In the Name of God

Amirkabir University of Technology
(Tehran Polytechnic)
Department of Electrical Engineering

Modern Control
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Homework 3
Due: Nov 11, 2024

Please notice the following:

- Write the answers to the exercises in a neat and readable manner and create a PDF file for it using the CamScanner. You can also type the answers if you prefer.
- You may use MATLAB and Simulink or Python only to solve exercises that are marked with the [**M**] symbol and the rest of the exercises must be solved through manual steps. It should be noted that there is no problem with using MATLAB to verify your answers for other exercises, but *do not use MATLAB first*. You will not gain any intuition by looking at results — you need to learn how the method works and improve your problem-solving acumen by solving the problems by hand.
- For exercises that require the use of MATLAB and Simulink, prepare a *maximum of a 3-minute* recorded video to answer each question and reduce the size of the file as much as possible. In the recorded video, describe the activities performed to obtain the solution and deliver your analysis of the results. To solve each question, it is *mandatory* to submit the written code along with the recorded video. Note that answers without a video or code will not be graded.
- Submit a compressed file with the naming format MC_HW1_FullName on the Courses platform. The file should include the answers PDF file along with any MATLAB files and videos (if applicable). Please ensure that the files are organized in their corresponding folders for each question.
- Students are expected to submit homework by 11:59 pm on the due date. However, If you are unable to submit the homework by the deadline due to any circumstances, you may still submit it up to one week late with a 20% penalty deducted from the earned grade. Submissions after one week past the due date will not be accepted. Please plan your time carefully to avoid needing this extension.
- The homework assignments are meant to be completed *individually*. While getting guidance from friends is acceptable, it is expected that you have sufficiently thought about the problem beforehand. However, any form of collaboration beyond seeking advice, such as exchanging solutions or copying code is strictly prohibited, and submitting similar answers will result in a grade of zero.
- The use of AI tools such as ChatGPT to write code is not allowed, and even if you modify the code generated by the AI, it is still detectable and will not be given any grades.
- If you have any questions regarding the exercises, please ask your questions through the Telegram group, as your question is likely a question that other friends may have as well.

Questions

1. The state-space model of the system is given as:

$$\dot{x}(t) = \begin{bmatrix} 1 & \alpha & -2 \\ 0 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} u(t), \quad y(t) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x(t)$$

   a) Find the value of α such that the system becomes uncontrollable.
   b) Using the value of α obtained in part (a), derive the transfer function of the system and show that the system exhibits reduced-order dynamics.

2. Consider an oscillatory second-order system with the following matrices:

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   Check the controllability of the system if the observability Gramian is obtained as follows:

$$W_o(t) = \int_0^t e^{A^T\tau} C^T C e^{A\tau} d\tau = \begin{bmatrix} \frac{5}{2}t - \frac{3\sin(4t)}{8} & \frac{3\cos^2(2t)}{8} \\ \frac{3\cos^2(2t)}{8} & \frac{5}{8}t + \frac{3\sin(4t)}{32} \end{bmatrix}$$

3. Check the controllability and observability of the following system. (Hint: Use the PBH test or the Jordan form of the system.)

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 1 \\ 0 & 5 & -1 & 0 & 2 & 0 \\ 4 & -1 & 0 & 6 & 4 & 0 \end{bmatrix} x(t)$$

4. Check the output controllability and functional controllability for the following system. You may use MATLAB to find the transfer functions matrix.

$$\dot{x}(t) = \begin{bmatrix} -3 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -4 \end{bmatrix} x(t) + \begin{bmatrix} 0 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} u(t), \quad y(t) = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & 0 \end{bmatrix} x(t)$$

5. The system equation is given as:

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t)$$

a) Determine the vectors that span the controllable subspace of the system.

b) Using the defined vectors from part (a) and by selecting appropriate additional vectors, express the system equation in the controllability canonical form. Afterwards, identify the controllable and uncontrollable modes.

c) Is this system stabilizable?

6. Consider the following system:

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & -2 & -2 \\ 0 & -1 & 2 & 2 \\ 0 & 0 & -1 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t), \quad y(t) = \begin{bmatrix} 0 & 0 & 2 & 1 \end{bmatrix} x(t)$$

a) Check the controllability and observability of the system. [**M**]

b) Find the Kalman decomposition for the system and determine the reduced-order subsystems. [**M**]

c) Determine the uncontrollable and unobservable modes (if any).

d) Is the system stabilizable and detectable?

7. Consider a communication satellite orbiting around the earth. The position of the satellite is specified by r, θ and ϕ as shown below:

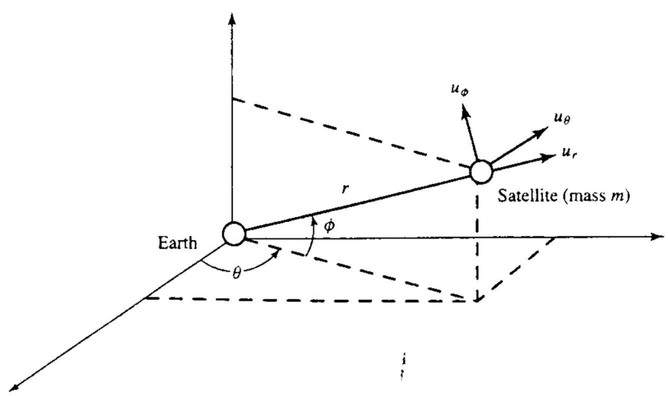

Figure 1: Schematic of communication satellite orbiting

The orbit can be controlled by three orthogonal thrusts u_r, u_θ and u_ϕ. Consider $x = \begin{bmatrix} r & \dot{r} & \theta & \dot{\theta} & \phi & \dot{\phi} \end{bmatrix}^T$ and $u = \begin{bmatrix} u_r & u_\theta & u_\phi \end{bmatrix}^T$ as state vector and input vector, respectively. The linearized model of the system if the satellite is orbiting in a circular equatorial orbit is obtained as follows:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0.0075 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -0.0025 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 \\ 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix} u(t)$$

a) Evaluate the controllability of the system under three separate scenarios where each of the thrusters fails. Are there any scenarios in which the system remains controllable despite the actuator failure? [**M**]

b) (Bonus) Compare the controllability of the system across these scenarios to identify which thruster failure has a more significant impact on system controllability (resulting in a less controllable system) and which failure has a lesser effect (resulting in a more controllable system). You may refer to singular values (and condition number) of the controllability matrix for this part. [**M**]

Good Luck
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