Shahir Ghani

2.

2. Given the following coordinates for measurement points, solve the problem of fitting a line to these points using linear least square estimation. You are to solve the problem **two** ways: (1) by hand, (2) with a Python program. Be sure to set up the problem in the form $Aq = b$ by defining A and b. Then compute the parameter vector q using least squares estimation. *Note: Fill in the sections of the Problem_2.ipynb file and submit.*

Points

x	y
-1.1	8.6
2.3	2.9
5.8	-5.8
-3.5	13.0
3.0	0.1
8.5	-9.9

$$A = \begin{bmatrix} -1.1 & 1 \\ 2.3 & 1 \\ 5.8 & 1 \\ -3.5 & 1 \\ 3.0 & 1 \\ 8.5 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 8.6 \\ 2.9 \\ -5.8 \\ 13.0 \\ .1 \\ -9.9 \end{bmatrix}$$

$$(A^TA) = \begin{bmatrix} 133.64 & 15 \\ 15 & 6 \end{bmatrix} \quad (A^TA)^{-1} = \frac{1}{133.64(6) - 15^2} \begin{bmatrix} 6 & -15 \\ -15 & 133.64 \end{bmatrix}$$

$$(A^TA)^{-1}A^T = \begin{bmatrix} -0.037 & -0.002 & 0.034 & -0.062 & 0.005 & 0.062 \\ 0.260 & 0.171 & 0.080 & 0.322 & 0.153 & 0.010 \end{bmatrix}$$

$$q = (A^TA)^{-1}A^Tb = [-1.95, 6.37]$$

3. As we discussed in class the following are basis vectors for $\mathbb{R}^3$ (3D Cartesian coordinate system).

$$w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, w_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Are the following vectors basis vectors for $\mathbb{R}^3$? Why or why not?

$$w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, w_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The following vectors do form a basis since they reduce to I

4. Do the following vectors form a basis for $\mathbb{R}^3$? Why or why not?

$$t_1 = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix}, t_2 = \begin{bmatrix} 0.707 \\ 0.707 \\ 0 \end{bmatrix}, t_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} .707 & .707 & 0 \\ -.707 & .707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xRightarrow{r_2 = r_2 + r_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

These vectors also form a basis in $\mathbb{R}^3$ since they reduce to I which we know forms a basis

5. Do the following vectors span $\mathbb{R}^3$? Why or why not? Do these vectors form a basis?

$$w_1 = \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{matrix}1\\2\\3\end{matrix}, w_2 = \begin{bmatrix}1\\1\\0\end{bmatrix} \begin{matrix}-1\\2\\3\end{matrix}, w_3 = \begin{bmatrix}0\\1\\1\end{bmatrix} \begin{matrix}0\\3\\3\end{matrix}, w_4 = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$w_4 = 3w_1 - 2w_2 + 2w_1$ The followins vectors do span $\mathbb{R}^3$ since you can form any vector in $\mathbb{R}^3$ from w_1, w_2, w_3 but they do not form a basis since w_4 is linearly dependent