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Algebra & Trigonometry (Final Exam)

Question no. 1;

Matrices;

$$A = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}_{(3\times3)}, \quad B = \begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \end{vmatrix}_{(3\times2)}$$

Evaluate if Possible;

(i) $(A+B)$;

Solution;

$\therefore$ $A+B$ is not possible, it is impossible, because;

$\therefore$ Columns of Matrice $A \neq$ Columns of Matrice B.

$\therefore$ i.e; $\boxed{\text{ORDER of } A \neq \text{Order of } B}$ Ans.

(ii) AB;

Solution;

$$\therefore AB = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}_{(3\times3)} \times \begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 3 \end{vmatrix}_{(3\times2)}$$

$$\Rightarrow \begin{vmatrix} 2\times1+3\times2+1\times0 & 2\times2+3\times1+1\times3 \\ 1\times1+0\times2+2\times0 & 1\times2+0\times1+2\times3 \\ 3\times1+1\times2+1\times0 & 3\times2+1\times1+1\times3 \end{vmatrix}$$

$$\boxed{AB = \begin{vmatrix} 8 & 10 \\ 1 & 8 \\ 5 & 10 \end{vmatrix}} \text{ Ans.}$$

(iii) $B^T A$.

Solution;

- First B^T;

$$\therefore \boxed{B^T = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \end{vmatrix}}$$

- Now $B^T A$;

$$B^T A = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \end{vmatrix} \times \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1\times2+2\times1+0\times3 & 1\times3+2\times0+0\times1 & 1\times1+2\times2+0\times1 \\ 2\times2+1\times1+3\times3 & 2\times3+1\times0+3\times1 & 2\times1+1\times2+3\times1 \end{vmatrix}$$

$$\boxed{B^T A = \begin{vmatrix} 4 & 3 & 5 \\ 14 & 9 & 7 \end{vmatrix}}$$ Ans.

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Question no. 2;

By synthetic Division, show that $(x+1)$ is a factor or not of the polynomial:

$$P(x) = (x^6 - 5x^3 + 4x - 2).$$

Solution;

- $\therefore x + 1 = 0$

$$\boxed{x = -1}$$

-

$$\begin{array}{r|rrrrrrr}
-1 & 1 & 0 & 0 & -5 & 0 & 4 & -2 \\
 & \downarrow & -1 & 1 & -1 & 6 & -6 & 2 \\
\hline
 & 1 & -1 & 1 & -6 & 6 & -2 & \boxed{0} \rightarrow R
\end{array}$$

Ans

- Remainder $\Rightarrow \boxed{R = 0}$ Ans

- $\therefore$ Since the Remainder is 0, we have shown that;

$\therefore$ $\boxed{(x+1) \text{ is a factor of } P(x)}$ Ans

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Algebra and Trignometry (Final-Exam).

Question no. 4;

Solve the Simultaneous Equations;

$$x^2+y^2=13 \quad , \quad y=2x$$

Solution;

$\therefore$ eq.1 $\Rightarrow x^2+y^2=13$

$\therefore$ eq.2 $\Rightarrow y=2x$

⊙ Considering eq.1;

$$x^2+y^2=13 \text{ --- eq.1}$$

$$x^2+y^2-13=0$$

Since eq.2;

$$\boxed{y=2x}$$

So;

$$x^2+(2x)^2-13=0$$

$$x^2+4x^2-13=0$$

$$5x^2-13=0$$

$$5x^2=13$$

$$x^2=\frac{13}{5}$$

$$\boxed{x=\pm\frac{\sqrt{15}}{5}}$$

⊙ Considering eq.2;

$$y=2x \text{ ---- eq.2}$$

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(i) First putting $\boxed{x = \frac{\sqrt{15}}{5}}$ in eq. 2;

$$y = 2\left(\frac{\sqrt{15}}{5}\right)$$

$$\boxed{y = \frac{2\sqrt{15}}{5}}$$

(ii) Now putting $\boxed{x = -\frac{\sqrt{15}}{5}}$ in eq. 2;

$$y = 2\left(-\frac{\sqrt{15}}{5}\right)$$

$$\boxed{y = -\frac{2\sqrt{15}}{5}}$$

So;

$$\therefore \boxed{y = \pm\frac{2\sqrt{15}}{5}} \text{ Ar}$$

$$\therefore \boxed{x = \pm\frac{\sqrt{15}}{5}} \text{ Ar}$$

$$\therefore \boxed{(x, y) = \left(\pm\frac{\sqrt{15}}{5}, \pm\frac{2\sqrt{15}}{5}\right)}$$

$\therefore$ Set 1;

$$\boxed{(x, y) = \left(\frac{\sqrt{15}}{5}, \frac{2\sqrt{15}}{5}\right)} \text{ As}$$

$\therefore$ Set 2;

$$\boxed{(x, y) = \left(-\frac{\sqrt{15}}{5}, -\frac{2\sqrt{15}}{5}\right)} \text{ As}$$

Algebra and Trignometry (Final Exam)

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Question no. 5;

Express the Complex Number Z in the form $(a+ib)$ and show by Graph:

$$Z = \frac{(3+2i)^2}{(1+3i)}$$

Solution;

$$Z = \frac{(9+12i+4i^2)}{1+3i}$$

$$\Rightarrow \frac{9+12i+4(-1)}{1+3i}$$

$$\Rightarrow \frac{9-4+12i}{1+3i}$$

$$\Rightarrow \frac{5+12i}{1+3i}$$

$$\Rightarrow \frac{5+12i}{1+3i} \times \frac{(1-3i)}{(1-3i)}$$

$$\Rightarrow \frac{(5+12i)(1-3i)}{1^2-(3i)^2}$$

$$\Rightarrow \frac{5-15i+12i-36i^2}{1-9i^2}$$

$$\Rightarrow \frac{5-3i+36}{1+9}$$

$$\Rightarrow \frac{-3i+41}{10}$$

$$Z = -\frac{3}{10}i + \frac{41}{10}$$

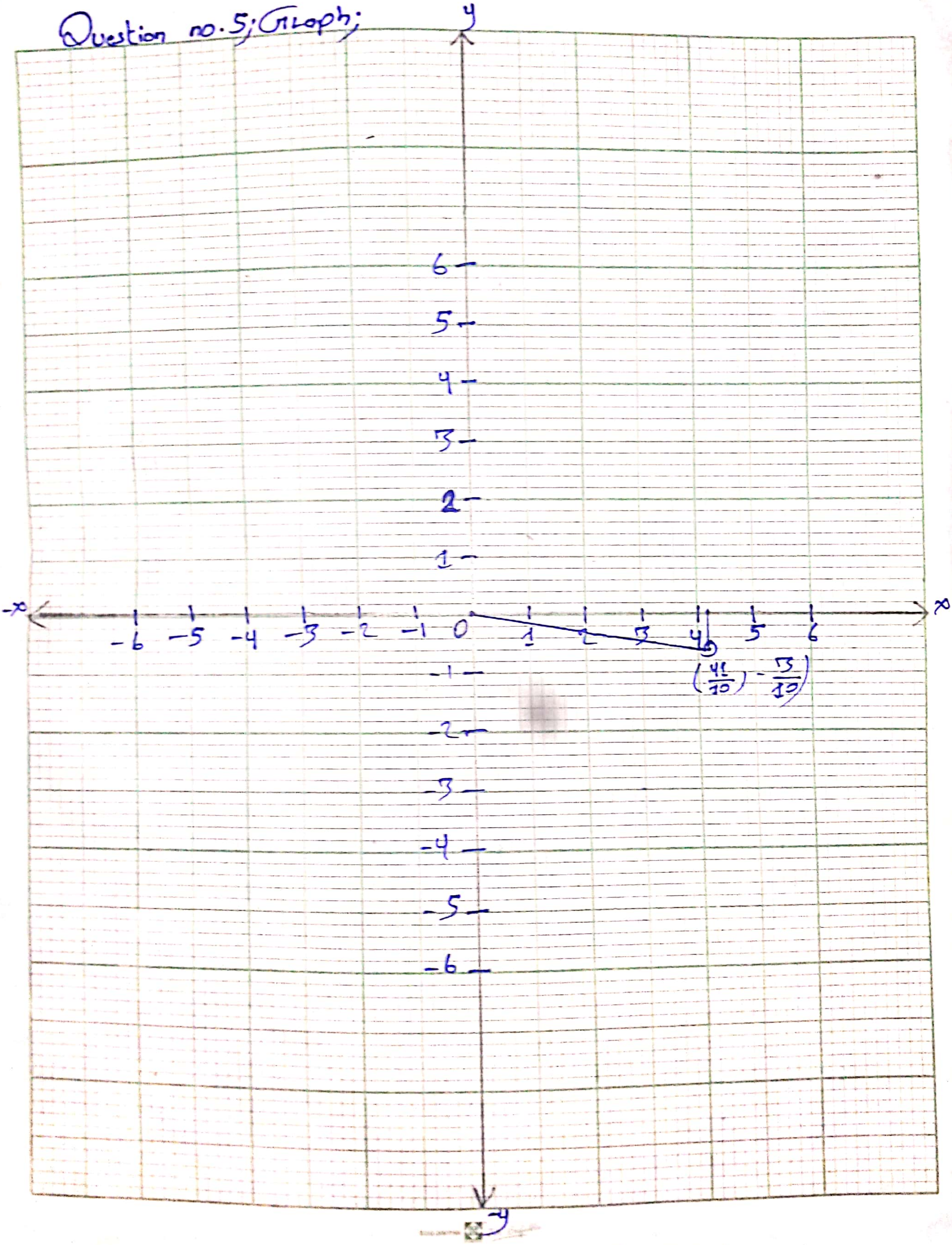
$$\boxed{Z = \frac{41}{10} - \frac{3}{10}i}$$ ∴ in $(a+bi)$ form.

As

∴ Here; $\boxed{a = \frac{41}{10}}$, $\boxed{b = -\frac{3}{10}}$

∴ Graph is on graph paper.

Question no. 5; Graph;



Algebra & trigonometry (Final Exam)

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Question no. 6;

Solve by Gauss Elimination;

$$x_1 - 2x_2 + x_3 = 4$$
$$2x_1 - 3x_2 - 4x_3 = 1$$
$$-x_1 + x_2 + x_3 = -1$$

Solution;

⊙ Co-effecient of matrix;

$$A = \begin{vmatrix} 1 & -2 & 1 \\ 2 & -3 & -4 \\ -1 & 1 & 1 \end{vmatrix}, \quad B = \begin{vmatrix} 4 \\ 1 \\ -1 \end{vmatrix}$$

⊙ Unknown;

$$X = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} ? \\ ? \\ ? \end{vmatrix}$$

⊙ Guass's Elimination Rule:

Augmented matrix system;

A			B	Row Operations (Proposed).
1	-2	1	4	Shifting $R_1 \to R_2$
2	-3	-4	1	Shifting $R_2 \to R_1$
-1	1	1	-1	
2	-3	-4	1	(Applied)
1	-2	1	4	$R_2 \to R_2 + R_3$
-1	1	1	-1	$R_3 \to R_3 + R_2$
2	-3	-4	1	(Applied)
0	-1	2	3	
0	-1	2	3	$R_3 \to R_3 - R_2$

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$$\left[\begin{array}{ccc|c} 2 & -3 & -4 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ (Applied) (10)

No. Solution; (further more...)

$\therefore$ $\boxed{0 = 0 \text{ is not possible}}$

$\boxed{\therefore \text{The System of Linear equations is Inconsistent.}}$

Algebra & Trignometry (Final Exam)

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Question no. 7;

Resolve into Partial Fractions;

$$f(x) = \frac{3x^2 - 5}{(x+1)(x^2-2)}$$

Solution;

$$f(x) = \frac{3x^2 - 5}{(x+1)(x^2-2)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-2)} \quad \text{--- eq. A}$$

$$\Rightarrow \frac{3x^2-5}{(x+1)(x^2-2)} = \frac{A(x^2-2)+(Bx+C)(x+1)}{(x+1)(x^2-2)}$$

$$\Rightarrow 3x^2 - 5 = Ax^2 - 2A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 3x^2 - 5 = A(x^2-2) + B(x^2+x) + C(x+1) \quad \text{--- eq. B}$$

$$\Rightarrow 3x^2 - 5 = A(x^2-2) + Bx(x+1) + C(x+1)$$

For "A" considering;

$$3x^2 = A(x^2-2) \quad \text{--- eq. 1}$$

∴ substituting the value of "x" from eq. 1 b/c eq. 1 is identity;

$$\therefore x+1=0$$

$$x = -1$$

∴ Putting $x = -1$ in eq. 1

$$3(-1)^2 = A((-1)^2-2)$$

$$3(1) = A(1-2)$$

$$3 = A(-1)$$

$$A = -3$$

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⊙ Considering (eq. B) for "B" and "C".

$$3x^2-5 = A(x^2-2) + Bx(x+1) + C(x+1) \quad ---- \text{(eq. B)}$$

$$3x^2-5 = Ax^2 - 2A + Bx^2 + Bx + Cx + C$$

$$3x^2-5 = Ax^2 + Bx^2 + Bx + Cx - 2A + C$$

$$3x^2-5 = x^2(A+B) + x(B+C) - (2A+C)$$

⊙ $\therefore A+B=3$

$-3+B=3 \qquad \therefore A=-3$

$B=3+3$

$\boxed{B=6}$

⊙ $\therefore B+C=0$

$6+C=0$

$\boxed{C=-6}$

$\therefore$ Putting $A=-3$, $B=6$, $C=-6$ in (eq. A)

$$\boxed{f(x) = \frac{3x^2-5}{(x+1)(x^2-2)} = \frac{-3}{(x+1)} + \frac{6x-6}{(x^2-2)}} \text{ Ans.}$$

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Question no. 8;

Expand in series by Binomial Theorem;

$$f(x) = (2-x)^5.$$

Solution;

$f(x) = (2-x)^5$

$(2-x)^5$;

Here;

$a = 5$

$\therefore$ By Using formula;

$$(a+b)^n = a + \frac{n}{1!} a^{n-1} \cdot b + \frac{n(n-1)}{2!} a^{n-2} \cdot b + \frac{n(n-1)(n-2)}{3!} a^{n-3} \cdot b + \frac{n(n-1)(n-2)(n-3)}{4!} a^{n-4} \cdot b$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} \cdot a^{n-5} \cdot b$$

$$\Rightarrow (2-x)^5 = 2^5 + \frac{5}{1} 2^{5-1} \cdot (-x) + \frac{5(5-1)}{2\times1} 2^{(5-2)} \cdot (-x)^2 + \frac{5(5-1)(5-2)}{3\times2\times1} 2^{5-3} \cdot (-x)^3 + \frac{5(5-1)(5-2)(5-3)}{4\times3\times2\times1} 2^{5-4} \cdot (-x)^4$$

$$+ \frac{5(5-1)(5-2)(5-3)(5-4)}{5\times4\times3\times2\times1} 2^{(5-5)} \cdot (-x)^5$$

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$$\Rightarrow (2-x)^5 = 32 + 5(16)\cdot(-x) + \frac{5(4)}{2}(8)\cdot(-x)^2 + \frac{5(4)(3)(4)}{6}(-x)^3 + \frac{5(4)(3)(2)}{24}(2)\cdot(-x)^4 + \frac{5(4)(3)(2)(1)}{120}(1)\cdot(-x)^5$$

$$\Rightarrow (2-x)^5 = 32 + 80(-x) + \frac{160}{2}(-x)^2 + \frac{240}{6}(-x)^3 + \frac{240}{24}(-x)^4 + \frac{120}{120}(-x)^5$$

$$\therefore \boxed{(2-x)^5 = 32 - 8x + 8x^2 - 40x^3 + 10x^4 - x^5} \text{ Ans.}$$