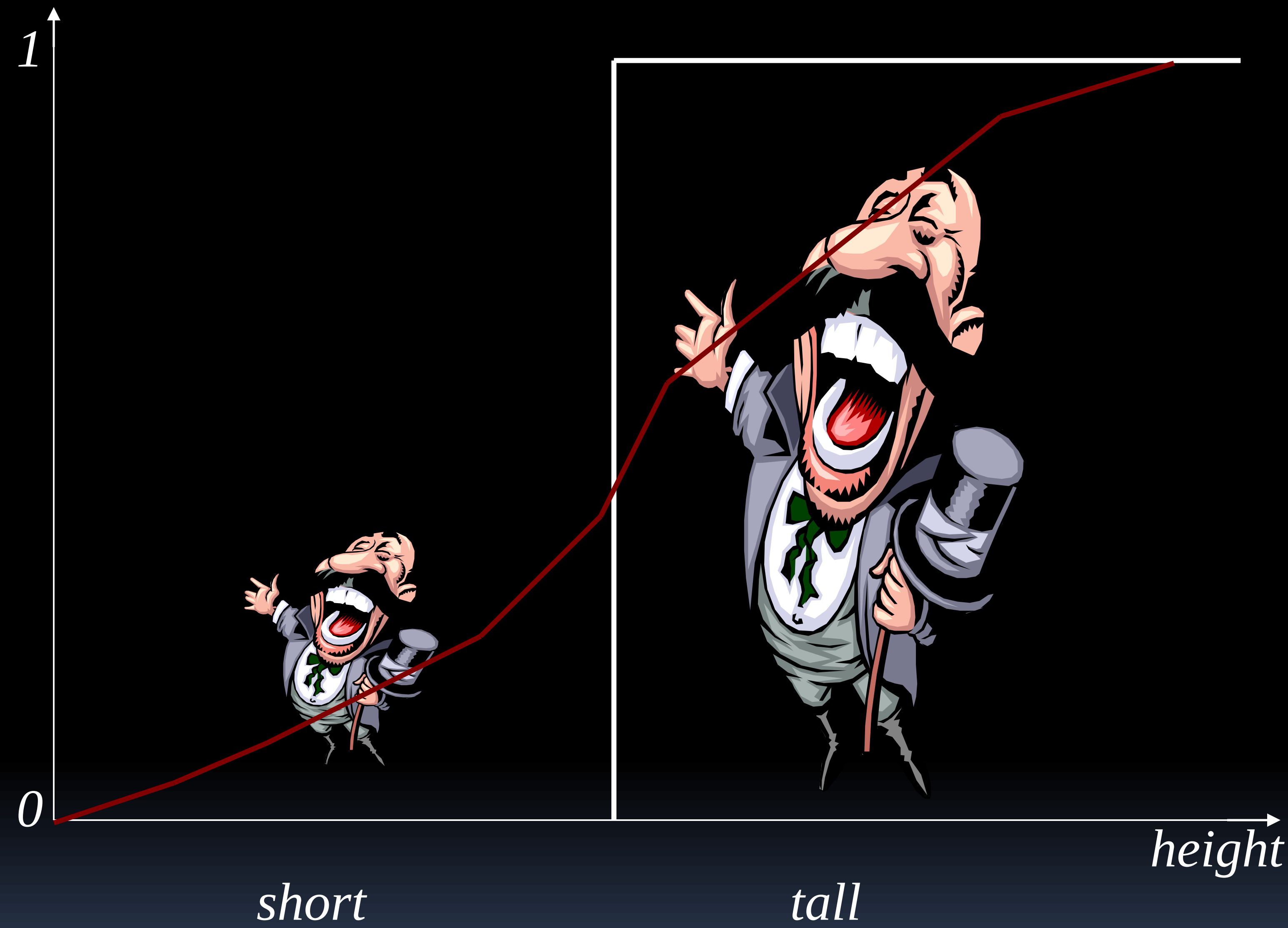
FUZZY SETS

Fuzzy sets is fully defined by its membership functions.

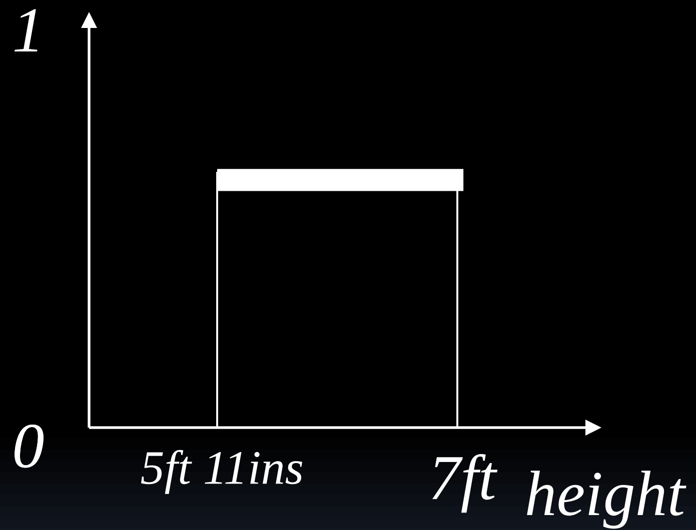
Membership function is a function in [0,1] that represents the degree of belonging.

Let’s start with a simple example

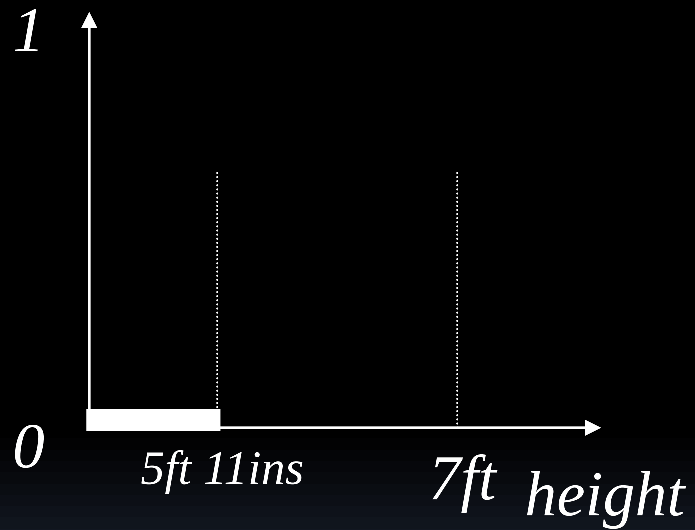
- Ask yourself how tall you are?
- Would you classify yourself as a tall person?
- What is the limit that determines tall and short people?
- Let’s collect some figures from you


1
0
height
short
tall

Let’s consider the first example
(How tall/short we are?)

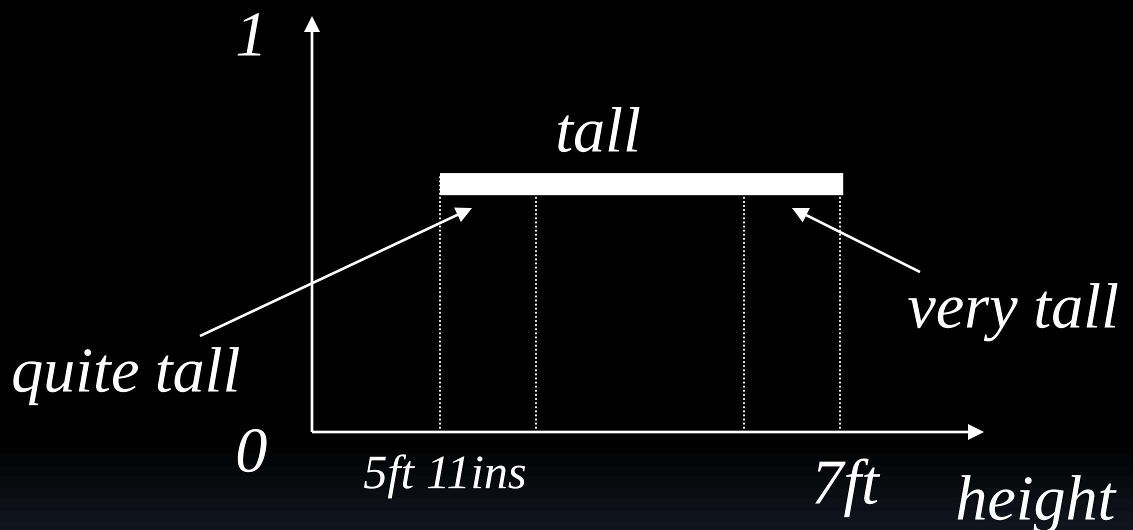


A crisp way of modelling tallness



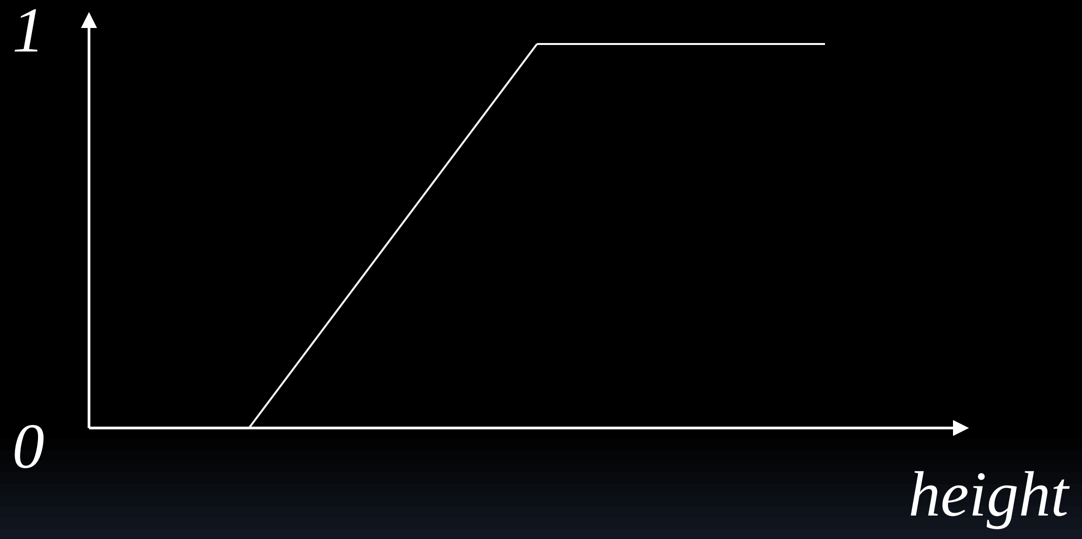
A crisp version of short

Let’s consider the first example (How tall/short we are?)

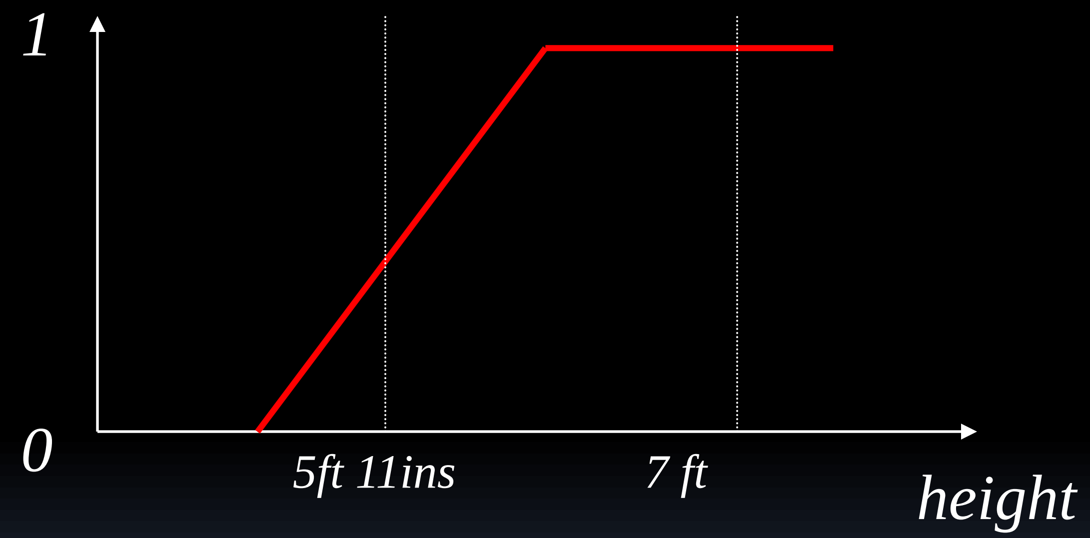


crisp definitions for tallness

Definition in a Fuzzy Set (How tall/short we are?)

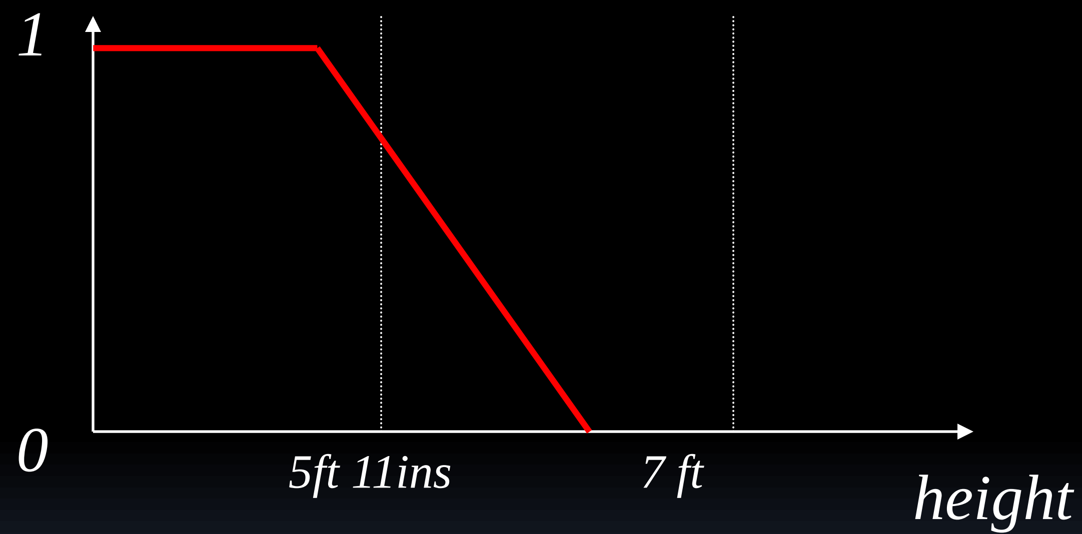


Definition in a Fuzzy Set (How tall/short we are?)



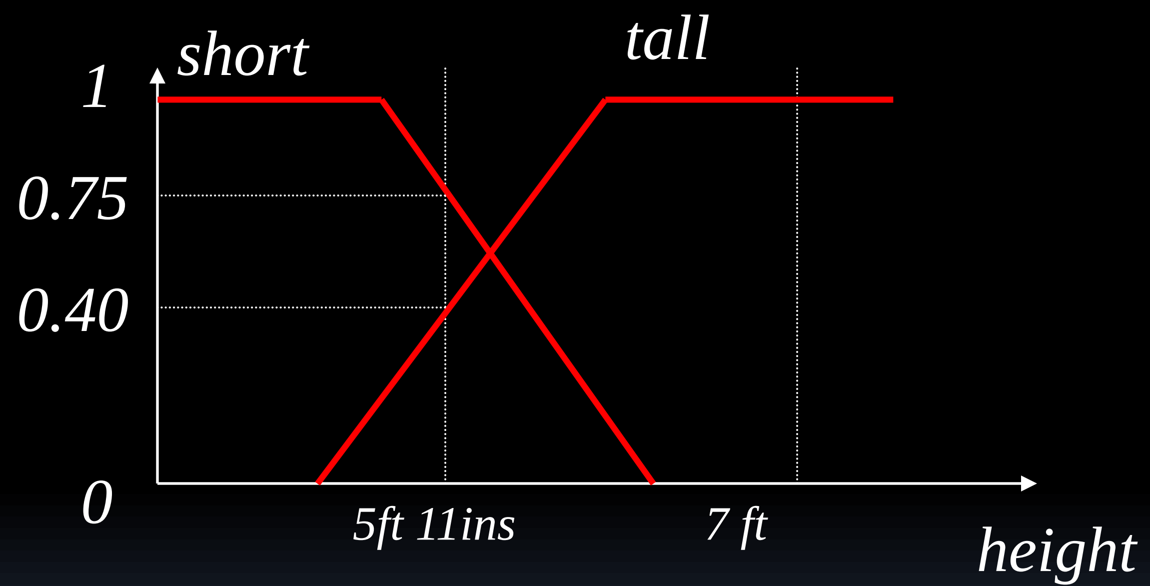
A possible fuzzy set tall

Definition in a Fuzzy Set (How tall/short we are?)



A possible fuzzy set short

Definition in a Fuzzy Set (How tall/short we are?)



Membership functions that represent tallness and short

Some maths!
Formal definitions of a fuzzy set

- For any fuzzy set, (let's say) A, the function μ_A represents the membership function for which $\mu_A(x)$ indicates the degree of membership that x, of the universal set X, belongs to set A and is, usually, expressed as a number between 0 and 1

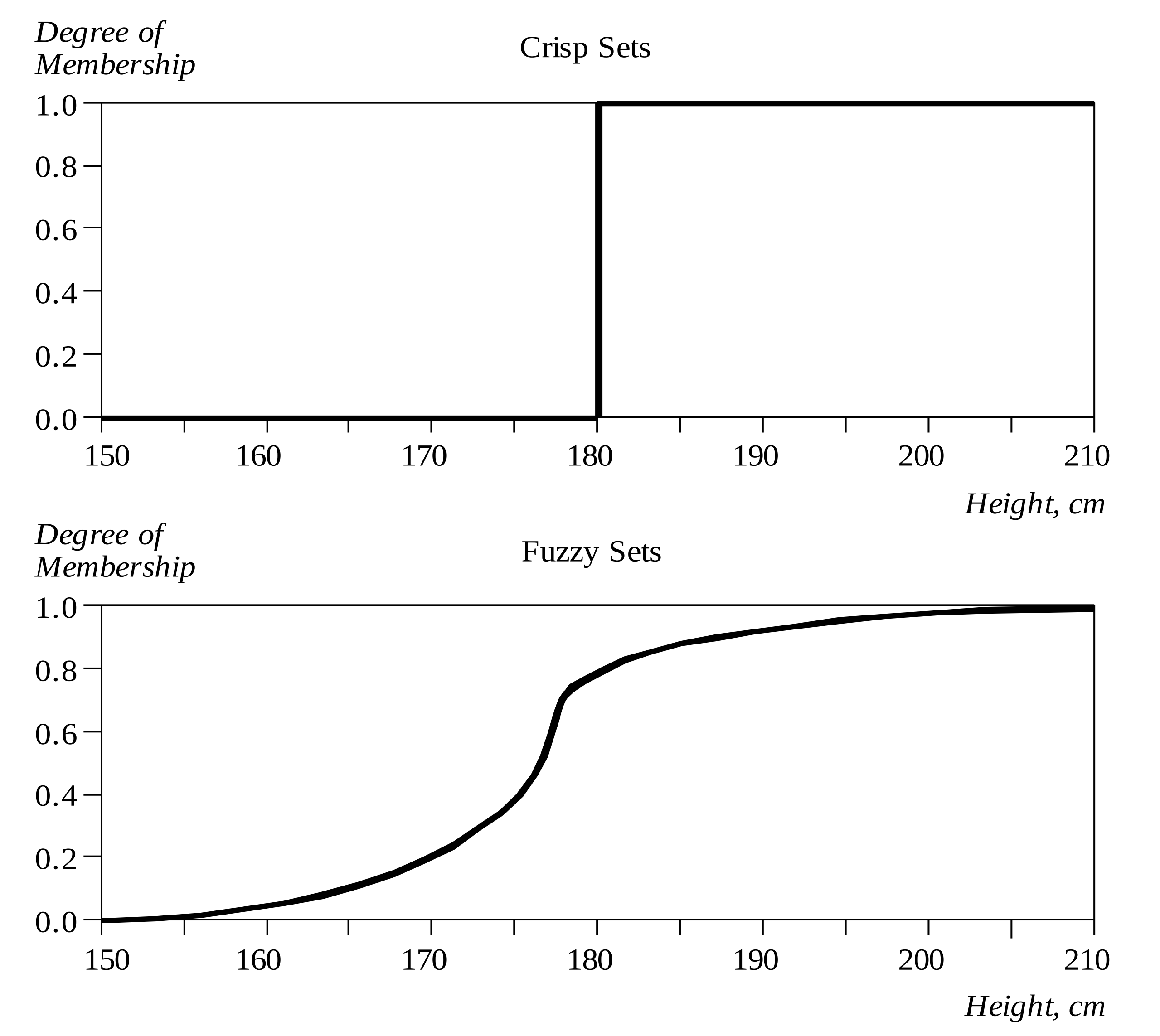
$$\mu_A(x) : X \rightarrow [0,1]$$

- Fuzzy sets can be either discrete or continuous

- The classical example in fuzzy sets is *tall men*. The elements of the fuzzy set "tall men" are all men, but their degrees of membership depend on their height.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

Crisp and fuzzy sets of “*tall men*”



- The *x*-axis represents the **universe of discourse** – the range of all possible values applicable to a chosen variable. In our case, the variable is the man height. According to this representation, the universe of men's heights consists of all tall men.
- The *y*-axis represents the **membership value of the fuzzy set**. In our case, the fuzzy set of "*tall men*" maps height values into corresponding membership values.

A fuzzy set is a set with fuzzy boundaries.

- Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, **crisp set A of X is defined as function f_A(x) called the characteristic function of A**

$f_A(x)$: $X \rightarrow \{0, 1\}$, where $$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

This set maps universe X to a set of two elements. For any element x of universe X, characteristic function $f_A(x)$ is equal to 1 if x is an element of set A, and is equal to 0 if x is not an element of A.

- In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the *membership function* of set A

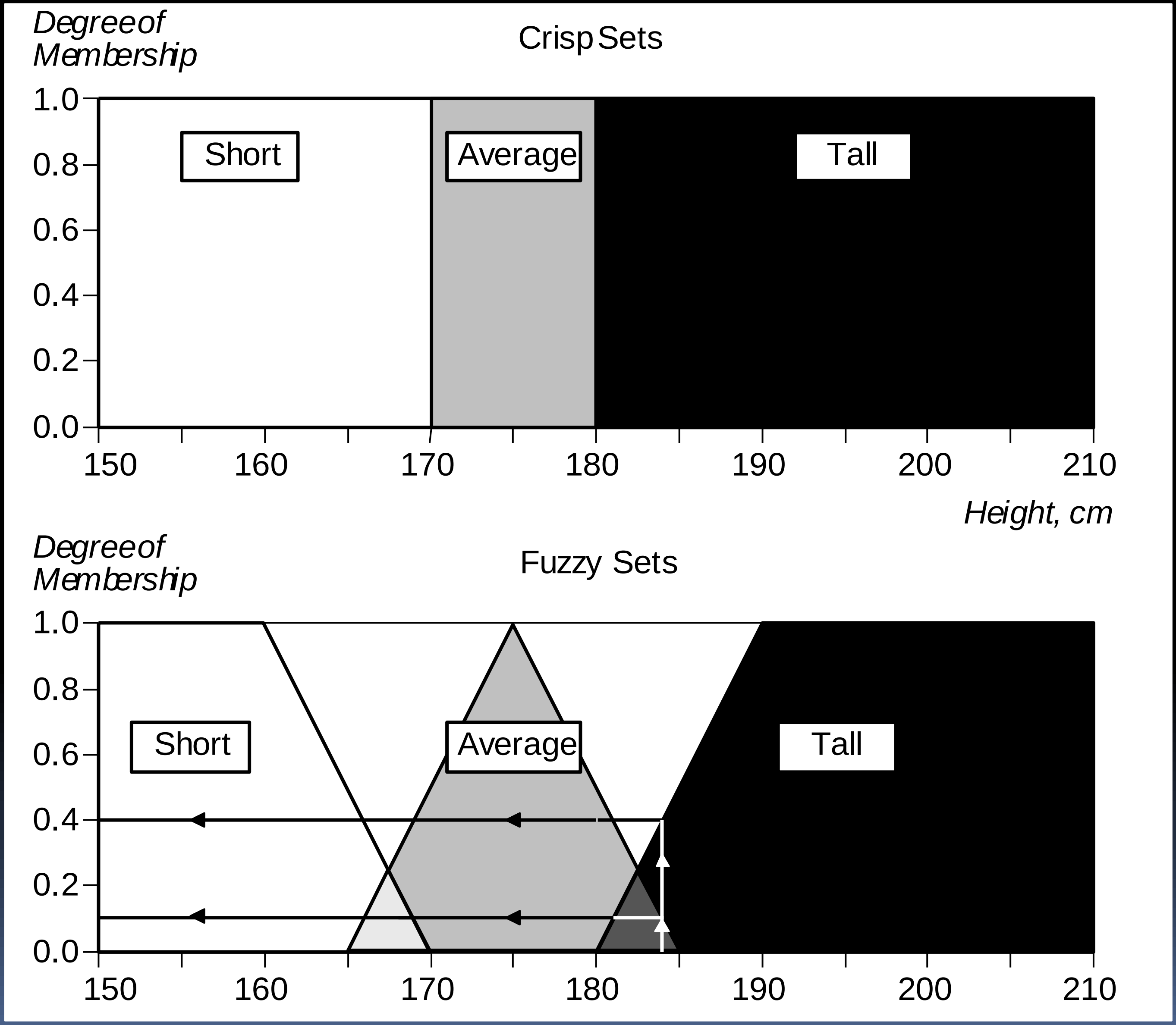
$$\mu_A(x): X \to [0, 1], \text{ where } \mu_A(x) = 1 \text{ if } x \text{ is totally in } A;$$
$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A;$$
$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A.$$

This set allows a continuum of possible choices. For any element x of universe X, membership function $\mu_A(x)$ equals the degree to which x is an element of set A. This degree, a value between 0 and 1, represents the **degree of membership**, also called **membership value**, of element x in set A.

How to represent a fuzzy set in a computer?

- First, we determine the membership functions. In our "*tall men*" example, we can obtain fuzzy sets of *tall*, *short* and *average* men.
- The universe of discourse – the men's heights – consists of three sets: *short*, *average* and *tall men*. As you will see, a man who is 184 cm tall is a member of the *average men* set with a degree of membership of 0.1, and at the same time, he is also a member of the *tall men* set with a degree of 0.4.

Crisp and fuzzy sets of short, average and tall men



- **The notation for fuzzy sets**: for the member, *x*, of a discrete set with membership *μ*, we use the notation *μ/x* . In other words, x is a member of the set to degree *μ*.
- **Discrete sets** are defined as:

$$A = \mu_1 / x_1 + \mu_2 / x_2 + \ldots\ldots + \mu_n / x_n$$

- or (in a more compact form)

$$A = \sum_{i=1 \ldots n} \mu_i / x_i$$

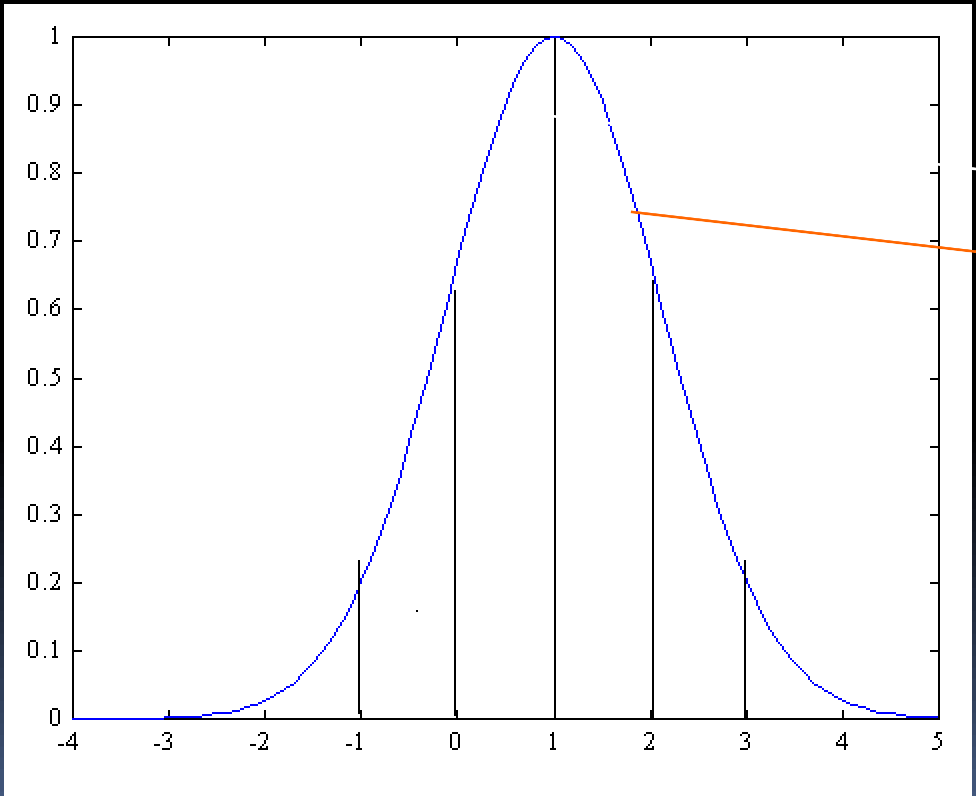
$x_1, x_2, \ldots\ldots x_n$: members of the set *A*

$\mu_1, \mu_2, \ldots\ldots \mu_n$: $x_1, x_2 \ldots\ldots x_n$'s degree of membership.

- **A continuous fuzzy set** *A can be* defined as:

$$A = \int_X \mu(x) / x$$

μ



Example:
Discrete and Continuous fuzzy sets to represent the set of numbers "close to 1"

Probability Vs Fuzzy Logic

- Probabilities on a finite universal set must add to 1 while there is no such requirement for membership grades.
- Fuzzy set theory differs from conventional set theory as it allows each element of a given set to belong to that set to some degree.
- In contrast to classical set theory each element either fully belongs to the set or is completely excluded from the set.
- In other words, classical set theory represents a special case of the more general fuzzy set theory.
- Elements in a fuzzy set X posses membership values between 0 and 1.The degree to which an element belongs to given set is called Grade of Membership.

Probability Vs Fuzzy Logic

- Represent “Helen is old” using probability theory and fuzzy set. Assume that Helen’s age is 75.

Probability approach:

- We may assign the statement “Helen is old” the truth value of 0.95. The interpretation is that there is 95% chance of Helen is old

Fuzzy approach:

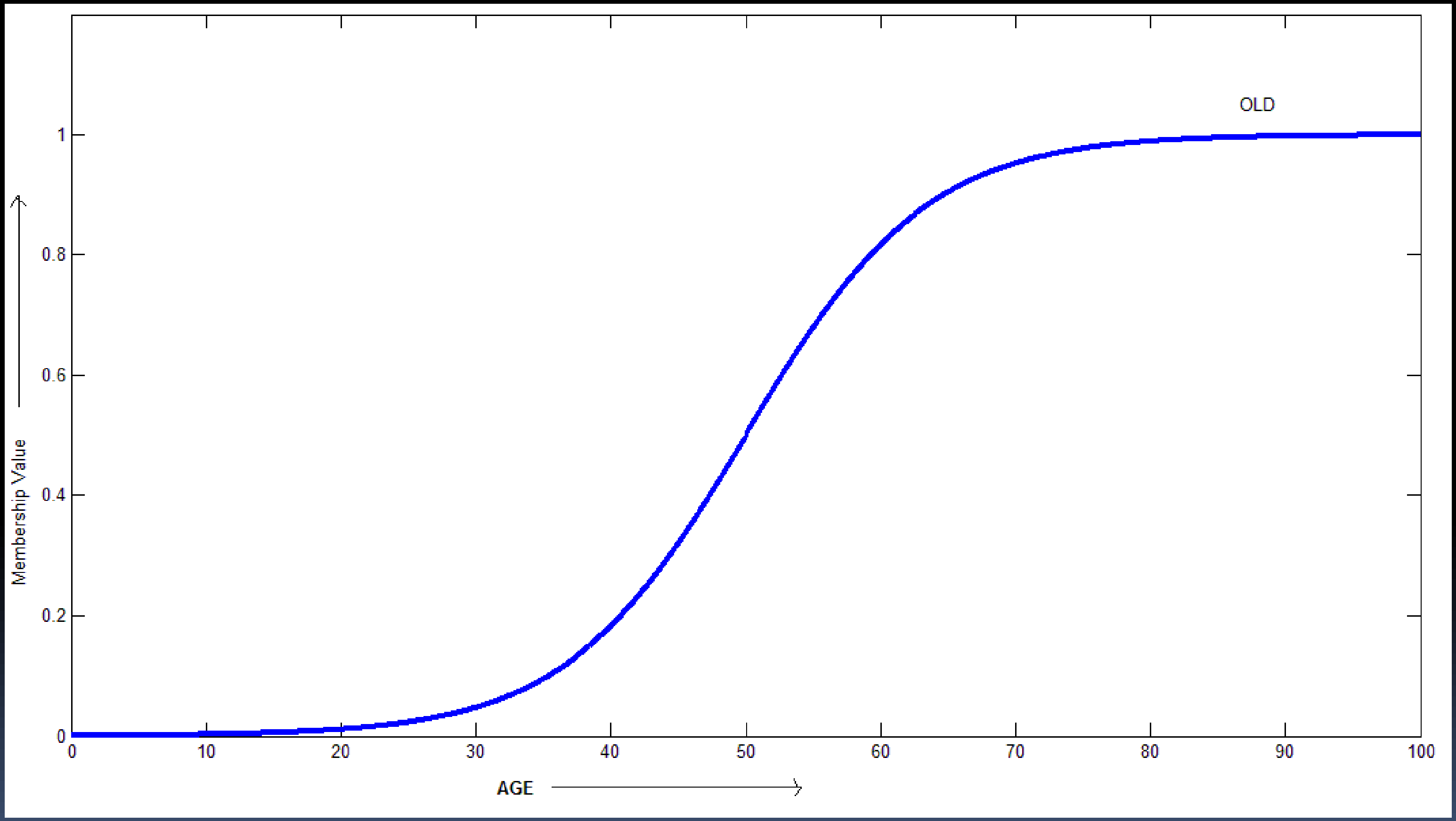
- The statement could be translated into fuzzy set terminology as follows:
- Helen is a member of the set of old people.
- It could be expressed in symbolic notation of fuzzy set as λ_{OLD}(Helen) = 0.95 i.e., Helen’s degree of membership within the set of old people = 0.95

Distinction in two views

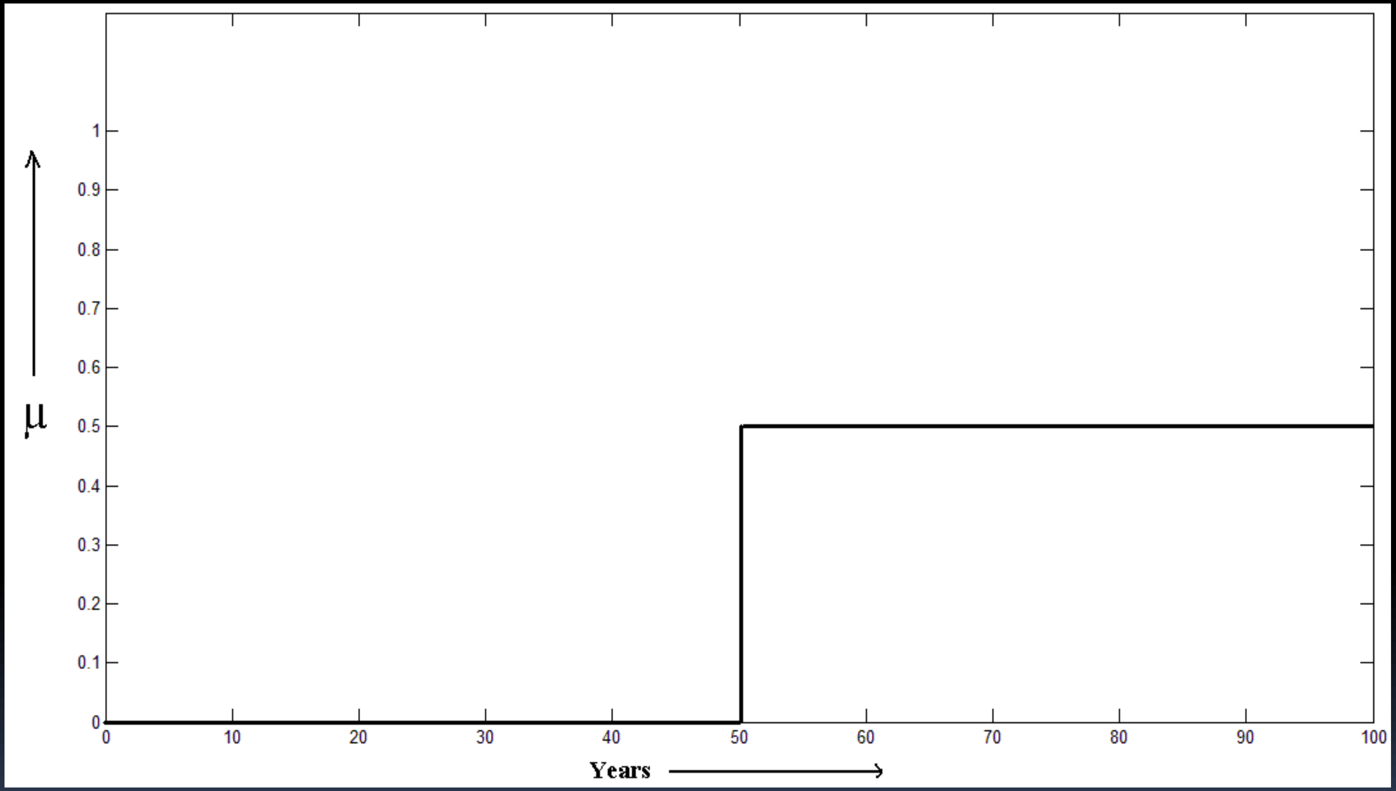
Important distinction between fuzzy systems and probability.

- Although these two statements seem similar but they actually carry different meanings.
- **First view:** There are 5% chances that Helen may not old
- **Second view**: There is no chance of Helen being young and she is more or less old.
  - Here μ_{OLD} is a membership function operation on the fuzzy set of old people (denoted OLD) which return a value between 0 and 1.

- *Membership function* μ_{OLD} *for the fuzzy set OLD is represented as*



• *Membership function for crisp (conventional) set older than 50 years is represented as:*



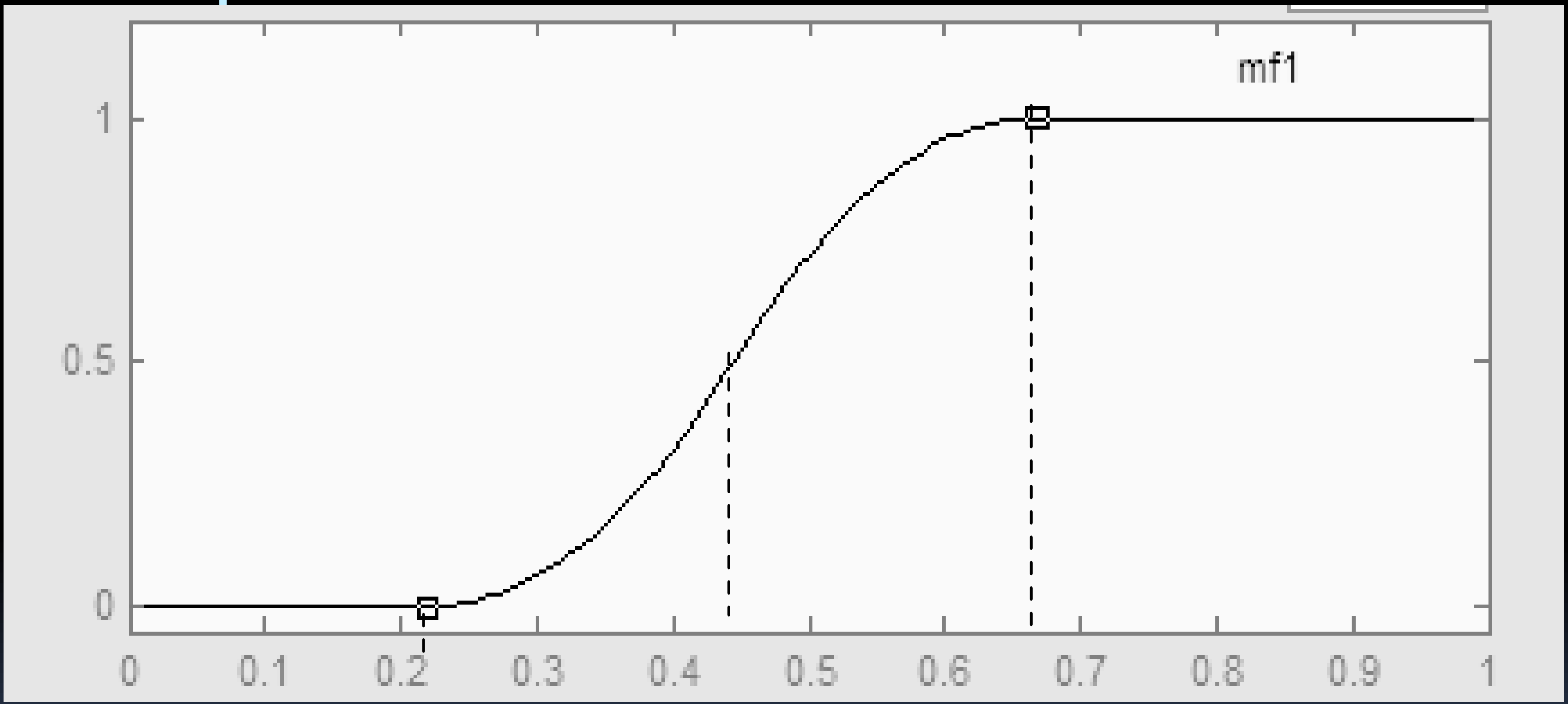
Various Types of Membership Functions

- *S-shaped function*
- *Z-shaped function*
- *Triangular Membership Function*
- *Trapezoidal Membership Function*
- *Gaussian Distribution Function*
- *Pi function*
- *Vicinity function*

S-shaped function

$$
\mu_S(x, a, b, c) = \begin{cases} 0, & \text{for } x \le a \\ 2[(x-a) / (c-a)]^2, & \text{for } a \le x \le b \\ 1- 2[(x-c) / (c-a)]^2, & \text{for } b < x \le c \\ 1, & \text{for } x \ge c \end{cases}
$$

Graphical Representation of S-Shaped Function



a b c

Figure S-shaped Membership Function

Z-Shaped Function

- It represents an asymmetrical polynomial curve open to the left.
- Z-membership function may be defined as follows:

$$\mu_Z(x, a, b, c) = \begin{cases} 1, & \text{for } x \le a \\ 1- 2[(x-a) / (c-a)]^2, & \text{for } a \le x \le b \\ 2[(x-c) / (c-a)]^2, & \text{for } b < x \le c \\ 0, & \text{for } x \ge c \end{cases}$$

Graphical Representation

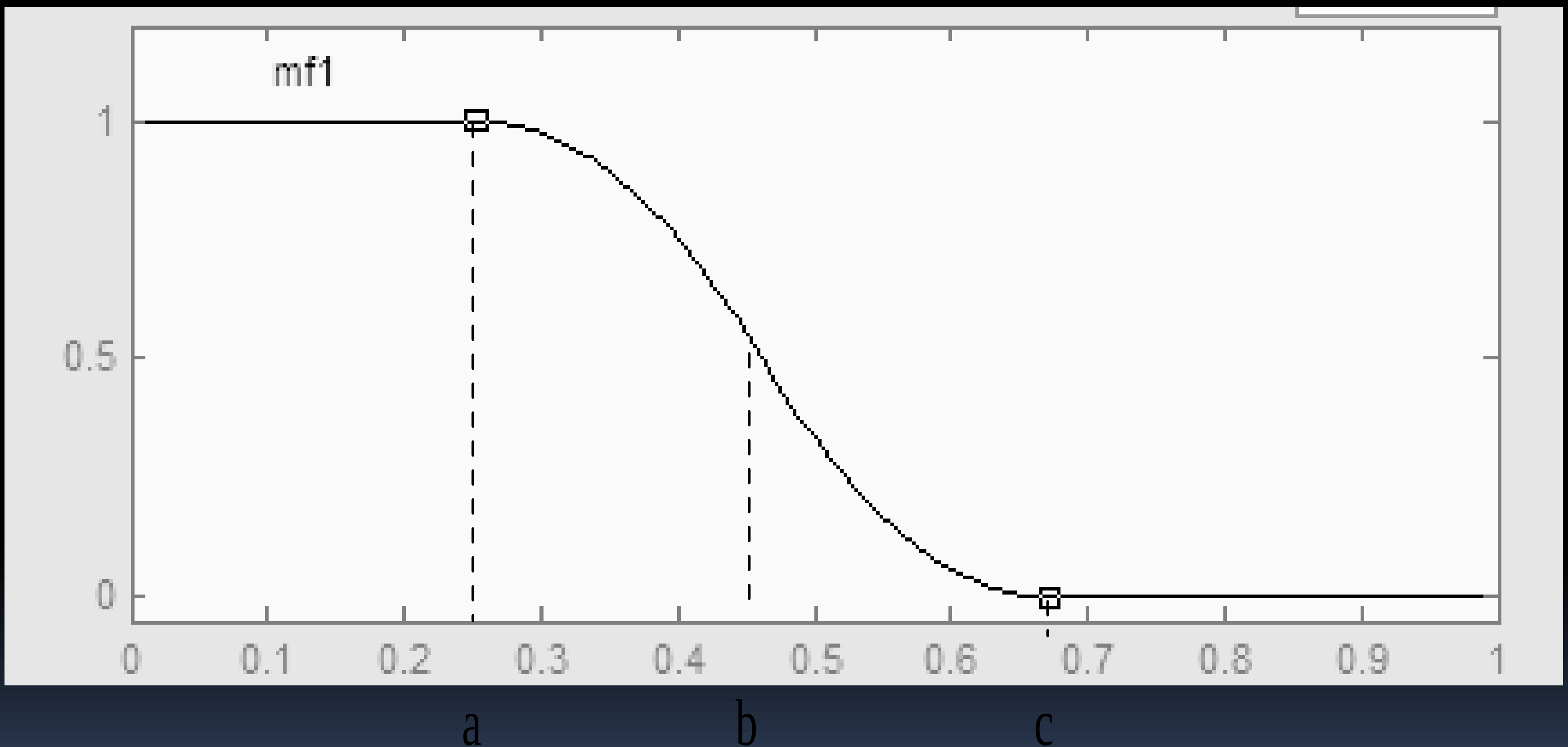


Figure Z membership function

Triangular membership functions

$$\mu_F(x, a, b, c) \quad = \begin{cases} 0, & \text{if } \; x < a \\ (x - a) / (b - a), & \text{if } a \le x \le b \\ (c - x) / (c - b), & \text{if } \; b \le x \; \le \\ 0, & \text{if } c < x \end{cases}$$

Cont…

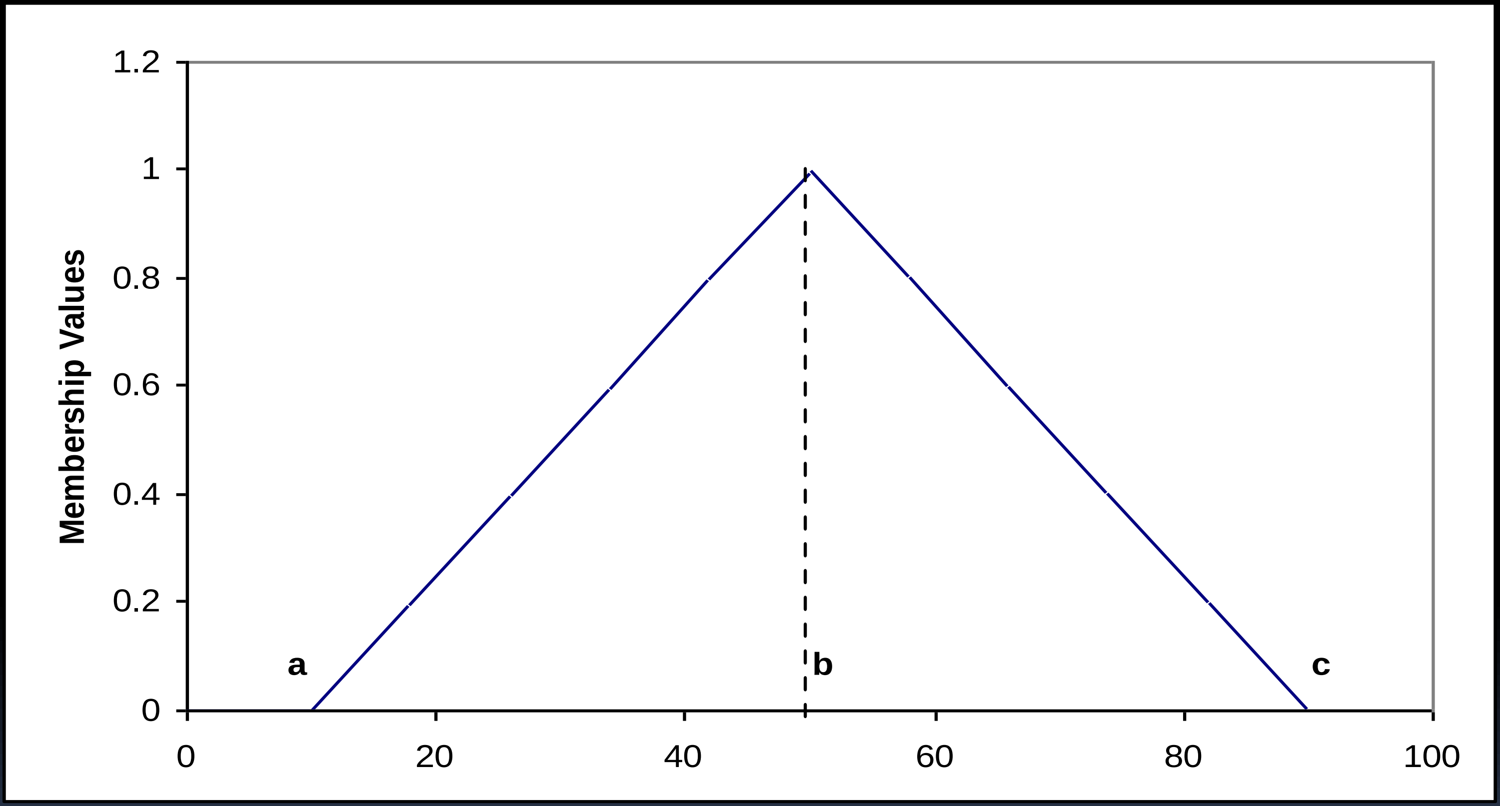


Figure Triangular Function

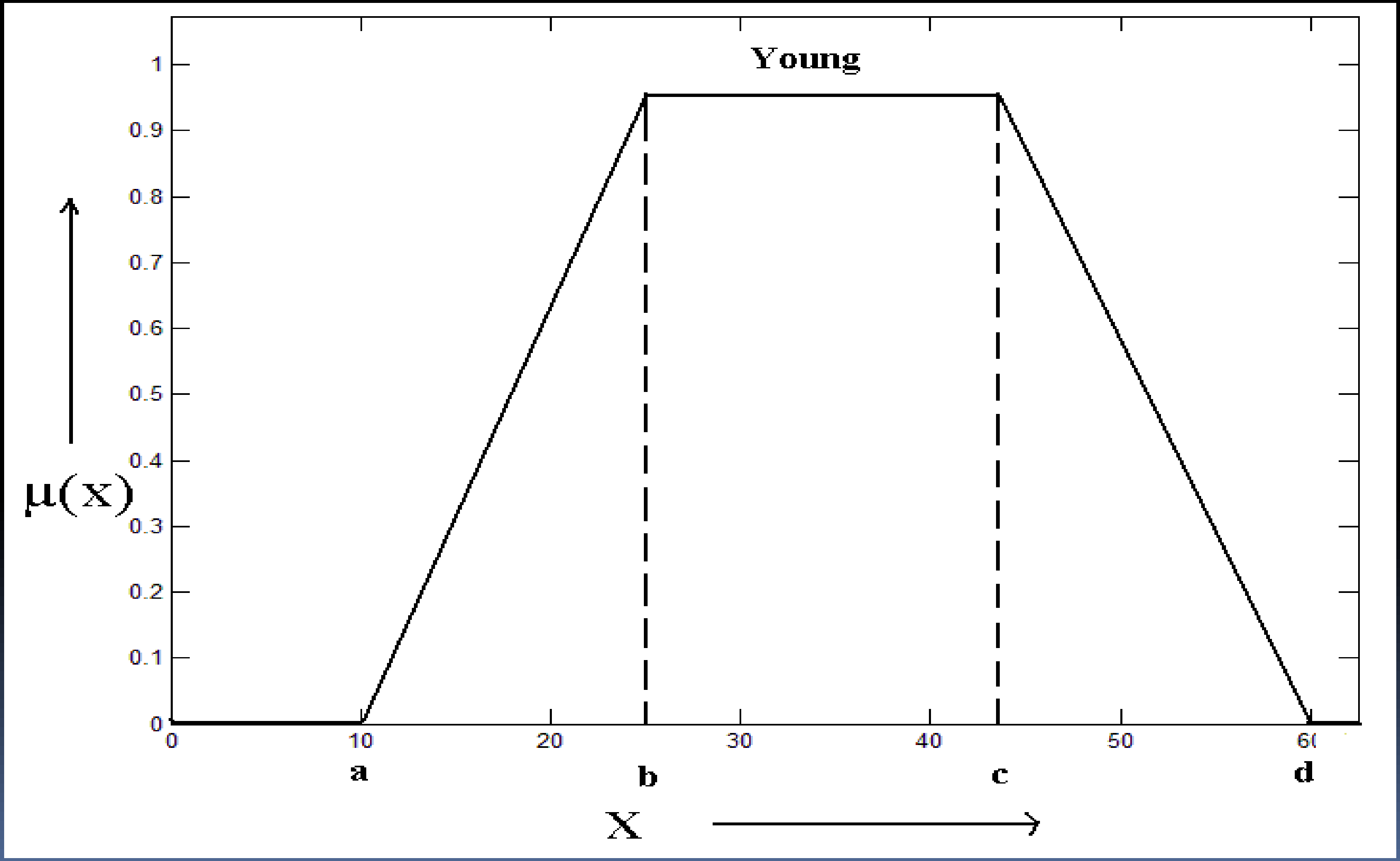
- ***Trapezoidal membership function***

$$\mu_F(x, a, b, c, d) = \begin{cases} 0, & \text{if } x < a \\ (x - a) / (b - a), & \text{if } a \le x \le b \\ 1, & \text{if } b < x < c \\ (d - x) / (d - c), & \text{if } c \le x \le d \\ 0, & \text{if } d < x \end{cases}$$

Trapezoidal membership function

$$\mu_F(x, a, b, c, d) = \begin{cases} 0, & \text{if } x < a \\ (x - a) / (b - a), & \text{if } a \le x \le b \\ 1, & \text{if } b < x < c \\ (d - x) / (d - c), & \text{if } c \le x \le d \\ 0, & \text{if } d < x \end{cases}$$

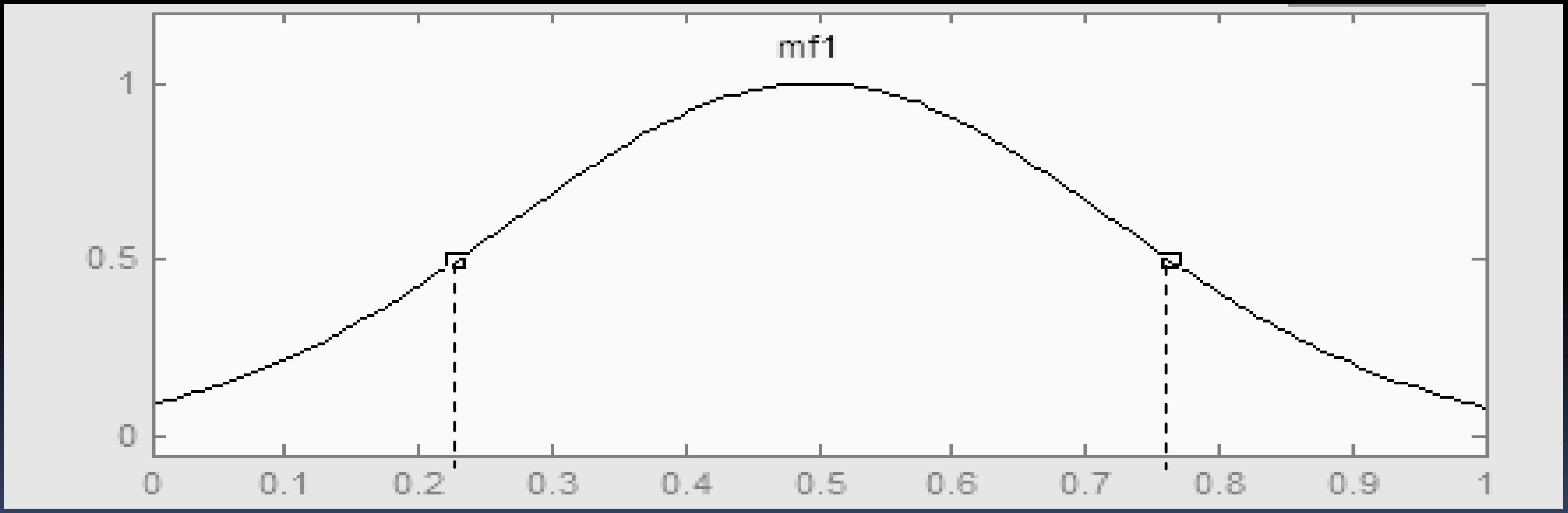
Cont…



Gaussian membership function

$$\mu(x,a,b) = e^{\frac{-(x-b)^2}{2a^2}}$$

The graph given in Fig. 10.6 is for parameters a = 0.22, b = 0.78



a b

Figure Gaussian Membership Function

Pi Function

- Pi-shaped curve is a spline-based curve which is named so because of its shape.
- This membership function is evaluated at four points namely *a*, *b*, *c*, and *d*.
- The parameters *a* and *d* locate the 'feet' of the curve, while *b* and *c* locate its 'shoulders'. In the graph given in Fig. 10.14, *a* = 2, *b* = 4, *c* = 5, and *d* = 9.

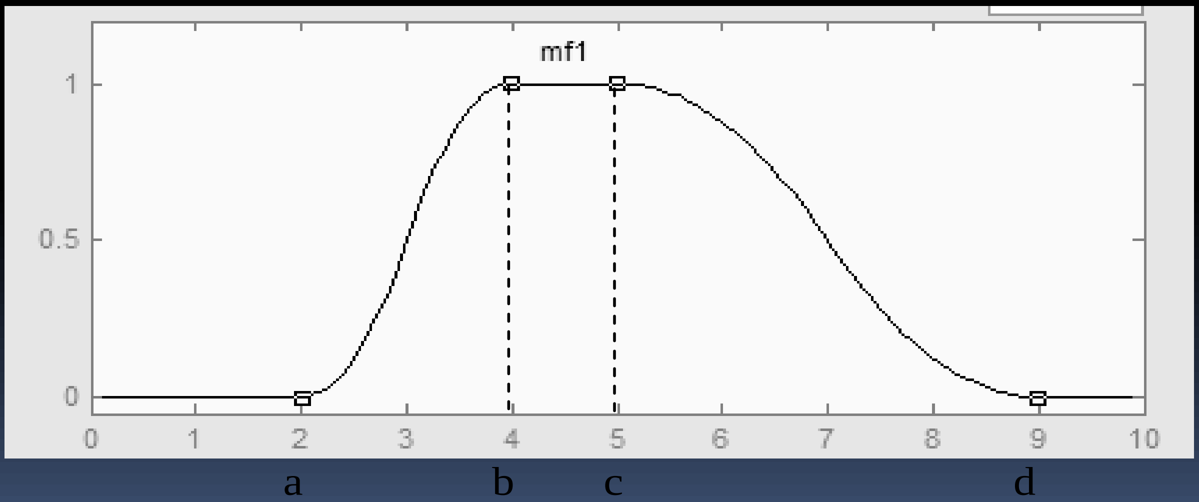


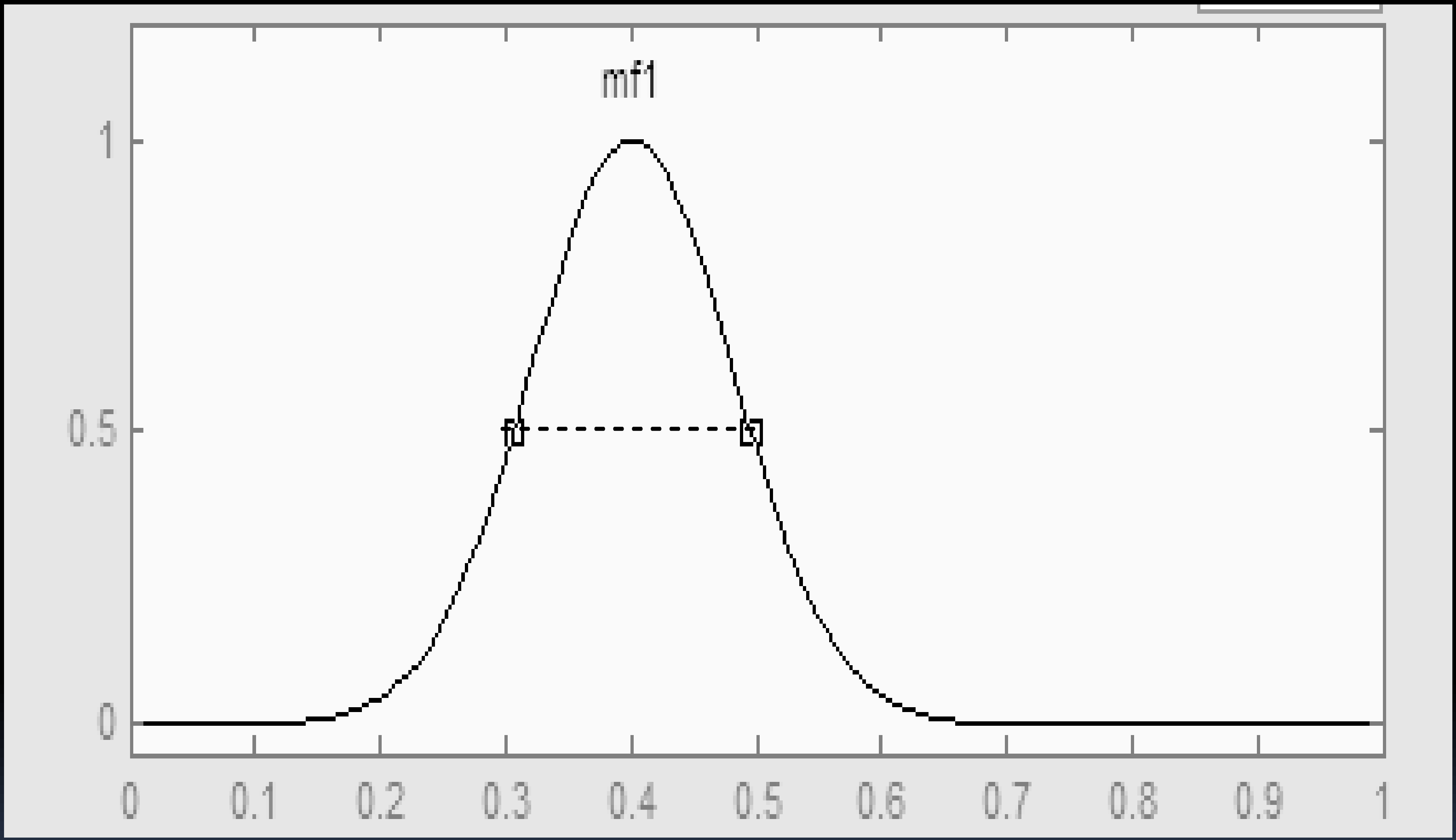
Figure Pi-shaped Membership Function

Vicinity function

- To represent the statement "x is close to xo, where xo is any fixed value of x", vicinity function using S function as follows can be used:

$$\mu_V(x, b, a) = \begin{cases} S(x, a - b, x - b/2, a), & \text{for } x \leq a \\ 1 - S(x, a, x + b/2, a + b), & \text{for } x \geq a \end{cases}$$

- This is also called π function. Here 'b' is called bandwidth.
- Total width of the function between two zero points is equal to '2b'.



a

Figure Vicinity Function

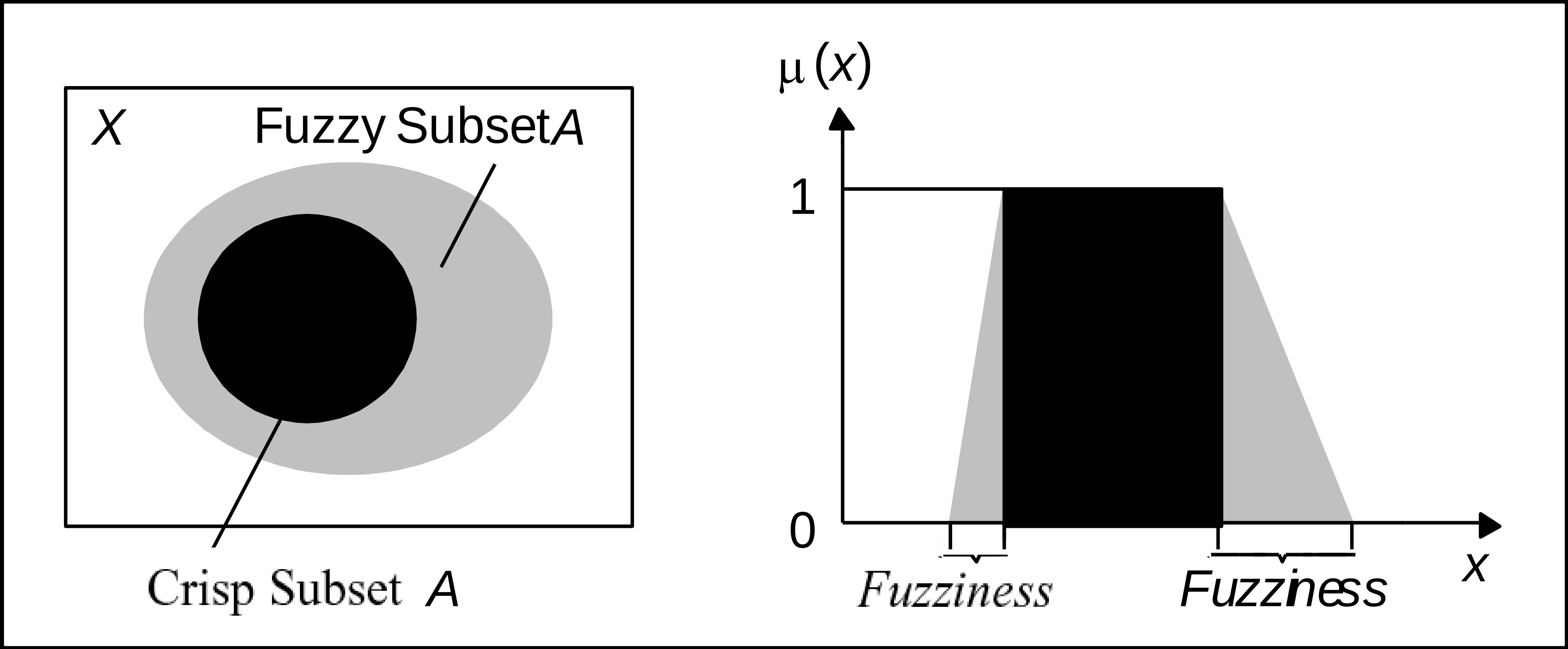
Basic Operations

- For reshaping the memebership functions, following three operations can be used.
  - **Dilation (DIL) :** It increases the degree of membership of all members by spreading out the curve. For example, DIL(OLD) = more or less OLD. Its membership function is defined as: $(\mu A\ (x))\ 0.5$
  - **Concentration (CON):** It decreases the degree of membership of all members. For example CONS(OLD) = very OLD. Its membership function is defined as: $(\mu A\ (x))\ 2$
  - **Normalization (NORM)** : It discriminates all membership degree in the same order unless maximum value of any member is 1. Its membership function is defined as: $\mu A(x) / \max (\mu A(x)), x \in X$
  - A fuzzy set is called normalized when at least one of its elements attains the maximum possible membership grade i.e., 1.

Methods for Determining Membership Functions

- Membership functions can be designed by analyzing the problem in hand.
- There are many possible forms of membership functions.
- Most of the actual fuzzy control operations are drawn from a small set of different curves.
- The methods for determining membership functions may be broadly classified into the following categories which are explained briefly as follows:
  - Trail and Error Basis
  - Subjective evaluation and elicitation
  - Converted frequencies or probabilities
  - Physical measurement
  - Learning and adaptation

Representation of crisp and fuzzy subsets



Typical functions that can be used to represent a fuzzy set are sigmoid, gaussian and pi. However, these functions increase the time of computation. Therefore, in practice, most applications use **linear fit functions**.

Linguistic variables and hedges

- At the root of fuzzy set theory lies the idea of linguistic variables.
- **A linguistic variable is a fuzzy variable.** For example, the statement "John is tall" implies that the linguistic variable *John* takes the linguistic value *tall*.

In fuzzy expert systems, linguistic variables are used in fuzzy rules. For example:

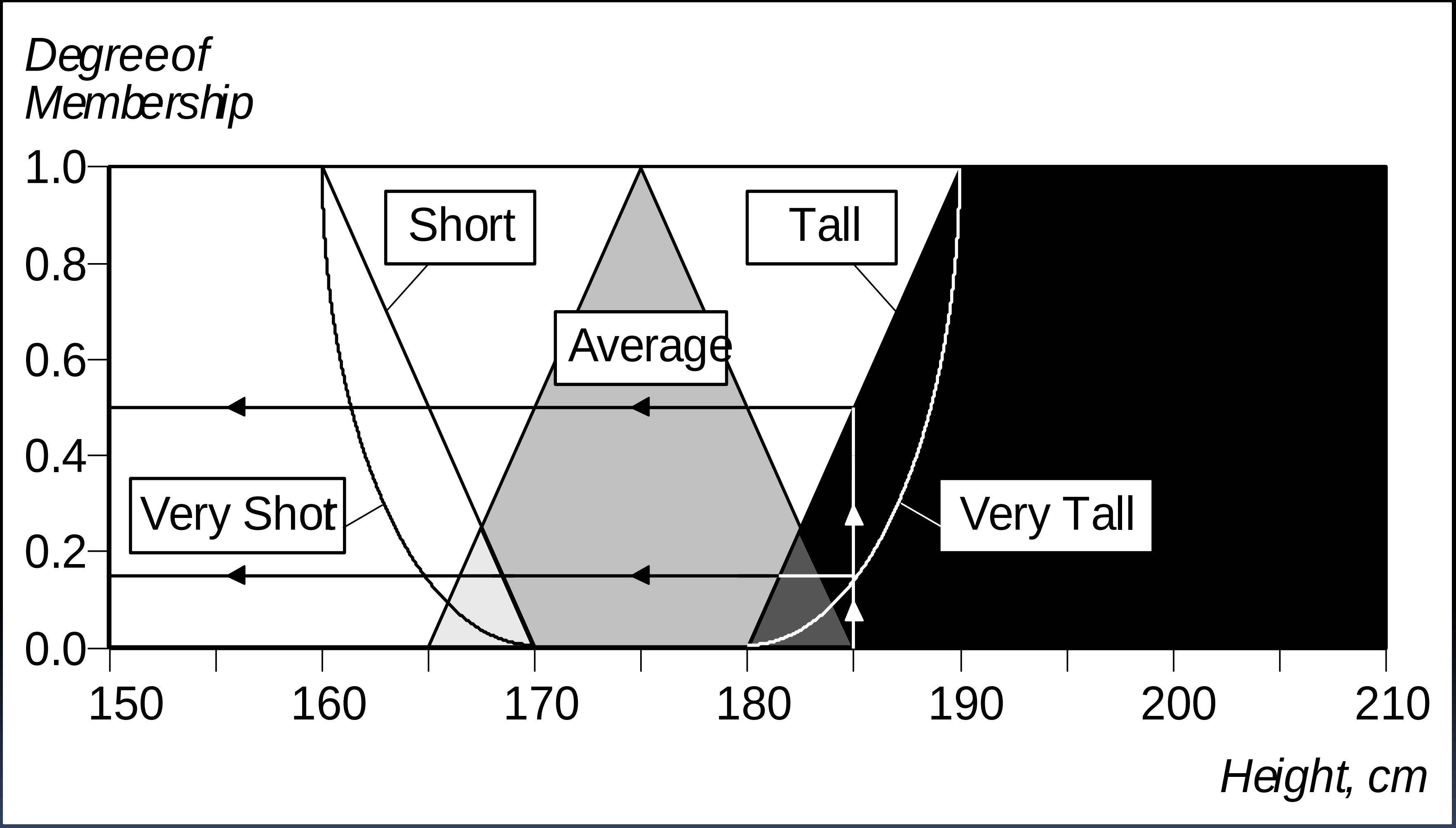
IF wind is strong
THEN sailing is good

IF project_duration is long
THEN completion_risk is high

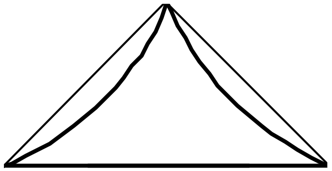
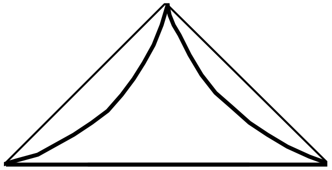
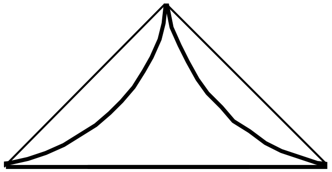
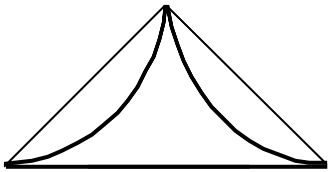
IF speed is slow
THEN stopping_distance is short

- The range of possible values of a linguistic variable represents the universe of discourse of that variable. For example, the universe of discourse of the linguistic variable *speed* might have the range between 0 and 220 km/h and may include such fuzzy subsets as *very slow*, *slow*, *medium*, *fast*, and *very fast*.
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called *hedges*.
- **Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as *very*, *somewhat*, *quite*, *more or less* and *slightly*.**

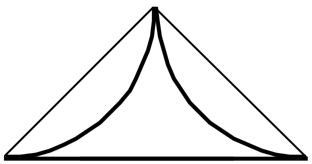
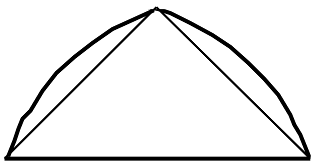
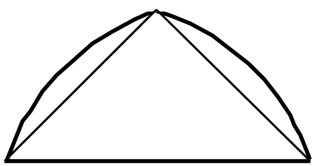
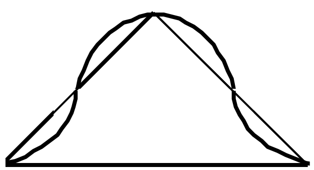
Fuzzy sets with the hedge *very*



Representation of hedges in fuzzy logic

Hedge	*Mathematical Expression*	*Graphical Representation*
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

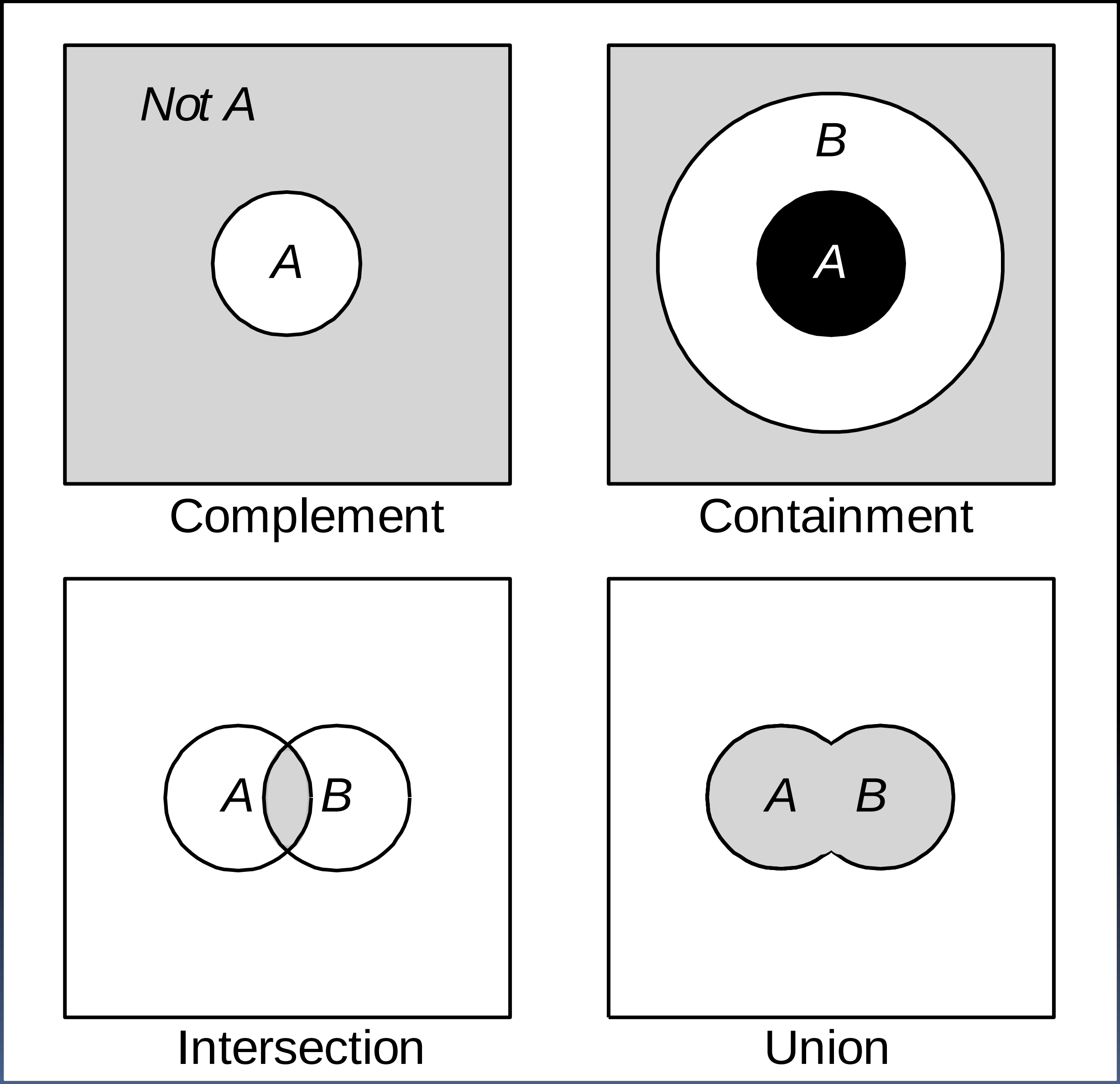
Representation of hedges in fuzzy logic (continued)

Hedge	*Mathematical Expression*	*Graphical Representation*
Very very	$[\mu_A(x)]^4$	
More or less	$\sqrt{\mu_A(x)}$	
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2\,[\mu_A(x)]^2$ if $0 \leq \mu_A \leq 0.5$ $1 - 2\,[1 - \mu_A(x)]^2$ if $0.5 < \mu_A \leq 1$	

Operations of fuzzy sets

The classical set theory developed in the late 19th century by Georg Cantor describes how crisp sets can interact. These interactions are called **operations**.

Cantor's sets



Complement

Crisp Sets: *Who does not belong to the set?*

Fuzzy Sets: *How much do elements not belong to the set?*

The complement of a set is an opposite of this set. For example, if we have the set of *tall men*, its complement is the set of *NOT tall men*. When we remove the tall men set from the universe of discourse, we obtain the complement. If A is the fuzzy set, its complement $\neg A$ can be found as follows:

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$

Containment

Crisp Sets: Which sets belong to which other sets?

Fuzzy Sets: Which sets belong to other sets?

Similar to a Chinese box, a set can contain other sets. The smaller set is called the **subset**. For example, the set of *tall men* contains all tall men; *very tall men* is a subset of *tall men*. However, the *tall men* set is just a subset of the set of *men*. In crisp sets, all elements of a subset entirely belong to a larger set. In fuzzy sets, however, each element can belong less to the subset than to the larger set. Elements of the fuzzy subset have smaller memberships in it than in the larger set.

Intersection

Crisp Sets: Which element belongs to both sets?

Fuzzy Sets: How much of the element is in both sets?

In classical set theory, an intersection between two sets contains the elements shared by these sets. For example, the intersection of the set of *tall men* and the set of *fat men* is the area where these sets overlap. In fuzzy sets, an element may partly belong to both sets with different memberships. A fuzzy intersection is the lower membership in both sets of each element. The fuzzy intersection of two fuzzy sets A and B on universe of discourse X:

$\mu_{A \cap B}(x) = \min\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x)$, where $x \in X$

Union

<u>*Crisp Sets:*</u> *Which element belongs to either set?*

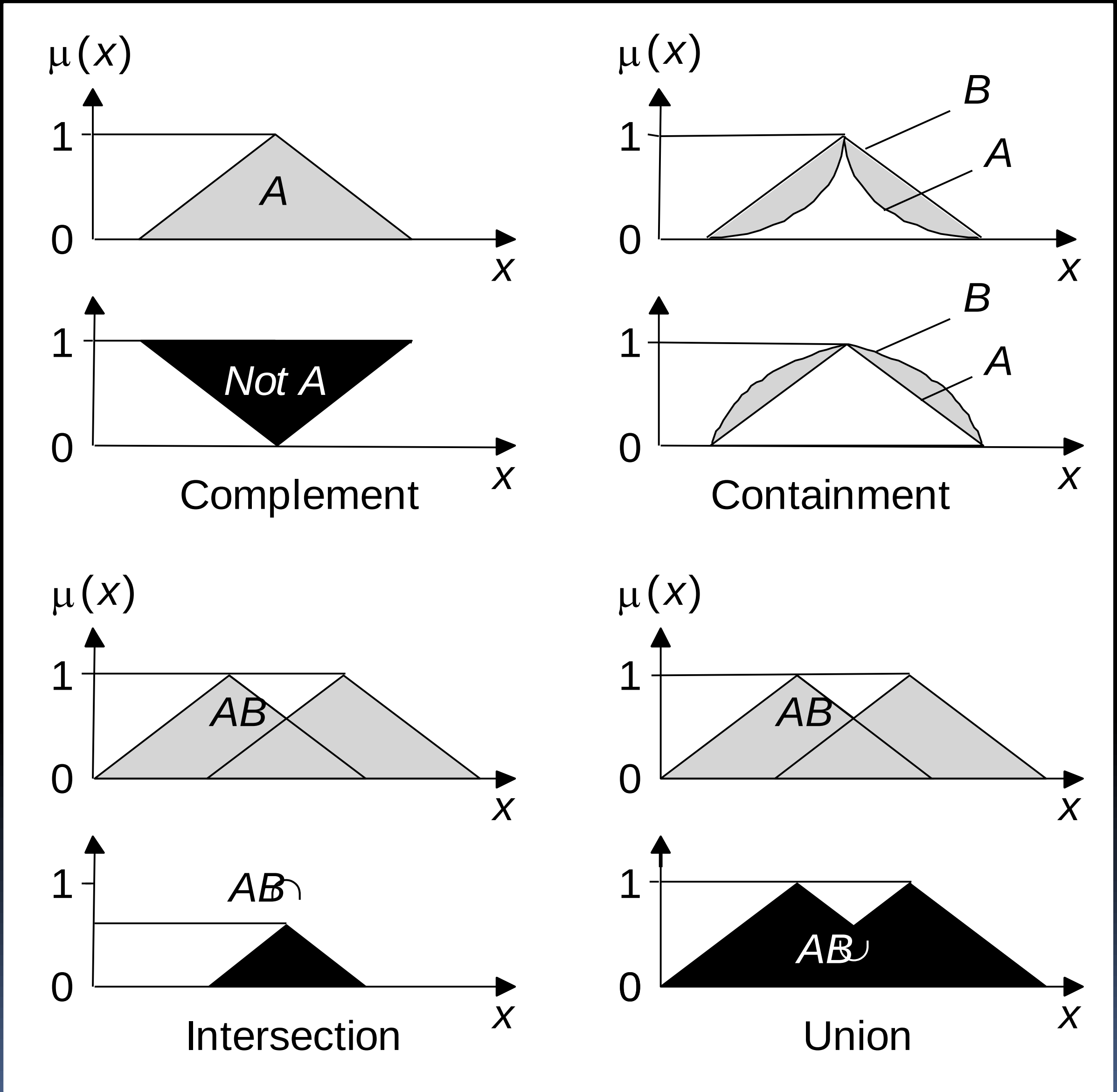
<u>*Fuzzy Sets:*</u> *How much of the element is in either set?*

The union of two crisp sets consists of every element that falls into either set. For example, the union of *tall men* and *fat men* contains all men who are tall **OR** fat. In fuzzy sets, the union is the reverse of the intersection. That is, the union is the largest membership value of the element in either set. The fuzzy operation for forming the union of two fuzzy sets *A* and *B* on universe *X* can be given as:

$$\mu_{A \cup B}(x) = \max\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x),$$

where $x \in X$

Operations of fuzzy sets



Additional operations

1. Equality: $A = B$, if $\mu_{A(x)} = \mu_{B(x)}, \quad \forall x \in X$
2. Not equal: $A \neq B$, if $\mu_{A(x)} \neq \mu_{B(x)}$ for at least one $x \in X$
3. Containment: $A \subseteq B$ if and only if $\mu_{A(x)} \leq \mu_{B(x)}, \quad \forall x \in X$
4. Proper subset: If $A \subseteq B$ and $A \neq B$
5. Product: A.B is defined as $\mu_{A.B(x)} = \mu_{A(x)} \cdot \mu_{B(x)}$
6. Power : A^N is defined as: $\mu_A{}^{N(x)} = (\mu_{A(x)})^N$
7. Bold union : $A \oplus B$ is defined as:

   $$\mu_{A \oplus B(x)} = \text{Min}\,[1, \mu_{A(X)} + \mu_{B(x)}]$$

8. Bold intersection: $A \odot B$ is defined as:

   $$\mu_{A \odot B(x)} = \text{Max}\,[0, \mu_{A(x)} + \mu_{B(x)} - 1]$$

Fuzzy rules

In 1973, **Lotfi Zadeh** published his second most influential paper. This paper outlined a new approach to analysis of complex systems, in which Zadeh suggested capturing human knowledge in fuzzy rules.

What is a fuzzy rule?

A fuzzy rule can be defined as a conditional statement in the form:

IF x **is** A
THEN y **is** B

where x and y are linguistic variables; and A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

What is the difference between classical and fuzzy rules?

A classical IF-THEN rule uses binary logic, for example,

Rule: 1
IF speed is > 100
THEN stopping_distance is long

Rule: 2
IF speed is < 40
THEN stopping_distance is short

The variable *speed* can have any numerical value between 0 and 220 km/h, but the linguistic variable *stopping_distance* can take either value *long* or *short*. In other words, classical rules are expressed in the black-and-white language of Boolean logic.

We can also represent the stopping distance rules in a fuzzy form:

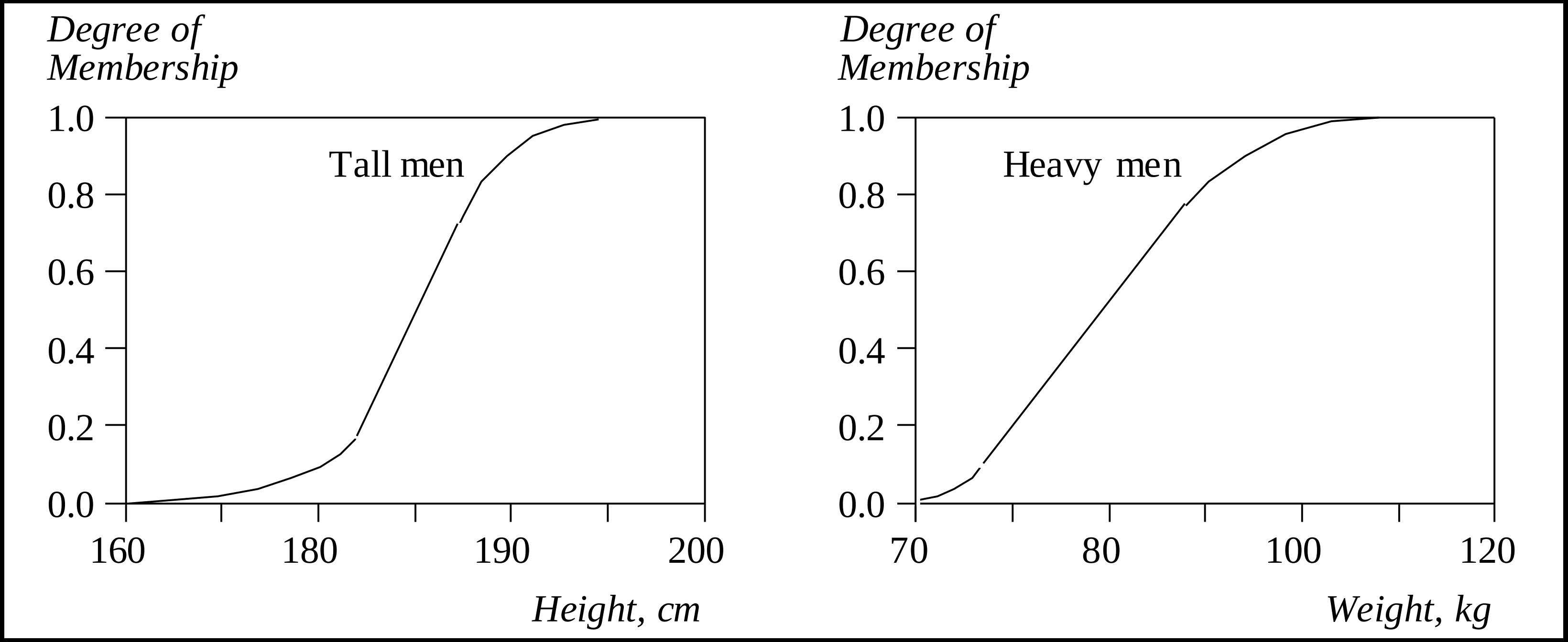
Rule: 1
IF speed is fast
THEN stopping_distance is long

Rule: 2
IF speed is slow
THEN stopping_distance is short

In fuzzy rules, the linguistic variable *speed* also has the range (the universe of discourse) between 0 and 220 km/h, but this range includes fuzzy sets, such as *slow*, *medium* and *fast*. The universe of discourse of the linguistic variable *stopping_distance* can be between 0 and 300 m and may include such fuzzy sets as *short*, *medium* and *long*.

- Fuzzy rules relate fuzzy sets.
- In a fuzzy system, all rules fire to some extent, or in other words they fire partially. If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

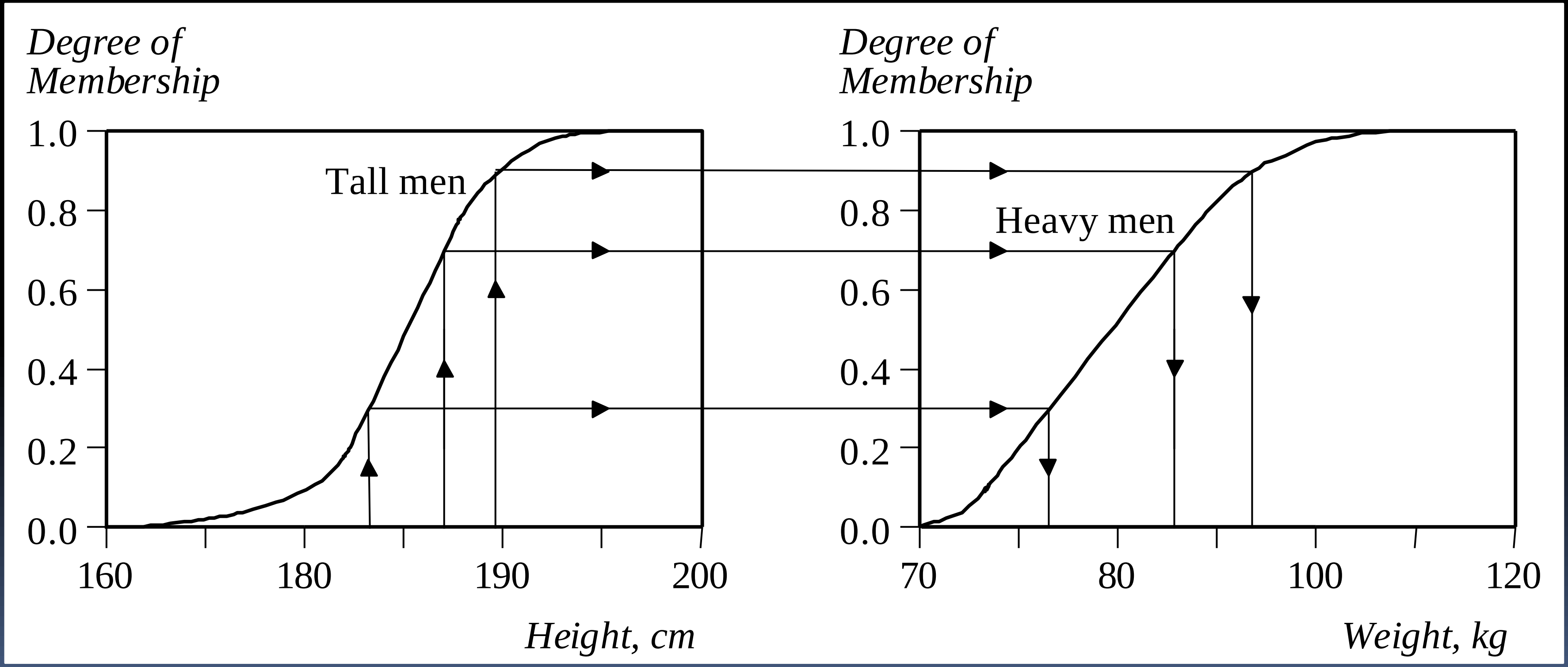
Fuzzy sets of *tall* and *heavy* men



These fuzzy sets provide the basis for a weight estimation model. The model is based on a relationship between a man's height and his weight:

IF height is *tall*
THEN weight is *heavy*

The value of the output or a truth membership grade of the rule consequent can be estimated directly from a corresponding truth membership grade in the antecedent. This form of fuzzy inference uses a method called **monotonic selection**.



A fuzzy rule can have multiple antecedents, for example:

IF	project_duration is long
AND	project_staffing is large
AND	project_funding is inadequate
THEN	risk is high

IF	service is excellent
OR	food is delicious
THEN	tip is generous

The consequent of a fuzzy rule can also include multiple parts, for instance:

IF temperature is hot
THEN hot_water is reduced;
cold_water is increased