Homework Assignment

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Problem 7.2:

a.

$$p_k = \frac{N-1}{N}\delta(k-1) + \frac{1}{N}\delta(k-(N-1))$$

b.

$$q_k = \frac{\delta(k-1) + \delta(k-(N-1))}{2}$$

c.

$$e_{jk} = \frac{\delta_{j,1}\delta_{k,N-1} + \delta_{j,N-1}\delta_{k,1}}{2}$$

$$r = \sum_{j,k} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}$$

$$\sigma^2 = \sum_k k^2 q_k - \left(\sum_k k q_k\right)^2$$

$$\Rightarrow r = \frac{(N-1) - \left(\frac{N^2}{4}\right)}{\frac{1+(N-1)^2}{2} - \left(\frac{N^2}{4}\right)} = -1$$

d.

All of the nodes with small degree are connected to the nodes with large degree. Therefore, the network is disassortative.

Problem 7.4:

a.

In the Erdős-Rényi $G(N, L)$ model, the probability that there is a link between nodes i and j is given by:

$$e_{ij} = \frac{L}{\binom{N}{2}}$$

Where $\binom{N}{2}$ is the total number of possible links.

The conditional probability that there is a link between i and j given there is a link between l and s is:

$$e_{ij}|e_{ls} = \frac{L-1}{\binom{N}{2}-1}$$

b.

For small networks, the ratio of these probabilities is:

$$\frac{e_{ij}|e_{ls}}{e_{ij}} = \frac{\frac{L-1}{\binom{N}{2}-1}}{\frac{L}{\binom{N}{2}}} \approx 1$$

For large networks (as $N \to \infty$), this ratio approaches 1.

c.

In the Erdős-Rényi $G(N, p)$ model, the probabilities are:

$$e_{ij} = p$$

$$e_{ij}|e_{ls} = p$$

These probabilities are independent.

The ratio is always 1, regardless of network size.

Implications for small networks:

- Using the $G(N, L)$ model introduces correlations between link existence probabilities that are not present in the $G(N, p)$ model.
- For model validation or simulation of small real-world networks, the $G(N, p)$ model may not capture these correlations appropriately.
- The ratio of conditional to unconditional probabilities being ≈ 1 in $G(N, L)$ for small N indicates significant correlations.
- The $G(N, L)$ model may better represent the linking process in small real networks.

So for modeling and analyzing small networks, using the $G(N, L)$ over $G(N, p)$ model is likely more appropriate to account for degree correlations not present in $G(N, p)$.