Number System:

1. Natural numbers (স্বাভাবিক সংখ্যা):

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$[\{0\} \cup \mathbb{N} \rightarrow \{0, 1, 2, 3, 4, \ldots\}]$

2. Integers or whole numbers (পূর্ণসংখ্যা):

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Z}^+ \rightarrow \{1, 2, 3, \ldots\} = \mathbb{N}$

3. Rational numbers (মূলদ সংখ্যা):

Numbers that can be expressed as quotient (ভগ্নাংশ).

$\mathbb{Q} = \left\{\frac{p}{q},\ p, q \in \mathbb{Z},\ q \neq 0\right\}$

$\frac{p}{q}$ → numerator, denominator

Example: $0.5 = \frac{5}{10}$, $5 = \frac{5}{1}$, $0.\dot{1} = 0.1111\ldots = \frac{1}{9}$ etc.

[Repeated decimal: $0.\dot{1} = \frac{1}{9}$, $0.\dot{2}\dot{3} = \frac{23}{99}$, $0.\dot{4}5\dot{6} = \frac{456}{999}$ etc]

Irrational numbers:

Numbers that can not be expressed as quotient of two integers such that the denominator is not zero.

Example: $e = 2.718281\ldots$, $\pi = 3.14159\ldots$, $\sqrt{5} = 2.236067\ldots$

Non-repeating, non-ending decimal numbers are also irrational numbers. Example: $0.303003000300003\ldots$

5. Real numbers:

The set of all rational and irrational numbers is the set of real numbers.

$\mathbb{R} = \mathbb{Q} \cup \mathbb{I} \rightarrow$ [irrational]

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Functions:

A function is a rule that takes certain numbers as inputs and assigns to each input exactly one output.

The set of all input numbers is called 'domain' and the set of resulting output numbers is called 'range' of the function.

The input is called independent variable and output is called dependent variable.

Exm: 1. $y = f(x) = x^2 + 2$

→ x is independent variable and y is the dependent variable.

→ y is the function of x.

2. $x = f(y) = y - 4$

→ y independent and x dependent.

→ x is the function of y.

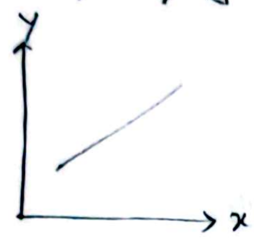
The rule of four:

Functions can be represented by tables, graphs, formulas and description in words.

1. Numerically by tables:

Year	1990	1991	1992	1993
Rain fall	30	25	45	50

2. Geometrically by graphs:



3. Algebraically by formulas:

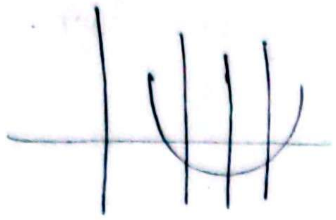
$$y = x^2 + 3$$

4. Verbally or description

Vertical line test:

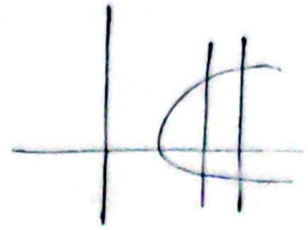
(To know if a graphs represents a function or not)

A curve in the xy plain is the graph of some function if and only if no vertical line intersects the curve more than once.



↓

function



↓

not function

[If not function, then the graph is called 'relation']

Given, $x = y^2$ ——— (i)

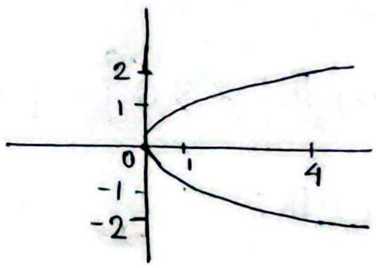
$\Rightarrow y = \pm\sqrt{x}$

Here,

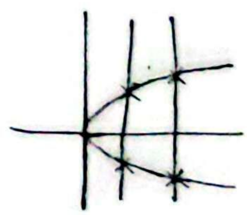
x	0	1	4
y	0	± 1	± 2

Since for each input we are not getting $\boxed{\text{exactly one}}$ output, so (i) is not a function.

Now, the graph will be:



If we perform vertical line test, we see that the lines intersect the curve more than once.



So, it is confirmed that (i) is not a function.

Intervals:

#

(a,b)

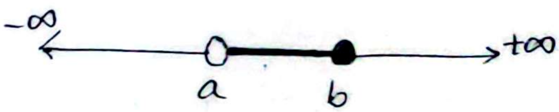
$= \{x \in \mathbb{R} : a < x < b\}$

#

$[a,b]$

$= \{x \in \mathbb{R} : a \leq x \leq b\}$

#



$(a, b]$

$= \{x \in \mathbb{R} : a < x \leq b\}$

#

$[a, b)$

$= \{x \in \mathbb{R} : a \leq x < b\}$

#

$(-\infty, +\infty)$

$= \mathbb{R}$

#

$(a, +\infty)$

#

$(-\infty, b]$

Domain and Range:

$f(x)=x$

$\Rightarrow y=x$

dom $f=\mathbb{R}=(-\infty,+\infty)$

Range $f=\mathbb{R}=(-\infty,+\infty)$

$f(x)=x^2$

$\Rightarrow y=x^2$

dom $f=\mathbb{R}=(-\infty,+\infty)$

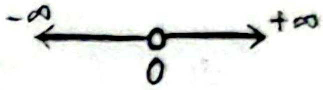
Range $f=[0,+\infty)$

$=\{0\}\cup\mathbb{R}^+$

$=\{y\in\mathbb{R}: y\geqslant 0\}$

$f(x)=\frac{1}{x} \Rightarrow y=\frac{1}{x}$

For domain, $x\neq 0$



$\therefore$ dom $f=\mathbb{R}\setminus\{0\}=\{x\in\mathbb{R}: x\neq 0\}=(-\infty,0)\cup(0,+\infty)$

Also,

Range $f=\mathbb{R}\setminus\{0\}=\{y\in\mathbb{R}: y\neq 0\}=(-\infty,0)\cup(0,+\infty)$

$f(x)=\sqrt{x} \Rightarrow y=\sqrt{x}$

dom $f=\{x\in\mathbb{R}: x\geqslant 0\}=\{0\}\cup\mathbb{R}^+=[0,+\infty)$

Range $f=\{y\in\mathbb{R}: y\geqslant 0\}=\{0\}\cup\mathbb{R}^+=[0,+\infty)$

$f(x)=-\sqrt{x} \Rightarrow y=-\sqrt{x}$

dom $f=\{x\in\mathbb{R}: x\geqslant 0\}=\{0\}\cup\mathbb{R}^+=[0,+\infty)$

Range $f=\{y\in\mathbb{R}: y\leqslant 0\}=\{0\}\cup\mathbb{R}^-=(-\infty,0]$

Example 6. Find the natural domain of

(a) $f(x)=x^3$

(b) $f(x)=\dfrac{1}{(x-1)(x-3)}$

(c) $f(x)=\tan x$

(d) $f(x)=\sqrt{x^2-5x+6}$

Soln:

(a) $f(x)=x^3$

$\therefore \operatorname{dom} f = \mathbb{R} = (-\infty,+\infty)$

[Also, Range $f = \mathbb{R} = (-\infty,+\infty)$]

(b) $f(x)=\dfrac{1}{(x-1)(x-3)}$

Here, $(x-1)(x-3) \neq 0$

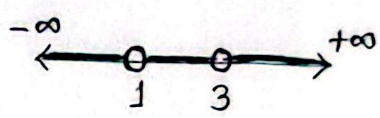
$\Rightarrow (x-1) \neq 0 \quad , \quad (x-3) \neq 0$

$\Rightarrow x \neq 1 \qquad\qquad \Rightarrow x \neq 3$

$\therefore \operatorname{dom} f = \{x \in \mathbb{R} : x \neq 1 \text{ and } x \neq 3\}$

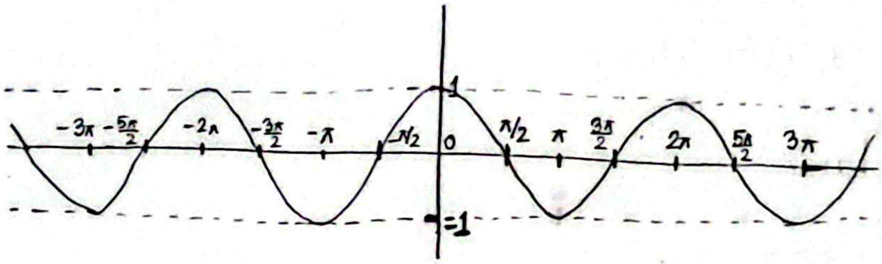
$= \mathbb{R} \setminus \{1,3\}$

$= (-\infty,1) \cup (1,3) \cup (3,+\infty)$



(c) $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

So, $\cos x \neq 0$



When $\cos x \neq 0$

$\Rightarrow x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\ldots$ [x is not an odd integer multiple of $\pi/2$]

$\therefore \operatorname{dom} f = \{x \in \mathbb{R} : x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\}$

$= \{x \in \mathbb{R} : x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$ and n is odd$\}$

$= \{x \in \mathbb{R} : x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}\}$

(d) $f(x) = \sqrt{x^2 - 5x + 6}$

$x^2 - 5x + 6 \geqslant 0$

$\Rightarrow x^2 - 3x - 2x + 6 \geqslant 0$

$\Rightarrow x(x-3) - 2(x-3) \geqslant 0$

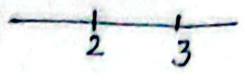
$\Rightarrow (x-3)(x-2) \geqslant 0$

So, $(x-3)(x-2) = 0$ or, $(x-3)(x-2) > 0$

Now, $(x-3)(x-2) = 0$

$\Rightarrow x = 2, 3$

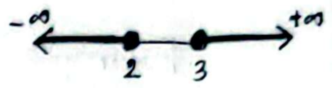
Again, $(x-3)(x-2)>0$



Intervals	Sign of $(x-3)(x-2)$	
$x<2$	$- - \Rightarrow +$	✓
$2<x<3$	$- + \Rightarrow -$	
$x>3$	$+ + \Rightarrow +$	✓

$\therefore$ dom $f = \{x \in \mathbb{R} : x \leq 2 \text{ or } x \geq 3\}$

$= (-\infty, 2] \cup [3, +\infty)$



Example 7.

Find the natural domain of $f(x) = \dfrac{x^2-4}{x-2}$.

Sol^n: $f(x) = \dfrac{x^2-4}{x-2}$

$\therefore x-2 \neq 0 \Rightarrow x \neq 2$

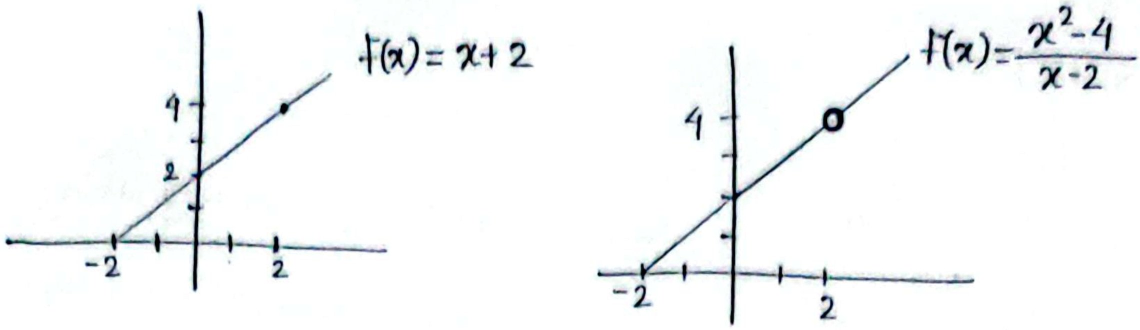
$\therefore$ dom $f = \{x \in \mathbb{R} : x \neq 2\} = \mathbb{R} \setminus \{2\}$

Now,

$$f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{(x-2)} = x+2$$

This simplified function has value at $x=2$, which is, $f(2) = 2+2 = 4$.

But the original function does not have value at $x=2$, i.e., $f(2)$ is undefined.



So, if we simplify we should write,

$f(x) = x + 2, \; x \neq 2.$

Example 8. Find the domain and range of

(a) $f(x) = 2 + \sqrt{x-1}$ (b) $f(x) = (x+1)/(x-1)$

Solⁿ:

(a) $f(x) = 2 + \sqrt{x-1} \Rightarrow y = 2 + \sqrt{x-1}$

For domain, $x - 1 \geq 0$

$\Rightarrow x \geq 1$

$\therefore$ dom $f = [1, +\infty)$

For range, $x \geq 1$

$\Rightarrow x - 1 \geq 0$

$\Rightarrow \sqrt{x-1} \geq 0$

$\Rightarrow 2 + \sqrt{x-1} \geq 2$

$\Rightarrow f(x) \geq 2 \Rightarrow y \geq 2$

$\therefore$ Range $f = [2, +\infty)$

(b) $f(x) = \frac{x+1}{x-1}$

$\Rightarrow y = \frac{x+1}{x-1}$

For domain, $x-1 \neq 0 \Rightarrow x \neq 1$

$\therefore$ dom $f = \{x \in \mathbb{R} : x \neq 1\} = (-\infty, 1) \cup (1, +\infty) = \mathbb{R} \setminus \{1\}$

For range, $y = \frac{x+1}{x-1}$

$\Rightarrow xy - y = x + 1$

$\Rightarrow xy - x = 1 + y$

$\Rightarrow x(y-1) = y+1$

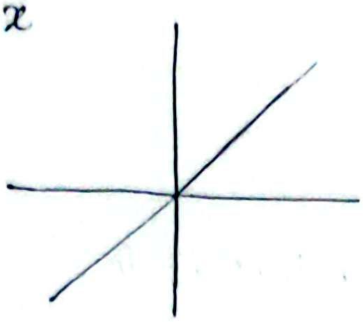
$\therefore x = \frac{y+1}{y-1}$

So, $y - 1 \neq 0 \Rightarrow y \neq 1$

$\therefore$ Range $f = \{y \in \mathbb{R} : y \neq 1\} = (-\infty, 1) \cup (1, +\infty) = \mathbb{R} \setminus \{1\}$

Graphical representation of functions:

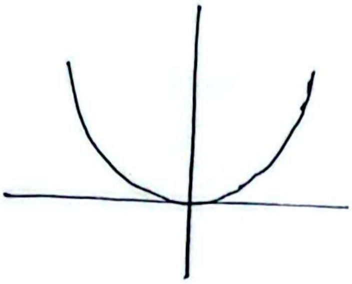
1) $y=x$



dom $=\mathbb{R}$.

range $=\mathbb{R}$

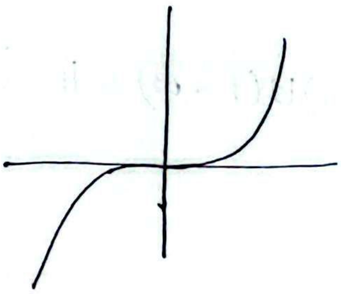
2) $y=x^2$



dom $=\mathbb{R}$

range $=[0,+\infty)$

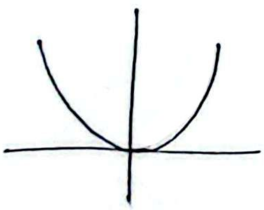
3) $y=x^3$



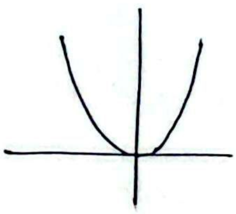
dom $=\mathbb{R}$

range $=\mathbb{R}$

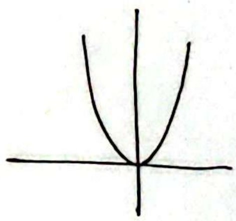
4) $y=x^{\text{even}}$



$y=x^2$



$y=x^4$

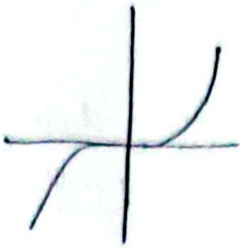


$y=x^6$

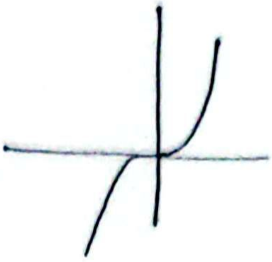
dom $=\mathbb{R}$

range $=[0,+\infty)$

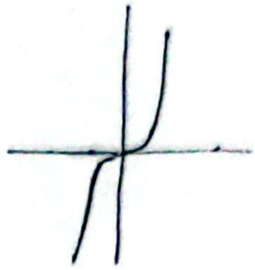
5) $y = x^{odd}$



$y = x^3$



$y = x^5$

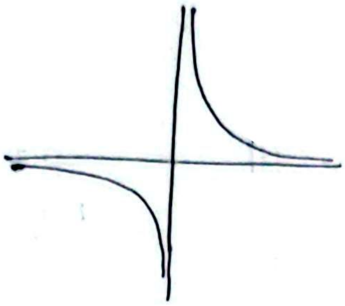


$y = x^7$

dom $= \mathbb{R}$

range $= \mathbb{R}$

6) $y = \frac{1}{x}$



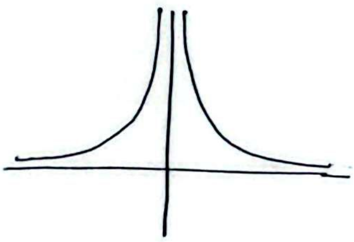
dom $= \mathbb{R} - \{0\}$

range $= \mathbb{R} - \{0\}$

vertical asymptote : $x = 0$ (y axis)

horizontal asymptote : $y = 0$ (x axis)

7) $y = \frac{1}{x^2}$



dom $= \mathbb{R} - \{0\}$

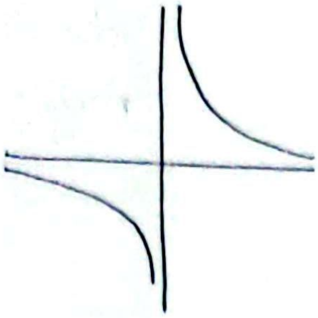
range $= (0, +\infty)$

vertical asymptote : $x = 0$ (y axis)

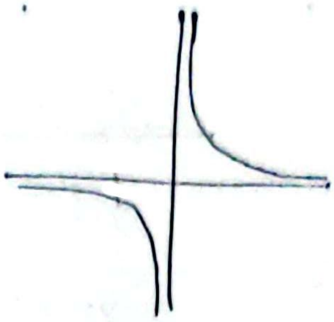
horizontal asymptote : $y = 0$ (x axis)

[Asymptote:
An asymptote is a line that a graph of a function approaches but never actually touches or crosses.]

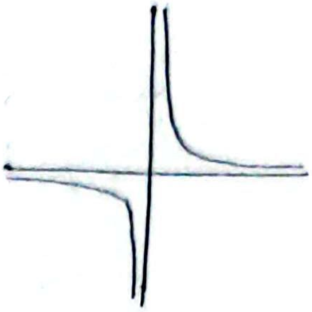
8) $y = \frac{1}{x^{odd}}$



$y = \frac{1}{x}$



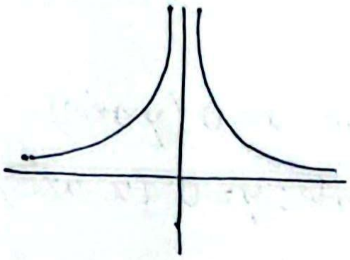
$y = \frac{1}{x^3}$



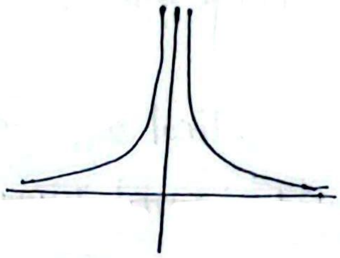
$y = \frac{1}{x^5}$

dom $= \mathbb{R} - \{0\}$
range $= \mathbb{R} - \{0\}$

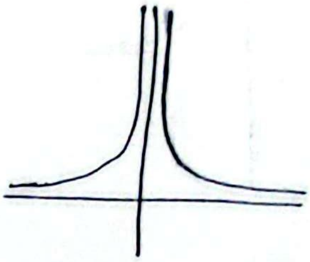
9) $y = \frac{1}{x^{even}}$



$y = \frac{1}{x^2}$



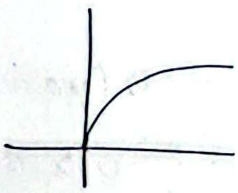
$y = \frac{1}{x^4}$



$y = \frac{1}{x^6}$

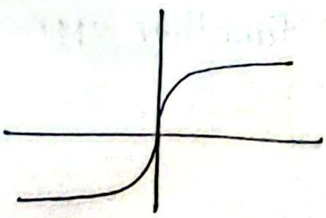
dom $= \mathbb{R} - \{0\}$
range $= (0, +\infty)$

10) $y = \sqrt{x}$



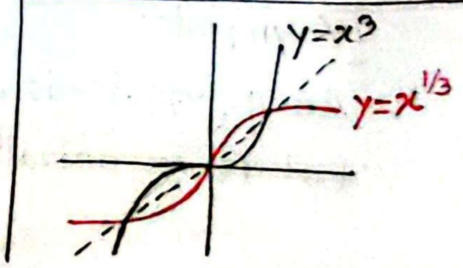
dom $= [0, +\infty)$
range $= [0, +\infty)$

11) $y = \sqrt[3]{x} = x^{1/3}$



dom $= \mathbb{R}$
range $= \mathbb{R}$

NB

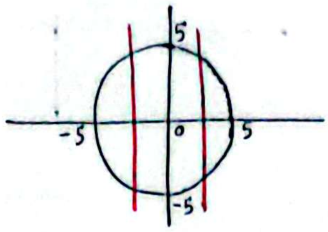


General equation of circle: $(x-h)^2+(y-k)^2=r^2$

center (h,k)

radius r

$\therefore x^2+y^2=25 \rightarrow$ equation of circle with center $(0,0)$ and radius 5.



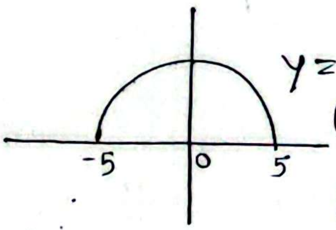
$\rightarrow$ Fails vertical line test.

$\therefore$ Not a function.

Now, $x^2+y^2=25$

$\Rightarrow y^2=25-x^2$

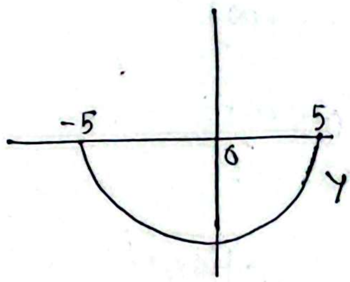
$\Rightarrow y=\pm\sqrt{25-x^2}$.



$y=\sqrt{25-x^2}$ (Half circle) or (semi circle)

$\downarrow$

Function



$y=-\sqrt{25-x^2}$ (Half circle) or (semi circle)

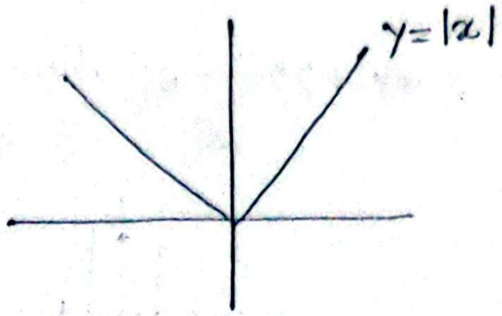
$\downarrow$

Function

Absolute Value Function:

The function $f(x) = |x|$ is called absolute value function.

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

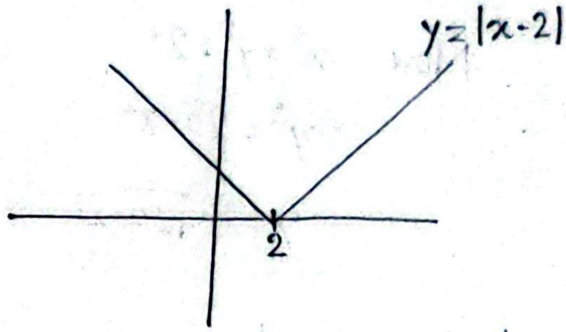


Dom $f = (-\infty, +\infty)$

Range $f = [0, +\infty)$

$f(x) = |x-2| = \begin{cases} x-2, & (x-2) \geq 0 \\ -(x-2), & (x-2) < 0 \end{cases}$

$$= \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$$



Dom $f = (-\infty, +\infty)$

Range $f = [0, +\infty)$

Piecewise Function:

$$f(x) = \begin{cases} \text{Expression } 1, & \text{Condition } 1 \\ \text{Expression } 2, & \text{Condition } 2 \\ \vdots & \vdots \\ \text{Expression } n, & \text{Condition } n \end{cases}$$

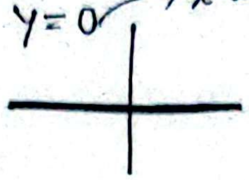
* $f(x) = |x|$ is an example of piecewise function.

Example 4. Sketch the graph of the function defined piecewise by the formula

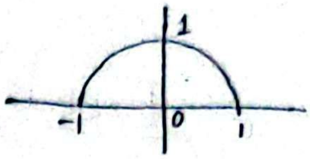
$$f(x)=\begin{cases} 0 & , \quad x\leqslant -1 \\ \sqrt{1-x^2} & , \quad -1<x<1 \\ x & , \quad x\geqslant 1 \end{cases}$$

Sol^n:

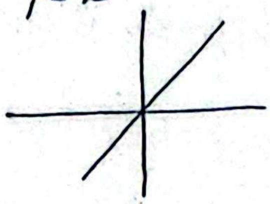
$y=0 \rightarrow$ x-axis



$y=\sqrt{1-x^2} \rightarrow$ semi-circle with radius 1



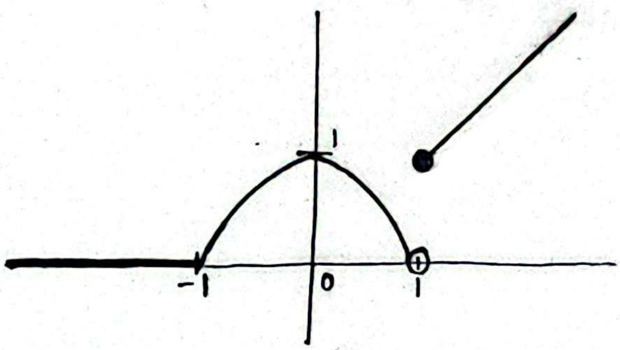
$y=x$



Graph:

Here, the formula for f changes at the points $x=-1$ and $x=1$. These are called the breakpoints.

$y=0$ is on the interval $(-\infty,-1]$, $y=\sqrt{1-x^2}$ is on the interval $(-1,1)$, $y=x$ is on the interval $[1,+\infty)$



$\rightarrow$ Domain: $(-\infty,+\infty)$

Range: $[0,+\infty)$

$$\# \; f(x) = \begin{cases} 0 \;, & x \leq -2 \\ x^2+1 \;, & -2 < x < 7 \\ \sqrt{x} \;, & x \geq 7 \end{cases}$$

$\therefore f(1) = (1)^2 + 1 = 2$

$f(-10) = 0$

$f(7) = \sqrt{7}$

Dom $f = \mathbb{R}$

Exercise 0.1 → 1,3,7,9

1. Use the accompanying graph to answer the following questions, making reasonable approximations where needed.

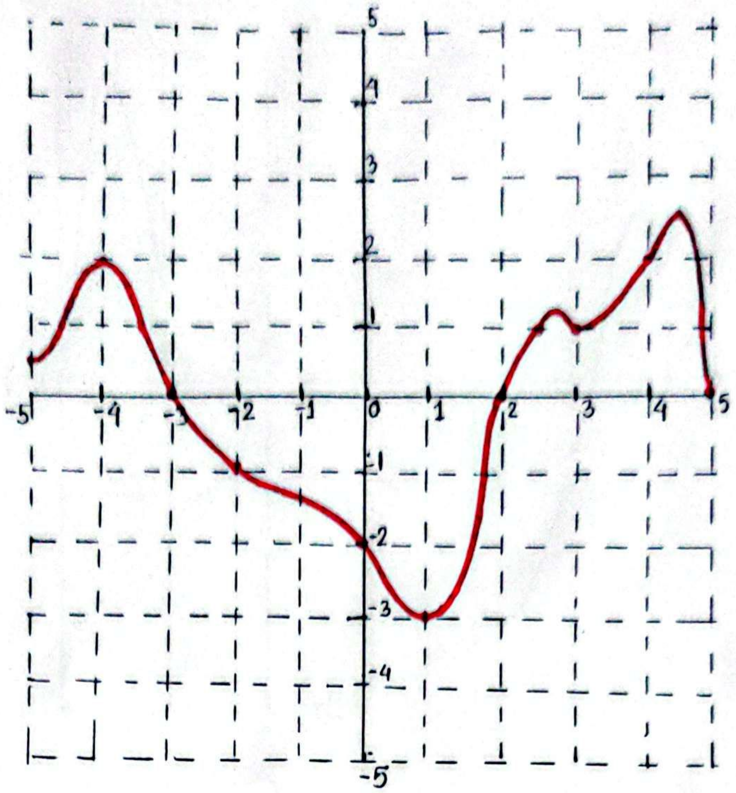
(a) For what values of x is $y = 1$?

(b) " " " " x is $y = 3$?

(c) " " " " y is $x = 4$?

(d) " " " " x is $y \leq 0$?

(e) What are the maximum and minimum values of y, for what values of x do they occur?



Sol^n:

(a) $y = 1$ at $x = -4.5, -3.5, 2.5, 3, 4.9$

(b) None

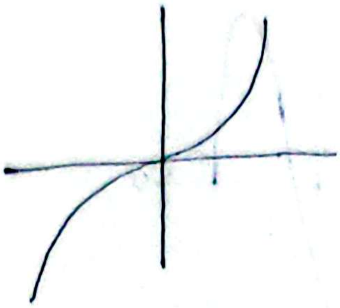
(c) $x = 4$ at $y = 2$

(d) $y \leqslant 0$ at $-3 \leqslant x \leqslant 2$, also $y = 0$ at $x = 5$.

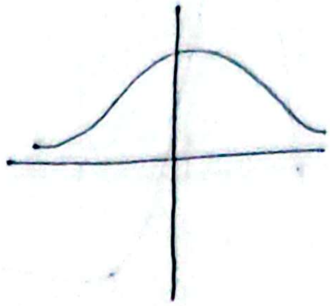
(e) $y_{max} = 2.5$ at $x = 4.5$.

$y_{min} = -3$ at $x = 1$.

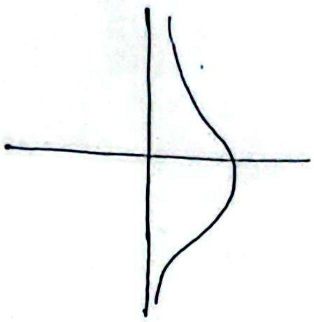
3. In each part of the accompanying figure, determine whether the graph defines y as a function of x.



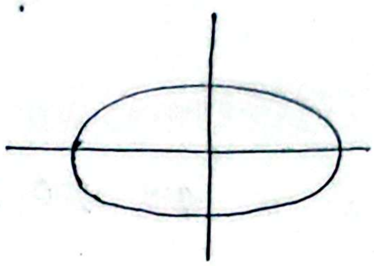
(a)



(b)

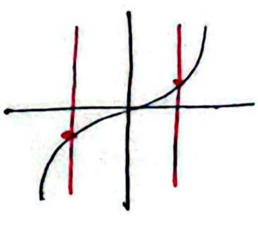


(c)



(d)

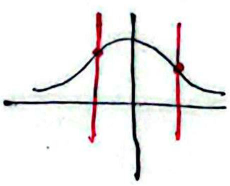
Soln: Using vertical line test:



(a)

↓

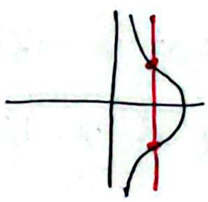
Function



(b)

↓

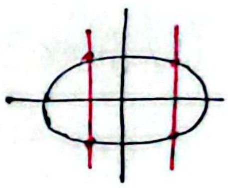
Function



(c)

↓

Not Function



(d)

↓

Not Function

7. Find $f(0)$, $f(2)$, $f(-2)$, $f(3)$, $f(\sqrt{2})$ and $f(3t)$

(a) $f(x) = 3x^2 - 2$ (b) $f(x) = \begin{cases} 1/x, & x > 3 \\ 2x, & x \leq 3 \end{cases}$

Soln:

(a) $f(x) = 3x^2 - 2$

$f(0) = 3.0^2 - 2 = -2$

$f(2) = 3.2^2 - 2 = 10$

$f(-2) = 3.(-2)^2 - 2 = 10$

$f(3) = 3.3^2 - 2 = 25$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$

$f(3t) = 3.(3t)^2 - 2 = 27t^2 - 2$

(b) $f(x) = \begin{cases} 1/x, & x > 3 \\ 2x, & x \leq 3 \end{cases}$

$f(0) = 2.0 = 0$

$f(2) = 2.2 = 4$

$f(-2) = 2(-2) = -4$

$f(3) = 2.3 = 6$

$f(\sqrt{2}) = 2.\sqrt{2} = 2\sqrt{2}$

$f(3t) = \begin{cases} 1/3t, & 3t > 3 \\ 2.3t, & 3t \leq 3 \end{cases}$

$= \begin{cases} \frac{1}{3t}, & t > 1 \\ 6t, & t \leq 1 \end{cases}$

9. Find the natural domain and determine the range of each function.

(a) $f(x) = \frac{1}{x-3}$

(b) $F(x) = \frac{x}{|x|}$

(c) $g(x) = \sqrt{x^2-3}$

(d) $G(x) = \sqrt{x^2-2x+5}$

(e) $h(x) = \frac{1}{1-\sin x}$

(f) $H(x) = \sqrt{\frac{x^2-4}{x-2}}$

Solⁿ:

(a) $f(x) = \frac{1}{x-3}$

Domain: $x-3 \neq 0 \Rightarrow x \neq 3$

$\therefore \text{dom} f = \mathbb{R} \setminus \{3\}$

Range: $\mathbb{R} \setminus \{0\}$

(b) $F(x) : \frac{x}{|x|}$

We know, $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain: $|x| \neq 0 \Rightarrow x \neq 0$

$\therefore \text{Dom} F = \mathbb{R} \setminus \{0\}$.

Range: $\frac{x}{|x|} = \begin{cases} x/x, & x > 0 \\ x/-x, & x < 0 \end{cases} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ [Not $x \geq 0$ since $x = 0$ is not in the domain]

$\therefore$ Range $F = \{1, -1\}$.

(c) $g(x)=\sqrt{x^2-3}$

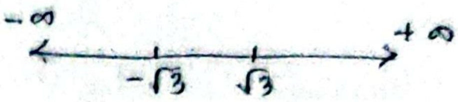
Domain: $x^2-3 \geq 0$

$\Rightarrow (x+\sqrt{3})(x-\sqrt{3}) \geq 0$

$\Rightarrow (x+\sqrt{3})(x-\sqrt{3}) > 0$ or $(x+\sqrt{3})(x-\sqrt{3}) = 0$

Now, $(x+\sqrt{3})(x-\sqrt{3}) = 0 \Rightarrow x = \sqrt{3}, -\sqrt{3}$

Again, for $(x+\sqrt{3})(x-\sqrt{3}) > 0$,



$\boxed{\sqrt{3} = 1{\cdot}732\ldots}$

Intervals	Sign of $(x+\sqrt{3})(x-\sqrt{3})$	
$x < -\sqrt{3}$	$(-)(-) \Rightarrow (+)$	✓
$-\sqrt{3} < x < \sqrt{3}$	$(+)(-) \Rightarrow (-)$	
$x > \sqrt{3}$	$(+)(+) \Rightarrow (+)$	✓

$\therefore$ dom $g = \{x \in \mathbb{R} : x \leq -\sqrt{3}$ or $x \geq \sqrt{3}\} = (-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$

Range: Range $g = [0, +\infty)$

[NB:

Usually,

If the expression under the square root has real roots (i.e. it can be zero), then range is $[0, +\infty)$.

If the expression under the square root has no real roots, then the range is $[\sqrt{\min}, +\infty)$.]

(d) $G(x) = \sqrt{x^2 - 2x + 5}$

$= \sqrt{x^2 - 2.x.1 + 1^2 + 5 - 1}$

$= \sqrt{(x-1)^2 + 4}$

Note:
$x^2 - 2x + 5 \geqslant 0$
→ middle term factorization is not possible.

Also, $x^2 - 2x + 5 = 0$
$\Rightarrow x = 1 \pm 2i$
→ complex roots

For domain, The condition that needs to be satisfied is $(x-1)^2 + 4 \geqslant 0$.

We know, the value of square function is always $\geqslant 0$.

So, $(x-1)^2 \geqslant 0$

Hence, $(x-1)^2 + 4 \geqslant 4$ —— (i)

∴ Automatically the value of $(x-1)^2+4$ is greater than or equal to 0.

So, For all x, $(x-1)^2 + 4 \geqslant 0$

Hence, $\text{dom}\, G = \mathbb{R}$

Again, for range, From (i),

$(x-1)^2 + 4 \geqslant 4$

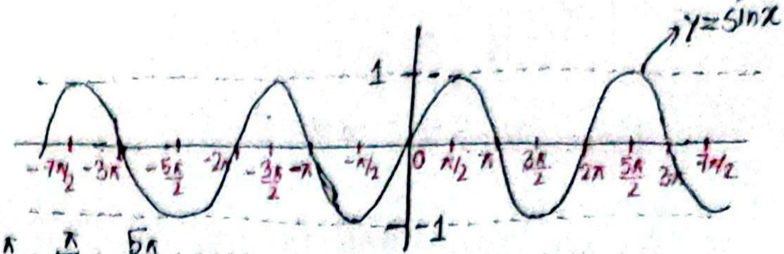
$\Rightarrow \sqrt{(x-1)^2 + 4} \geqslant 2$ → [Not ±2, because the output of a square root function is non-negative.]

$\Rightarrow G(x) \geqslant 2$

∴ Range $G = [2, +\infty)$

(c) $h(x) = \dfrac{1}{1-\sin x}$

So, $1 - \sin x \neq 0$



$\Rightarrow \sin x \neq 1$

$\Rightarrow x \neq \ldots -\frac{7\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$

$\Rightarrow x \neq (2n+\frac{1}{2})\pi, \ n = \mathbb{Z}$

$\therefore \text{dom } h = \{x \in \mathbb{R} : x \neq \ldots -\frac{7\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \cdots\}$

$= \{x \in \mathbb{R} : x \neq (2n+\frac{1}{2})\pi, n \in \mathbb{Z}\}$

$= \{x \in \mathbb{R} : x \neq 2n\pi + \frac{\pi}{2}, n \in \mathbb{Z}\}$

Again,

Range of $\sin x$ is $[-1, 1]$

So, $-1 \leqslant \sin x \leqslant 1$

$\Rightarrow 1 \geqslant -\sin x \geqslant -1$

$\Rightarrow 2 \geqslant 1 - \sin x \geqslant 0$

$\Rightarrow \frac{1}{2} \leqslant \frac{1}{1-\sin x} \leqslant \frac{1}{0}$

$\Rightarrow \frac{1}{2} \leqslant h(x) \leqslant \frac{1}{0}$ and Hence, $h(x) \geqslant \frac{1}{2}$

$\therefore$ Range $h = [\frac{1}{2}, \infty)$

f) $H(x) = \sqrt{\dfrac{x^2-4}{x-2}}$

Here, $x-2 \neq 0 \Rightarrow x \neq 2$

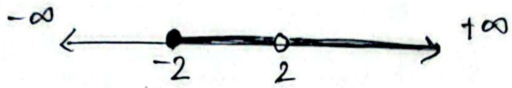
So, we can simplify as,

$$H(x) = \sqrt{\frac{(x+2)(x-2)}{(x-2)}} = \sqrt{x+2}\ ,\ x \neq 2$$

Now, $H(x) = \sqrt{x+2}, x \neq 2$

So, $x+2 \geq 0$

$\Rightarrow x \geq -2$



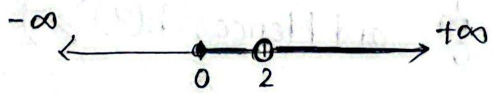
$\therefore$ dom $H = [-2, 2) \cup (2, +\infty)$

Again,

Range of $\sqrt{x+2}$ is $[0, +\infty)$. But we must exclude $x=2$.

When $x=2$, $H(x) = \sqrt{2+2} = \sqrt{4} = 2$

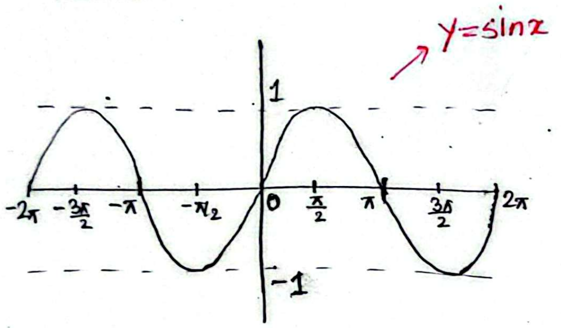
So, $y = H(x) = 2$ will be excluded from the range.

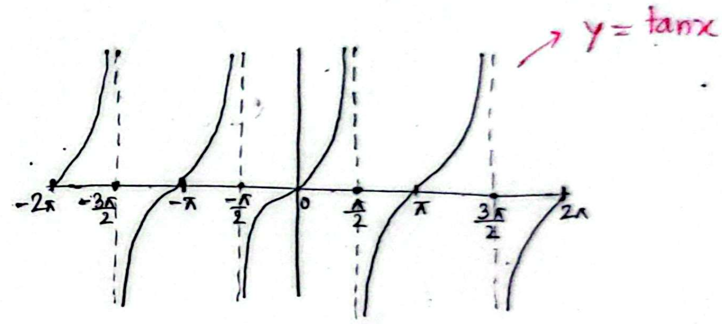


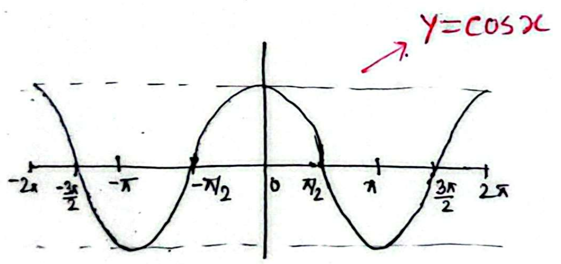
$\therefore$ Range $H = [0, 2) \cup (2, +\infty)$

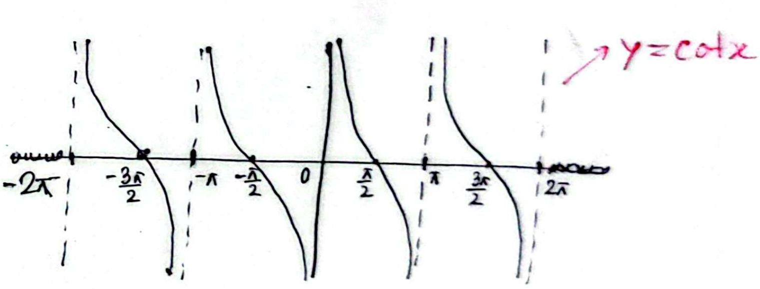
# Functions	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\{x \in \mathbb{R} : x \neq (2n+1)\pi/2, n \in \mathbb{Z}\}$	$\mathbb{R}$
$\cot x$	$\{x \in \mathbb{R} : x \neq n\pi, n \in \mathbb{Z}\}$	$\mathbb{R}$
$\sec x$	$\{x \in \mathbb{R} : x \neq (2n+1)\pi/2, n \in \mathbb{Z}\}$	$(-\infty, -1] \cup [1, +\infty)$
$\operatorname{cosec} x$	$\{x \in \mathbb{R} : x \neq n\pi, n \in \mathbb{Z}\}$	$(-\infty, -1] \cup [1, +\infty)$

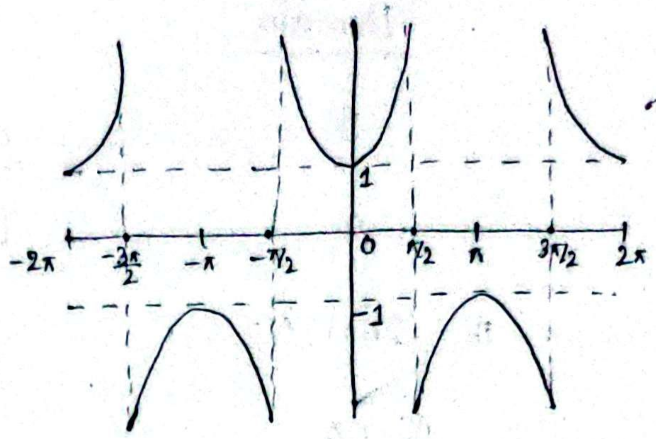
Graphs:



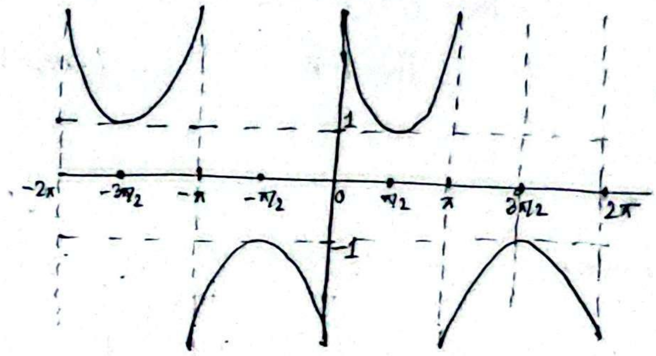








→ $y = \sec x$



→ $y = \operatorname{cosec} x$