Ordinal Multi-class Classification Problem on Harvard GLOPOP Data

Country of choice: Tajikistan

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Data Acquisition

The GLOPOP-S dataset is a comprehensive global synthetic population database hosted on the Harvard Dataverse. The data was obtained from global synthetic population database <u>GLOPOP-S</u>. They took the Demographic and Health Surveys for 45 countries including Tajikistan, categorized and harmonized it, as well as went through other data processing steps including creating regional marginal distributions.

In case of missing data in the regions, to impute missing regional data, new values were modeled based on *k* number of regions with similar subnational human development index (SHDI) or SHDI relative to national HDI.

Then, marginals were scaled to match the region's population size in 2015 to conserve the shape of the marginal distributions.

The following methods were used to generate a synthetic population from DHS and LIS: synthetic reconstruction, combinatorial optimization, statistical learning.

All the columns of the data, although represented as numbers, are of categorical type. Below is the description of the columns taken from the article <u>"A global dataset of 7 billion individuals with socio-economic characteristics"</u> Nature journal.

Attributes	HH/I	Levels
Income	Individual	l: poorest 20%, 2: poorer 20%, 3: middle 20%, 4: richer 20%, 5: richest 20%, -1 : unavailable for country
Wealth	Individual	l: poorest 20%, 2: poorer 20%, 3: middle 20%, 4: richer 20%, 5: richest 20%, -1 : unavailable for country
Settlement type	Household	0: urban, 1: rural
Age	Individual	1: 0-4, 2: 5-14, 3: 15-24, 4: 25-34, 5: 35-44, 6: 45-54, 7: 55-64, 8: 65+
Gender	Individual	1: male, 0: female
Education	Individual	1: less than primary, 2: complete primary, 3: incomplete secondary, 4: complete secondary or tertiary, 5: higher
Household type	Household	1: single, 2: couple, 3: couple with children, 4: one parent with children, 5: couple with (non-) relatives, 6: couple with children and (non-) relatives, 8: other
Household ID	Household	1,,
Relationship to head	Individual	1: head, 2: partner, 3: child, 4: relative, 5: non-relative
Household size	Household	1: 1, 2: 2, 3: 3-4, 4: 5-6, 5: 7-10, 6: 10+
Ownership of agricultural land (DHS)	Household	1: yes, 2: no, -1: unavailable for country
Floor material (DHS)	Household	1: natural, 2: rudimentary, 3: finished, -1 : unavailable for country
Wall material (DHS)	Household	1: natural, 2: rudimentary, 3: finished, —1: unavailable for country
Roof material (DHS)	Household	1: natural, 2: rudimentary, 3: finished, -1, unavailable for country
Source	Country	1: LIS, 2: LIS survey, 3: LIS marginals, 4: Modeled by LIS data, 5: DHS, 6: Modeled by DHS data

Table 2. Attributes in GLOPOP-S.

Exploratory Data Analysis

General Observations

- The dataset contains **9,269,799 records** with **16 variables**, all of which are categorical (encoded as integers).
- No missing values are present in any column (Miss = 0 for all variables).

Key Variable Insights

1. Household ID (HID)

 High duplication: ~20 records per household on average, suggesting longitudinal or multi-member sampling.

2. Household Relationships (RELATE_HEAD)

• Majority are neither head nor primary dependents (median=3), hinting at extended family structures.

3. Economic Indicators (INCOME , WEALTH)

- INCOME: The column is constant entirely masked (all 1) required removal.
- WEALTH: Slight right skew (Q75=4), but few reach the highest tier (Q95+=5).

4. Geographic/Rural Status (RURAL)

• Only 19.5% rural.

5. Demographics (AGE , GENDER , EDUC)

- AGE: 8 age groups. Mean ~3.6 (median = 3), suggesting a younger population.
- GENDER: Near-balanced (mean = 0.496, ~49.6% female).
- EDUC: 5-tiered education levels. Mean ~2.96 (median = 3), indicating most have mid-level education.

6. Household Structure (HHTYPE , HHSIZE_CAT)

- HHTYPE: Has complex non-ordinal categories. No conclusions based on numeric representation of groups.
- HHSIZE_CAT: Median = 5 out of 6 categories, mean = 4.54, suggesting larger households.

7. Housing Quality (FLOOR, WALL, ROOF)

- FLOOR: Ordinal category of 3 types. Mean = 2.28 favoring "finished".
- WALL: Mean = 2.08, suggesting mixed types.
- Roof: Strong skew toward category 3 (median = 3, mean = 2.96), indicating majority has "finished".

8. Agriculture (AGR_CONNERSHIP)

• Binary (0/1): 52.1% have agricultural connections (mean = 0.521), reflecting Tajikistan's agrarian economy.

9. Region (REGION)

• 5 regions: Mean = 2.86 (median = 3), with values spread across categories.

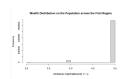


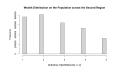
Data Split

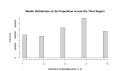
To develop the machine learning model, the dataset—comprising over 9 million observations—was randomly shuffled to eliminate any potential ordering bias. Following this, the data was partitioned into training and testing subsets using a 70:30 ratio based on index-based slicing. Given the large volume of data, stratified sampling was deemed unnecessary, as random shuffling was sufficient to ensure representative distribution across both subsets.

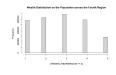
Homogeneity Across Regions and Data Splits Test

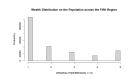
A homogeneity check was conducted to verify the consistency of data distribution across different regions and between the training and test subsets. This was done by constructing and analyzing histograms for key variables within each region and dataset split. The goal was to ensure that the random shuffling and partitioning process preserved the underlying statistical properties and regional characteristics of the data. The first row shows distribution of WEALTH variable across all five regions in the Population Set, while the second and the third rows — Train and Test Sets accordingly.

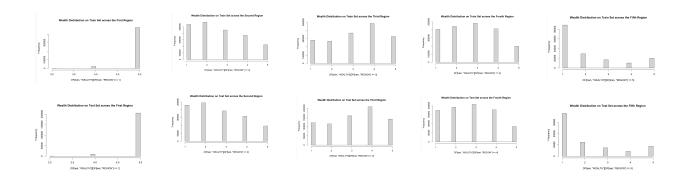




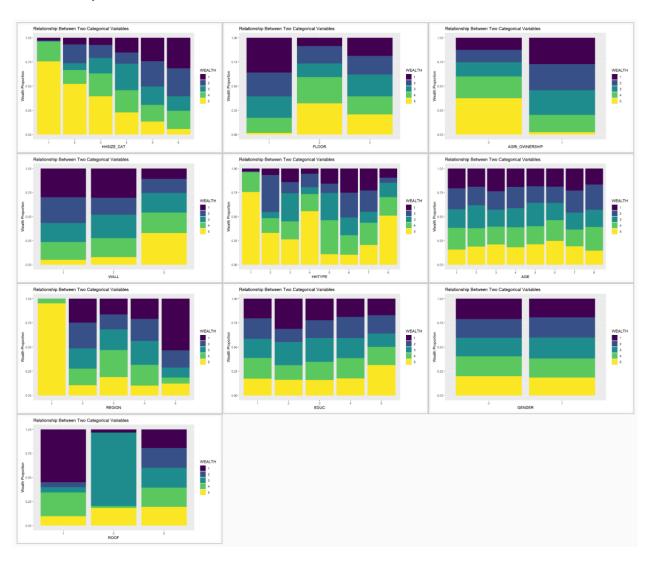








Relationships between WEALTH and other variables



Ordinal Logistic Regression Model

Model: MASS::polr()

Model Assumptions

- 1. **Proportional odds**: The relationship between each predictor and the outcome is consistent across all outcome thresholds.
- 2. No multicollinearity: Check VIF (<5).
- 3. Adequate sample size: ~10-20 cases per predictor per outcome level.
- 4. Meaningful ordinal outcome: Categories must follow a logical order.

Correlation Matrix

AGE	GENDER	EDUC	HHTYPE	HHSIZE_CAT	AGRI_OWNERSHIP	FLOOR	WALL	ROOF	REGION
1.00	-0.02	0.72	-0.04	-0.15	-0.02	-0.01	0.02	-0.07	0.02
-0.02	1.00	0.11	-0.07	-0.02	0.04	0.03	0.00	-0.02	0.00
0.72	0.11	1.00	-0.10	-0.20	-0.01	0.01	0.13	-0.08	-0.03
-0.04	-0.07	-0.10	1.00	0.50	0.04	-0.03	-0.01	0.15	0.02
-0.15	-0.02	-0.20	0.50	1.00	-0.04	-0.12	-0.18	0.36	0.00
-0.02	0.04	-0.01	0.04	-0.04	1.00	0.04	-0.14	-0.32	0.22
-0.01	0.03	0.01	-0.03	-0.12	0.04	1.00	0.06	-0.01	0.09
0.02	0.00	0.13	-0.01	-0.18	-0.14	0.06	1.00	0.14	-0.32
-0.07	-0.02	-0.08	0.15	0.36	-0.32	-0.01	0.14	1.00	-0.31
0.02	0.00	-0.03	0.02	0.00	0.22	0.09	-0.32	-0.31	1.00

Mutual Information

```
## AGE 0.0066949491
## GENDER 0.008184556
## EDUC 0.0125136209
## HHTYPE 0.0659387786
## HHSIZE_CAT 0.0727038513
## AGRI_ONNERSHIP 0.1289645686
## FLOOR 0.0719929378
## WALL 0.0874742434
## ROOF 0.0108648835
## REGION 0.1495840274
```

Conclusion: Based on the Mutual Information table and Correlation matrix the following features can be removed: AGE and HHTYPE.

Model

A classical statistical model for ordinal outcomes that assumes proportional odds - the effect of predictors is consistent across all outcome thresholds. It provides interpretable coefficients but requires strict assumptions about data structure and linearity.

```
## Call:
## MASS::polr(formula = WEALTH ~ EDUC + HHSIZE_CAT + WALL + ROOF +
## FLOOR + AGRI_OWNERSHIP, data = DF[1:1000000, c(features,
## "WEALTH")])
##
## Coefficients:
## EDUC.L EDUC.Q EDUC.C EDUC^4 HHSIZE_CAT.L
## 0.1077399 0.2283148 -0.3086780 0.2799945 -2.2316220
## HHSIZE_CAT.Q HHSIZE_CAT.C HHSIZE_CAT^4 HHSIZE_CAT^5 WALL2
## -0.2713608 -0.5475459 0.4753573 -0.1532835 0.3280669
## WALL3 ROOF2 ROOF3 FLOOR2 FLOOR3
## 1.3573465 0.4965558 1.2079452 1.7566316 1.2148722
## AGRI_OWNERSHIP1
## -2.0293344
##
## Intercepts:
## 1|2 2|3 3|4 4|5
## -0.4812981 0.8579052 2.0798659 3.6655017
##
## Residual Deviance: 2626380.86
## AIC: 2626440.86
```

The Proportionality of Odds Assumption Test

ull hypothesis regarding the proportional odds assumption is rejected for all predictors. Hence, non of the predictors are acceptable for the modeling.

Multicollinearity Test

Hence, no variable is significantly multicollinear.

Model Evaluation

→ Train Set

Metric	Value
Multi-class area under the curve	0.7417
Gini	0.483420297892295

→ Test Set

```
# Confusion Matrix
    Actual

Predicted 1 2 3 4 5
    0 0 0 0 0 0
    1 46554 46585 46966 45594 44588
    2 32089 32299 32909 31801 30709
    3 30592 29850 30082 29661 28696
    4 53309 53203 53395 52380 51139
```

Multi-class area under the curve	0.5002
Gini on test set	0.000441979905060563
Cohen's Kappa	-0.000370885631921676
Weighted Kappa	0.0007372
Ordinal Concordance	0.00036373355246857
Accuracy	0.199767946450715
Precision	0.287673601462297
Recall	0.202291054206273
F1 Score	0.237542832667047



Conclusion on the model: Since the proportionality of odds assumption does not hold for this model, even though no other assumptions are violated, this model is in appropriate to be used.

Generalized Linear (Logistic) Regression Model

VGAMS::vglm()

A flexible framework for fitting various regression models, including extended ordinal logistic models that can relax the proportional odds assumption. This allows more flexibility and higher quality in compare to poir.

Offers sophisticated modeling options like partial proportional odds and continuation ratio models.

As the Thomas W. Yee suggests in his article [The VGAM Package for Categorical Data Analysis] (file:///C:/Users/SNurubloeva/Downloads/v32i10.pdf), "the framework is very well suited to many 'classical' regression models for categorical responses".

Model Assumptions

- 1. **Proportional odds**: The assumption of Ordinal Logistic Regression is relaxed on a feature if it's included in family = cumulative(parallel = FALSE ~ .) of VGLM model.
- 2. No multicollinearity: Check VIF (<5).
- 3. Adequate sample size: ~10-20 cases per predictor per outcome level.
- 4. **Meaningful ordinal outcome**: Categories must follow a logical order.

Modification of predictors' bins due to non-homogenneity

```
DF$HHSIZE_CAT \leftarrow fct_collapse(DF$HHSIZE_CAT,

"1_2" = c("1", "2"),

"3_4" = c("3", "4"),

"5_6" = c("5", "6")) %>% factor(., ordered = TRUE)

DF$ROOF \leftarrow fct_collapse(DF$ROOF,

"1_2" = c("1", "2"),

"3" = c("3")) %>% factor(., ordered = TRUE)
```

```
model ← vglm(
```

 $\label{eq:wealth} Wealth \sim EDUC + AGRI_OWNERSHIP + FLOOR + Wall*ROOF + REGION + HHSIZE_CAT+ EDUC*AGRI_OW \\ family = cumulative(parallel = FALSE \sim AGRI_OWNERSHIP + FLOOR + WALL), \ \# TRUE \sim EDUC + ROOF +$

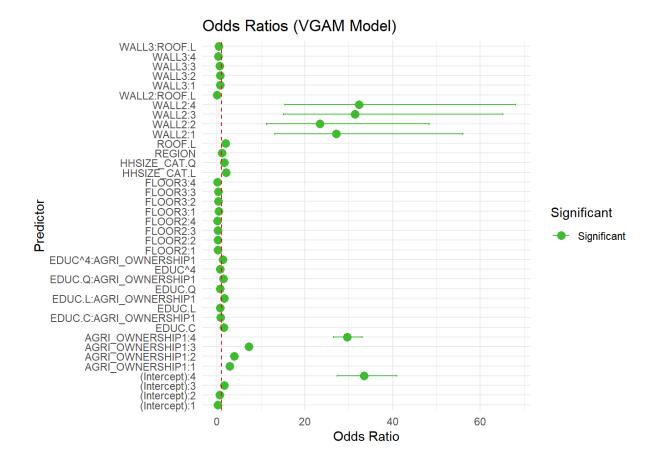
```
data = DF[1:1000000, ]
)
```

Relaxation of proportionality of odds assumption was performed not to all predictors but only to half of them.

Possible reasons:

- The model becomes overly complex when too many predictors are allowed to have non-proportional effects (each relaxed predictor gets **K-1 coefficients**, where **K = number of outcome categories**).
- If the data lacks sufficient variation or has small cell counts, the model may fail to converge.

Odds Ratio Test



Model Evaluation

→ Train Set

Multi-class area under the curve 0.759
--

Gini 0.518007

→ Test Set

Confuction Matrix

Actual

Predicted 1 2 3 4 5

1 278925 156001 87674 59444 1980

2 65302 113456 65225 33085 10692

3 121724 106729 156987 107103 19277

4 85771 166849 214373 255119 71785

5 10087 17973 41309 102781 431290

Model Performance Metrics						
Test Set Evaluation Results						
Metric Value Interpretation						
Gini 0.547 Fair (0.5-0.7: Moderate)						
Weighted Kappa	0.497	Fair (0.4-0.6: Moderate)				
Weighted MAE	0.815	Poor (>0.5)				
ORC	0.665	Fair (70-84%)				



Conclusion on the model: The model performs significantly better then Ordinal Logistic Regression die to relaxation of the proportionality of odds assumption on several predictors. Metrics of model evaluation show fair results. The difference of Gini coefficient between the train and test sets are insignificant, indicating absence of overfitting.

Light Gradient Boosting Machine

A high-performance gradient boosting framework optimized for speed and efficiency, using histogram-based algorithms and leaf-wise growth. Excels with large datasets, handling non-linear relationships automatically while offering GPU support and categorical feature handling. Requires careful tuning but delivers strong predictive accuracy with computational efficiency.

Model Assumptions

1. No strict linearity assumption: Handles non-linear relationships.

- 2. Preprocess categoricals: Use categorical_feature or one-hot encoding.
- 3. **Hyperparameter tuning**: Critical for performance (e.g., num_leaves , learning_rate).
- 4. Robust to outliers: But extreme values may affect splits.

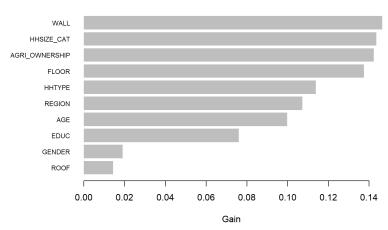
Hyperparameter Tuning

The following results were obtained from Random Search hyperparameter tuning:

```
lgb_params ← list(
 num_leaves = 73,
 learning_rate = 0.1603422,
 feature_fraction = 0.8519221,
 min_data_in_leaf = 20,
 lambda_11 = 2.903614,
 lambda_12 = 1.354935,
 max_depth = 12,
 min_gain_to_split = 0.005324554,
 bagging_fraction = 0.97738,
 bagging_freq = 3,
 objective = "multiclass",
 metric = "multi_logloss",
 num_class = 5,
 min_split_gain = 0.01,
 feature_pre_filter = FALSE,
 verbose = 1
```

Feature Importance

Feature Importance



Feature Interaction Table

```
Parent
               Child sumGain frequency
     <char>
               <char>
                         <num> <int>
1:
      WALL
                FLOOR 825052.5453
                                     271
2:
      FLOOR HHSIZE_CAT 796903.7084
                                        382
3:
      HHTYPE AGRI_OWNERSHIP 791467.2841
                                           101
4:
       EDUC
                  AGE 712466.4063
                                    830
5: AGRI_OWNERSHIP HHSIZE_CAT 659599.6116
                                             158
6:
      FLOOR
                 WALL 573482.6579
                                     320
7:
      HHTYPE HHSIZE_CAT 531039.8572
                                        371
      GENDER
8:
                   AGE 494531.4966
                                     411
9:
      HHTYPE
                  FLOOR 474359.7659
                                       271
       WALL
10:
                 HHTYPE 470625.8279
                                       249
11:
   HHSIZE_CAT AGRI_OWNERSHIP 457892.8371
                                             122
                 EDUC 453433.4267
12:
        AGE
13:
       WALL HHSIZE_CAT 449109.9422
                                        322
14:
    HHSIZE_CAT
                    FLOOR 436838.3284
                                         367
15: AGRI_OWNERSHIP
                       WALL 416599.2166
                                          145
16:
    HHSIZE_CAT
                   HHTYPE 342684.7520
                                          356
17:
       FLOOR
                 HHTYPE 342495.2700
                                       317
```

Model Evaluation

→ Train Set

Metric	Value
Multi-class area under the curve	0.9584
Gini	0.91677

→ Test Set

Model Performance Metrics				
Test Set Evaluation Results				
Metric Value Interpretation				
Gini	0.917	Excellent (≥0.7: Strong discrimination)		
Weighted Kappa	0.798	Excellent (≥0.6: Substantial agreement)		
Accuracy	0.792	Fair (70-79%)		
Precision	0.764	Excellent (≥75%)		
Recall	0.773	Fair (60-79%)		
F1 Score	0.768	Excellent (≥0.75)		



Conclusion on the model: The Light GBM performance is excellent.

CatBoost Model

An advanced gradient boosting implementation featuring native handling of categorical variables through ordered boosting and innovative approaches to missing data. Particularly robust for heterogeneous datasets. Shows high quality results in a moderate speed while not requiring preprocessing of categorical variables

Model Assumptions

- 1. Handles categoricals natively: No need for one-hot encoding.
- 2. Requires GPU for large data: Optimal performance with GPU acceleration.
- 3. Hyperparameter sensitivity: Tune depth, iterations, and learning_rate.
- 4. Automatic handling of missing values: But imputation may still help.

Feature Independence Test

CatBoost, like most gradient boosting algorithms, does not require features to be statistically independent. It can model interactions and dependencies between features through the construction of decision trees. However, it is robust and effective even when features are independent.

```
## # A tibble: 55 × 6
## Variable1
              Variable2
                           ChiSq df p_value Warning
## <chr>
              <chr>
                          <dbl> <int>
                                      <dbl> <chr>
## 1 WEALTH
                          124259. 28 0
                                           OK
                AGE
## 2 WEALTH
                GENDER
                             16294.
                                     40
                                            OK
## 3 WEALTH
                EDUC
                           244927. 16 0
                                            OK
## 4 WEALTH
                HHTYPE
                            1251902. 28 0
                                              OK
## 5 WEALTH
                HHSIZE_CAT
                             1343130. 200
                                               OK
## 6 WEALTH
                AGRI_OWNERSHIP 2117631.
                                          40
                                                 OK
## 7 WEALTH
                FLOOR
                           1156970.
                                     8 0
                                            OK
## 8 WEALTH
                WALL
                           1504440. 80
                                            OK
## 9 WEALTH
                ROOF
                           221873.
                                    8 0
                                           OK
                                             OK
## 10 WEALTH
                REGION
                            3355114. 16 0
## 11 AGE
              GENDER
                           28498.
                                  7 0
                                          OK
              EDUC
## 12 AGE
                         7686730. 28 0
                                           OK
## 13 AGE
              HHTYPE
                          1196934. 49 0
                                            OK
                            731593. 35 0
## 14 AGE
              HHSIZE_CAT
                                             OK
```

```
## 15 AGE
              AGRI_OWNERSHIP 7797.
                                       70
                                              OK
## 16 AGE
              FLOOR
                           54476. 140
                                          OK
## 17 AGE
              WALL
                          52964. 140
                                          OK
## 18 AGE
              ROOF
                          31848. 140
                                          OK
## 19 AGE
              REGION
                           83454. 28 0
                                           OK
## 20 GENDER
                                             OK
                 EDUC
                            227923.
                                     40
## 21 GENDER
                HHTYPE
                              67903.
                                     70
                                             OK
## 22 GENDER
                 HHSIZE_CAT
                               22148.
                                       5 0
                                               OK
## 23 GENDER
                 AGRI_OWNERSHIP
                                   5926.
                                          10
                                                 OK
## 24 GENDER
                 FLOOR
                              5738.
                                     20
                                            OK
## 25 GENDER
                 WALL
                             5736.
                                    20
                                           OK
## 26 GENDER
                 REGION
                              6278.
                                     40
                                            OK
## 27 EDUC
               HHTYPE
                            360103. 28 0
                                             OK
## 28 EDUC
                HHSIZE_CAT
                              466861. 200
                                               OK
                                                OK
## 29 EDUC
                AGRI_OWNERSHIP 41507.
                                         4 0
## 30 EDUC
                FLOOR
                           150129.
                                           OK
                                    80
## 31 EDUC
               WALL
                          161483.
                                   80
                                          OK
## 32 EDUC
                ROOF
                           35670.
                                    80
                                           OK
## 33 EDUC
                REGION
                            263861. 16 0
                                            OK
## 34 HHTYPE
                 HHSIZE_CAT
                              19763144. 35 0
                                                OK
                 AGRI_OWNERSHIP 170289.
                                                  OK
## 35 HHTYPE
## 36 HHTYPE
                 FLOOR
                            189676. 14 0
                                             OK
## 37 HHTYPE
                 WALL
                            378964. 140
                                             OK
## 38 HHTYPE
                 ROOF
                             88110. 14 0
                                            OK
                 REGION
## 39 HHTYPE
                             453920. 28 0
                                              OK
                  AGRI_OWNERSHIP 223841.
## 40 HHSIZE_CAT
                                            50
                                                    OK
## 41 HHSIZE_CAT
                  FLOOR
                              431892. 10 0
                                              OK
## 42 HHSIZE_CAT
                  WALL
                              355496.
                                      10 0
                                              OK
## 43 HHSIZE_CAT
                  ROOF
                              215479. 10 0
                                              OK
## 44 HHSIZE_CAT
                              910990. 200
                                               OK
                  REGION
## 45 AGRI_OWNERSHIP FLOOR
                                 20100.
                                         20
                                                 OK
## 46 AGRI_OWNERSHIP WALL
                                114080.
                                         20
                                                OK
## 47 AGRI_OWNERSHIP ROOF
                                 71304.
                                         20
                                                OK
                                                  OK
## 48 AGRI_OWNERSHIP REGION
                                 897582.
                                           40
## 49 FLOOR
                WALL
                           249740.
                                     40
                                            OK
## 50 FLOOR
                ROOF
                            62535.
                                    40
                                           OK
## 51 FLOOR
                REGION
                           1685502.
                                     80
                                             OK
## 52 WALL
               ROOF
                           593653.
                                    40
                                           OK
## 53 WALL
               REGION
                           2363488.
                                     80
                                             OK
## 54 ROOF
                REGION
                           1148568.
                                     80
                                            OK
## 55 GENDER
                 ROOF
                             1061.
                                    2 4.13e-231 OK
```

From the above Chi-Square Test we can conclude that there is no significance dependence among the chosen predictors.

Catboost Feature Importance

To understand how each feature contributes to the model's prediction let's compute Feature Importance.

To get the following table, CatBoost traverses trees, recording how much each feature contributes to prediction shifts. It averages over all trees in the ensemble. The result is a vector of importances (same length as number of features), often normalized so the sum is 100.

```
##
            [,1]
## AGE
            8.442758
              3.263070
## GENDER
## EDUC
             8.034356
## HHTYPE
              11.754859
## HHSIZE_CAT 15.877825
## AGRI_OWNERSHIP 16.961526
## FLOOR
            15.599579
## WALL
            13.335130
## ROOF
            1.723804
## REGION
              5.007093
```

Model Evaluation

Metric	Value
Gini (Train)	0.91578
Gini (Test)	0.9154
Weighted Kappa (Test)	0.7965
Accuracy	0.79137



Conclusion on the model: Catboost performs excellent results. Time of compilation is long: 9 hours.

Comparison of the models

Metric	OLR	GLR	Light GBM	Catboost
Gini (Train)	0.483	0.548	0.916	0.915
Gini (Test)	0.000441979905060563	0.547	0.917	0.915
Weighted Kappa (Test)	0.0007372	0.497	0.798	0.796
ORC (Test)	0.00036373355246857	0.665	0.870	
Accuracy	0.199767946450715	0.469	0.792	0.791

Precision	0.287673601462297	0.179	0.764	
Recall	0.202291054206273	0.449	0.773	
F1 Score	0.237542832667047	0.256	0.768	

Conclusion

Based on the evaluation metrics, **LightGBM** and **CatBoost** significantly outperform **Ordinal Logistic Regression (OLR)** and **Generalized Linear Regression (GLR)** across all metrics. Both gradient boosting models achieve the highest values in Gini (Train and Test), Accuracy, Precision, Recall, and F1 Score, with LightGBM showing a slight edge overall. In contrast, OLR and GLR exhibit poor generalization, with extremely low Gini (Test) and near-zero Weighted Kappa and ORC, indicating weak predictive power. Among traditional models, GLR performs better than OLR but still falls short of acceptable thresholds. Therefore, **LightGBM** and **CatBoost** are the most suitable models for deployment in this context.