A&T Assignment#02

Name: Shahmeer Khan

Class-ID: 106195

Student-ID: 12113.

Question no: 1;

$$A = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 2 & 3 & 2 \end{vmatrix}_{(3\times3)} , \quad B = \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 4 & 5 \end{vmatrix}_{(3\times2)}$$

Evaluate if Possible:

(i) A+B.

Solution:

⊙ ∴ A+B is impossible. because;

∴ columns of Matrice A ≠ columns of Matrice B

∴ i.e; $\boxed{\text{Order of } A \neq \text{Order of } B}$.

(ii) AB.

Solution;

$$AB = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 2 & 3 & 2 \end{vmatrix}_{(3\times3)} \times \begin{vmatrix} 1 & 3 \\ 2 & 1 \\ 4 & 5 \end{vmatrix}_{(3\times2)}$$

$$\Rightarrow \begin{vmatrix} 3\times1+2\times2+1\times4 & 3\times3+2\times1+1\times5 \\ 1\times1+2\times2+0\times4 & 1\times3+2\times1+0\times5 \\ 2\times1+3\times2+2\times4 & 2\times3+3\times1+2\times5 \end{vmatrix}$$

$$\boxed{AB = \begin{vmatrix} 11 & 16 \\ 5 & 5 \\ 16 & 19 \end{vmatrix}} \text{ Ans.}$$

(iii) $B^T A$.

Solution:

⊙ First B^T;

$$\therefore \boxed{B^T = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \end{vmatrix}}$$

⊙ Now $B^T A$;

$$B^T A = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 1 & 5 \end{vmatrix} \times \begin{vmatrix} 3 & 2 & 1 \\ 1 & 2 & 0 \\ 2 & 3 & 2 \end{vmatrix}$$

$$B^T A = \begin{vmatrix} 1\times3+2\times1+4\times2 & 1\times2+2\times2+4\times3 & 1\times1+2\times0+4\times2 \\ 3\times3+1\times1+5\times2 & 3\times2+1\times2+5\times3 & 3\times1+1\times0+5\times2 \end{vmatrix}$$

$$\boxed{B^T A = \begin{vmatrix} 13 & 18 & 9 \\ 20 & 23 & 13 \end{vmatrix}} \text{ Ans.}$$

Question no. 2;

Solve by Gauss's Elimination Rule:

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 - x_3 = 8$$
$$x_1 - x_2 + 2x_3 = -1$$

Solution;

⊙ Co-efficient of Matrix:

$$A = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix}, \quad B = \begin{vmatrix} 3 \\ 8 \\ -1 \end{vmatrix}$$

⊙ Unknowns

$$X = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} ? \\ ? \\ ? \end{vmatrix}$$

⊙ Gauss Elimination Rule.

Augmented Matrix System:

A			B	Row Operations (Proposed)
1	2	1	3	
2	3	-1	8	Shifting $R_2 \to R_3$
1	-1	2	-1	Shifting $R_3 \to R_2$
1	2	1	3	(Applied)
1	-1	2	-1	$R_2 \to R_2 - R_1$ (R_2 will be subtracted by R_1).
2	3	-1	8	$R_3 \to R_3 - 2R_1$ (R_3 will be subtracted by R_1's double means $2 \times R_1$)
1	2	1	3	(Applied)
0	3	-1	4	
0	1	3	-2	$R_3 \to 3R_3 - R_2$ (R_3's Thrice means $3 \times R_3$ will be subtracted by R_2)
1	2	1	3	(Applied)
0	3	-1	4	
0	0	-10	10	

∴ $x_1 + 2x_2 + x_3 = 3$ ---- eq i

∴ $3x_2 - x_3 = 4$ --- eq. ii

∴ $-10x_3 = 10$ --- eq. iii

∵ eq. iii $\Rightarrow$ $-10x_3 = 10$

∴ $\boxed{x_3 = -1}$

∵ eq. ii $\Rightarrow$ $3x_2 - x_3 = 4$

∴ $3x_2 - (-1) = 4$

$\Rightarrow$ $3x_2 + 1 = 4$

∴ $\boxed{x_2 = 1}$

∵ eq. $\Rightarrow$ $x_1 + 2x_2 + x_3 = 3$

∴ $x_1 + 2(1) + (-1) = 3$

$x_1 + 1 = 3$

∴ $\boxed{x_1 = 2}$

∴ $\boxed{X = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 2 \\ 1 \\ -1 \end{vmatrix}}$ Ans.

Scanned with CamScanner

Question no. 3

Evaluate by Gauss-Jordan Rule:

$$A = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

Verify $A^{-1}A = I$.

Solution;

$\therefore A^{-1}A = I$ --- eq. A (Proving it)

$\Rightarrow \frac{A}{A} = I$

$\Rightarrow \boxed{A = AI}$

$\therefore$ R.H.S.

$$\therefore \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$\therefore$ Interchanging them;

$$\therefore \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$\therefore R_2 - 2R_1$

$$= \begin{vmatrix} 1 & 4 \\ 0 & -5 \end{vmatrix} \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} \quad \therefore R_2 - 2R_1$$

$\therefore$ Dividing R_2 by -5.

$$\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 0 & 1 \\ -1/5 & 2/5 \end{vmatrix}$$

$\therefore R_1 - 4R_2;$

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{vmatrix}$$

$$\therefore A^{-1} = \begin{vmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{vmatrix}$$

$\therefore A^{-1}A = I$ --- eq. A (Proving it).

$\therefore$ L.H.S; $\therefore \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} \times \begin{vmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} (2\times 4/5)+(3\times -1/5) & 2(-3/5)+3(2/5) \\ 1(4/5)+4(-1/5) & 1(-3/5)+4(2/5) \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 8/5 - 3/5 & -6/5 + 6/5 \\ 4/5 - 4/5 & -3/5 + 8/5 \end{vmatrix}$$

$$\boxed{A^{-1}A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \text{ Ans}}$$

$\therefore$ $\boxed{L.H.S = R.H.S}$ (Proved) $(A^{-1}A = I)$ Ans.