Submitted by: Shahram Najam Syed **Dated: 5th October, 20223**
Homework # 2 (16-665 Robot Mobility)

Q1.1.1:

Ans: Since, we know that the expression for compaction resistance for a rigid wheel is given by:

$$R_{c(rw)} = \frac{\left(3\ \frac{F_z}{\sqrt{d_w}}\right)^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}}\ (n+1)(k_c + b\ k_\Phi)^{\frac{1}{(2n+1)}}}$$

Where,

F_z = Normal force at the wheel-terrain interface

For a 4-wheeled vehicle:

$$F_{z4} = \frac{W_G}{4}$$

And for a 6-wheeled vehicle:

$$F_{z6} = \frac{W_G}{6}$$

Given that n=0.5, the formula for compaction resistance for a rigid wheel becomes:

$$R_{c(rw)} = \frac{\left(3\ \frac{F_z}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{(2.5)^{\frac{3}{2}}\ (1.5)(k_c + b\ k_\Phi)^{\frac{1}{3}}}$$

Computing the compaction resistance for the 4-wheeled and 6-wheeled vehicle:

$$R_{c(rw)4} = 4. \frac{\left(3\ \frac{\frac{W_G}{4}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{(2.5)^{\frac{3}{2}}\ (1.5)(k_c + b\ k_\Phi)^{\frac{1}{3}}}$$

$$R_{c(rw)6} = 6. \frac{\left(3\ \frac{\frac{W_G}{6}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{(2.5)^{\frac{3}{2}}\ (1.5)(k_c + b\ k_\Phi)^{\frac{1}{3}}}$$

The ratio of total compaction resistance of the 4-wheeled vehicle to the 6-wheeled vehicle is:

$$Ratio = \frac{R_{c4}}{R_{c6}}$$

i.e.

$$Ratio = \frac{4.\frac{\left(3\frac{\frac{W_G}{4}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{(2.5)^{\frac{3}{2}}\,(1.5)(k_c + b\,k_\Phi)^{\frac{1}{3}}}}{6.\frac{\left(3\frac{\frac{W_G}{6}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{(2.5)^{\frac{3}{2}}\,(1.5)(k_c + b\,k_\Phi)^{\frac{1}{3}}}}$$

$$Ratio = \frac{4\,.\left(3\frac{\frac{W_G}{4}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}{6\,.\left(3\frac{\frac{W_G}{6}}{\sqrt{d_w}}\right)^{\frac{3}{2}}}$$

$$Ratio = \frac{4\,.\left(\frac{1}{4}\right)^{\frac{3}{2}}}{6\,.\left(\frac{1}{6}\right)^{\frac{3}{2}}} = \frac{\sqrt{6}}{2} \sim 1.2$$

Which comes out to:

$$Ratio > 1$$

This means that the compaction resistance for the 4 wheeled vehicle is greater than that of the 6-wheeled vehicle, which means that the 6-wheeled vehicle is better in terms of minimizing compaction resistance compared to the 4-wheeled vehicle. Intuitively this makes sense since the vehicle's weight over a greater number of wheels reduces the compaction resistance experienced by each wheel, which in turns lowers the overall resistance for the vehicle.

Q1.1.2:

Ans: We first start by calculating the soil thrust for each vehicle concept. Since we are asked to determine the ideal soil thrust (with no slip), hence:

$$\mathrm{H} = \mathrm{cA} + \mathrm{W} \tan \Phi$$

Where:

- c is the soil cohesion
- A is the contact area of the tire with the soil
- W is the normal force at the wheel-terrain interface
- Φ is the soil angle of internal shearing resistance (friction)

Since,

$$W = \sigma \mathrm{A}$$

Where, σ is the normal stress.

For a 4-wheeled vehicle:

$A_4 = 1.2\ A_6$

$W_{G4} = W_G$
$\sigma_4 = \frac{W_{G4}}{4\ \mathrm{A}_4}$
$W_4 = \sigma_4\ \mathrm{A}_4$

For a 6-wheeled vehicle:

$W_{G6} = W_G$
$\sigma_4 = \frac{W_{G6}}{6\ \mathrm{A}_6}$
$W_6 = \sigma_6\ \mathrm{A}_6$

Now, computing the soil thrust for each concept:

For the 4-wheeled concept:

$\mathrm{H}_4 = 4(\mathrm{cA}_4 + \mathrm{W}_4 \tan \Phi)$

$\mathrm{H}_4 = 4(\mathrm{c}\ (1.2\ \mathrm{A}_6) + \sigma_4(1.2\ \mathrm{A}_6) \tan \Phi)$

$\mathrm{H}_4 = 4.8\ \mathrm{A}_6\ (\mathrm{c} + \sigma_4 \tan \Phi)$

Similarly, for the 5-wheeled concept:

$\mathrm{H}_6 = 6(\mathrm{cA}_6 + \mathrm{W}_6 \tan \Phi)$

$\mathrm{H}_6 = 6(\mathrm{cA}_6 + \sigma_6\ \mathrm{A}_6 \tan \Phi)$

$\mathrm{H}_6 = 6\ \mathrm{A}_6(\mathrm{c} + \sigma_6 \tan \Phi)$

The ratio of total thrust of the 4-wheeled concept over the 6-wheeled concept is represented as:

$$Ratio = \frac{\mathrm{H}_4}{\mathrm{H}_6}$$

$$Ratio = \frac{4.8\ \mathrm{A}_6\ (\mathrm{c}\ + \sigma_4 \tan \Phi)}{6\ \mathrm{A}_6(\mathrm{c}\ +\ \sigma_6 \tan \Phi)}$$

$$Ratio = \frac{4.8(\mathrm{c}\ + \sigma_4 \tan \Phi)}{6(\mathrm{c}\ +\ \sigma_6 \tan \Phi)}$$

Since, we know that:

$\sigma_4 = \frac{W_{G4}}{4\ \mathrm{A}_4}$

$\sigma_6 = \frac{W_{G6}}{6\ \mathrm{A}_6}$

Then,

$$Ratio = \frac{4.8(\mathrm{c}\ + \frac{W_{G4}}{4\ \mathrm{A}_4} \tan \Phi)}{6(\mathrm{c}\ +\ \frac{W_{G6}}{6\ \mathrm{A}_6} \tan \Phi)}$$

$$Ratio = \frac{4.8(\mathrm{c}\ + \frac{W_{G}}{4.8\ \mathrm{A}_6} \tan \Phi)}{6(\mathrm{c}\ +\ \frac{W_{G}}{6\ \mathrm{A}_6} \tan \Phi)}$$

$$Ratio = \frac{4.8\ \mathrm{A}_6\ \mathrm{c}\ + W_G \tan \Phi}{6\ \mathrm{A}_6\ \mathrm{c}\ +\ W_G \tan \Phi}$$

Which is the expression for the ratio of ideal thrust for the 4-wheeled concept to the ratio of the 6-wheeled concept. Since, c(cohesion) represents the soil's ability to stick together and resistance to being pulled apart, it cannot be negative. Similarly, Φ is the angle of internal friction is usually between 0-45 degrees. Consequently, W_G (gross vehicle weight) and A_6 (contact area) can also never be negative. **It can be inferred from the above expression that the ideal thrust for the 6-wheeled concept will be greater than the ideal thrust for the 4-wheeled concept.** Conceptually this makes sense. Although for each wheel of the 4-wheel vehicle concept has a 20% larger contact area as compared to that of the 6-wheeled vehicle, but the 6-wheeled vehicle has a total of 2 more wheels as compared to the 4-wheeled vehicle concept, hence the total cohesive thrust from all six wheels is greater than the total of the four wheels of the 4-wheeled vehicle.

Q 1.1.3:

Ans: Since the drawbar pull (DP) is given by:

$$DP = H - \Sigma R$$

Where, H is the ideal soil thrust and ΣR is the total motion resistance.

Since we are given that the only significant form of motion resistance is compaction, so:

$$\Sigma R = R_{c(rw)} = \frac{\left(3 \frac{F_z}{\sqrt{d_w}}\right)^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}} (n+1)(k_c + b\, k_\Phi)^{\frac{1}{(2n+1)}}}$$

And the ideal soil thrust (H) is given by:

$$\mathrm{H}_6 = \mathrm{cA}_6 + \mathrm{W} \tan \Phi$$

Since, we are considering only a single wheel of the 6-wheeled vehicle hence:

$\sigma_6 = \frac{W_{G6}}{6\,\mathrm{A}_6}$

$W_{G6} = \sigma_6 \mathrm{A}_6$

$F_z = W_{G6}$

So, the total motion resistance becomes:

$$\Sigma R = R_{c(rw)} = \frac{\left(3 \frac{\sigma_6 \mathrm{A}_6}{\sqrt{d_w}}\right)^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}} (n+1)(k_c + b\, k_\Phi)^{\frac{1}{(2n+1)}}}$$

Given in Q 1.1.1, n = 0.5:

$$\Sigma R = R_{c(rw)} = \frac{\left(3 \frac{\sigma_6 \mathrm{A}_6}{\sqrt{d_w}}\right)^2}{(2.5)^2\,(1.5)(k_c + b\, k_\Phi)}$$

And, the ideal soil thrust (H) becomes:

$$\mathrm{H}_6 = \mathrm{cA}_6 + \sigma_6 \mathrm{A}_6 \tan \Phi$$

So finally the drawbar pull for a single wheel becomes:

$$DP = H - \Sigma R$$

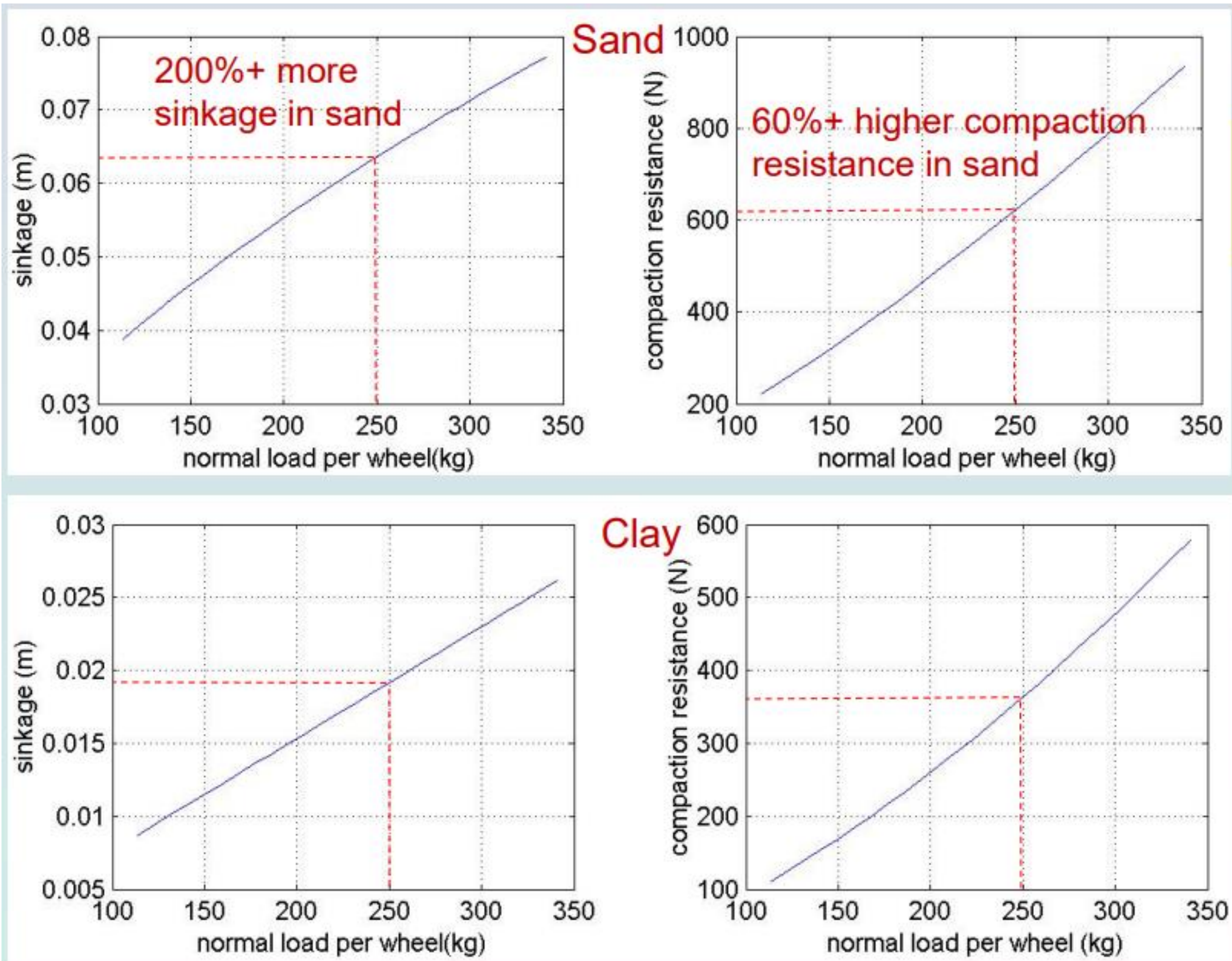
$$DP = H - R_{c(rw)}$$

$$DP = \mathrm{cA}_6 + \sigma_6 \mathrm{A}_6 \tan\Phi - \frac{\left(3\,\frac{\sigma_6 \mathrm{A}_6}{\sqrt{d_w}}\right)^2}{(2.5)^2\,(1.5)(k_c + b\,k_\Phi)}$$

Which is the analytical equation for the drawbar pull generated by a single wheel of the 6-wheeled vehicle considering only compaction as a form of motion resistance.

Q 1.1.4:

Ans: For this question, I referenced the following graph provided during the classes:



Using these graphs, one could effectively observe trends related to sinkage and compaction resistances of a rover operating in both loose sand and packed clay. Although these trends don't directly discuss maximum thrust, but the insights from the trends of sinkage and compaction resistance are pivotal in making a cogent reasoning for the thrust.

Following are my observations:

- **Sinkage vs Normal Load:**
  The sinkage in sand increases more rapidly with an increase in the normal load per wheel as compared to clay. Specifically, there is a 200% or more increase in sinkage in sand when comparing the range of normal loads as shown in the graph. On the contrary, the increase in sinkage with normal load is more gradual in clay compared to sand, indicating that packed clay can better support increased load with lesser sinkage.
- **Compaction resistance vs. Normal Load:**
  Referencing the graphs above, the compaction resistance in sand rises sharply with an increase in normal load, with a 60% or more increase in compaction resistance in sand within the given range

of normal loads. Conversely, packed clay exhibits a similar trend but in a more linear and steady fashion.

Based on these observations, one could reason regarding the maximum thrust for packed clay vs loose sand as follows:

- For the case of loose sand, increased sinkage due to higher normal loads would lead to decrease in the effective contact area of the wheel with solid ground, thus reducing its ability to generate thrust. Added onto that the sharp increase in compaction resistance as normal load increases aggregates to reduce the maximum thrust making it harder to traverse the loose sand terrain.
- For the case of packed clay, it still exhibits a trend of increase in sinkage and compaction resistance with increased normal load, but the change is more linear and predictable. This means that even though there will be a reduction in maximum thrust it might not be as drastic as in the case of loose sand.

Now discussing the impact of vehicle weight and contact area. Both vehicle weight and contact area have a direct impact on the ground pressure, sinkage, and compaction resistance. Increasing the weight of the vehicle (i.e. the normal load per wheel) without a proportional increase in contact area would result in higher sinkage and compaction resistance particularly in loose sand thus leading to reduced thrust. Consequently, increasing the contact area can help distribute the vehicle's weight more evenly over a wider surface, reducing the depth to which the wheel sinks, and thereby potentially increasing the maximum thrust the vehicle can generate.

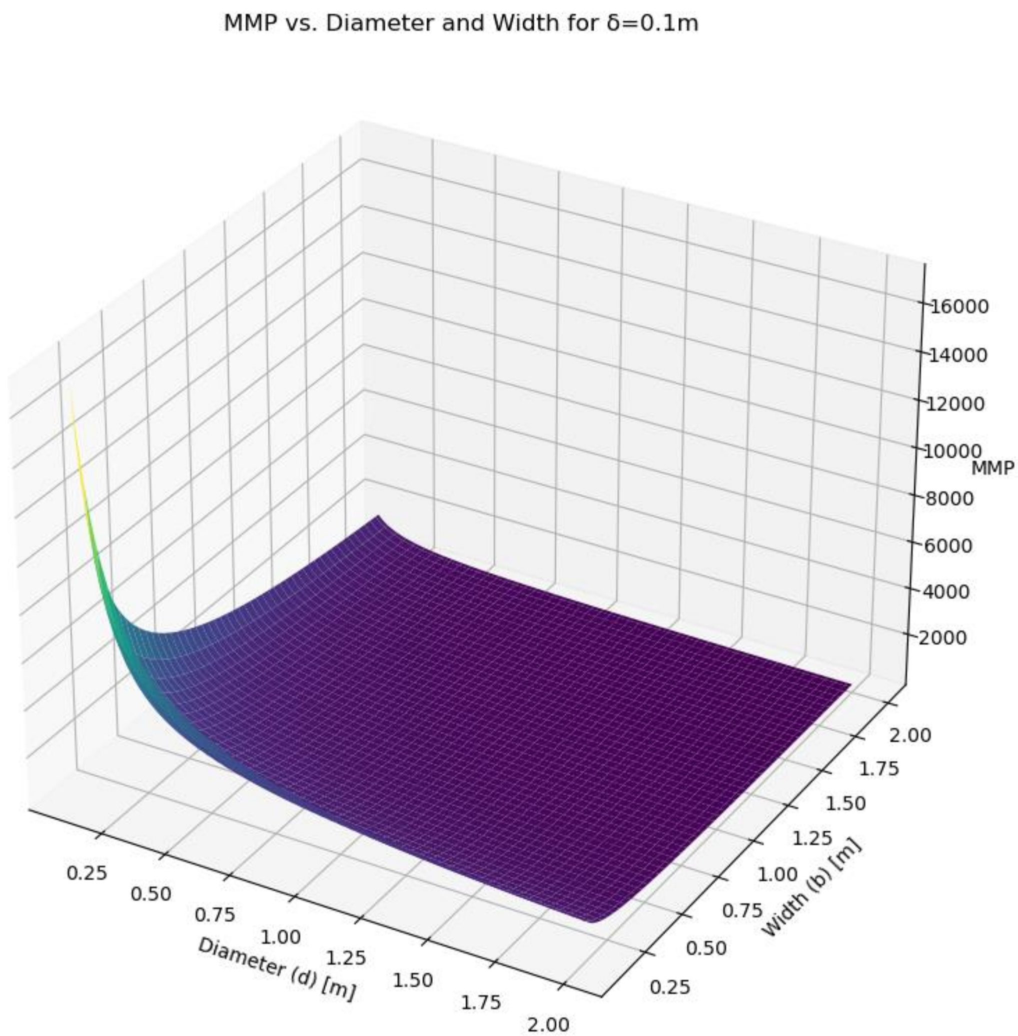
Q 1.2.1:

Ans: The Mean Maximum Pressure (MMP) is a measure that often relates to a vehicle's ability to traverse a certain terrain. A lower MMP implies that for the given weight, tire dimension, and deflection the vehicle is designed to distribute its weight more effectively thus less pressure exerted between the wheel-terrain interface, making the vehicle concept more beneficial for softer terrains.

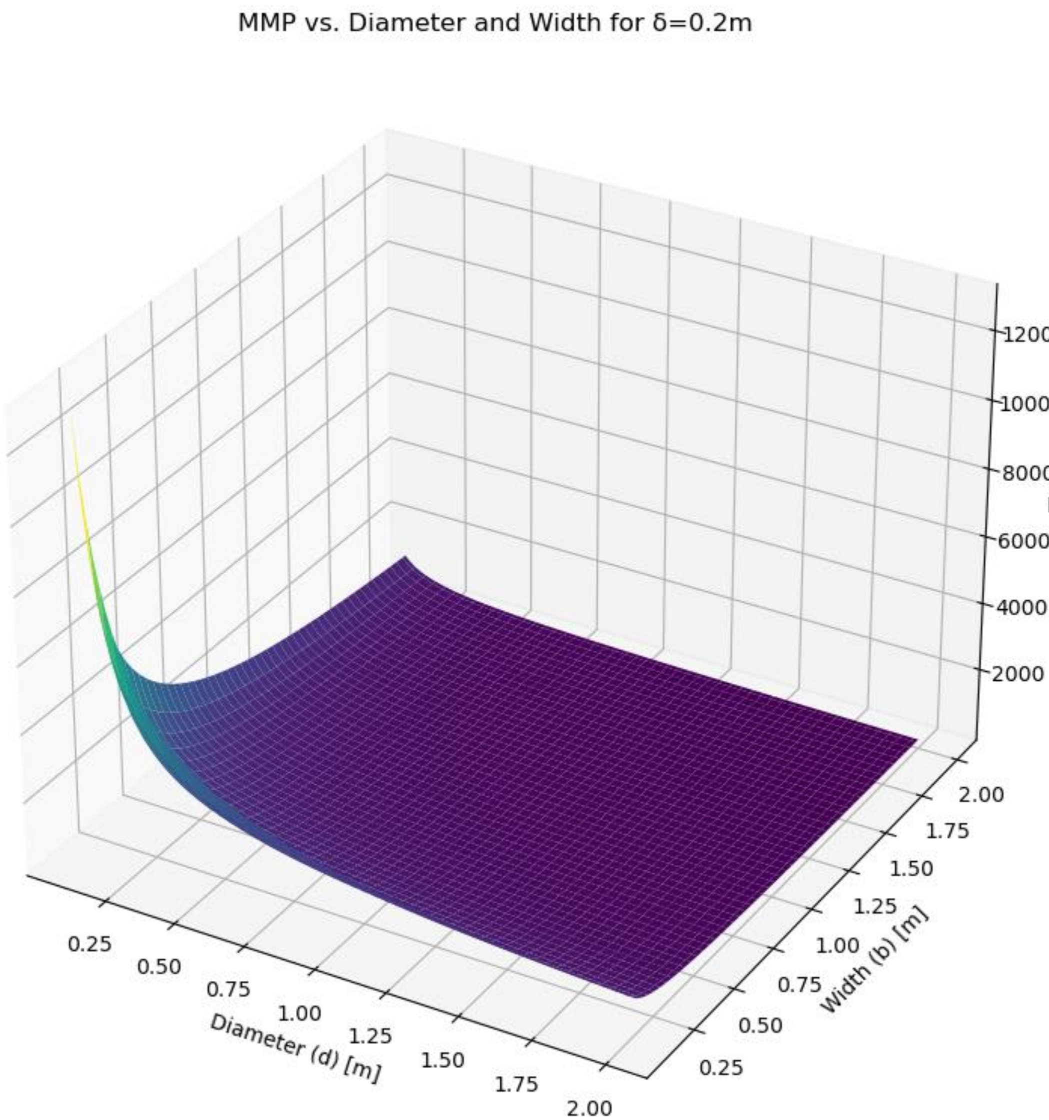
Given the choice to choose between the MMP values of 40 and 100 the obvious choice would be to choose an MMP value of 40, especially for softer terrains.

Q 1.2.2:

Ans: (The code for this question is annexed at the end of the document) Following are the 3D parametric plot of MMP as a function of d and b for W_G= 100kN, a= 2, n= 2, and δ= 0.1m.



From the above graph, it can be observed that there is a decreasing non-linear trend of MMP as the diameter (d) and width (b) increases, which is a trivial observation which can be inferred from the given formula itself. The steepest decline for MMP can be observed when both the diameter and width are smaller, indicating that for smaller dimensions higher pressure is observed between the wheel-terrain interface. Similarly, for larger dimensions of d and b the change in MMP tappers off.

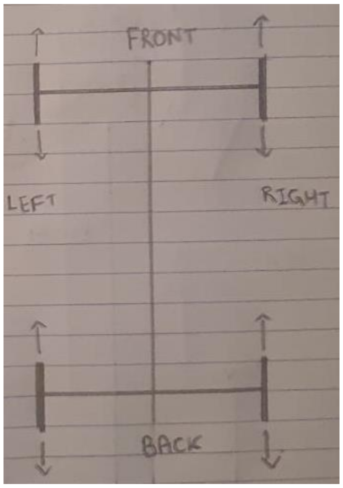


Changing the value of δ= 0.2m, an overall decrease in the MMP across the same range of diameter and width values can be observed. The overall trend and shape between the two plots remain the same, with just the graph for δ= 0.2m shifting downwards on the axis of MMP indicating its inversely proportional relationship with MMP.

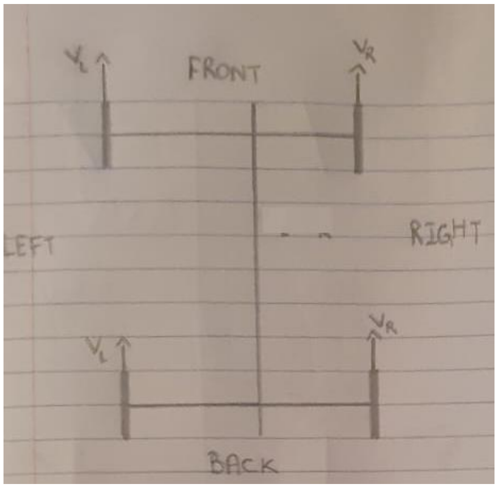
Q 2.1.1.:

Ans: In this question we are asked to draw diagrams for the following steering configuration depicting the portion of the vehicle that moves. The answer is referenced from [Steering and Control of a Passively Articulated Robot by Shamah etal.]() and from the class lecture slides. These are as follows:

a. **Skid (differential) steering:** For skid steering, the wheels on the left side of the vehicle (both at the front and back) are synchronized in terms of linear velocity and direction. Subsequently is the same for the wheels on the right side of the vehicle as well. Following is the diagram depicting the same.



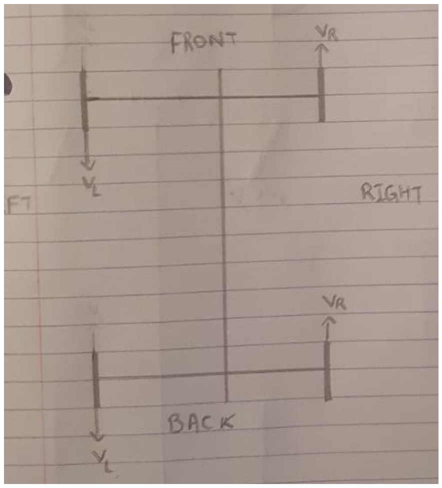
To depict the maneuverability of the vehicle more explicitly following image can be used as reference:

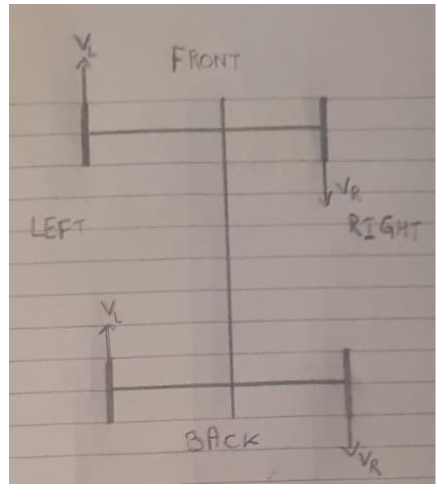


For the diagram above,

i) $V_L = V_R$: Moves forward
ii) $V_L > V_R$: Turns left
iii) $V_L < V_R$: Turns right

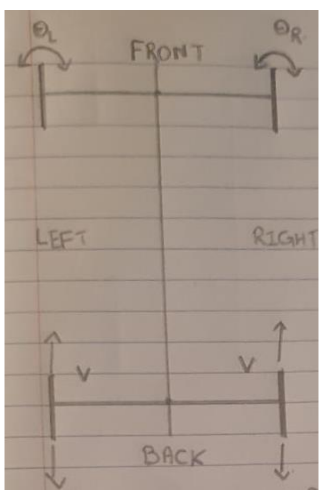
The following images exhibit the full pivot turn (both left and right):

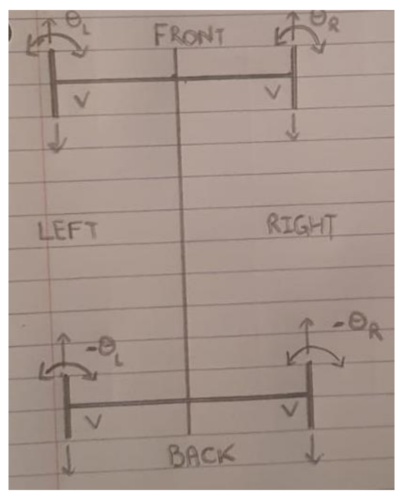




b. **Ackermann Steering:**

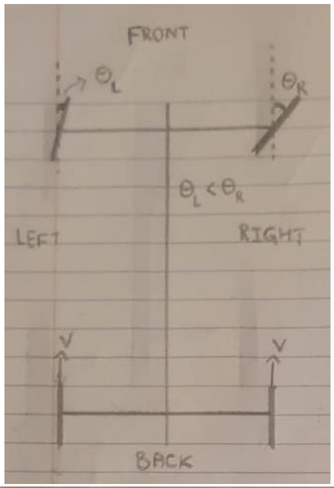
For the sake of simplicity (and to be in line with the lecture slides) only single axle Ackermann steering and four-wheel Ackermann steering will be discussed when answering this question. For an Ackermann steering, the inner wheel and outer wheel track a different radius of curvature for a smooth turning effect. Following are the depictions of the plausible direction of motion of the parts of the vehicle with single-axle Ackermann Steering & four wheel Ackermann steering.

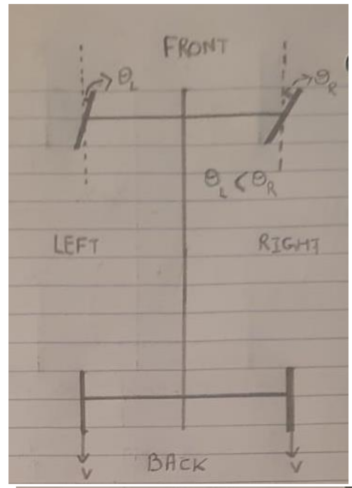


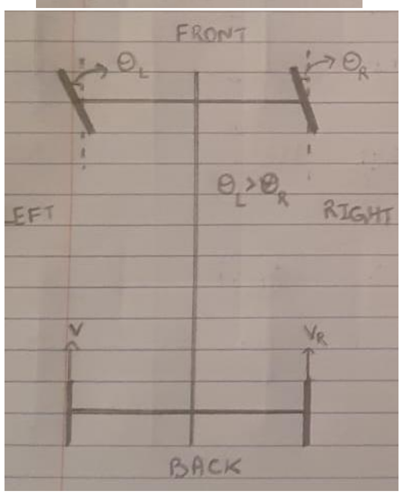


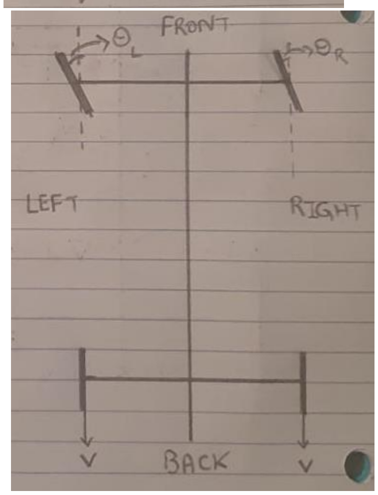
As can be seen, for the single-axle Ackermann configuration (left image above) the front wheels can turn laterally to the left or to the right (with the inner wheel turning more as compared to the outer wheel), while the rear wheels provide propulsion. As for the four wheel Ackermann, both the front and rear axle wheel can provide lateral turn (to the left or the right). Depending on whether it is a 2WD, AWD, or 4WD system, the propulsion force maybe provided by either the front or the back set of wheels or by both (i.e. both the front and the back). It is worth qualifying that unlike the explicitly independent configuration, the lateral rotation of the wheels on a common axle are coupled (i.e. you cannot just turn the front left wheel without subsequently turning the front right wheel) for an Ackermann steering configuration.

Following are some of the turning maneuvers that can be performed using the single-axle Ackermann:



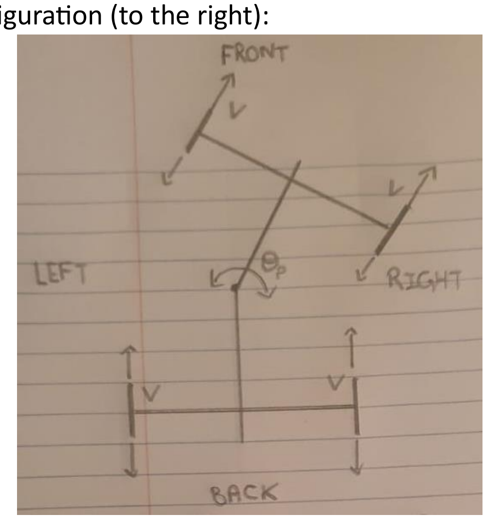


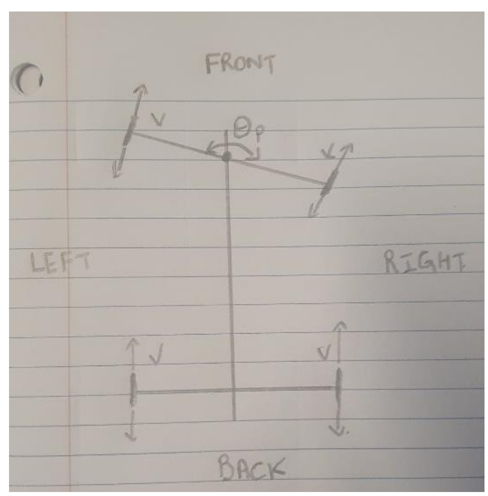




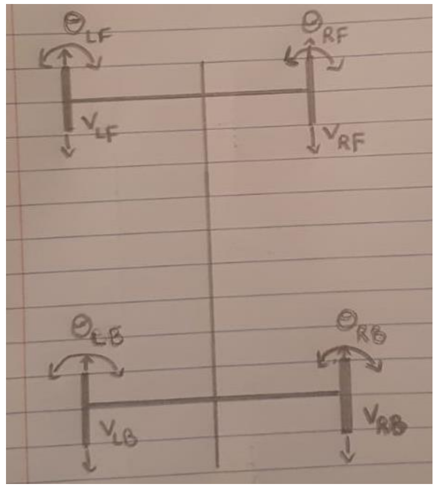
c. Articulated Steering:

For Articulated steering the lateral motion of the vehicle happens around a pivot point, while the wheels (both front and back) provide longitudinal propulsion. Following is the diagrams depict the examples of frame articulated steering configuration (to the left) and axle articulated steering configuration (to the right):





d. **Explicit Independent wheel articulation:**
For explicit independent wheel articulation, each wheel has its own motor drive, controlling both the lateral yaw and velocity at each wheel independently. Following is a simplistic 2D rendition of the same.



Q 2.1.2.:

Ans: Following are the positive and negative attributes (added 2 points for each in case any point was not distinctive) for the type of steering discussed in Q 2.1.2:

1. **Skid (Differential) Steering:**
   i) **Positive Attribute:**
      a. This configuration allows for in-place turns (i.e. zero turn radius) thus providing high maneuverability in a constrained environment.
      b. With a simplistic mechanical design and few moving parts (as compared to traditional steering systems) low effort is required for maintenance due to reduced failure points.
   ii) **Negative Attribute:**
      a. For turning maneuvers there is an increase in rolling resistance which detrimentally effects the operational time, thus making them energy inefficient.
      b. The skidding motion induces wear and tear on the times causing frequent replacements and associated costs.

2. **Ackermann Steering:**
   i) **Positive Attribute:**
      a. By optimizing the wheel angles for reduced slipping the tire wear is reduced during turning.
      b. With a linear steering response, this type of steering provides more accurate control particularly during turning.
   ii) **Negative Attributes:**
      a. Complex linkage design and the requirement of precision in angle alignment of the wheels makes it convoluted and involved when designing and tuning the steering configuration.
      b. Ackermann steering provides limited maneuverability in constrained environments (as compared to skid steering).

3. **Articulated Steering:**
   i) **Positive Attributes:**
      a. The articulating action (particularly for frame articulated configuration) provides a better load distribution during uneven terrain navigation.
      b. The pivot-based turning mechanism provides an increased maneuverability for compact or constrained space turning.
   ii) **Negative Attributes:**
      a. For heavier normal loads, the central pivot point accumulates significant mechanical stress leading to wear and more frequent maintenance.
      b. For high speeds, articulated vehicles exhibit reduced stability.

4. **Explicit (Independent) Wheel Articulation:**
   i) **Positive Attributes:**
      a. Independent movement of each wheel facilitates ground contact for better terrain adaptability.

b. This sort of configuration provides better distributed normal load management, thus decreasing the possibility of the vehicle getting stuck.

ii) Negative Attributes:

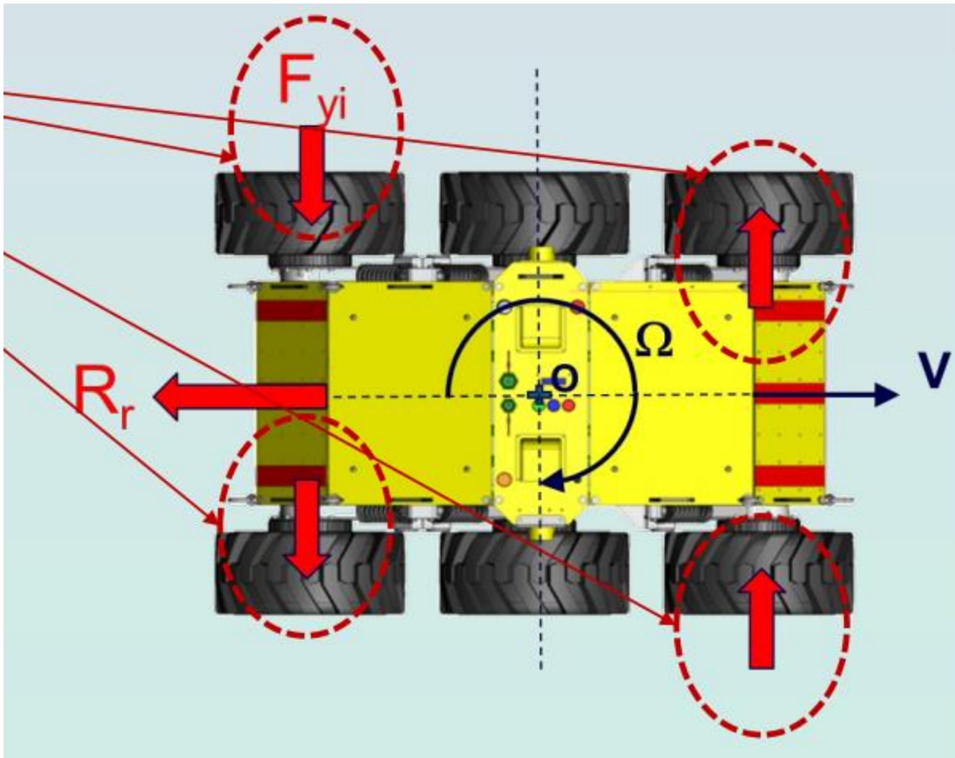
a. There is a tradeoff involved with increased independence of DOF for each wheel, making the control mechanism and algorithms more complex.

b. With more moving mechanisms, there is an increase in the potential number of failure points, and thus maintenance requirements.

Submitted by: Shahram Najam Syed **Dated: 5th October, 20223**
Homework # 2 (16-665 Robot Mobility)

Q 2.1.3.:

Ans: Following image is being alluded in this particular question, which is provided in the slide # 9 of lecture # 4:



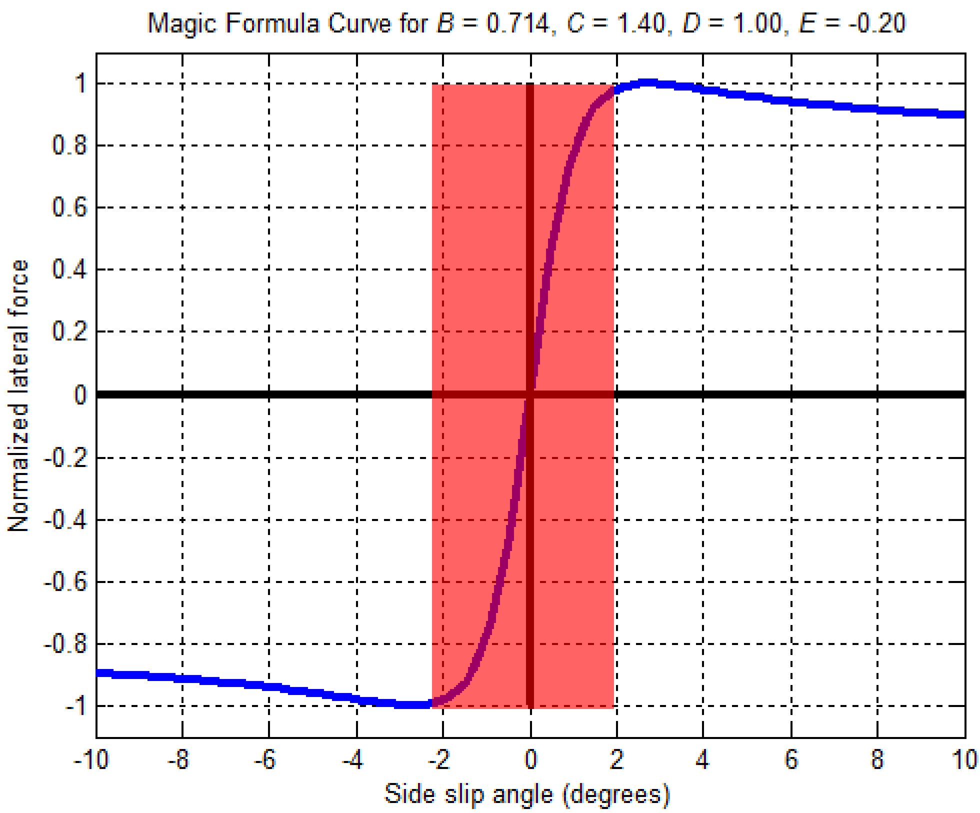
Following is the analysis of each force:

1. **F_y (Lateral Force):** This force primarily resists the side-to-side motion of the wheel (perpendicular to the direction of each wheel's movement) and is involved when a vehicle is turning, during which tires generates a lateral force opposite to the direction of the turn. This is engendered by the tire's deformation and the interaction between the wheel-terrain interface. This force plays a vital role in generating necessary centripetal force during the turning maneuver of the vehicle.

2. **F_x (Longitudinal Force):** Although it is not directly labeled on the slide, F_x represent the longitudinal force in the direction of motion. This force is responsible for propelling the vehicle forward (or resist forward motion on a declining slope). This type of force is generated and dependent on the traction between the tire and the terrain, and can be affected by the acceleration, braking, or driving up or down a slope.

3. **R_r (Rolling Resistance):** Rolling resistance is a reactive force that opposes the rolling motion of a tire on a surface. This type of resistance is generally caused by hysteresis or deformation of the tire as it contacts and bulldozes through the terrain during motion.

For estimating the Lateral Force F_y accurately, one must know the slip angle, tire (material) properties, and the normal load on each tire. A common method for estimating the lateral force is to use something called "Pacejka Magic Formula Curve" as shown below. This formula is an empirical model to approximate the complex relationship between the longitudinal and lateral tire forces for various influencing factors such as slip angle, normal load, and inflation pressure.



As for estimating rolling resistance, it can be calculated through the formula $R_r = C_{rr} \times N$, where N is the normal load on each wheel and C_{rr} is the coefficient of rolling resistance determined through experiments for specific tire and terrain combination.

Q 2.2.:

Ans: The choice of suspension system plays an instrumental role in determining the ride quality, handling, and overall performance of a vehicle. Following are my three reasons for choosing a semi-active suspension over passive suspensions:

a. Semi-active suspensions have the ability to adjust their damping coefficients by varying properties of the damping fluid or mechanically adjusting the value in the damper providing an increased dynamic adaptability in operations and handling at high speeds for a range of different terrains.

b. By dynamically adjusting the suspension damping, a semi-active suspension maintains a more consistent tire-to-ground contact leading to better traction and braking control.

c. The ability to dynamically adjust the damping coefficient can assist in minimizing oscillations and overshoots during quick maneuvers, thus providing quicker stabilization, and enhancing vehicle agility and safety.

Now coming to the discussion as to why, an active suspension would be more beneficial as opposed to a semi-active suspension is the total suspension control. Active suspensions employ sensors, control units, and actuators preemptively predict and respond to disturbances, as opposed to the semi-active suspension limitation to only modulate the resistance to motion. This provides a leg up for vehicles with active suspensions in extreme terrains by providing the ability to proactively manage and adjust the vehicle's suspension characteristics for a smooth ride with lesser disturbances.

Codes:

Q 1.2.2.

```
import numpy as np
import matplotlib.pyplot as plt


# Given constants
WG = 100  # kN
a = 2
n = 2
delta = 0.1  # First case

# Generate a range of values for d and b

d = np.linspace(0.1, 2, 100)  # Example range from 0.1 to 2 meters
b = np.linspace(0.1, 2, 100)  # Example range from 0.1 to 2 meters

# Create meshgrid for d and b

D, B = np.meshgrid(d, b)


# Calculate MMP using the given formula

MMP = (6.896 * WG) / (n * a * B**0.8 * D**0.8 * delta**0.4)

# Plotting for δ=0.1

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf = ax.plot_surface(D, B, MMP, cmap='viridis', label='δ=0.1')
fig.colorbar(surf, ax=ax, pad=0.2)


# Set labels and title


ax.set_xlabel('Diameter (d) [m]')
ax.set_ylabel('Width (b) [m]')
ax.set_zlabel('MMP')
ax.set_title('MMP vs. Diameter and Width for δ=0.1m')


plt.show()
```

```
# Now for δ=0.2

delta = 0.2
MMP = (6.896 * WG) / (n * a * B**0.8 * D**0.8 * delta**0.4)

# Plotting for δ=0.2

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
surf = ax.plot_surface(D, B, MMP, cmap='viridis', label='δ=0.2')
fig.colorbar(surf, ax=ax, pad=0.2)


# Set labels and title

ax.set_xlabel('Diameter (d) [m]')
ax.set_ylabel('Width (b) [m]')
ax.set_zlabel('MMP')
ax.set_title('MMP vs. Diameter and Width for δ=0.2m')

plt.show()
```