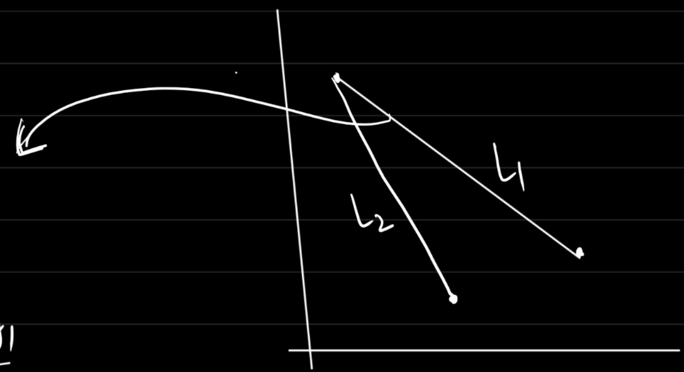
Maths

eq^n of a line

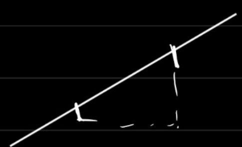


$\Rightarrow y = mx + c$

$\downarrow$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

→ The major diff b/w L_1 & L_2 is Slope



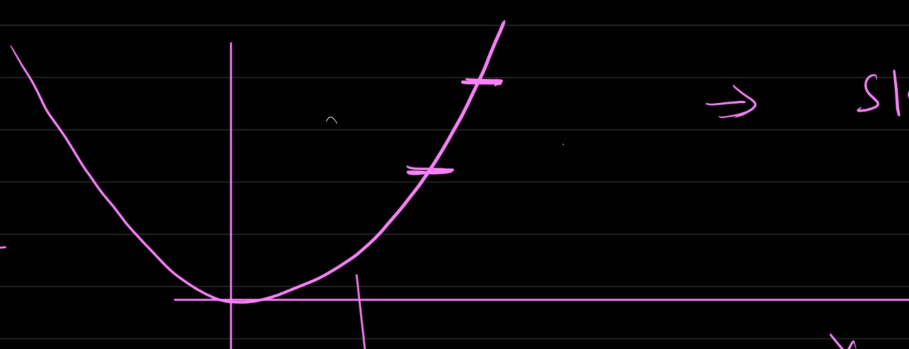
$$\text{Slope} = \frac{\text{Change is } y}{\text{Change in } x}$$

Gradient → how steep the line is

$y = \underline{\underline{mx + c}}$

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

* $y = x^2$



$\Rightarrow$ Slope = ??

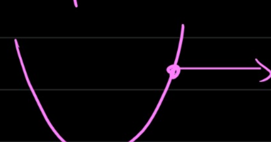
→ in case of line, the change had constant rate, that's $\frac{y_2 - y_1}{x_2 - x_1}$

In this case slope is varying.

$\Downarrow$

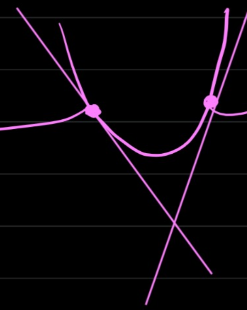
Steepness varies.

X	X^2
1	1
2	4
3	9
4	16



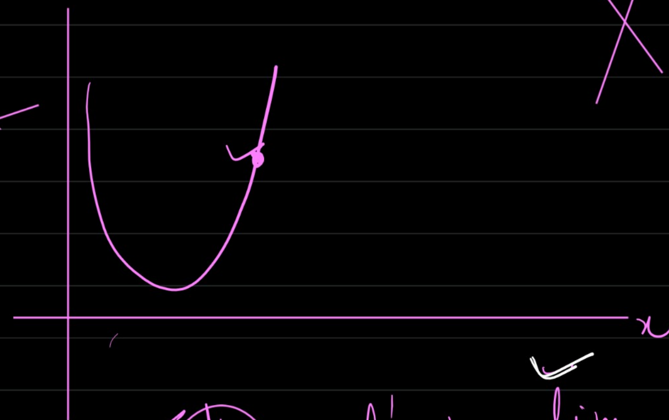
→ instantanoous rate of change wrt y.

tangent / slope



tangent is a line that touches a curve at one point.

$y = \cancel{\dfrac{y_2 - y_1}{x_2 - x_1}}$



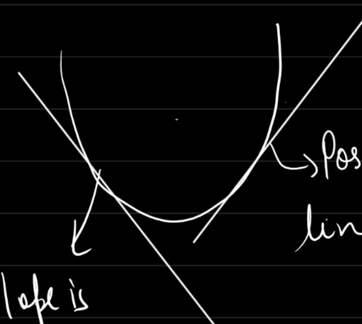
first principle

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$\dfrac{dy}{dx}$ (wrt to x)

(instantaneous rate of change in y wrt)

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$



Slope is neg. (line is down ward)

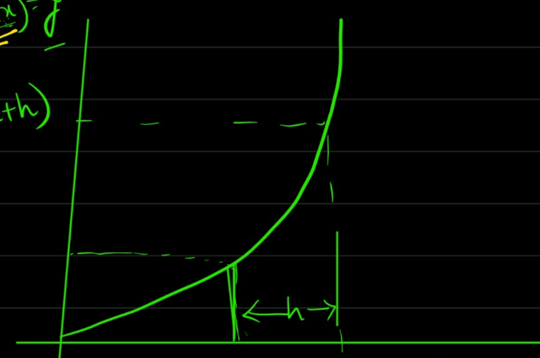
Positive line is upward

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(x) = y$

f(a+h)

f(a)



$x = a$ $\quad$ $x = a+h$ $\quad$ x

$f(x) = x^2$ $\qquad$ $x^n = n x^{n-1}$

$$\frac{d f(x)}{dx} = \frac{d}{dx} x^2 = 2x^{2-1} = 2x$$

with first principle $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

Since x^2:

$x+h$

$(x+h)^2$

$(a+b)^2 = a^2 + b^2 + 2ab$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h = \underline{2x}$$

Proved

* $\frac{d}{dx} x^n = n x^{n-1}$

$$\frac{d}{dx} x^3 = 3x^2$$

* $\frac{d}{dx}(1) = 0$

* $\frac{d}{dx}(x) = 1 x^{1-1} = 1 x^0 = 1$

* $\frac{d}{dx} \ln(x) = \frac{1}{x}$, $\frac{d}{dx} \log_a x = \frac{1}{x \ln(a)}$

* $\frac{d}{dx} e^x = e^x$

$e = 2.37$

$$\frac{d}{dx} a^x = a^x \ln(a)$$

* $\frac{d}{dx}(\sin x) = \cos(x)$

$\frac{d}{dx}(\cos x) = -\sin(x)$

$\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

* important derivative rule

① Constant rule , $\frac{d}{dx}(c) = 0$

② Constant multiple rule : $\frac{d}{dx}(c\, f(x)) = c \cdot f'(x)$

③ Power rule – $\frac{d}{dx} x^n = n x^{n-1}$

④ Sum rule

$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$

⑤ difference rule

$$\frac{d}{dx}\left(f(x) - g(x)\right) = f'(x) - g'(x)$$

⑥ Product rule

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f(x) g'(x) + g(x) \cdot f'(x)$$

⑦ Quotient rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot f'(x) - f(x)\, g'(x)}{\left(g(x)\right)^2}$$

⑧ Chain rule → $\frac{d}{dx} f(g(x)) = f' g(x) \cdot g'(x)$

* $x^2 = 2x$

* $\frac{d}{dy} y^5 = 5y^4$

* $\frac{d}{d}(x) = 1$

* $\frac{d}{dx} 2x^3 = \underline{6x^2}$

* $\frac{d}{dx}(4x) = 4$

* $\frac{d}{dx}(4+x) = \frac{d}{d} 4 + \frac{d}{dx} x$

$0 + 1 = 1$

* $x^3 + x^2 = 3x^2 + 2x.$

* $x^3 - x^2 = 3x^2 - 2x.$

* $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$

$(uv)' = uv' + vu'$

* $\frac{d}{dx}(\underset{f(x)}{x}\, \underset{g(x)}{\sin x}) = x \cdot \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx} x$

$= x \cdot \cos x + \sin x \cdot 1$

$= \sin x + x \cos x$ Ans

* $\frac{d}{dx} x^2 \ln(x) = x^2 \frac{d}{dx}(\ln(x)) + \ln x \frac{d}{dx} x^2$

$= \cancel{x^2} \cdot \frac{1}{\cancel{x}} + \ln x \; 2x$

$= x + 2x \ln x.$

* Quotient rule $\left(\frac{u}{v}\right)' = \frac{v u' - u v'}{v^2}$

$\frac{d}{dx} \frac{x}{\sin x} = \frac{\sin x \cdot 1 - x \cdot \cos x}{\sin^2 x} = \frac{\sin x - x \cos x}{\sin^2 x}$

$\frac{d}{dx}\left(\frac{x}{\ln(x)}\right) = \frac{\ln(x) - 1}{(\ln(x))^2}$

$x^n = n x^{n-1}$

* Chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$\sin(x) = \cos x$

$\sin x^2 = \cos x^2$

$$\frac{d}{dx}(\sin(x^2)) = \cos x^2 \cdot 2x$$

$$= 2x \cos x^2$$

$f + g(x)$

$f(g(x))$

$$\frac{d}{dx} \tan(\ln(x)) = \sec^2(\ln(x)) \cdot \frac{1}{x}$$

* $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$y = 8(6xe^x)^{-4}$$

Say $u = 6xe^x$

$u' = 6e^x + 6xe^x$

$\frac{d x^2}{dx} = 2x$

$\frac{d \cdot u^2}{dy}$ $2u \cdot u'$

$$y = 8(u)^{-4}$$

$$y' = 8 \times -4\, u^{-4-1} \cdot u' = -32\, u^{-5} \cdot u'$$

$$= -32 \cdot (6xe^x)^{-5} \cdot 6e^x + 6xe^x$$

HW $y = (x^2+1)^7\, 9x^4$

Ans $18x^3(x^2+1)^6(9x^2+2)$

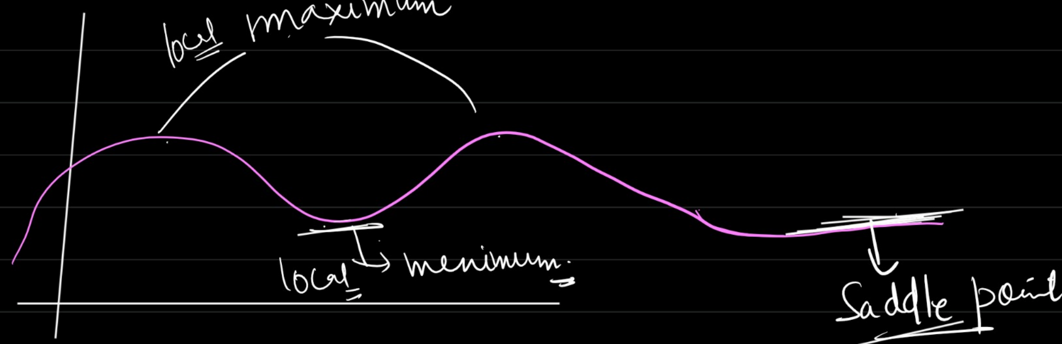
$$y' = \frac{-32\,(6e^x + 6xe^x)}{(6xe^x)^5}$$

$$y' = \frac{-192\, e^x\,(x+1)}{7776\, x^5 e^{5x}}$$

Use

↳ gradient descent

*



* maximum | minimum in a fn.
* slope = 0 (flatten → In case of maxima | minima)

$\frac{dy}{dx} = 0$

example → A trajectory of a ball is followed by $y = 4 + 10t - 6t^2$. What is max ht that ball can reach?

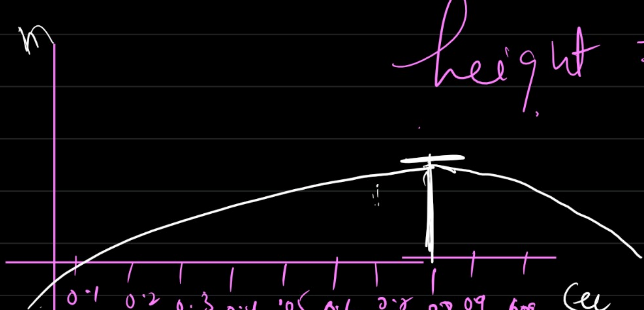
→ $\frac{dy}{dt} = \frac{d}{dt}(4 + 10t - 6t^2) = 0 + 10 - 12t$
$= 10 - 12t$

To get

$10 - 12t = 0$

$10 = 12t \Rightarrow t = \frac{10}{12} = 0.83$

at 0.83 sec, it will reach max ht.

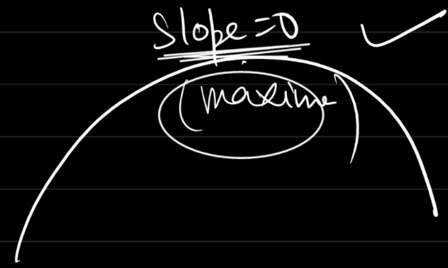


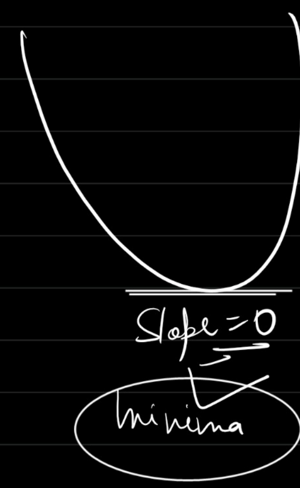
height $= 4 + 10t - 6t^2$
$= 4 + 10 \times 0.83 - 6 \times (0.83)^2$
$= 8.166$

So max ht 8.166 m at t = 0.83

$\frac{dy}{dt} = 0$

$t = 0.83$





Second order derivative test

$10 - 12t$

When a fn slope is 0 at x and the second order derivative at x is :-

→ less than 0, it is local maximum

→ greater than 0, it local minimum

→ equal to 0, you can not say anything.

$\frac{d}{dt}(10-12t) = -12 \longrightarrow$ negative $< 0 \longrightarrow$ the $\frac{dy}{dx} = 0$ is of local maximum