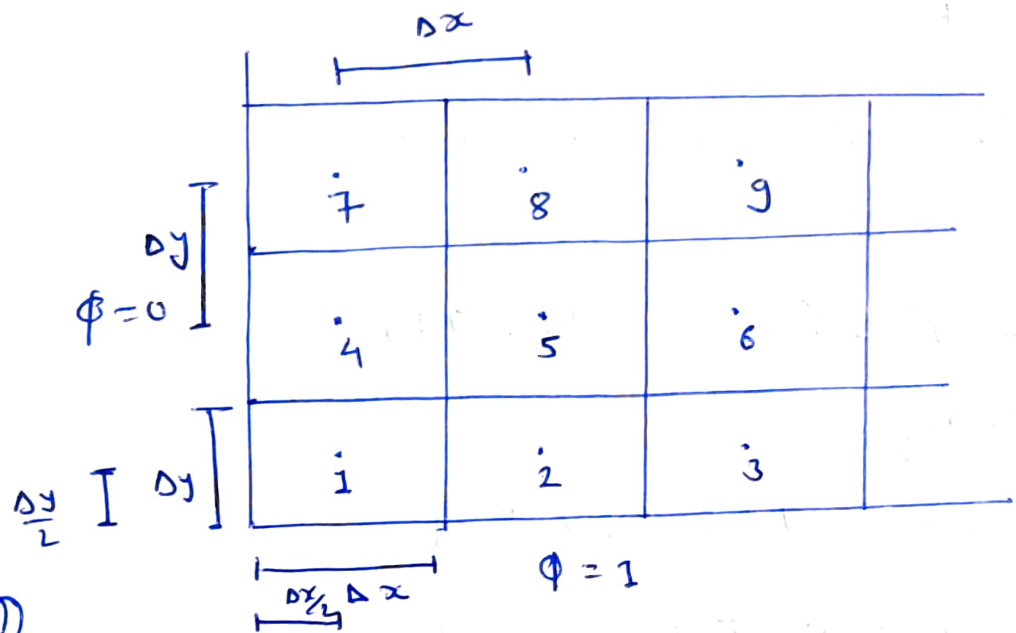
Que 1



(a) Scalar transport eqn

$$\frac{\partial \rho \phi}{\partial t} + \nabla(\rho \vec{u} \phi) = \nabla(\Gamma \nabla \phi) + S_\phi$$

$\phi = 1$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{u}) = \nabla(0) + S_{\phi m}$$

steady incompressible flow $\rho = c$

$$\rho \nabla \vec{u} = S_{\phi m}$$

$$\text{LHS.} = \rho \nabla \vec{u} = \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$u = x^2+1 \qquad v = y^2+1$

$$\text{LHS} = \rho(2x+2y) = 2(2x+2y)$$

$$\text{LHS} = 4(x+y)$$

$RHS = S_\phi = 4(x+y)$

$LHS = RHS$

Continuity eqn is satisfied

(b) $\cancel{\frac{\partial \rho \phi}{\partial t}}^{\,no} + \nabla(\rho \vec{u} \phi) = \nabla(\Gamma \nabla \phi) + S_\phi$

steady

$\nabla(\rho \vec{u} \phi) = \nabla(\Gamma \nabla \phi) + S_\phi$

$\nabla(\rho \vec{u} \phi) - \nabla(\Gamma \nabla \phi) = S_\phi$

Integrating

$\int \nabla(\rho \vec{u} \cdot \phi)\, dv - \int \nabla(\Gamma \nabla \phi)\, dv = \int S_\phi\, dv$

$\int_{cs} (\rho \vec{u} \phi) \cdot d\vec{A} - \int_{cs} (\Gamma \nabla \phi) \cdot d\vec{A} = \int_{cv} S_\phi dv$

$\sum_{f=e,w,n,s} (\rho \vec{u} \phi)_f \cdot \vec{A_f} - \sum_{f=e,w,n,s} (\Gamma \nabla \phi)_f \cdot \vec{A_f} = \int_{cv} S_\phi dv$

~~for~~ considering only convection

$\sum_{f=e,w,n,s} (\rho \vec{u} \phi)_f \cdot \vec{A_f} = S_\phi\, dV$

$\sum (\rho \vec{u} \phi)_f \cdot \vec{A_f} = 4(\bar{x}+\bar{y})\, \phi_s\, V$

$$(\rho u \phi)_e \Delta y - (\rho u \phi)_w \Delta y + (\rho v \phi)_n \Delta x - (\rho v \phi)_s \Delta x$$

$$= 4 (\bar{x} + \bar{y}) \times \phi_s \times \Delta x \, \Delta y$$

$\rho = 2 \qquad \Delta x = \Delta y = 1$

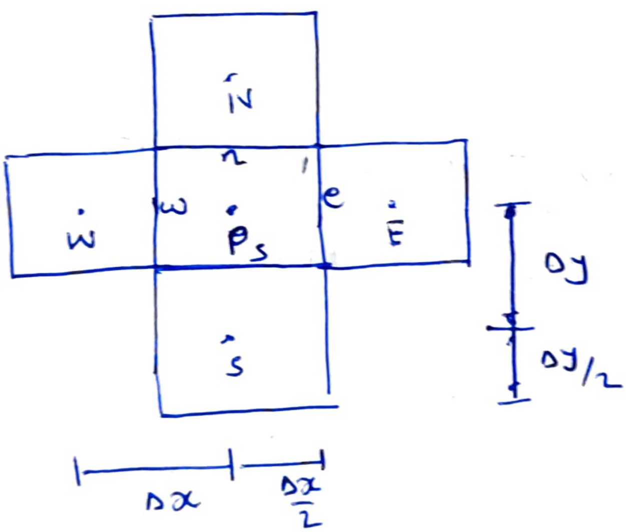
$$(u \Delta y)_e \phi_e - (u \Delta y)_w \phi_w + \cancel{(v \Delta x)}$$

$$+ (v \Delta x)_n \phi_n - (v \Delta x)_s \phi_s = \frac{4}{2} (\bar{x} + \bar{y}) \times \frac{1}{2}$$

upwind scheme

$$(u \Delta y)_e \phi_P - (u \Delta y)_w \phi_W + (v \Delta x)_n \phi_P - (v \Delta x)_s \phi_S$$

$$= (\bar{x} + \bar{y})$$



$$\phi_P = \frac{(u \Delta y)_w \phi_W + (v \Delta x)_s \phi_S + (\bar{x} + \bar{y})}{(u \Delta y)_e + (v \Delta x)_n}$$

$$\phi_P = \frac{(1+x^2)_\omega \phi_\omega + (1+y^2)_s \phi_s + (\bar{x}+\bar{y})}{(1+x^2)_e + (1+y^2)_n}$$

$$\phi_1 = \frac{(1+0)(0) + (1+0)(1) + 2(\frac{1}{2}+\frac{1}{2}) \times 0.5}{(1+1) + (1+1)}$$

$$\phi_1 = 0.5$$

$$\phi_2 = \frac{(1+1)(0.5) + (1+0)(1) + 2}{5+2}$$

$$\phi_2 = 4/7$$

$$\phi_3 = \frac{(1+4)(4/7) + (1+0)(1) + (5/2+\frac{1}{2})}{(1+9) + (1+1)}$$

$$\phi_3 = \frac{5 \times 4/7 + 1 + 3}{12} = \frac{\frac{20}{7}+4}{12}$$

$$\phi_3 = \cancel{48} \quad \phi_3 = \frac{48}{7 \times 12} = 4/7$$

$$\Phi_4 = \frac{(1+0)(0) + (1+1)\frac{1}{2} + \left(\frac{1}{2} + \frac{3}{2}\right)}{(1+1) + (1+4)}$$

$$\Phi_4 = \frac{3}{7}$$

$$\Phi_5 = \frac{(1+1)\left(3/7\right) + (1+1)\frac{4}{7} + \left(3/2 + 3/2\right)}{(1+4) + (1+4)}$$

$$\Phi_5 = \frac{\frac{6}{7} + \frac{8}{7} + 3}{10} = 0.5$$

$$\Phi_6 = \frac{(1+4)(0.5) + (1+1)\left(\frac{4}{7}\right) + \left(5/2 + 3/2\right)}{(9+1) + (4+1)}$$

$$\Phi_6 = \frac{5/2 + \frac{8}{7} + 4}{15} = \frac{107}{210}$$

$$\Phi_7 = \frac{(1+0)(0) + (1+4)\left(3/7\right) + \left(\frac{1}{2} + 5/2\right)}{(1+1) + (1+9)}$$

$$\Phi_7 = \frac{\frac{15}{7} + 3}{12} = \frac{3}{7}$$

$$\phi_8 = \frac{(1+1)\left(\frac{3}{7}\right) + (1+4)\frac{1}{2} + \left(\frac{3}{2}+\frac{5}{2}\right)}{(1+4)+(1+9)}$$

$$\phi_8 = \frac{\frac{6}{7} + \frac{5}{2} + 4}{15} = \frac{103}{210}$$

$$\phi_9 = \frac{(1+4)\left(\frac{103}{210}\right) + (1+4)\left(\frac{107}{210}\right) + 5}{(1+9)+(1+9)}$$

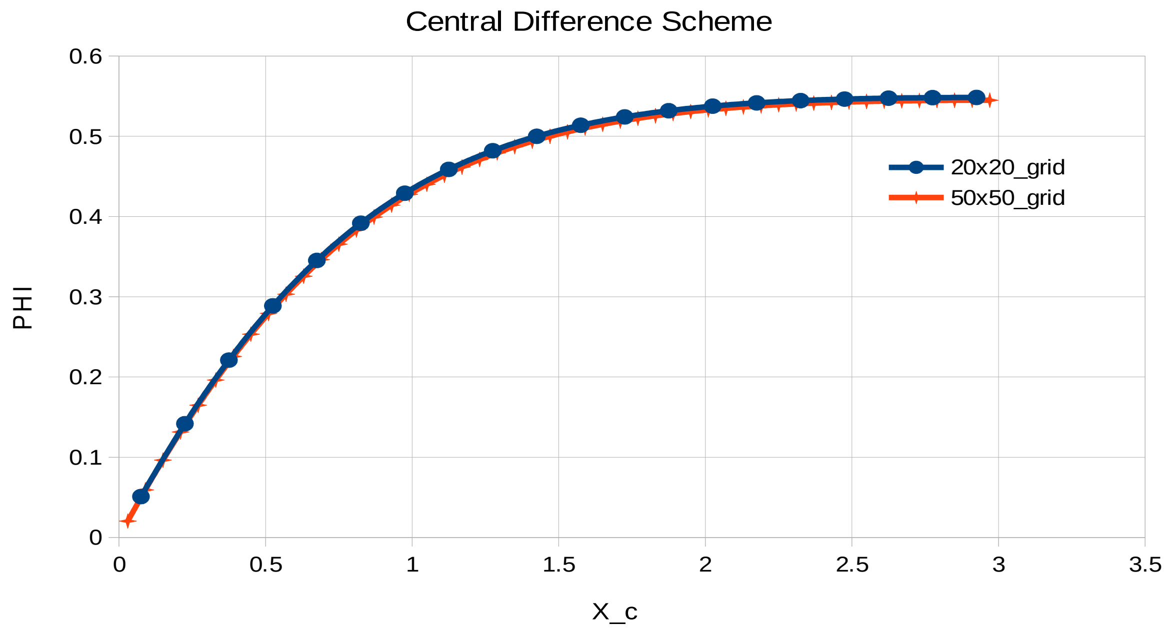
$$\phi_9 = 0.5$$

$\phi_1 = 0.5$ $\quad \phi_2 = \frac{4}{7}$ $\quad \phi_3 = 4/7$

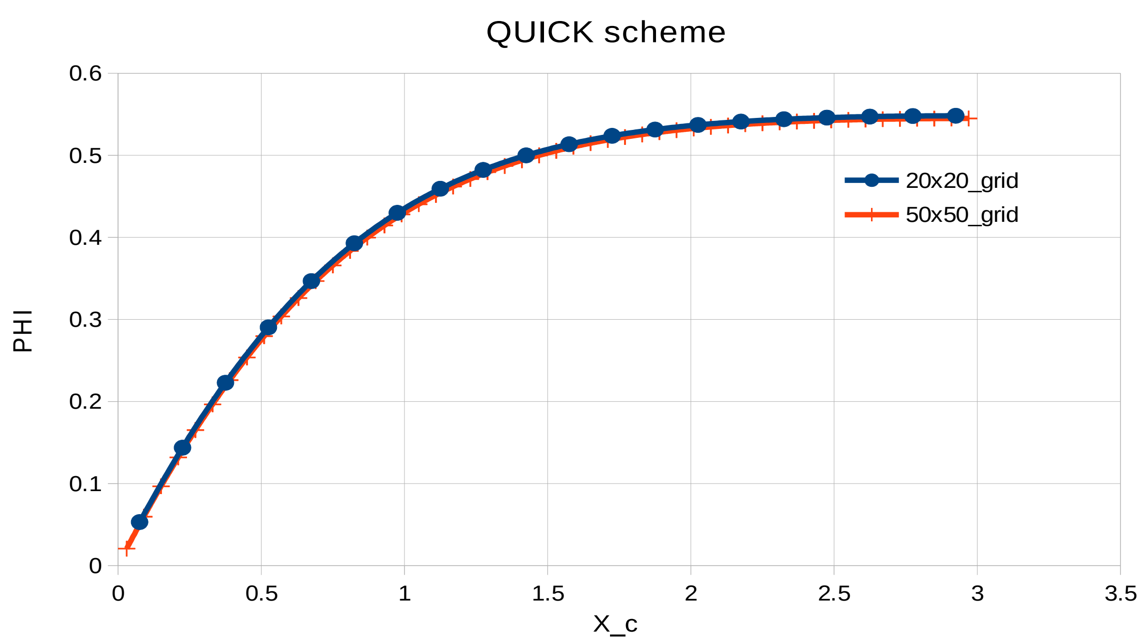
$\phi_4 = 3/7$ $\quad \phi_5 = 0.5$ $\quad \phi_6 = 107/210$

$\phi_7 = 3/7$ $\quad \phi_8 = \frac{103}{210}$ $\quad \phi_9 = 0.5$

Que 2



Central Difference Scheme
PHI
X_c
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
20x20_grid
50x50_grid



QUICK scheme
PHI
X_c
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
20x20_grid
50x50_grid