Till now

→ Different type of pmf

→ Z test

Agenda → Hypothesis testing

Z test (more question)

Q. A factory has a machine that fills 80 ml of baby medicine in a bottle. An employee believes the average amount of baby medicine is not 80ml. Using 40 sample, he measures the average amount dispersed by the machine to be 78 ml with a std devn of 2.5.

At a 95% CI, is there enough evidence to support machine is not working properly?

→

$\sigma = 2.5, \; n = 40 \quad \bar{x} = 78, \; \mu = 80$

① $H_0 : \mu = 80$

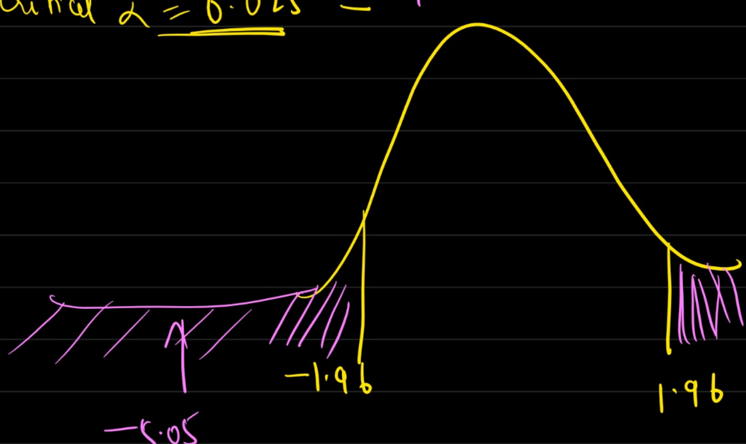
$H_A : \mu \neq 80$

② $\alpha = 0.05$, two tail test $\quad \alpha = \frac{0.05}{2} = 0.025$

③ Z test.

$Z_{statistic} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{78 - 80}{2.5 / \sqrt{40}} = -5.05$

④ $Z_{critical} \; \alpha = 0.025 = -1.96$



⑤ Conclusion → if $Z_{stats} = -5.05$ is less than -1.96 or greater than 1.96, then reject the H_0 with 95% CI.

→ There is some fault in the machine

Q A complain was registered, the boys in the a primary school underweight. Avg weight of boys of age 10 is 32 kg with S.D = 9 kg. A sample 40 boys was selected from the school and the avg weight was found to be 29.5 kg. With C.I 95%. Check whether it is true or false?

→ $\mu = 32$, $\sigma = 9kg$, $n = 40$, $\bar{x} = 29.5$
$\alpha = 0.05$

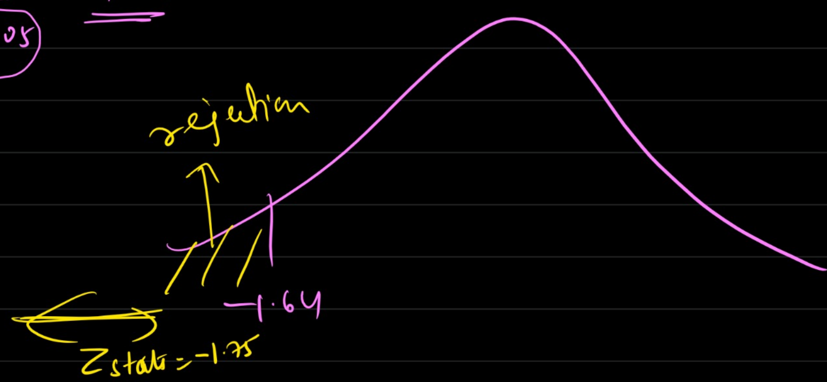
① $H_0 : \mu_{wt} = 32$
$H_A : \mu_{wt} < 32$

② $\alpha = 0.05$

③ $$Z_{statis} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29.5 - 32}{9/\sqrt{40}} = -1.75$$

Z critical approach

④ $Z_{critical} \xrightarrow{} \alpha = 0.05 = -1.64$

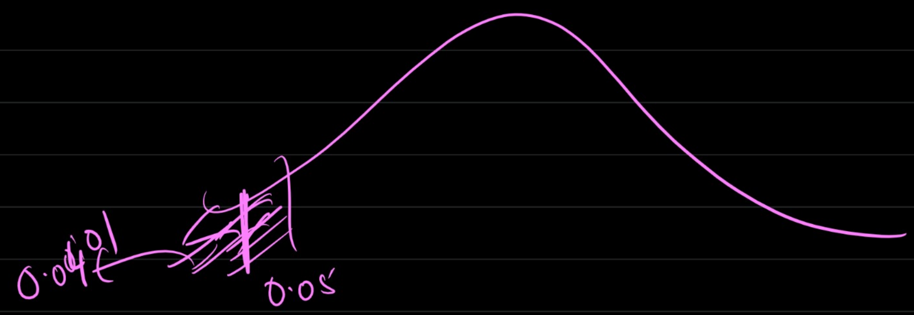


⑤ Conclusion → Reject the H_0. - boys are underwt

P-value approach

$Z_{stats} = -1.75$ to p-value

pvalue = 0.0401



p value < 0.05 → reject the H_0

Q. The avg wt of all residents in town XYZ is 168 kg. A nutritionist believes true mean to be different. He measured the wt of 36 individuals and found the mean to be 169.5 kg wt, the pop sd = 3.9

at 95% test the hypothesis.

→

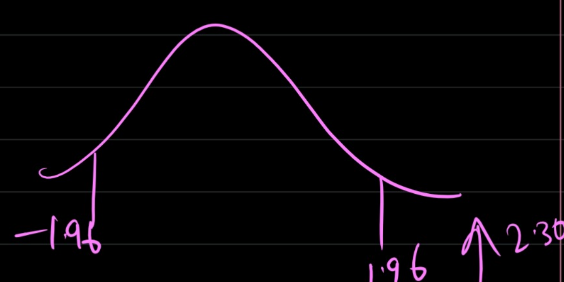
① $H_0: \mu = 168$
$H_1: \mu \neq 168$

② $\alpha = 0.05$, two tail test. $\alpha = 0.025$

③ Z test.

$$Z_{statist} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{169.5 - 168.0}{3.9/\sqrt{36}} = 2.30$$

④ $Z_{critical}$ $\alpha = 0.025$ $= -1.96$



⑤ Conclusion → reject the H_0

t-test

S.S < 30 or σ population $\rightarrow$ t-test.

Q. Suppose a child Psychologist says that the average time working mother spend talking to their children is upto 11 minutes per day. To test the hypothesis, you conducted an experiment with random sample of 20 working mother and find that they spend 11.5 minutes per daily talking with their children. The sample std deviation is 2.3 mins. Conduct the test with 5% level of significance. ($\alpha = 0.05$)

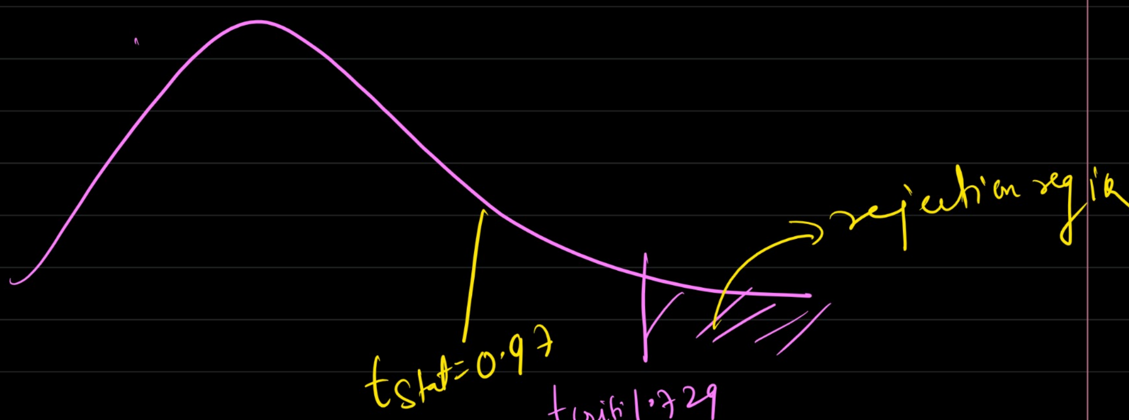
① $H_0: \mu <= 11$ $\quad H_A: \mu > 11$

② $\alpha = 5\%$ (one tail test)

③ t test

$\hookrightarrow$ t statistics (t score) $= \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{11.5 - 11}{2.3/\sqrt{20}} = 0.97$

④ t critical $\alpha = 0.05$, dof $= 20 - 1 = 19 = 1.729$



(5) fail to reject H_0

Q. Sachin's hypothesis: μ_{age} of Deloitte employee 45 years

Rahul conducted an experiment with sample 20, sd of sample came out to be 1. and mean 49 years.

Test this hypothesis $\alpha = 0.05$

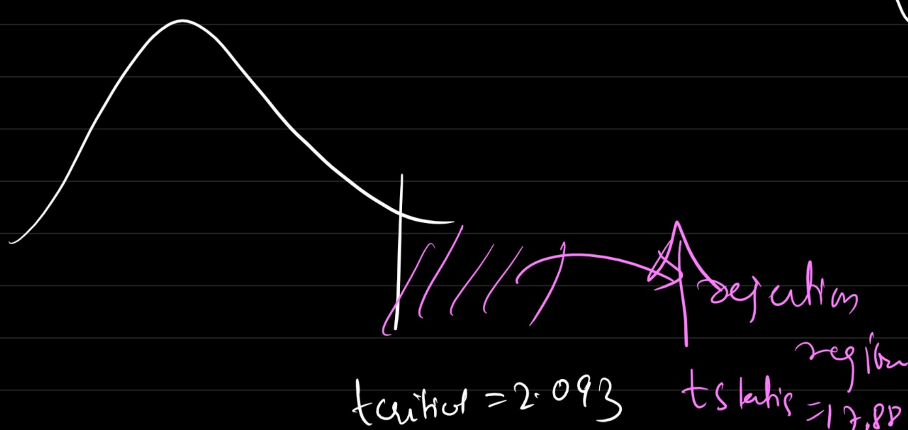
→ (1) $H_0: \mu_{age} = 45$, $H_A: \mu_{age} \neq 45$.

(2) t-test, two tail test

(3) tstatistic (tscore) $= \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{49-45}{1/\sqrt{20}} = 17.88$

(4) tcritical → $\alpha = 0.05$ two tail test and dof 19 $\Rightarrow 2.093$

(5) Conclusion



Note

In t-table do not do $\alpha = \alpha/2$ for two tail test, t table take care of this of a both one tail two tail values are given

reject the H_0

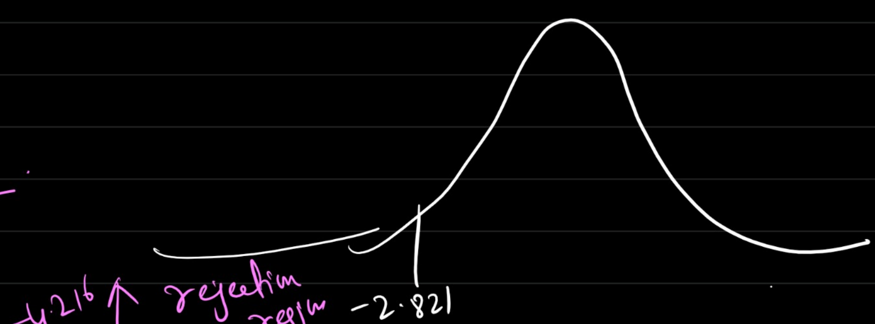
Q A battery manufacturer company manufactures batteries with an average life span of 2 or more years.
A engineer believes this value is less. 10 samples, he measured the avg life span to be 1.8 years with a std of 0.15. Test the hypothesis $\alpha = 0.01$

→ ① $H_0 : \mu \geq 2$, $H_1 : < 2$

② $\alpha = 0.01$ → one tail test

③ t - test , t stats (t score) $= \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{1.8 - 2}{0.15/\sqrt{10}} = \dfrac{-0.2}{0.15/31.622} = -4.216$

④ t critical $\alpha = 0.01$ and 1 tail test, dof 9 $= 2.821$



⑤ Conclusion $-4.216 < -2.821$ (Reject the H_0

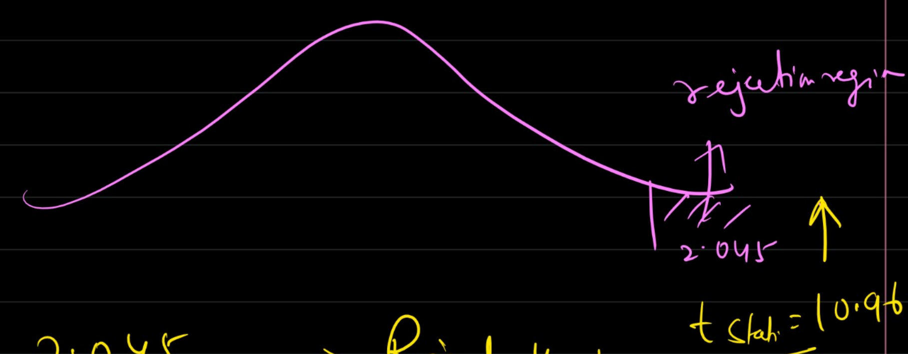
Q In the population the avg IQ is 100. A team of researchers want to test a medicine to check the +ve or -ve effect on intelligence.

A sample of 30 participants who took medicine has a mean IQ of 140 with std dev of 20. Did the medication affected intelligence? test hypothesis with 5% level of significance.

1. $H_0: \mu = 100$, $\mu \neq 100$
2. $\alpha = 5\%$ → 2 tail test   dof $= n-1 = 30-1 = 29.$
3. t test.

↳ t statistics (t score) $= \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{140-100}{20/\sqrt{30}} = \dfrac{40}{3.65} = 10.96$

4. $t_{critical}$ $\alpha = 0.05$, two tail $= 2.045.$



$t_{stas}(10.96) > 2.045$ → Reject the H_0

5. Conclusion → The medicine has no effect on intelligence

T-test Vs Z test

$\boxed{\text{if Standard deviation } (\sigma) \text{ of population is given}}$

Yes → $\boxed{\text{If the S.S greater than 30}}$

NO → Use t-test

yes → $\boxed{\text{Use Ztest}}$ NO → $\boxed{\text{Use t-test}}$

Z test → σ_{pop} is given and S.S > 30

t test — σ_{pop} is not given or S.S < 30.

Type of errors :-

① Type-I error — is the rejection of the null hypothesis when it is actually true.

Example :→ An Innocent person is convicted.

② Type-II error → fail to reject the null hypothesis that is actually false.

example: A guilty person is not convicted.

	H_0 is true	H_A is true
Support H_0	Correct conclusion	type-II error.
Support HA	type-I error.	Correct conclusion

⇒ Confusion matrix.

Actual decision → H_0 is true or H_0 is false

based on your analysis → H_0 is true or H_0 is false

Scen-1 → Reject the H_0, when actually it is false → good case

Scen-2 Reject the H_0 when actually it is true — Type-I error

Scen-3 → Retain the H_0 when actually it is False → Type-II error

Scenar-4 → Retain the H_0 when actually it is true → good.

Example

Ajay → has commited a crime

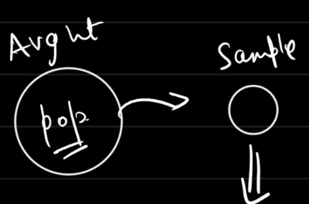
H_0 → Ajay is innocent.

Scen-1 Ajay has been charged for a crime and in the court he is convicted. — Type-I

Scen-2 A guilty person has been charged, but court didn't find him guilty — Type-II

* Confidence interval and margin of error

estimatee → An estimate of a population parameter is an approximation depending solely on sample information.



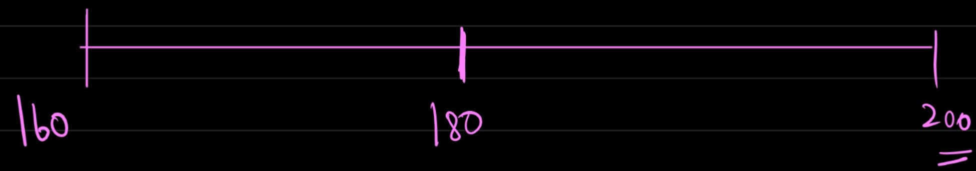
estimate the ht of peope of India for population.

estimation

- point → Single no.
- interval → Point estimate ± error

180 ± 20

180
↓
160 – 200



Confidence Interval (CI) = point estimate ± margin of error

eg People visiting a restaurant spends 1600 Rs on an avg.
Point

↳ It is much safer to say that people spend 800 – 1200

↳ Confidence interval

⇓

More accurate representation of reality

⇓

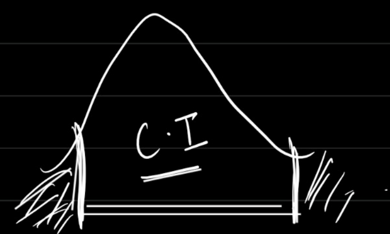
You can not be 100% confident unless you check entire population

$$C.I = \text{point estimate} \pm \text{margin of error}$$

Z test (two tail) ⇒ $C.I = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

t test (two tail) ⇒ $CI = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

C.I → What value the sample statistics with take can be known through C.I



Q In an exam the std devⁿ of marks is 100. A sample of 36 student has a mean of 500 marks. Calculate a 95% confidence interval about the mean?

→
$$C.I = \bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$
$$= 500 \pm Z_{\frac{0.05}{2}} \cdot \frac{100}{\sqrt{36}}$$
$$= 500 \pm 1.96 \cdot \frac{100}{6}$$

Lower $CI = 500 - 1.96 \times \frac{100}{6} = 467.33$

Upper $C.I = 500 + 1.96 \times \frac{100}{6} = 532.6$

I am 95% confident that the mean score in the exam lies b/w 467.3 and 532.6

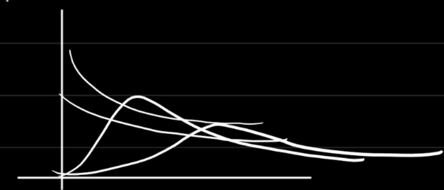
* Chi-square test

Chi-square dist^n — is a prob dist^n that describes the dist^n of a sum of squares of k random.

→ χ^2 dist^n is a non-neg.

→ right skewed data

→ depends on dof / k / df = n−1



$$\chi^2_{stats} = \sum \frac{(O-E)^2}{E}$$

χ^2 test

↳ Chi-sq dist^n

↳ goodness of fit test ⇒ Some claims have been made about population categorical proportion.

Types of car	theory (Expected)	observed (sample)
Sports car	1/3	22
SUV	1/3	17
Sedan	1/3	59

→ observed categorical dist^n

⇓

Using this observed sample dis^n you have to verify that the theoretical dist^n is true or not.

→ goodness of fit

→ observed data is as good as theoretical data and that's why it is known as goodness of fit test.

Q. 12% of people are left handed. To verify this theory, You took a sample of 75 students, 11 are left handed.
With 5% level of significan, conduct the test.

→

	Observed	Expected
left	11	9
Right	64	66

① $H_0 : \mu = 12\%$, $H_A : \mu \neq 12\%$

② $\alpha = 5\%$

③ Chisquar test

↳ Chisquare Statistics

$$\chi^2_{statics} = \sum \frac{(O-E)^2}{E}$$

$$\chi^2_{stats} = \frac{(11-9)^2}{9} + \frac{(64-66)^2}{66} = \frac{2^2}{9} + \frac{2^2}{66}$$

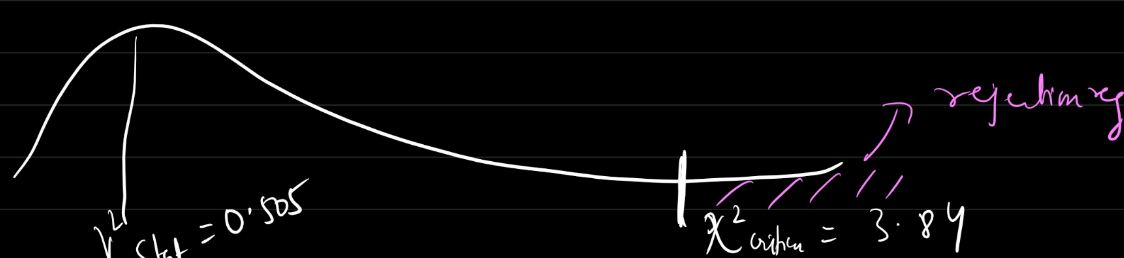
$$= 0.505$$

④ $\chi^2_{critical}$ for $\alpha = 0.05$

dof = 2-1 = 1 (No of categories is 2)

$\chi^2_{critical} = 3.84$

Step. 5



if $\chi^2_{stats} < \chi^2_{critical}$ → fail to reject H_0

$0.505 < 3.84$ → we fail to reject the H_0

12% of People are left handed with 95% confidence

Q A Casino states that the following is result when die is thrown.

Observed

Numbers	1	2	3	4	5	6
frequency	22	24	38	30	46	44

total throw = 204.

Could you figure it out whether the Casino is using Fair dice or not?

→

Expected

	1	2	3	4	5	6
$\frac{1}{6} \times 204$	34	34	34	34	34	34

① H_0: Dice is fair
H_A: Dice is not fair

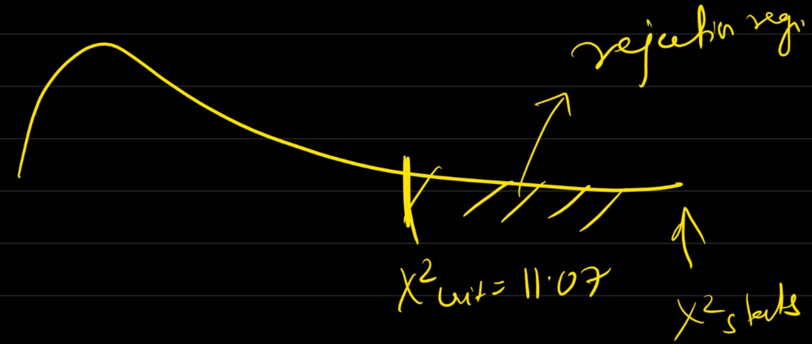
② $\alpha = 0.05$

③ $\chi^2_{statistic} = \frac{(O-E)^2}{E}$ $\Rightarrow \frac{(22-34)^2}{34} + \frac{(24-34)^2}{34} + \frac{(38-34)^2}{34} + \frac{(30-34)^2}{34} + \frac{(46-34)^2}{34} + \frac{(44-34)^2}{34}$

$\Rightarrow \chi^2_{stats} = 15.29$

④ χ^2 critical for $\alpha = 0.05$ dof $= 6-1 = 5$

$\chi^2_{cr} = 11.07$



Step-5.

Reject the H_0

↳ dice is not Fair.

Q. In 2010 census of the city, the weight of people in a city were found to be following

< 50kg	50-75kg	> 75kg
20%	30%	50%

if 2020 weight of 500 people were samples

< 50	50-75	> 75
140	160	200

$\alpha = 0.05$, can you conclude the population difference of wt has changed in last 10 year or not?

→

2010 →

< 50	50-75	> 75
20%	30%	50%

Observed (2020)	< 50	50-75	> 75
	140	160	200

Expected (2020)	< 50	50-75	> 75
	$0.2 \times 500 = 100$	0.3×500 150	$0.5 \times 500 = 250$

Step-1 H_0: the data is as per expectation

H_A: " " not as per "

Step-2 $\alpha = 0.05$

Step-3 χ^2 statis $= \sum \frac{(O-E)^2}{E}$

$$= \frac{(140-100)^2}{100} + \frac{(160-150)^2}{100} + \frac{(200-250)^2}{250}$$

$$= \frac{1600}{100} + \frac{100}{160} + \frac{250}{250} = \underline{26.66}$$

Step-4 χ^2 critical $\alpha = 0.05$

$$dof = \underline{3-1} = 2$$

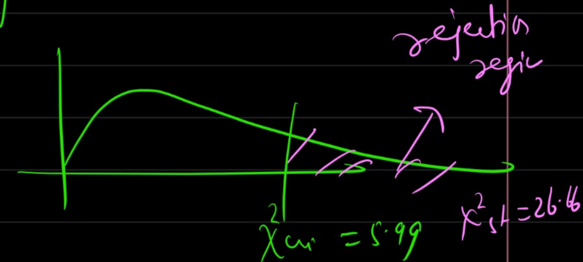
$$\chi^2_{criti} = 5.99$$

Conclusion

if $\chi^2_{stats} > \chi^2_{critical} \Rightarrow$ Reject the H_0

$$26.66 > 5.99$$

reject the H_0



Conclusion — The wt of 2020 population are different than those expected 2012 population.

* Assignment

Z test with proportion

Q. A tech company believes that the percentage of resident in town abc who owns a cell phone is 70% but the CEO believes that this value is different. He conducts a survey of 200 individuals and found that 130 responded yes to owning a cell phone.

Test the hypothesis with 95% Confidence Interval.

$\Rightarrow$ ① $H_0 : P_0 = 0.70$ (p_0 – Population proportion)

$H_A = P_0 \neq 0.70$

$\hat{p}$ – Sample Proportion

From experiment / sample

$n = 200$, $x = 130$

② $\alpha = 0.05 \rightarrow$ two tail test

$\alpha = 0.025$

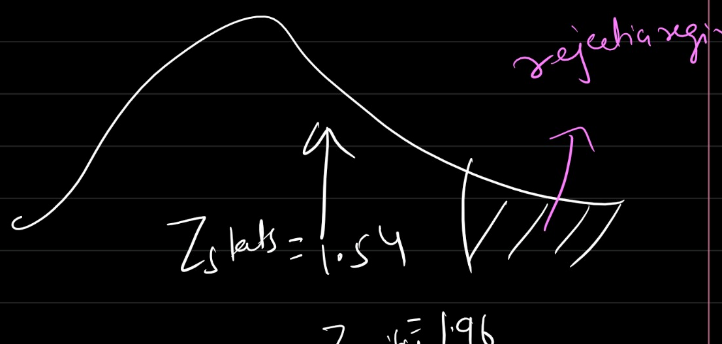
avg %age of people having cell phone in sample $\leftarrow$ $\hat{P} = \frac{x}{n} = \frac{130}{20} = 0.65$

$q_0 = 1 - P_0$

$= 1 - 0.65 = 0.35$

③ $$Z_{statistics} = \frac{\hat{P} - P_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.65 - 0.70}{\sqrt{\frac{0.7 \times 0.3}{200}}} = 1.54$$

④ $Z_{critical}\ \alpha = 0.025 = 1.96$



⑤ fail to reject H_0

The proportion of people is 0.70