Report - Problem Set No 6

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Problem 1

Basic description:

In this problem, we're going to find the answer to the integral of a function. So we will follow the instructions introduced by the book. We will deploy both methods of the Monte Carlo algorithm for solving an integration. As the problem told us:

$$f_{(x)} = \int e^{-x^2}\, dx \Rightarrow I = \int_0^2 e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}\mathrm{erf}_{(2)}$$

We solved the integration for different amounts of samples for both methods and showed the benchmark result of each one in the notebook (Q1-1DInt.ipynb). We also showed the distribution of answers for both methods. And finally, we will show the answers, errors, and performance for each amount of samples.

The results:

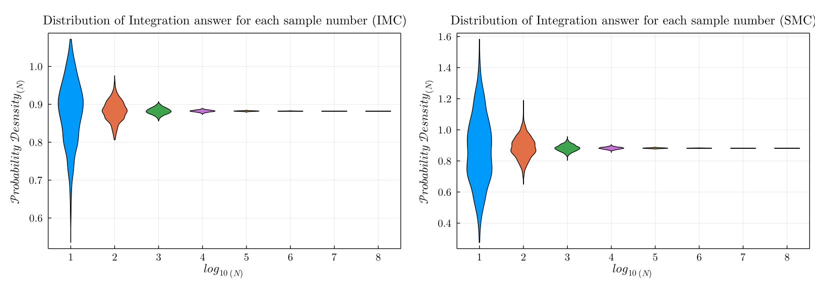

Figure 1: Distribution of answers for both methods.

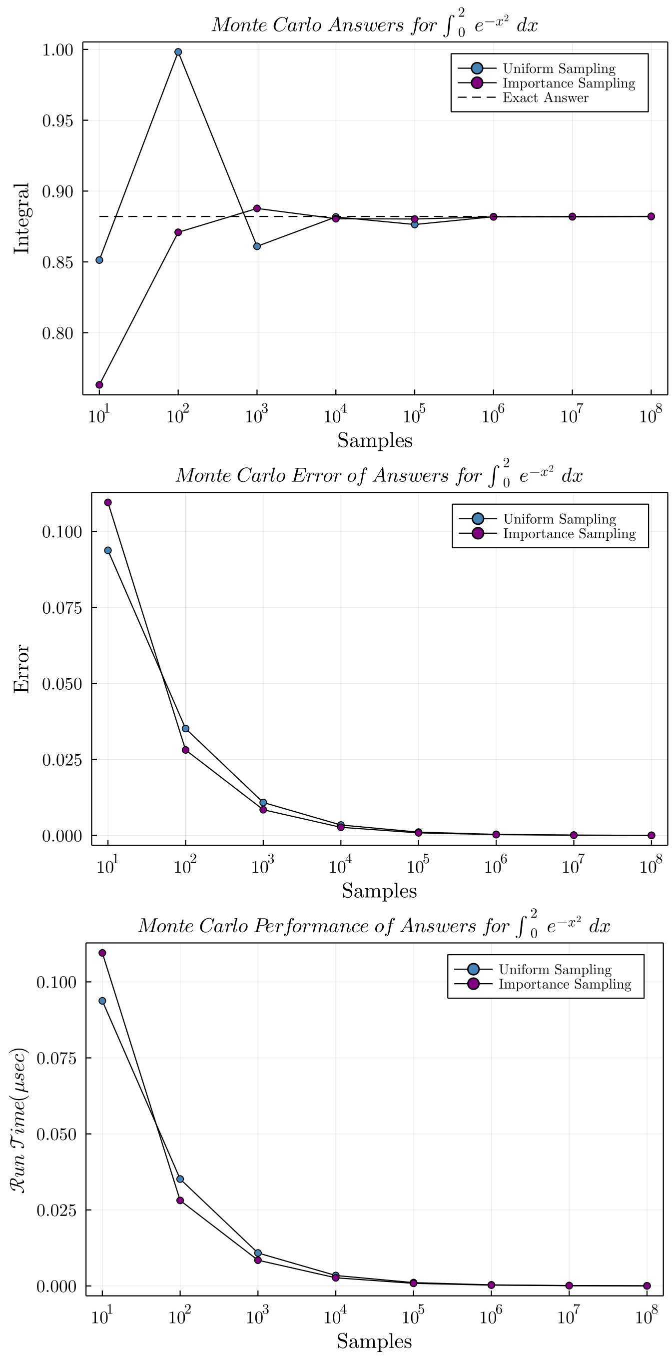

Figure 2: Plot of answers, errors, and performance for both methods.

Problem 2

Basic description:

In this problem, we want to find the center of mass of a sphere whit specific mass distribution. So we have to find the answer to a 2D integration. As we know:

$$R_{CoM} = \frac{I}{M} \Rightarrow R_{CoM} = \frac{\int_{Sphere} z\rho dV}{\int_{Sphere} \rho dV}, \;\; \rho = \rho_0(3 + \frac{r}{R}\cos\theta) \Rightarrow$$

$$R_{CoM} = \frac{\int_0^R \int_0^\pi (3 + \frac{r}{R}\cos\theta) r^3 \sin\theta \cos\theta d\theta dr}{\int_0^R \int_0^\pi (3 + \frac{r}{R}\cos\theta) r^2 \sin\theta d\theta dr}$$

That has the exact answer of $R_{CoM} = \frac{R}{15}$. We drew the distribution of a bunch of answers. Also, we calculated the values of errors. The final answer is $R_{CoM} = 0.0665806190158467$. All steps and values are available in the notebook.

The results:

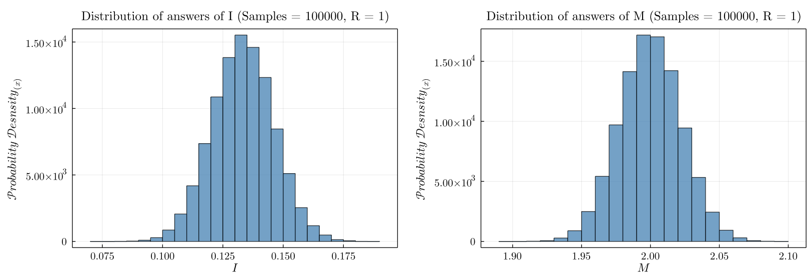

Figure 3: Distribution of answers of integration of I and M.

Problem 3

Basic description:

In this problem, we want to create the Normal distribution by moving samples of the Uniform distribution in a span with various chances of movement and deploying the Metropolis algorithm. As we know the best distribution comes out with $a_r \approx 0.5$. So we draw a distribution with $a_r \approx 0.5$. Then, we will show the relation between Δ and a_r. And finally, we will show the Auto Correlation and Correlation length. The Correlation length will come out by finding the exponent of the Auto Correlation.

The results:

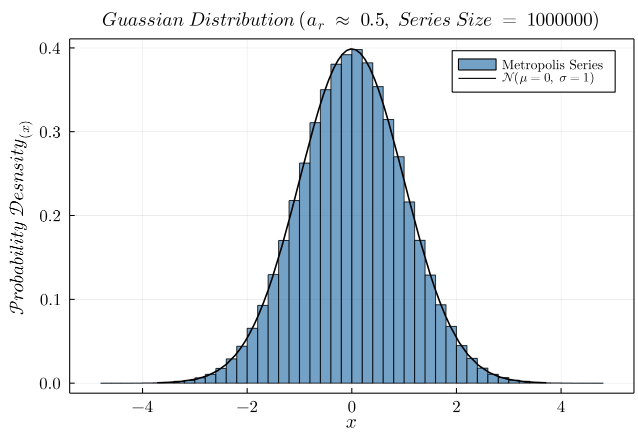

Figure 4: Metropolis Normal Distribution with $a_r \approx 0.5$.

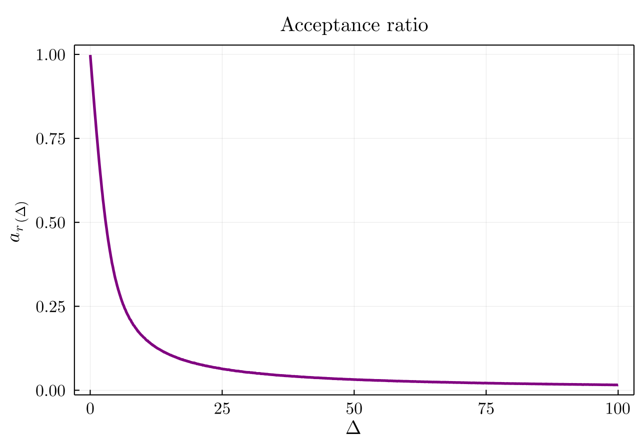

Figure 5: Acceptance Ratio plot.

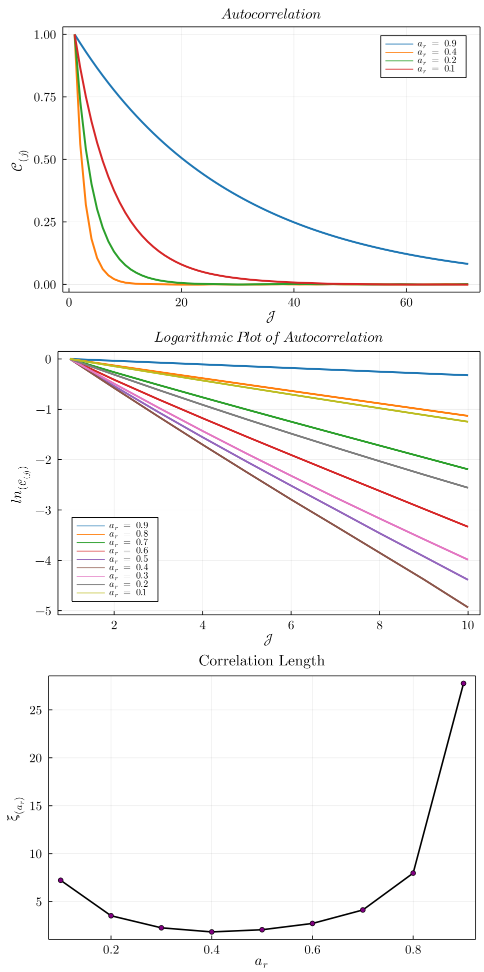

Figure 6: Auto correlation and Correlation Length Plot.

The whole data I gathered is in this link

Thanks for watching :)