Assignment 1

Assigned date: 8th February 2022; Due date: 18^{th} February 2022; Max marks: 100

1. (10 points) Simplify and obtain an expression for the dissipation function Φ, that occurs in the internal energy equation, by substituting expressions for viscous stresses in terms of strain rates for a Newtonian fluid. Show that the simplified dissipation function is non-negative and is a source of internal energy.

2. (10 points) Write the x-component of momentum equation for the unsteady flow of a compressible Newtonian fluid with variable density ρ and viscosity μ. Expand the Newtonian stress tensor in terms of the spatial derivatives of velocity. The objective is to compare this equations with the general scalar transport equation and obtain expressions for ϕ, Γ and S_ϕ for the following cases:

   (a) Both ρ and μ are variable.

   (b) Both ρ and μ are constant. (Hint: use the continuity equation).

3. (50 points) Consider the one-dimensional diffusion equation with in the calculation domain shown in figure 1. Assume that there is a source term $S = 8x$, and that the diffusion coefficient $\Gamma = 1$ is constant. Discretize the domain into five uniform cells with $\Delta x = 1$. Let the boundary values of ϕ be $\phi_A = 100$ and $\phi_B = 600$.

   (a) Write the discrete equations for each of the cells 1 to 5.

   (b) Solve the system of linear equations obtained after discretization using Gauss-Seidel iteration and report the resulting cell-centroid values.

   (c) Compute the diffusion flux $\Gamma\frac{\partial\phi}{\partial x}$ at each of the cell faces f_1 through f_6 using the same discretization approximations made in obtaining the discrete equations above.

   (d) Show that the conservation principle is satisfied on each discrete cell and on the whole domain.

   (e) Find the exact solution to this problem and calculate the percentage error in the computed solution at each cell centroid. Tabluate the exact solution, computed solution and the percentage error in the computed solution at each cell centroid. Notice that even though the computed solution is conservative, it is not exact.

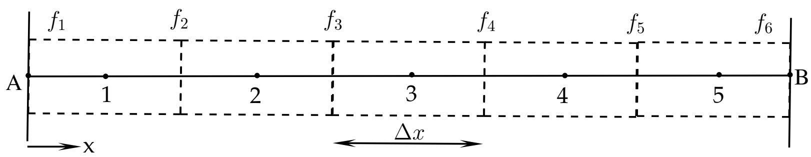

Figure 1: Domain and mesh for problem 2

4. (30 points) Consider one-dimensional conduction on a uniform mesh as shown in figure 2 with $S = 0$ and $\Gamma = 1 + 2x + 3x^2$.

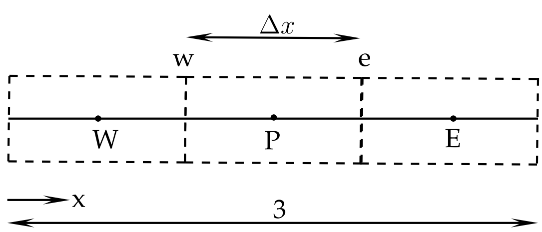

Figure 2: Domain and mesh for problem 3

(a) Starting with

$$\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right) = 0$$

Find the discrete equation for the cell P using finite-volume method. Find the value of ϕ_P for a given cell-centroid values for W and E as $\phi_W = 100$ and $\phi_E = 400$ respectively. Confirm that conservation is exactly satisfied for cell P.

(b) Let us now consider a non-conservative form of the same governing equation as follows:

$$\Gamma\frac{d^2\phi}{dx^2} + \frac{d\Gamma}{dx}\frac{d\phi}{dx} = 0.$$

Using

$$\Gamma\frac{d^2\phi}{dx^2} = \Gamma_P\frac{\phi_E + \phi_W - 2\phi_P}{(\Delta x)^2}$$

and

$$\frac{d\phi}{dx} = \frac{\phi_E - \phi_W}{2\Delta x}$$

write the discrete equation for cell-centroid value ϕ_P. Note that $\frac{d\Gamma}{dx}$ is known analytically. Solve the discrete equation to find the value of ϕ_P. Find the diffusive fluxes on the faces e and w using a straight line profile assumption for ϕ between cell centroids. Do the fluxes satisfy conservation?