project3

January 30, 2023

Image Compression using Clustering Algorithms

1) Using in built kmean clustering

[ ]:
```
from skimage.util import view_as_blocks
```

[ ]:
```
import numpy as np
import pandas as pd
import cv2
import matplotlib.pyplot as plt
from sklearn.cluster import KMeans
```

[19]:
```
def squared_error_distortion(img, compressed_img):
    return np.mean(np.square((img.astype(int) - compressed_img.astype(int))))

def find_entropy(l):
    d = {}
    for i in l:
        if i not in d:
            d[i] = 0
        d[i] += 1
    H = []
    prob = []
    for i in d:
        p = d[i]/len(l)
        prob.append(p)
        H.append(p*np.log2(p))
    return -1* sum(H)
```

[ ]:
```
def compress(img, P, R):
    M, N = img.shape
    patches = view_as_blocks(img, (P, P))
    X = patches.reshape(M//P,N//P,1,P**2).squeeze().reshape((M//P)**2,1,P**2).
↪squeeze()
    bits_fr_encoded = int(np.ceil(R*(P**2)))
    C = 2**bits_fr_encoded
    kmeans = KMeans(n_clusters=C, random_state=42).fit(X)
    labels = kmeans.labels_
```

```
    cc = kmeans.cluster_centers_
    temp = np.zeros(X.shape, dtype=np.uint8)
    for i in range(len(labels)):
        temp[i] = cc[labels[i]]
    temp = temp.reshape(M//P,N//P, 1,P**2).reshape(M//P,N//P,P,P)

    compress_img = np.zeros(img.shape, dtype=np.uint8)
    for i in range(temp.shape[0]):
        for j in range(temp.shape[1]):
            compress_img[i*P:P*(i+1), P*j:P*(j+1)] = temp[i][j]
    H = find_entropy(labels)
    r = H/(P**2)

    return compress_img, np.round(r, 2)
```

[ ]:
```
path = "./test_image_3.jpg"
img = cv2.imread(path, cv2.IMREAD_GRAYSCALE)
```

0.0.1 Original Image

[ ]:
```
plt.imshow(img, cmap='gray')
plt.axis(False)
```

[ ]: (-0.5, 2047.5, 2047.5, -0.5)

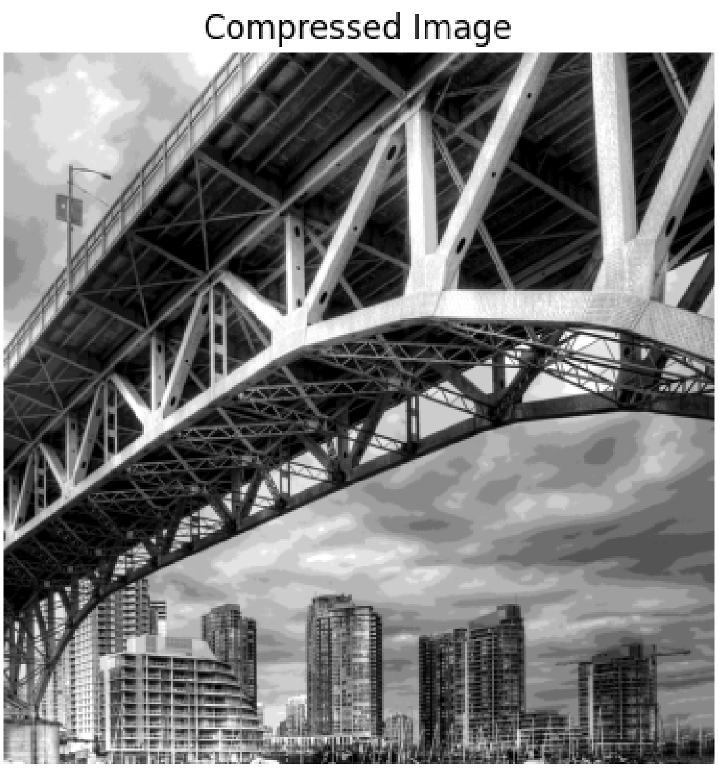
A greyscale image of dimensions 2048 x 2048 was loaded.

Image compression for patch size P = 2 x 2 and rate R = 1 is demonstrated below

[ ]:
```
compress_img, _ = compress(img, 2, 1)
```

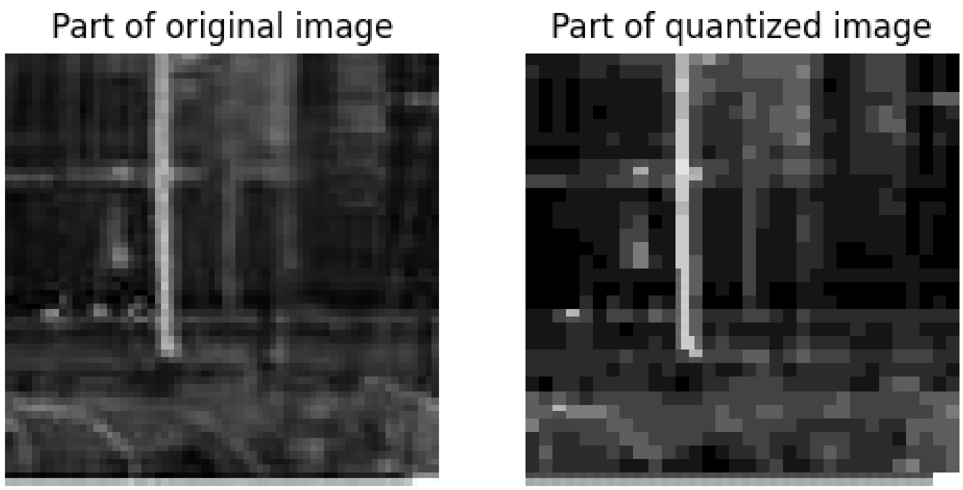
[ ]:
```
plt.imshow(compress_img, cmap='gray')
plt.title("Compressed Image")
plt.axis(False)
```

[ ]: (-0.5, 2047.5, 2047.5, -0.5)



[ ]:
```
plt.subplot(121)
cropped_img_original = img[-64:,-64:]
plt.imshow(cropped_img_original, cmap='gray')
plt.title("Part of original image")
plt.axis(False)
plt.subplot(122)
croppped_img = compress_img[-64:,-64:]
plt.imshow(croppped_img, cmap='gray')
plt.title("Part of quantized image")
plt.axis(False)
```

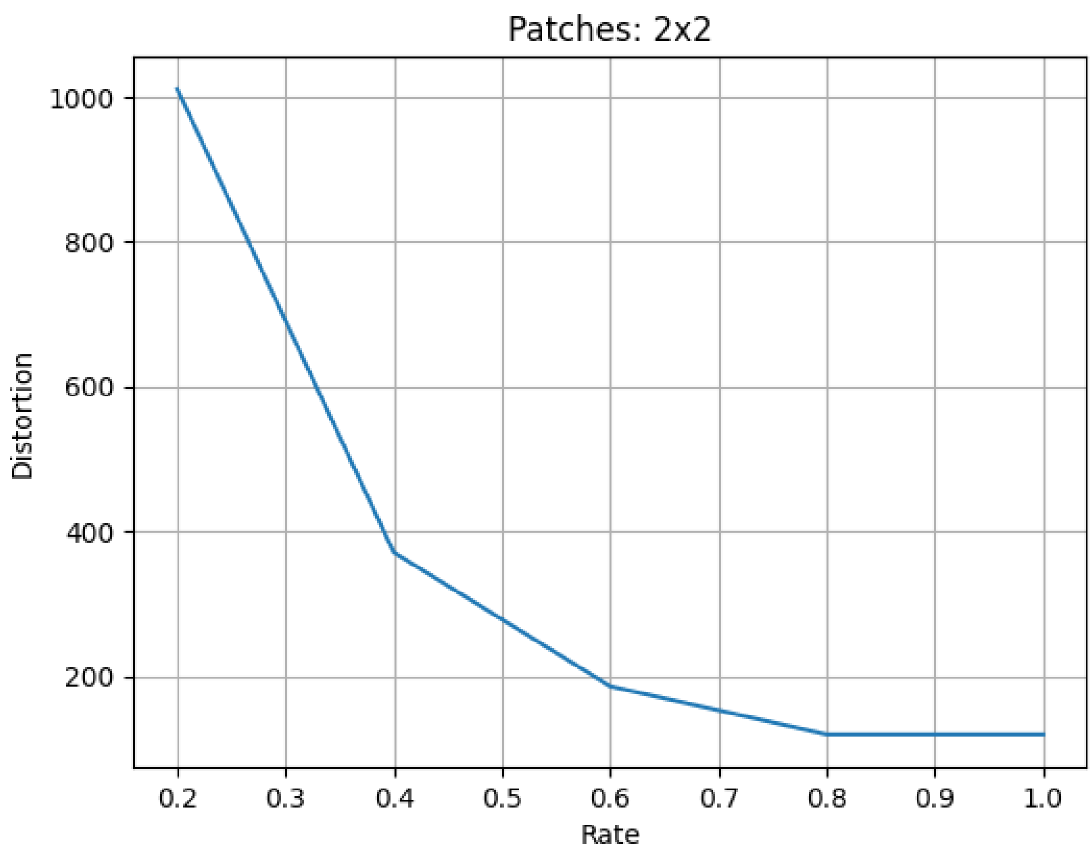
[ ]: (-0.5, 63.5, 63.5, -0.5)



0.1 Rate vs Distortion

[ ]:
```
R = [0.2, 0.4, 0.6, 0.8, 1]
D = []
P = 2
for r in R:
    compress_img, _ = compress(img, P, r)
    D.append(squared_error_distortion(img, compress_img))
```

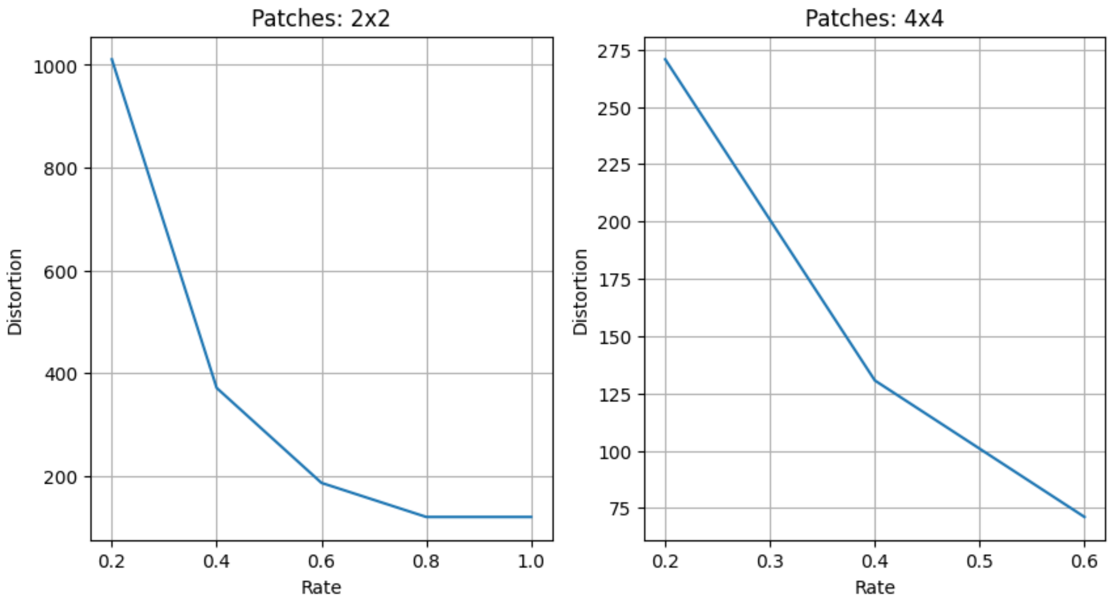
[ ]:
```
plt.plot(R, D)
plt.xlabel('Rate')
plt.ylabel('Distortion')
plt.title(f'Patches: {P}x{P}')
plt.grid()
```



Discussion As rate i.e., the average number of bits needed to encode each pixel increases, the distortion i.e., the loss in compression, reduces.

[ ]:
```
plt.figure(figsize=(10,5))
plt.subplot(121)

plt.plot(R, D)
plt.xlabel('Rate')
plt.ylabel('Distortion')
plt.title(f'Patches: {P}x{P}')
plt.grid()

plt.subplot(122)

R = [0.2, 0.4, 0.6]
D = []
P = 4
for r in R:
    compress_img, _ = compress(img, P, r)
    D.append(squared_error_distortion(img, compress_img))
```

```
plt.plot(R, D)
plt.xlabel('Rate')
plt.ylabel('Distortion')
plt.title(f'Patches: {P}x{P}')
plt.grid()
```



Discussion As per the formula, as the number of patches increases, number of cluster centers also increases and this causes less distortion. A smaller patch size captures the intricacies of the image and can fit better with other images as well. As patch size increases, the model overfits to that particular image. Hence, the distortion with respect to itself would be less. This wouldn't work well with other images. Distortion will be high when such a model is applied to other images. Ideally, one must find the best combination of patches to compress a set of images to obtain a model that delivers optimal/best performance for all the iamges of that set.

0.1.1 Entropy Coding

For patch size = 2, we obtain 8% reduction in coding rate.

[ ]:
```
_, r_2 = compress(img, 2, 1)
r_2
```

[ ]: 0.92

For a patch size = 4, we obtain 11.67% reduction in coding rate.

[ ]:
```
_, r_4 = compress(img, 4, 0.6)
r_4
```

[ ]: 0.53

0.2 K-means from scratch

[6]:
```
import matplotlib.pyplot as plt
from skimage.util import view_as_blocks
import numpy as np
import pandas as pd
import cv2
import time

path = "./test_image_3.jpg"
img = cv2.imread(path, cv2.IMREAD_GRAYSCALE)
```

[7]:
```
#function to calculate euclidean distance between points to cluster centers
def euclidean(point, data):
    return np.sqrt(np.sum((point - data)**2, axis=1))
```

[12]:
```
class kmeans:
    def __init__(self, n_clusters, max_iteration=20):
        self.num_clusters = n_clusters
        self.max_iter = max_iteration

    def fit(self, X):
        #taking min and max pixel values
        min, max = np.min(X, axis=0), np.max(X, axis=0)
        #randomly select n centroid points between min and max value␣
↪distributed uniformly
        self.centroids = [np.random.uniform(min, max) for _ in range(self.
↪num_clusters)]

        iteration = 0
        previous_centroids = None
        #condition to check if centroid points repeats then stop or iteration␣
↪more than max_ter then stop
        while np.not_equal(self.centroids, previous_centroids).any() and␣
↪iteration < self.max_iter:

            sorted_points = [[] for _ in range(self.num_clusters)]
            for x in X:
                dists = euclidean(x, self.centroids)
                centroid_idx = np.argmin(dists)                 #taking index␣
↪of minimum distance or nearest neighbor
                sorted_points[centroid_idx].append(x)
```

```
            previous_centroids = self.centroids                     #updating␣
↪centroid value in a list so that we can check if centroids are changing or␣
↪not
            self.centroids = [np.mean(cluster, axis=0) for cluster in␣
↪sorted_points]

            #to check cluster with no points/pixels
            for i, centroid in enumerate(self.centroids):
                if np.isnan(centroid).any():
                    self.centroids[i] = previous_centroids[i]
            iteration += 1
            print(iteration)

    def labels(self, X):
        centroid = []
        centroid_indxes = []
        for x in X:
            dists = euclidean(x, self.centroids)             #calculating the␣
↪distance between each points and final centroid value
            centroid_index = np.argmin(dists)
            centroid.append(self.centroids[centroid_index])
            centroid_indxes.append(centroid_index)           #labels of each␣
↪point
        return centroid_indxes, self.centroids
```

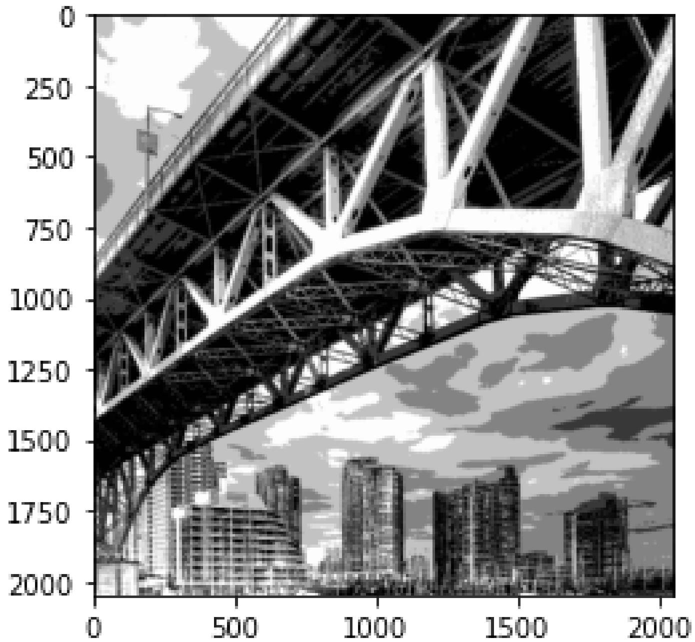
[13]:
```
def compress1(img, P, R):
    M, N = img.shape
    patches = view_as_blocks(img, (P, P))
    X = patches.reshape(M//P,N//P,1,P**2).squeeze().reshape((M//P)**2,1,P**2).
↪squeeze()
    bits_fr_encoded = int(R*(P**2))
    C = 2**bits_fr_encoded
    start_time = time.time()
    kmean = kmeans(n_clusters=C)
    kmean.fit(X)
    classification, points = kmean.labels(X)
    end_time = time.time()
    temp = np.zeros(X.shape, dtype=np.uint8)
    for i in range(len(classification)):
        temp[i] = points[classification[i]]
    temp = temp.reshape(M//P,N//P, 1,P**2).reshape(M//P,N//P,P,P)

    compress_img = np.zeros(img.shape, dtype=np.uint8)
    for i in range(temp.shape[0]):
        for j in range(temp.shape[1]):
            compress_img[i*P:P*(i+1), P*j:P*(j+1)] = temp[i][j]
```

```
    return compress_img, start_time, end_time
```

[ ]:
```
compress_img1, start_time, end_time = compress1(img, 4, 0.2)
print(end_time-start_time)
```

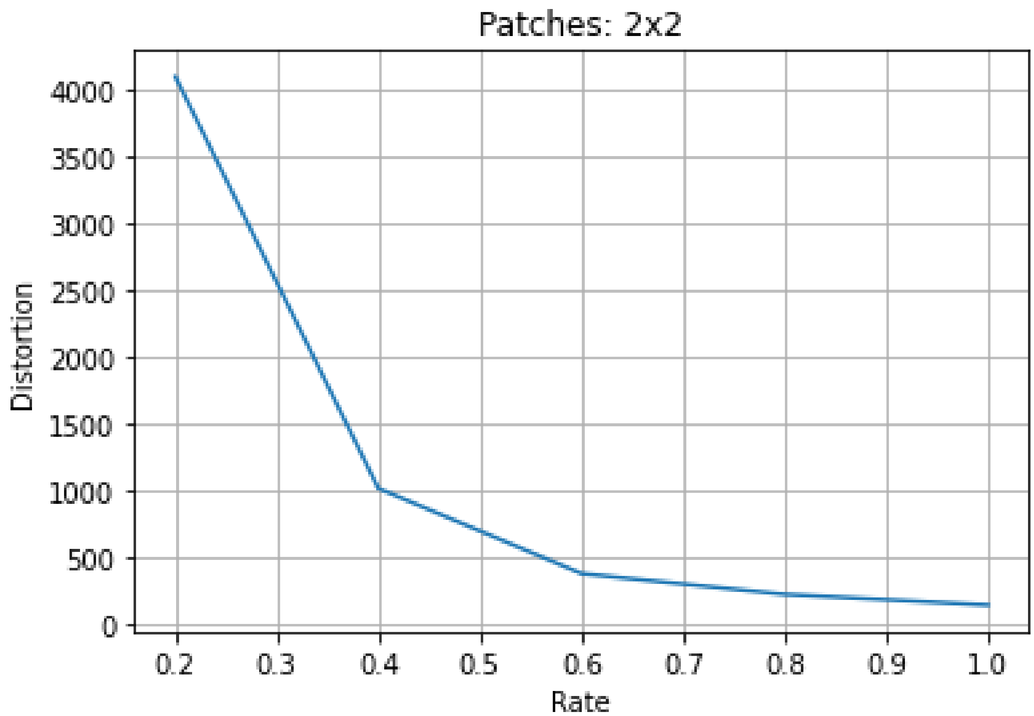
[15]:
```
plt.imshow(compress_img1, cmap='gray')
```

[15]: <matplotlib.image.AxesImage at 0x7f55166b4190>



[ ]:
```
R = [0.2, 0.4, 0.6, 0.8, 1]
D = []
P = 2
for r in R:
    compress_img, _, _ = compress1(img, P, r)
    D.append(squared_error_distortion(img, compress_img))
```

[21]:
```
plt.plot(R, D)
plt.xlabel('Rate')
plt.ylabel('Distortion')
plt.title(f'Patches: {P}x{P}')
plt.grid()
```



0.3 Another clustering algorithm

Fuzzy c means clustering

K mean is hard clustering but Fuzzy c means is a soft clustering.

In the case of the fuzzy c-means algorithm, the data point can belong to more than one cluster with a likelihood whereas in k means it belongs to only one cluster. Fuzzy c-means clustering gives comparatively better results for overlapped data sets.

[25]:
```
from fcmeans import FCM

def compress2(img, P, R):
    M, N = img.shape
    patches = view_as_blocks(img, (P, P))
    X = patches.reshape(M//P,N//P,1,P**2).squeeze().reshape((M//P)**2,1,P**2).
 ↪squeeze()
    bits_fr_encoded = int(R*(P**2))
    C = 2**bits_fr_encoded
    start_time = time.time()
    fcm = FCM(n_clusters=C)
    fcm.fit(X)
    fcm_centers = fcm.centers
    fcm_labels = fcm.predict(X)
    end_time = time.time()
```

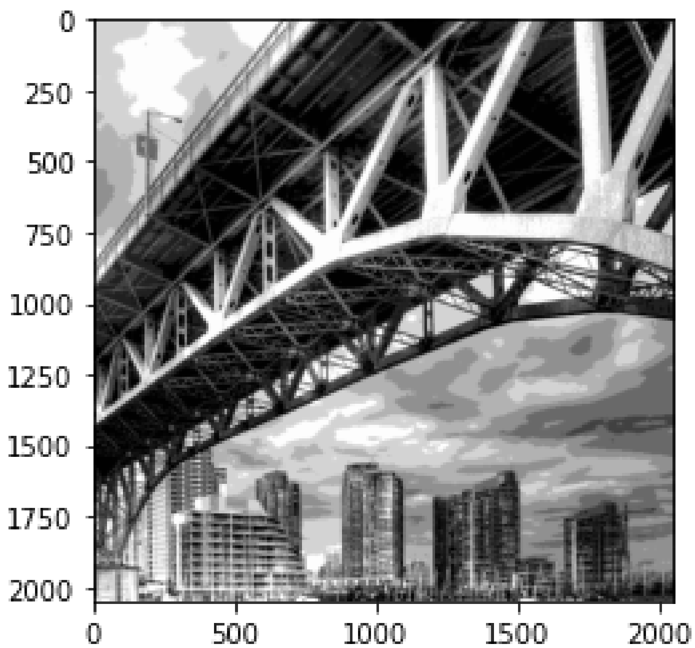
```python
    temp = np.zeros(X.shape, dtype=np.uint8)
    for i in range(len(fcm_labels)):
        temp[i] = fcm_centers[fcm_labels[i]]
    temp = temp.reshape(M//P,N//P, 1,P**2).reshape(M//P,N//P,P,P)

    compress_img = np.zeros(img.shape, dtype=np.uint8)
    for i in range(temp.shape[0]):
        for j in range(temp.shape[1]):
            compress_img[i*P:P*(i+1), P*j:P*(j+1)] = temp[i][j]

    return compress_img, start_time, end_time
```

[ ]:
```python
compress_img, start_time, end_time = compress2(img, 4, 0.2)
print(end_time-start_time)
```

47.111382484436035

[ ]:
```python
plt.imshow(compress_img, cmap='gray')
```

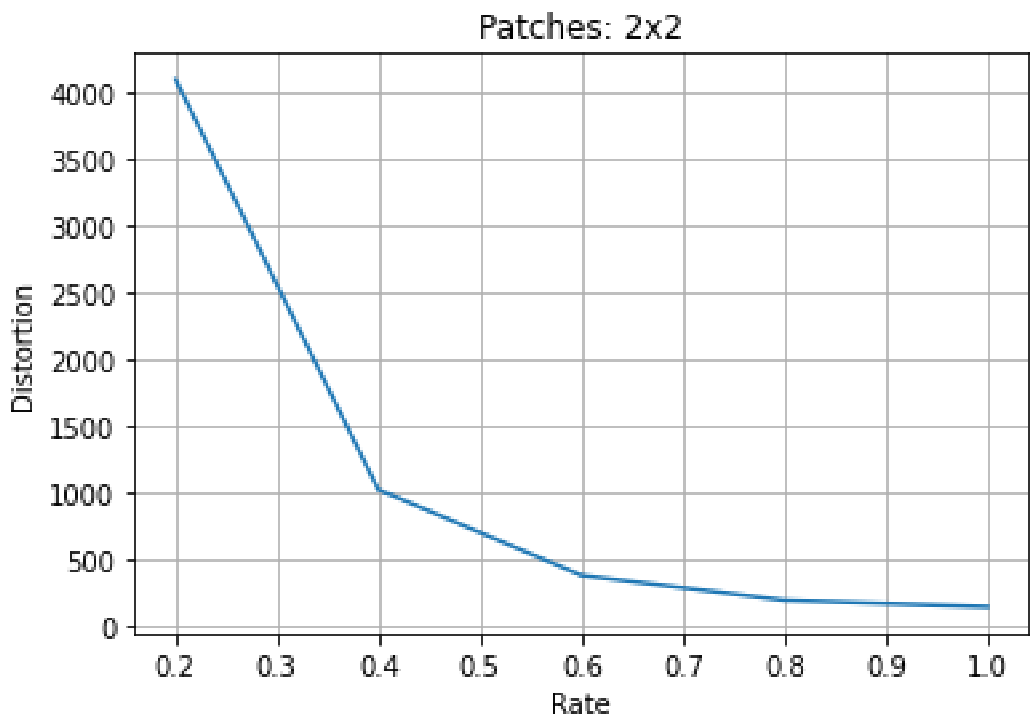
[ ]: <matplotlib.image.AxesImage at 0x7fecaa527690>



[27]:
```python
R = [0.2, 0.4, 0.6, 0.8, 1]
D = []
P = 2
for r in R:
```

```
    compress_img, _, _ = compress2(img, P, r)
    D.append(squared_error_distortion(img, compress_img))
```

[28]:
```
plt.plot(R, D)
plt.xlabel('Rate')
plt.ylabel('Distortion')
plt.title(f'Patches: {P}x{P}')
plt.grid()
```



There is a huge reduction in distortion as value of Rate change from 0.2 to 0.4 and so on. We can see that for the R value equal to 1 the distortion is close to 100. Its slightly better than Kmeans clustering technique

Run Time Comparison: (in seconds)

for P = 2 and R = 1:

1. in built k_means time: 32.736351013183594
2. k_means from scratch: 445.2068884372711
3. fcmeans: 33.11645817756653

for P = 4 and R = 0.2:

1. in built k_means time: 13.96139907836914
2. k_means from scratch: 126.62154030799866
3. fcmeans: 47.111382484436035

Time taken to run Fuzzy c means is more than kmeans