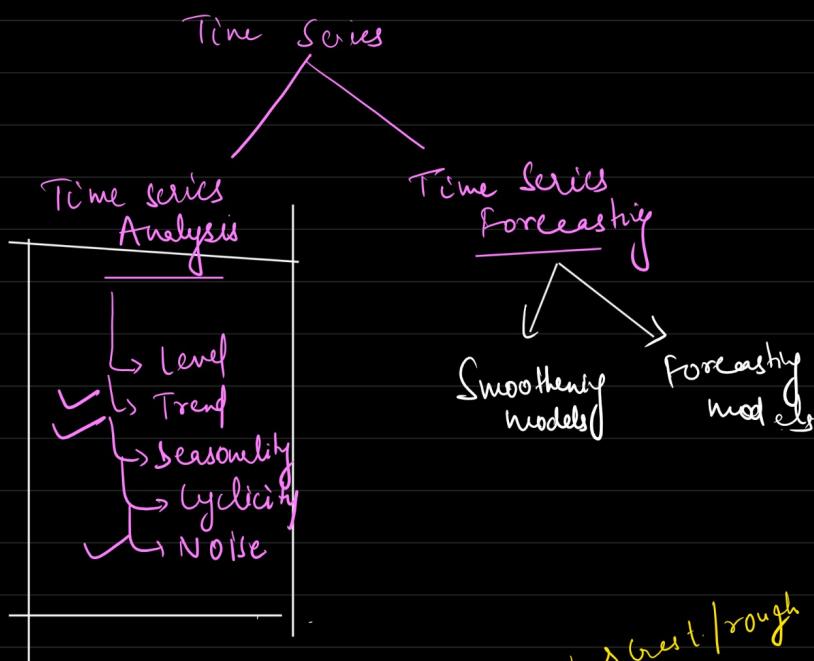


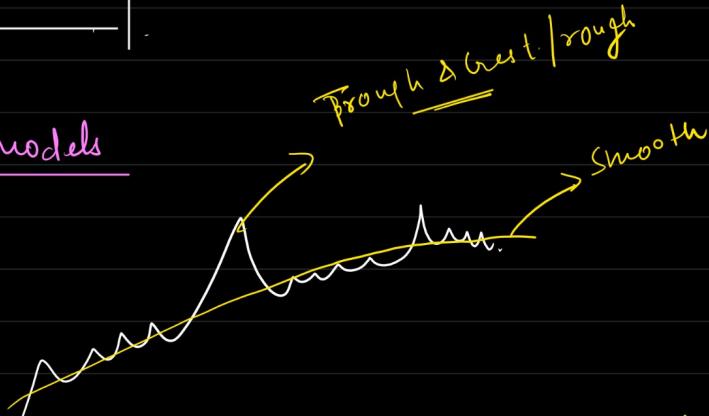
\* Time series  $\rightarrow$  Time component is present in the data.

\*



\* Smoothening models

Smoothing



Smoothing of time series  $\rightarrow$  removing fluctuations.

I'  $\rightarrow$  Naive Model  $\rightarrow$  last observed value is the forecast  
 I''  $\rightarrow$  Average Model  $\rightarrow$  Average value is the prediction.

- ① Simple moving Avg (SMA)
- ② Cumulative moving Avg (CMA)
- ③ Exponential weighted Moving Average (EWMA)

① SMA.

$$\text{Avg} = \frac{\text{Sum of all nos}}{\text{no of values}}$$

$$2, 3, 4, 5 \Rightarrow \frac{2+3+4+5}{4} = \frac{14}{4} = 3.5$$

\* MA  $\Rightarrow$  moved over the time axis in a specific window. (Window size, Avg value)

Window 3

Avg.

Jan	50
Feb	65
Mar	70
April	85
May	90
June	100
July	110

SMA = 3

Na

Na.

62

$$1^{\text{st}} \text{avg} = \frac{50 + 65 + 70}{3} = 62$$

$$2^{\text{nd}} \text{avg} = \frac{65 + 70 + 85}{3} = 74$$

$$3^{\text{rd}} \text{avg} = \frac{70 + 85 + 90}{3} = 82$$

W-S = 2

1	Na
2	$\frac{1+2}{2}$
3	$\frac{2+3}{2}$
4	$\frac{3+4}{2}$
5	$\frac{5+4}{2}$

80

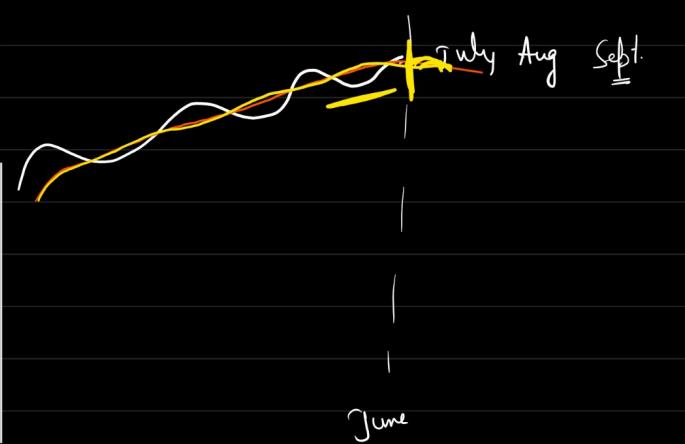
90

95

Aug

Sep

Oct



\* Why Smoothing

- To check trend of date
- To remove all the effect from date
- reduce the effect of outlier
- visualisation

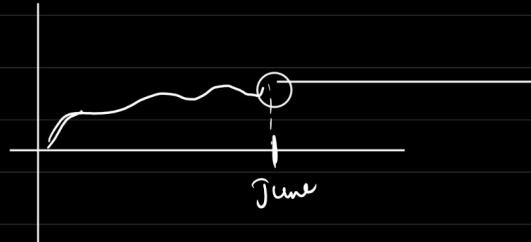


## ② Cumulative Moving Average

→ find all the avg of all the dp's upto given timestamp.

		CMA
Jan	10	10
Feb	12	$\frac{10+12}{2}$
Mar	15	$\frac{10+12+15}{3}$
Apr	14	$\frac{10+12+15+14}{4}$

→ for long period of time.  
→ Exponential trend



## ③ EMA or EW MA

$$\underline{\underline{V_t}} = \underline{\underline{0 \ 0 \ 1 \ 0 \ 1}} \rightarrow \underline{\underline{7.6}}$$

→ We give more weightage / importance / priority to the recent datapoint / time stamp

SMA, CMA, Naive, Avg

→ It gives equal priority to all the values.

→ In time series current observation is highly influenced by last few observations.  
∴ You need to give priority to recent observation

$$V_t = \beta V_{t-1} + (1-\beta) \theta_t$$

$V_t = \text{EMA at time } t$

$\beta = 0 < \beta < 1 \rightarrow$  generally its 0.9 (weight)

$V_{t-1} = \text{EMA at previous ts.}$

$\theta_t = \text{Date at current stamp}$

$V_{t-1}$	Month	Value	EMA.
$V_0$	$D_1$	25	25
$V_1$	$D_2$	13	13
$V_2$	$D_3$	17	17
$V_3$	$D_4$	31	2.87
$V_4$	$D_5$	43	-

$\beta = 0.9$  To reduce the prev time stamp effect  
 $\frac{1}{1-\beta}$

$$V_1 = \frac{\beta \cdot V_0 + (1-\beta) \theta_1}{\frac{1}{1-\beta}} = \frac{0.9 \cdot 0 + (1-0.9) \times 13}{\frac{1}{1-0.9}} = \frac{0.1 \times 13}{1} = 1.3$$

$$V_2 = \beta \cdot V_1 + (1-\beta) \theta_2 = 0.9 \times 1.3 + 0.1 \times 17 = 2.87$$

## Time Series

Additive

multiplicative

$$y_t = \text{Trend} + \text{Season} + \text{Noise}$$

→ Linear over time

→ Constant variance

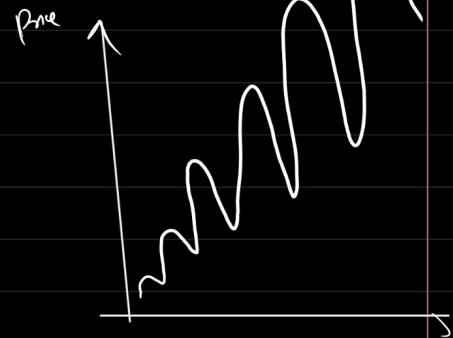
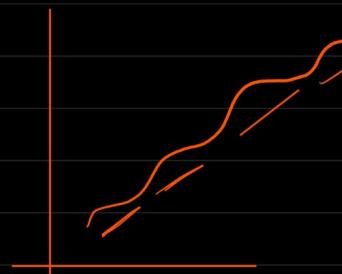
→ Increased trend at same difference

$$y_t = \text{Trend} \times (\text{Seasonality} \times \text{Noise})$$

→ Non linear

→ Non constant variance

Day 1	100
Day 2	200
Day 3	300
Day 4	400



## Time Series

Analysis

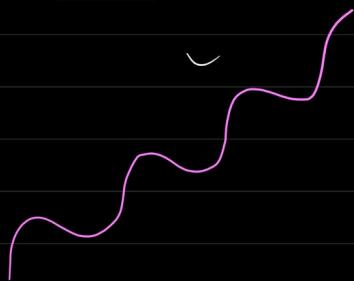
forecasting

smoothing      Autoregressive models

→ You can build a smoothing time series model on any time series data

\* To build Autoregressive model, statistical properties of a time series like mean / variance should be constant (not change over time)

Scenario 1



Scenario 2

→ It will be easy to build a forecasting model in SC-2  
→ less variance in data

Non stationarity → Mean - Variance will not be constant.  
Stationarity over the time, value is not varying (changing)

\* To build a Autoregressive model,  
make the time series stationary,

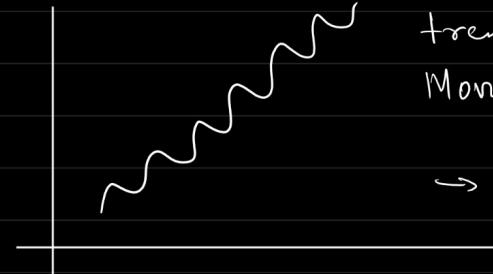
\* ML - Data ingestion → Analysis → preprocessing → Model building

\* Time Series → .. → TSA → Timeseries is stationary → build forecasting model.

↳ if nonstationary → convert to stationary + s

\* To check if a TS is stationary or not

## ① Visualisation



trend = upward

Moving avg is increasing over time

→ Variance is not const.

↓

TS is non stationary.



Moving avg

Const → time series

is stationary.

## ② Statistical test

### a) ADF (Augmented dickey fuller test)

$H_0$ : TS data is non

$H_A$ : TS " is stationary

p value < 0.05 → reject  $H_0$

Conclude - TS data is stationary

### b) KPSS test

(Kwiatowski -  
Philipps -  
Schmidt - Shin)  
test

$H_0$ : TS data is stationary

\* How to Convert non-st. time series to stationary ts

### ⇒ ① Differencing

② log transformation - take the log of value

③ root " → " " sq root of values

④ Seasonality adjustment

Box - Cox transformation (to make Variance constant → +ve)

⑤ Yeo - Johnson transfo. (All ts data)

Differencing → Difference ( $y_t - y_{t-1}$ )

Month	Price	1st diff	2nd orderd (curr) Previous	3rd
J	-5	NA	NA	1
F	10	$10 - 5 = 5$	NA	1
M	6	$6 - 10 = -4$	$-4 - (-5) = -1$	1
A	8	$8 - 6 = 2$	$2 - (-4) = 6$	1
M	18	?	-	-
J	7	-8	-	-

Check st → 3rd order

After differencing check if TS is stationary

Statistical test (ADP) Visualisation (w/ MA)

\* if stationary → build the forecast by model  
else → Again do difference.

\* ACF → Auto Correlation Function  
\* PACF → Partial Auto-correlation function

\* Autoregression.

\* ACF → Auto + Correlation  
 ↓  
 Correlation  
 itself in the feature.  
 Relationship b/w two variables

\* ACF measures the correlation b/w time series & its lag value.

Month	$y_t$	1st lag	2nd lag	3rd
→ Jan	10	NA		
→ feb	25	10 (Jan)	NA	
Mar	35	25 (Feb)	10 (Jan)	NA
APR	42	35 (Mar)	25 (Feb)	NA
May	50	42 (Apr)	35 (Mar)	NA
June	55	50 (May)	42 (Apr)	NA
July	62	55 (June)	50 (May)	NA

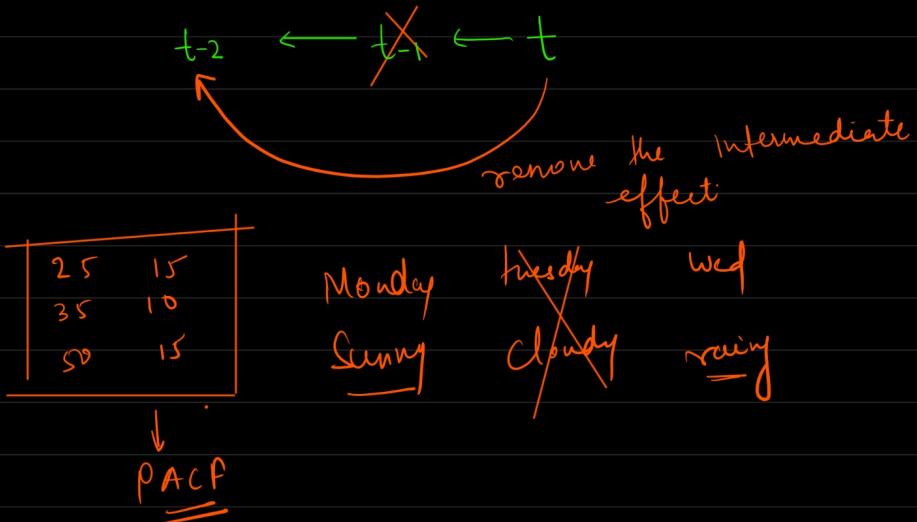
✓ ✓  
 corr ( $y_t, y_{t-1}$ )

corr ( $y_t, y_{t-2}$ )

cor ( $y_t, y_{t-3}$ )

## \* Partial Auto-correlation Function

Month	$y_t$	$y_t - y_{t-2}$
t-6 Jan	10	
t-5 F	18	x
t-4 M	25	15 (25-10)
t-3 A	29	x
t-2 May	35	10 (35-25)
t-1 J	45	x
t July	50	15 (50-35)



## Autoregression

Auto regression  
regression

regression with itself.

$$\underline{y_t} = \psi \underline{y_{t-1}} + c$$

$$y_t = \psi_{t-1} + \psi_2 y_{t-2} + c$$

$y_t$  - value at current ts

$\psi$  - coeff

$c$  - constant

$$X | Y$$

$$X - IV$$

$$Y - DV$$

$$y = m \cdot x + c$$

$$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

$$y_t = \psi_1 y_{t-1} + \psi_2 y_{t-2} + \psi_3 y_{t-3} + \dots + \psi_n y_{t-n} + c$$

## Autoregressive models

→ AR

→ MA

→ ARIMA

→ SARIMA

→ SARIMAX

A R  
(Autoregression)

$$\Downarrow p \\ (0, 1, 2, \dots, n)$$

lag value  
 $\Downarrow$   
PACF plot  
(correlogram)

$$\begin{aligned} \text{ACF (for MA)} & (q) \\ \text{Corr}(y_t, y_{t-1}) & \\ \text{Corr}(y_t, y_{t-2}) & \\ \text{Corr}(y_t, y_{t-3}) & \end{aligned}$$

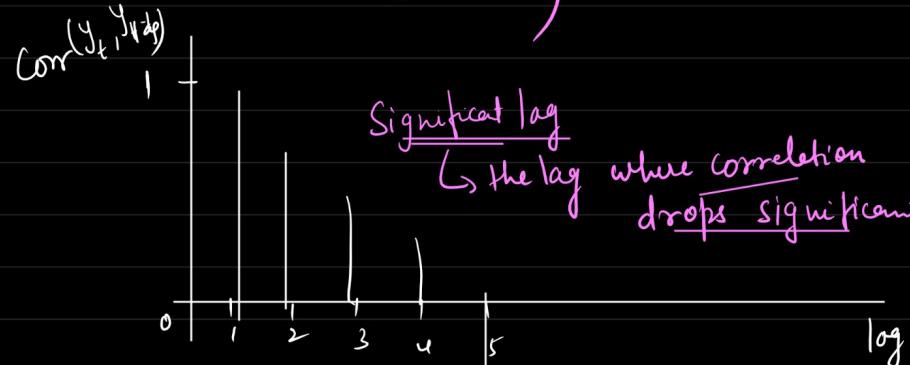
I  
Integrated  
 $\Downarrow d$   
(0, 1, 2, ..., n)  
lag value  
 $\Downarrow$

Differencing  
(stationary)

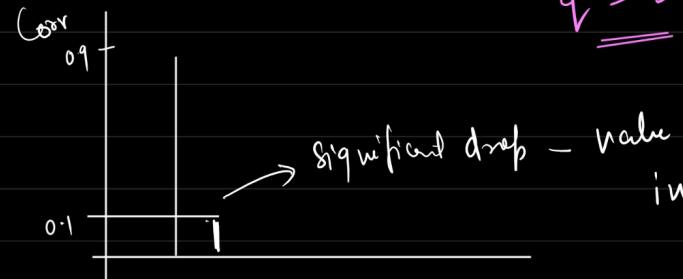
M A  
(Moving Average)

$$\Downarrow q \\ (0, 1, 2, \dots, n)$$

lag value  
 $\Downarrow$   
ACF Plot  
(correlogram)



$$\begin{aligned} \text{PACF} \\ \frac{1}{\sqrt{1 - \rho^2}} \\ y_{t-2} \rightarrow y_{t-1} - y_t \end{aligned}$$



MA —  $q(\underline{\text{ACF}})$

AR —  $p(\underline{\text{PACF}})$

(AR)

Autoregression

$$y_t = \psi_1 y_{t-1} + \psi_2 y_{t-2} + \dots + \psi_n y_{t-n} + \epsilon_t$$

\* we decide  $p(\text{PACF})$  of AR,  
by seeing PACF plot

and  $q(\text{ACF})$  of MA

by seeing ACF plot.

Till how much lag we have  
to model?

$\Downarrow$   
PACF  $\Rightarrow p$   
(significantly)  $\underline{\underline{=}}$

# Integrated (differencing)

$$y_t - y_{t-1} = D_1$$

$$y_t - y_{t-2} = D_2$$

\* MA. (Moving Average) (Not same as of Smoothing)  
 $q$  - ACF

→ It models the error.

$$\begin{aligned} y_t &= \epsilon_{t-1} \psi + c \\ &\quad \overbrace{\hspace{10em}}^{\text{(error)}} \\ y_t &= \epsilon_{t-1} y_{t-1} + \epsilon_{t-2} y_{t-2} + \epsilon_{t-3} y_{t-3} + \\ &\quad \cdots \epsilon_{t-n} y_{t-n} + c \end{aligned}$$

$$\text{ARIMA} = \frac{y_{t-1} \psi_{t-1} + y_{t-2} \psi_{t-2} + \cdots y_{t-n} \psi_{t-n} + y_{t-1} - y_{t-2} - y_{t-3} - \cdots - y_{t-n}}{\text{AR (p - PACF)}} + \frac{\int \text{differencing} - d}{\text{Integral (differencing - d)}} + \epsilon_{t-1} \psi_{t-1} + \epsilon_{t-2} \psi_{t-2} + \cdots + \epsilon_{t-n} \psi_{t-n} + c$$

$$\begin{aligned} \text{AR (1)} \\ \text{I (2)} \\ \text{MA (1)} \end{aligned} \left\{ \begin{array}{l} \text{ARIMA } (p, d, q) \\ (1, 2, 1) \end{array} \right\} \begin{aligned} y_t &= y_{t-1} \psi_{t-1} + y_t - y_{t-1} - y_{t-2} + \epsilon_{t-1} \psi_{t-1} \\ &\quad \overbrace{\hspace{10em}}^{\text{differencing of order 2}} \overbrace{\hspace{10em}}^{\text{MA or order 1}} \psi - \text{Coefficient} \\ &\quad \overbrace{\hspace{10em}}^{\text{AR of order 1}} \quad y_t - \text{obs} \\ &\quad \downarrow \quad \uparrow \\ &\quad (1, 2, 1) \end{aligned} \quad \begin{aligned} \epsilon_t &- \text{Error} \\ c &- \text{intercept} \end{aligned}$$

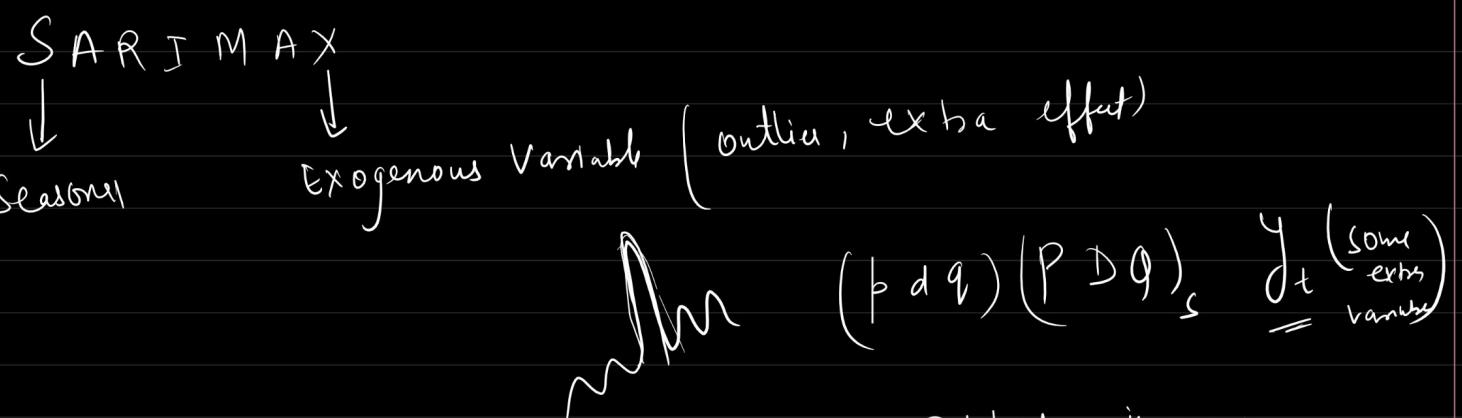
SARIMA  
 $\downarrow$

Seasonality

$$\text{ARIMA} = \underbrace{(p, d, q)}_{(1, 1, 2)} \underbrace{(P, D, Q)}_{(1, 0, 1)} S$$

→ Same  $p, d, q$  for seasonal component





Date	Sales	is_summer_val
Jan	150	0
Feb	160	0
Mar	170	0
Apr	180	0
May	190	1
Jun	200	1

## Time series

Smoothing model

→ Exponential smoothing

→ double Exponential Smoother (Holt's method)

→ triple .. " (Holt winter method)

↳ seasonality

trend

## Sources

→ git - time series

→ Kaggle glossary.

→ Scikit cheat sheet.

→ Books  
probabilistic Models by Murphy

→ Pattern recognition by Christopher Bishop.

## Stats

think stat

- Youtube channel → 3 Blue 1 Brown (ML, DL)  
 → Statquest (statistics)  
 → Python (Carrey Schaefer)

## Certifications

- Intro to ML (Andrew Ng)  
 courses  
 → Deep learning Specialization  
 → deeplearning.ai

Stanford Leec