Report - Problem Set No 6

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Problem 1

Basic description:

In this problem, we're going to collect samples of the sum of a specific number of particles. According to the central limit theorem, this distribution must be gaussian. As we can see the results approved the theorem.

The results:

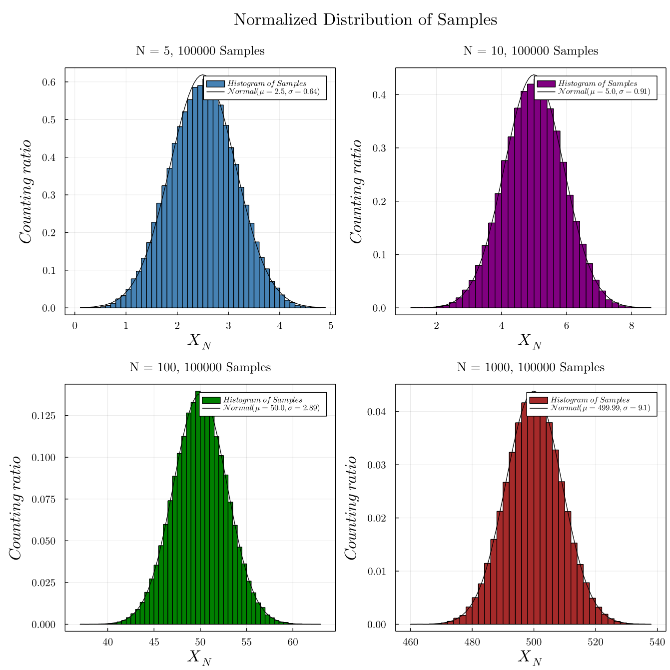

Figure 1: Histogram of the distribution of the samples.

Problem 2

Basic description:

In this problem, we're going to convert a uniform distribution to a gaussian. So we continue the book solution. We have x1 and x2 samples. To find y1 and y2 (that are from the gaussian distribution) we continue this solution:

$$f_{(x)}dx = g_{(y)}dy \Rightarrow \int f_{(x)}dx = \int g_{(y)}dy$$
$$\Rightarrow x = \int g_{(y)}dy = G_{(y)}$$
$$\Rightarrow x = G^{-1}{}_{(x)}$$
$$g(y) = \frac{e^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$$

We know G(y) can't be expressed as we want, so we use the box-muller solution to get what we want:

$$g_{(y_1,y_2)dy_1dy_2 = g_{(y_1)}g_{(y_1)}} = \frac{e^{-\frac{{y_1}^2+{y_2}^2}{2\sigma^2}}}{2\pi\sigma^2}dy_1dy_2$$

We transform into polar coordinates:

$$y_1 = \rho \sin_\theta, \; y_2 = \rho \cos_\theta$$
$$\Rightarrow g_{(y_1,y_2)}dy_1dy_2 = g_{(\rho,\theta)}\rho\theta = \frac{e^{-\frac{\rho^2}{2\sigma^2}}}{2\pi\sigma^2}\rho d\rho d\theta$$
$$\Rightarrow g_{1(\rho)} = \frac{e^{-\frac{\rho^2}{2\sigma^2}}}{\sigma^2}\rho d\rho d\theta, \; g_{2(\theta)} = \frac{1}{2\pi}$$
$$\Rightarrow G_{1(\rho)} = 1 - e^{-\frac{\rho^2}{2\sigma^2}}, \; g_{2(\theta)} = \frac{\theta}{2\pi}$$
$$\Rightarrow \; \rho = \sigma\sqrt{2ln(\frac{1}{1-x_1})}, \; \theta = 2\pi x_2$$

So we use the following formula to transform uniform distribution to gaussian distribution.

I also built a function to create a uniform distribution. So, initially I call my uniform distribution creator function and transform the distribution to the Gaussian distribution.

The results:

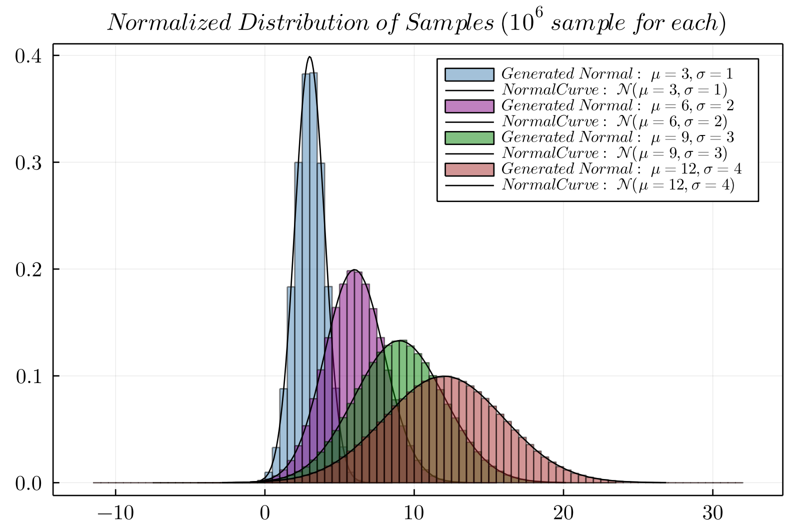

Figure 2: Histogram of the distribution of the samples.

The whole data I gathered is in this link
Thanks for watching :)