Cs501
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Homework5- week5

34)==> Prove by mathematical induction: The sum of consecutive cubes

Q-34

Mathematical induction : The sum of consecutive cubes

$$1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+3+\dots+n)^2$$

Initial: $f(1) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2 * (n+1)^2}{4} = 1$

Assume $f(n) = \frac{n^2 * (n+1)^2}{4}$

Prove $f(n+1) = \frac{(n+1)^2 * (n+2)^2}{4}$

$$f(n+1) = 1^3 + 2^3 + 3^3 + \dots + n^3 + (n+1)^3$$

$$= f(n) + (n+1)^3$$

$$= \frac{n^2 (n+1)^2}{4} + (n+1)^3$$

$$= \frac{1}{4}\left[n^2 (n+1)^2 + 4(n+1)^3\right]$$

$$= \frac{(n+1)^2}{4}\left[n^2 + 4(n+1)\right]$$

$$= \frac{(n+1)^2}{4}\left(n^2 + 4n + 4\right)$$

$$= \frac{(n+1)^2 (n+2)^2}{4}$$

Q32 ==> Please use Substitution method to find Big-O of this recursion

Q-32 Substitution Method

$T(n) = 0$ if $n = 1$

$T(n) = T(n-1) + n - 1$ if $n \geq 2$

$T(n) = T(n-1) + n - 1$

$T(n-1) = (T(n-1-1)] + (n-2) + (n-1)$

$T(n-2) = (T(n-2-1)) + (n-3) + (n-2) + (n-1)$

$T(n-3) = (T(n-3-1)) + (n-4) + (n-3) + (n-2) + (n-1)$

$= T(n-4) + (n-4) + (n-3) + (n-2) + (n-1)$

$\vdots$

~~T(n-K~~

$T(1) = 0 \Rightarrow O(1)$

$n - K = 1$

$K = n - 1$

$= T(n-K) + (n-K) + (n-(K-1)) + (n-(K-2)) + \ldots (n-1)$

$= T(n-(n-1)) + (n-(n-1)) + (n-(n-2) + \cdots (n-1)$

$= T(1) + 1 + 2 + 3 + \cdots + (n-1)$

$= 0 + (1 + 2 + 3 + \cdots + n) - 1$

$= \frac{n(n+1)}{2} - 1$

$= \frac{n^2 + n}{2} - 1$

Big-O-Notation : $O(n^2)$

Q8 ==> Please use Master Theroem to analyze the following recurrence

Q-8 using Master theorem to analyze following Recursion.

$$T(n) = 7T(n/2) + 500n^2$$

$$T(n) = aT(n/b) + f(n)$$
$$T(1) = c$$

where $a \geq 1$, $b \geq 2$, $c > 0$ If $f(n) \in \Theta(n^d)$, where $d \geq 0$ then.

so ~~let's~~ $a = 7$
$b = 2$
$d = 2$

$$T(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

In this given $a, b, \& d$ it satisfies
$a > b^d \rightarrow 7 > 2^2$ so

Big-O-Notation : $\Theta(n^{\log_2 7})$

$\boxed{\text{Ans : } \Theta(n^{\log_2 7})}$

Q5 ==> Please draw a diagram to show the search of 76 from the following numbers using Binary Search

Q.5 Binary Search $(m = l + (r-l)/2)$

Search of 76 from the following no using binary Search.

6 12 28 29 45 54 62 76

Step 1:

0	1	2	3	4	5	6	7
6	12	28	29	45	54	62	76

$l = 0$ $m = 3$ $r = 7$

$m = 0 + (7/2)$

$m = 3$

Step 2:

$76 > 29$

6	12	28	29	45	54	62	76

$l = 4$ $m = 5$ $r = 7$

$m = 4 + (3/2)$

$m = 5$

Step 3:

$76 > 54$

6	12	28	29	45	54	62	76

$l = 6$ $r = 7$

$m = 6$

$m = 6 + (1/2)$

$= 6$

step 4:

$76 > 62$

6	12	28	29	45	54	62	76

$l = m = r = 7$

$76 = 76$ So; we found the position of 76 in the given Array

Q10 ==> Please draw a diagram to show the Merge Sort of these number