Assignment 3

Assigned date: 25 March 2022; Due date: 8 April 2022; Max marks: 100

1. (50 points) Consider the steady transport of a scalar ϕ by pure-convection in a square domain of side $L = 3$ m with uniform cells and boundary conditions as shown in figure 1. The flow field in the domain is known and is given by:

$$u = x^2 + 1 \text{ m/s}$$
$$v = y^2 + 1 \text{ m/s}.$$

There is a mass-source in the domain given as follows:

$$S = 4(x + y) \text{ kg/m}^3\text{s}$$

which brings with it $\phi = \phi_s = 0.5$. The density of the fluid $\rho = 2$ kg/m^3.

(a) Write the continuity equation and confirm that the flow field satisfies mass conservation.

(b) Using the finite-volume-method with first-order upwind-difference schemes for discretizing the convection term, find the value of ϕ at all the cell-centroids in the domain. This problem can be solved entirely using hand-calculations or with the help of a computer program depending on your convenience.

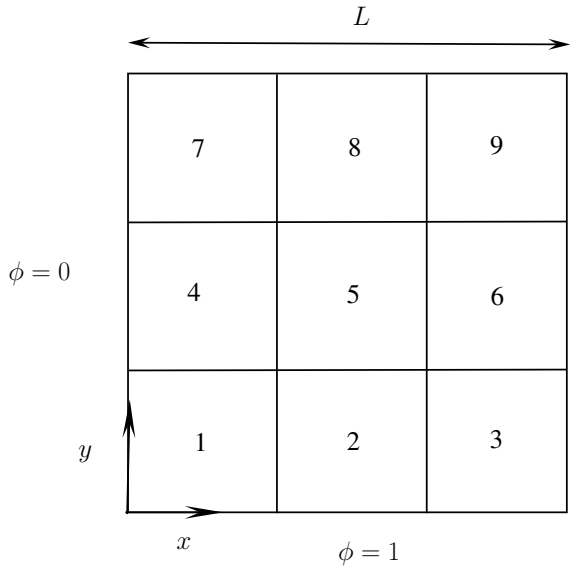

Figure 1: Computational domain and mesh for problem 1.

2. (50 points) Consider a steady convection-diffusion equation in the same domain as in problem 1, with $\Gamma = 3$ kg/ms. Calculate the value of ϕ in the domain using finite-volume method and the following schemes for convection: (a) central difference scheme (b) QUICK scheme. Plot the variation of ϕ along the horizontal center-line for 20×20 and 50×50 uniform meshes. If the mesh is not fine enough you may not satisfy the Scarborough criterion.

At outflow boundaries, you can make a high Peclet number approximation. That is neglect diffusive fluxes and upwind the face value of ϕ. QUICK scheme is a higher-order scheme and it requires more upwind points in the discretization, which are not available at the boundaries. At inflow boundary faces and near-boundary faces which do not have the required number of upwind grid points, use the upwind difference scheme. As QUICK scheme does not satisfy the Scarborough criterion, in order to use an iterative scheme, you need to cast the face value of ϕ as follows:

$$\phi_f = \phi_{\text{upwind}} + \Delta\phi_f$$

where $\Delta\phi$ contains the extra terms due to QUICK scheme when compared to an upwind difference scheme. The $\Delta\phi$ value could be absorbed in the b term in the discrete equation, and needs to be computed using the existing values of ϕ in that iteration.