SLAM: Inertial Navigation

Robot Localization and Mapping
16-833

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October 23, 2024

Inertial Navigation

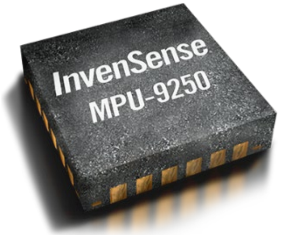
- Inertial Measurement Unit (IMU)
  - Gyroscope
  - Accelerometer
  - Magnetometer (optional)
  - Pressure (optional)
- Inertial Navigation System (INS)
  - Attitude Heading Reference System (AHRS)
  - GPS-Aided Inertial
  - DVL-Aided Inertial
  - Vision-Aided Inertial

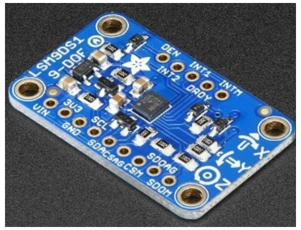
Inertial Sensors

>$10k 1deg/h

Around $1000

<$10 1deg/s

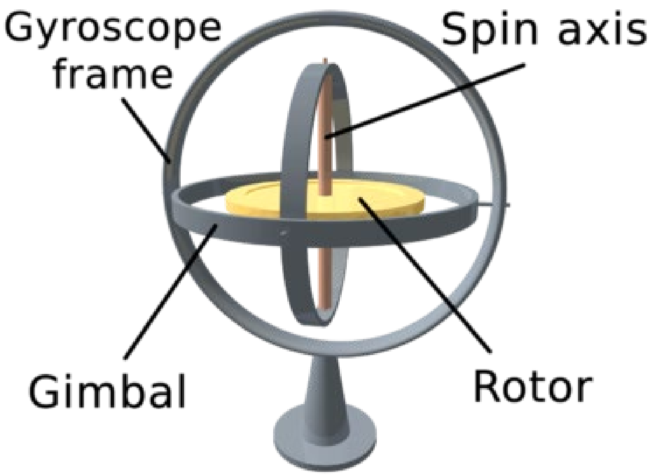




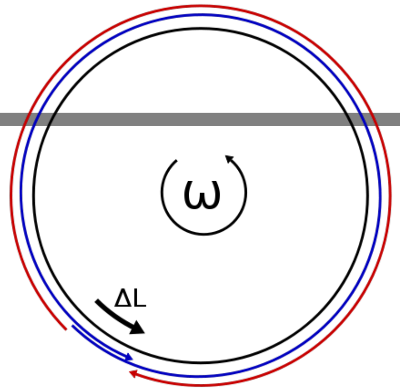
Gyroscopes

- Mechanical
  - Gimbal
  - Vibrating structure gyros
    - Tuning fork resonator
    - Hemispherical resonator
    - Cylindrical resonator
- MEMS
  - Vibrating structure gyros
- Optical
  - Ring-Laser Gyro (RLG)
  - Fiber Optic Gyro (FOG)

Leon Foucault, 1852



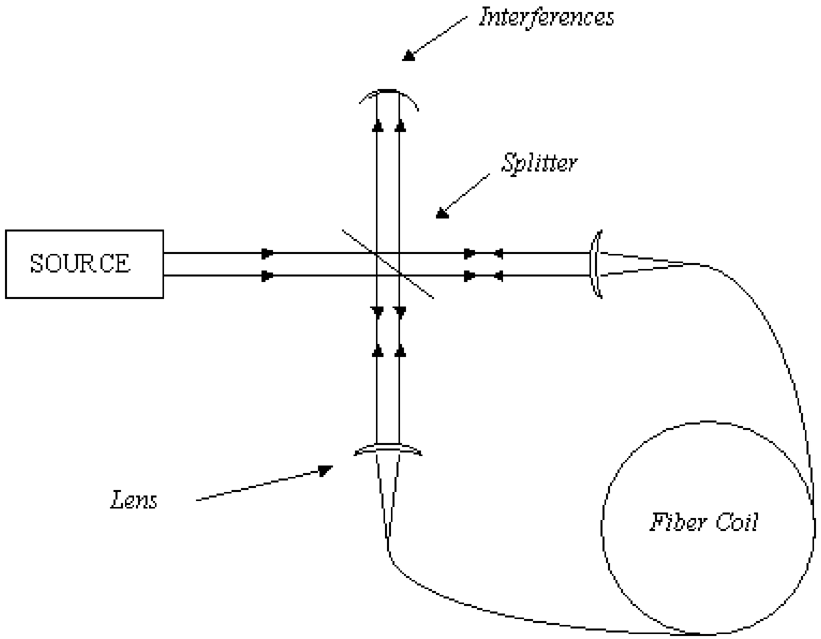
Fiber Optic Gyroscopes



- Use coherent light to estimate angular rate
- Sagnac effect
  - A single light beam split and sent in opposite directions around a loop will emerge with a phase difference proportional to the angular velocity of the loop

$$\Delta\phi = \frac{2\pi L D}{\lambda c}\Omega$$

  - A detector can be built to look at the interference pattern and measure the rate



Courtesy of Ryan Eustice

Fiber Optic Gyro

- Very accurate measurement of rotation rates
  - Use 3 coils, in three different planes, to calculate true heading
  - No moving parts, no magnetic biases
  - Will estimate its own latitude

$$\omega_{\text{observed}} \propto \sin(\text{lat})$$

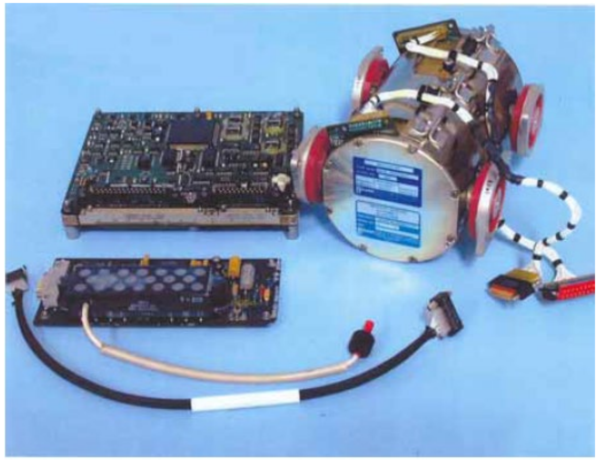
Heading	
Accuracy	0.1 deg secant latitude [1] [2]
Resolution	0.01 deg
Settling time (static conditions)	< 1 min
Full accuracy settling time (all conditions)	< 5 min
Heave / Surge / Sway	
Accuracy	5 cm or 5% (whichever is highest)
	Set-up free (SAFE-HEAVE ™)
Roll / Pitch	
Dynamic accuracy	0.01 deg (for ±90 deg amplitude)[2]
Range	No limitation (-180 deg to 180 deg)
Resolution	0.001 deg

Courtesy of Ryan Eustice

North-Seeking Gyrocompass/INS

- Mechanical Gyro
  - Obsolete
- Optical Gryo
  - Ring-Laser or Fiber-Optic
  - Accuracy
    - 0.1 Heading
    - 0.01 Pitch & Roll
  - Cold start in 5 minutes.
  - Power: 15-30W
  - Cost: $30K - $80K

Kearfott KI4920 INS.

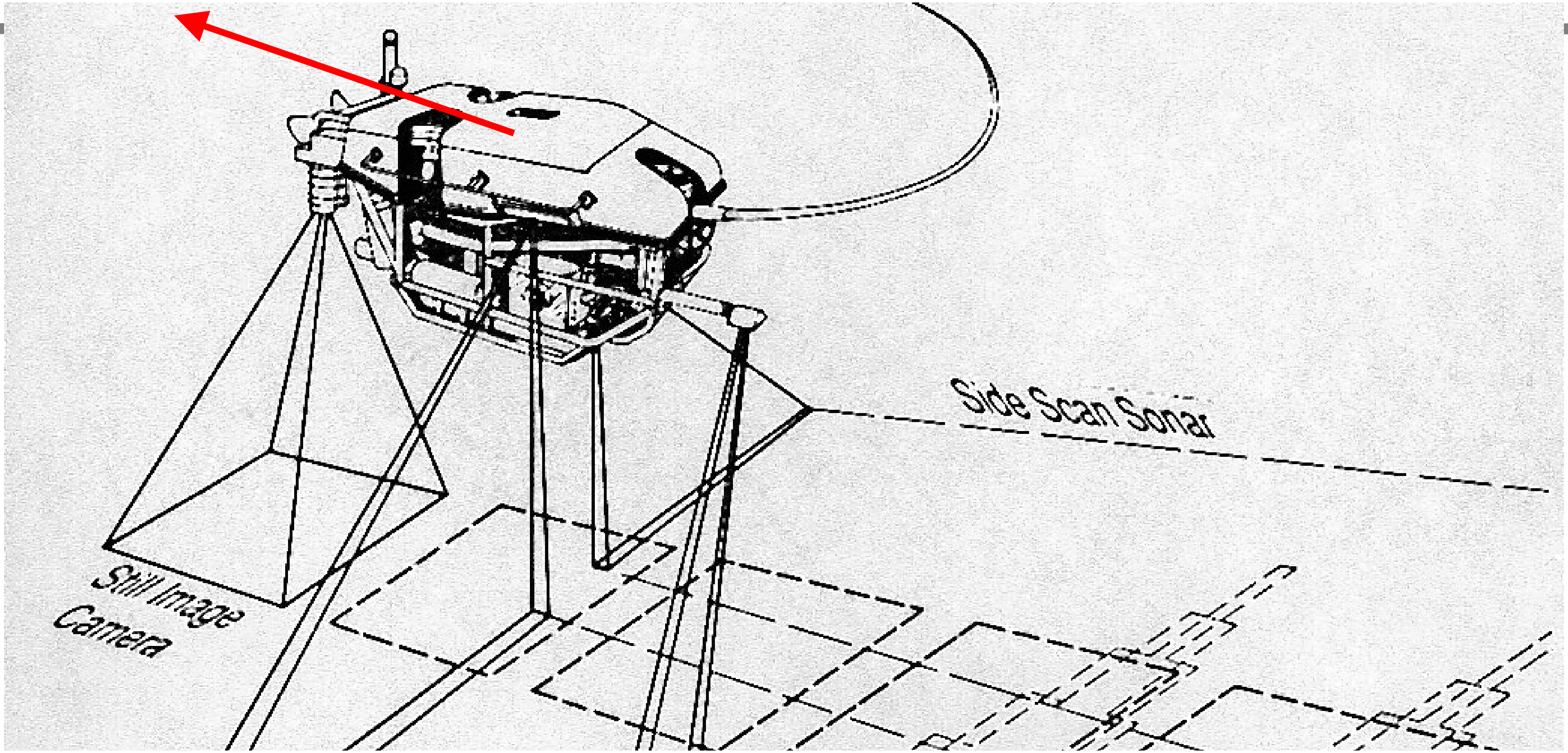


Ixsea Octans Gyrocompass

Courtesy of Ryan Eustice

Inertial Navigation

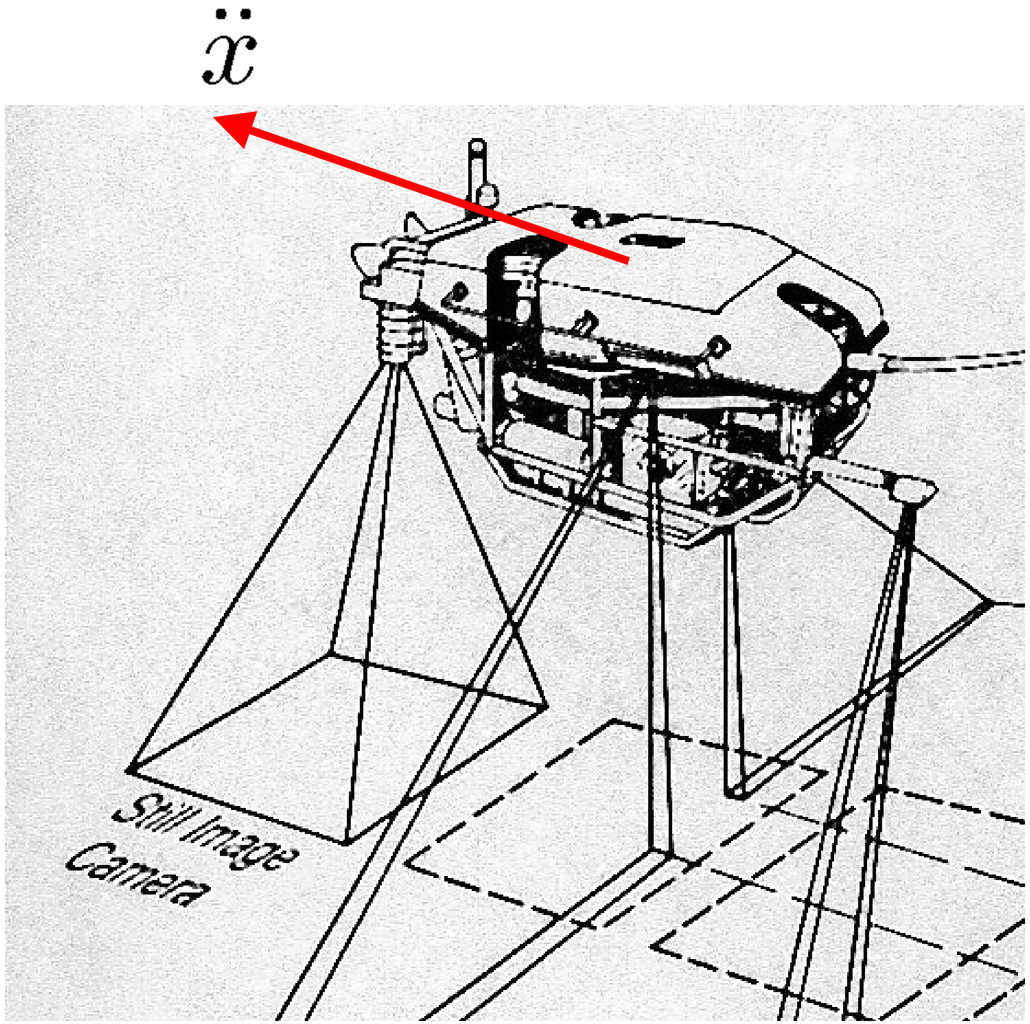
$\ddot{x}$



$$x(t) = x(t_o) + \int_{t_o}^{t} \dot{x}(t_o)t + \int_{t_o}^{t}\int_{t_o}^{t} \ddot{x}(\tau)\, d\tau$$

Courtesy of Ryan Eustice

Inertial Navigation



Advantages

- High precision local navigation
- High update rate

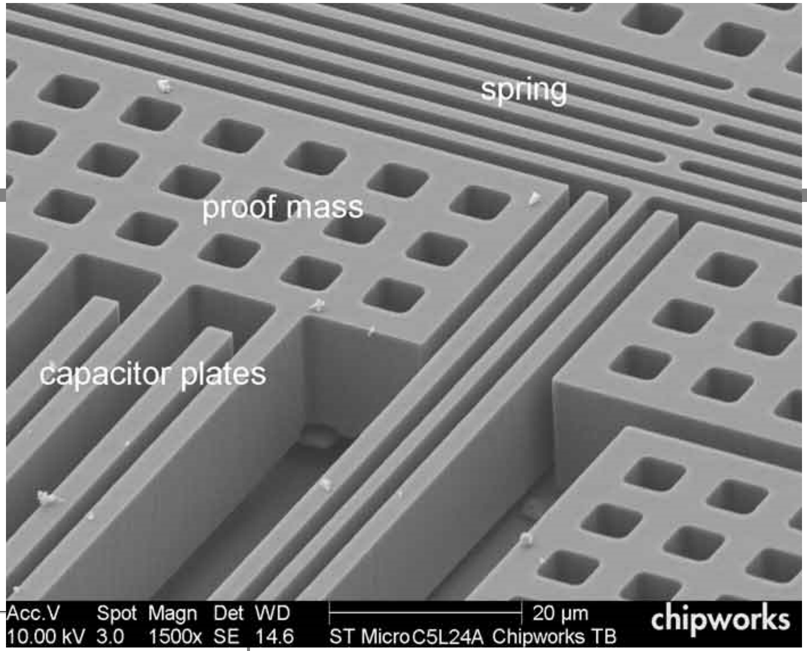
Disadvantages

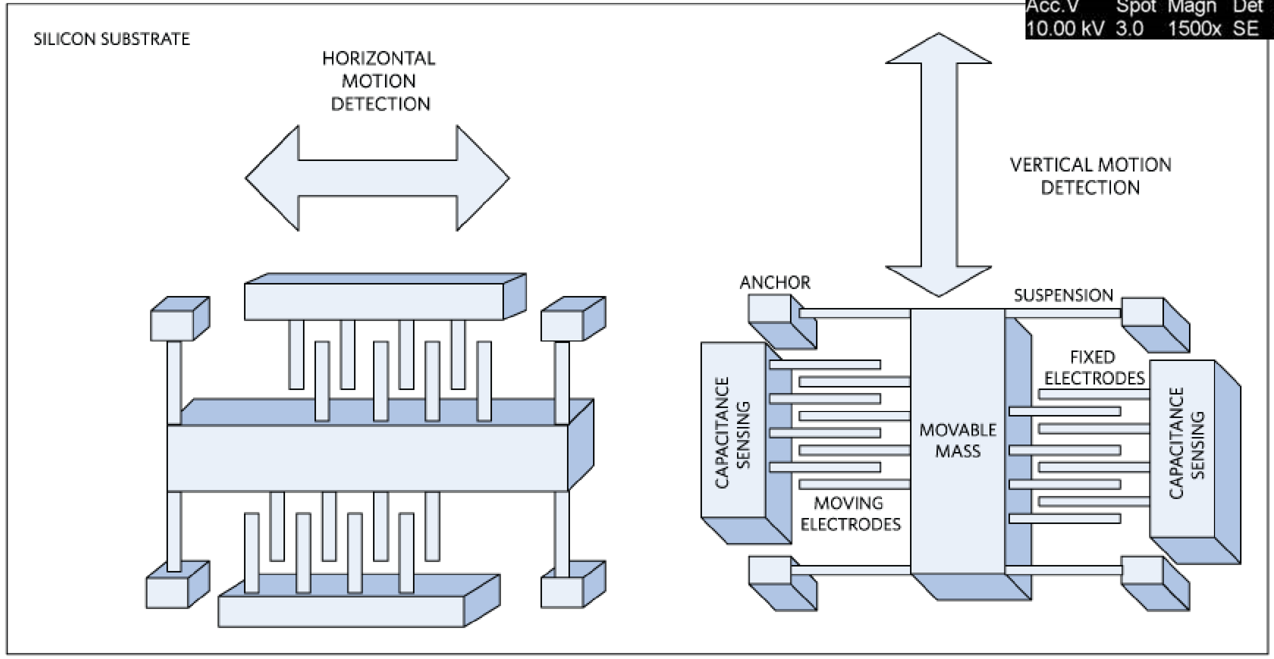
- Drift! Requires correction from position and velocity sensors:
  - XY position (LBL, USBL)
  - Z position (depth sensor)
  - Velocity correction (Doppler)
- Corrected accuracy ~0.1% - 2% of distance traveled reported.
- Cost: US$75K-$250K
- Power: 15-50 Watt

Courtesy of Ryan Eustice

Accelerometer

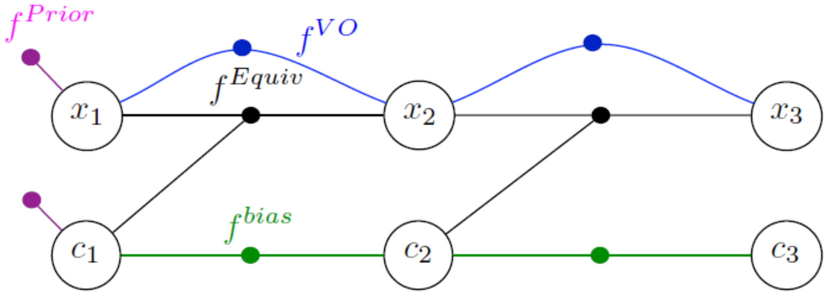
- Pendular Accelerometer
  - Open loop
  - Closed loop

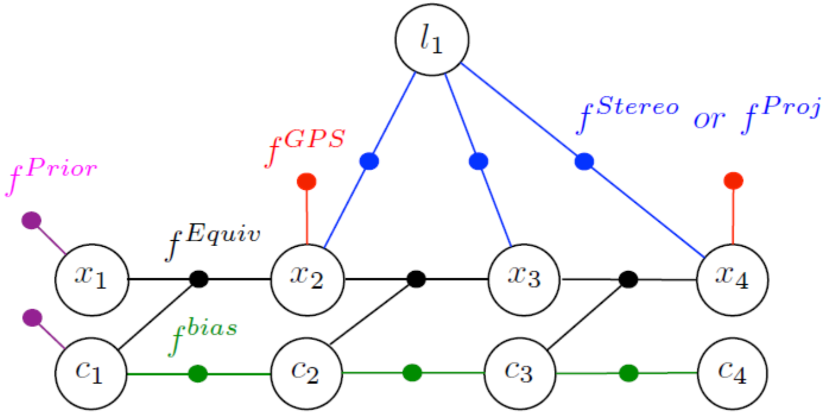




Estimation with Inertial Sensors

- Math on board
- Factor graph





“Information Fusion in Navigation Systems via Factor Graph Based Incremental Smoothing”, Vadim Indelman, Stephen Williams, Michael Kaess, and Frank Dellaert, J. of Robotics and Autonomous Systems, RAS, vol. 61, no. 8, Aug. 2013, pp. 721-738.

“On-Manifold Preintegration for Real-Time Visual-Inertial Odometry”, Forster, Carlone, Dellaert, Scaramuzza, IEEE Transactions on Robotics 2017

Implemented in gtsam library (https://github.com/borglab/gtsam)

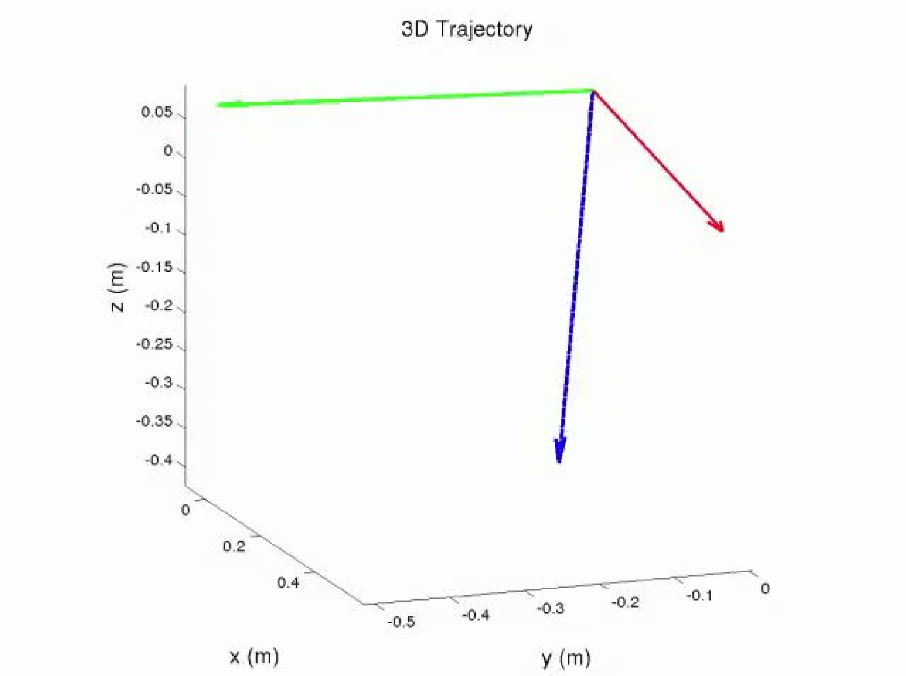
Example: GPS-Aided Inertial Navigation

Used in planes, robots…

[Indelman, Williams, Kaess, Dellaert, RAS 13]
[Huang, Kaess, Leonard, ICRA 14]

Example: Vision-Aided Inertial Navigation

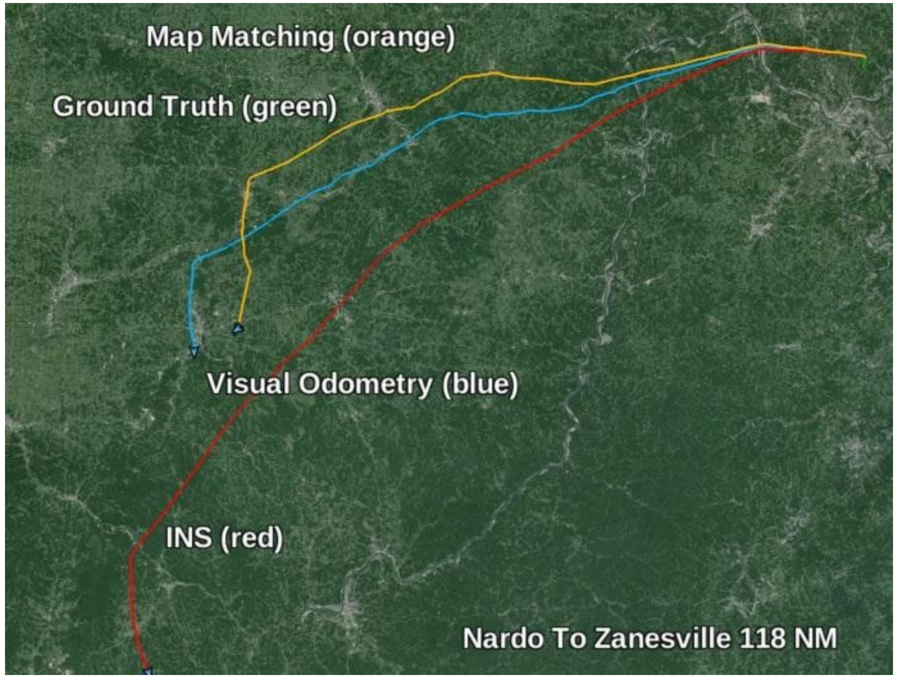
- Monocular camera + IMU

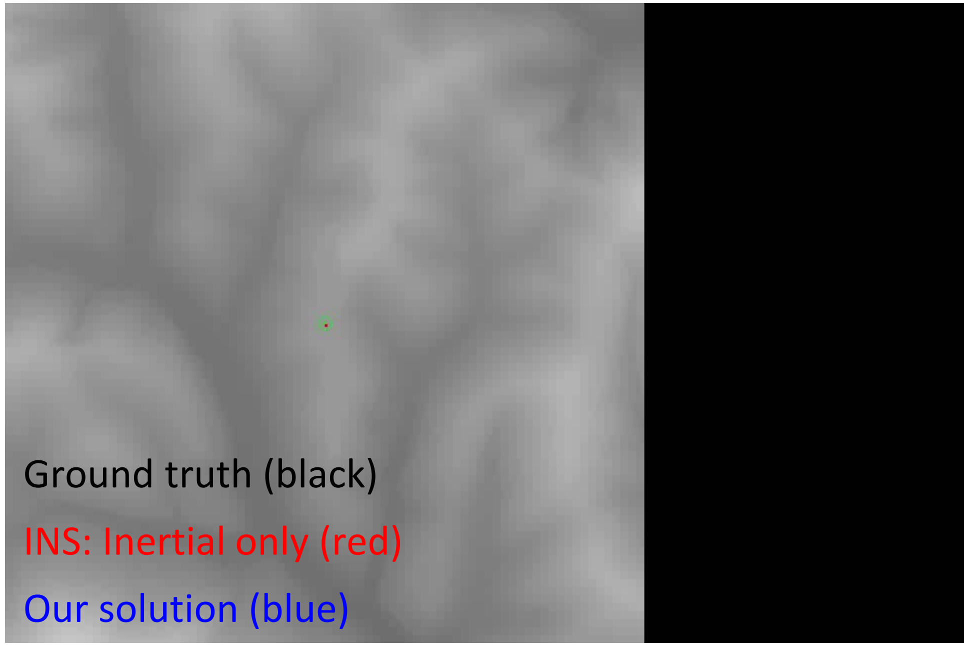


Example: Laser-Aided Inertial

Inertial plus any of:
camera, lidar, pressure, GPS, …

e.g.: GPS-denied flight, distance 118nm = **218km**, landing position error **< 40m**





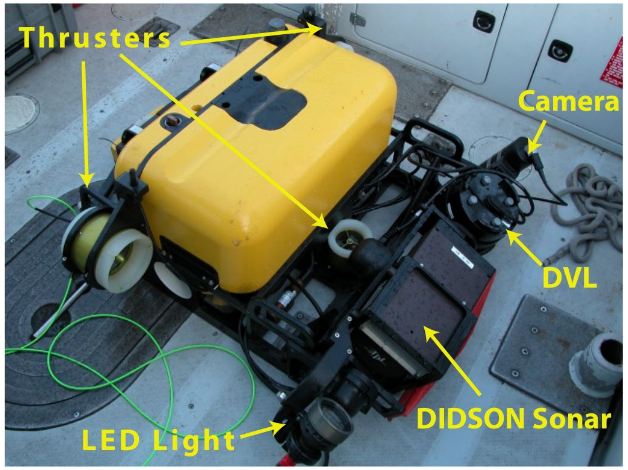
DEM

Lidar

Example: Doppler-Aided Inertial

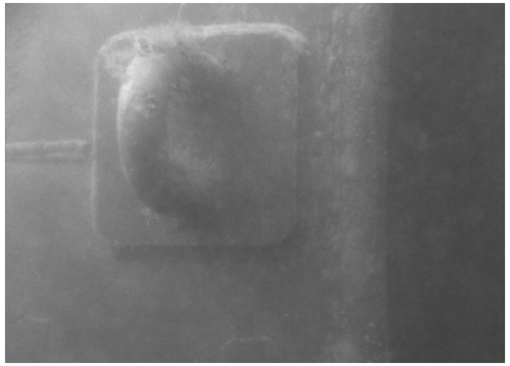
Challenges underwater:

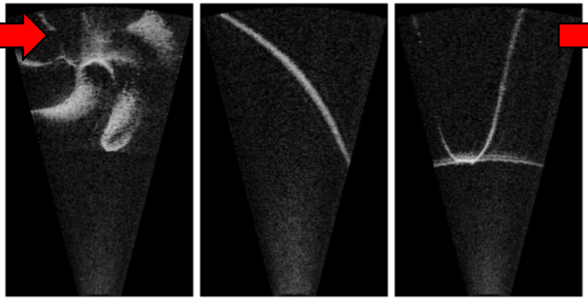
- No GPS
- No radio signals
- Low visibility

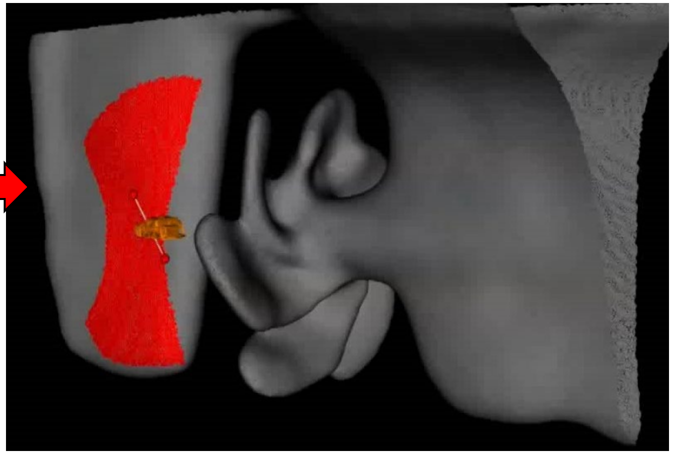


Bluefin Hovering Autonomous Underwater Vehicle (HAUV)

SS Curtiss, San Diego (180m)







Doppler Velocity Log

- Constantly measure Doppler shift of “pings” as they reflect off the ground beneath the AUV.
- Can measure body-frame velocity of vehicle in x, y, z.

Courtesy of Ryan Eustice

Doppler Effect

- *Change in observed sound pitch caused by relative motion of the sound source or observer*
  - Change in pitch is proportional to the speed of the source or observer
  - Dopper shift – difference in frequencies

$$F_{shift} = F_{still}\left(\frac{v}{c}\right)$$

  - Note:

$F_{shift} \uparrow$ For larger v and F

$F_{shift} \downarrow$ For sound speed increase (or -v)

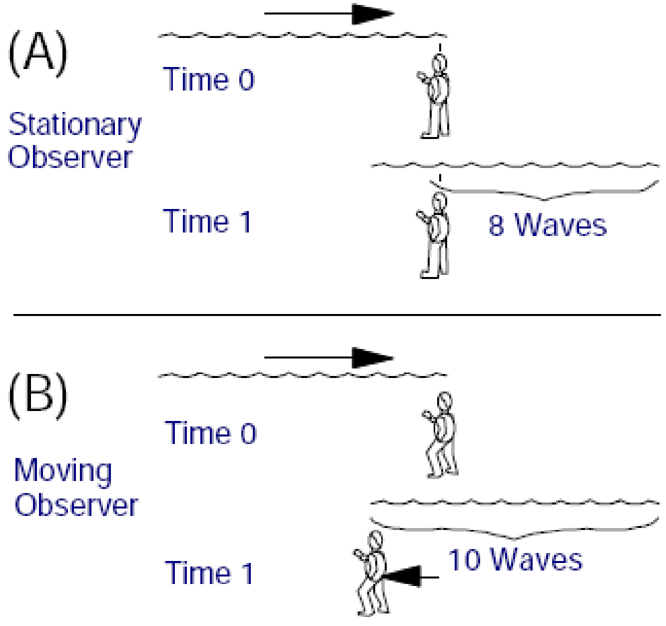


Figure 3. The Doppler effect. An observer walking into the waves will see more waves in a given time than will someone standing still.

$$f = \left(\frac{v + v_r}{c + v_s}\right) f_o$$

Courtesy of Ryan Eustice

Doppler Based Navigation

- 1200 kHz RDI Doppler provides velocity measurements at 0.03% standard deviations at 1-10Hz.

- The resulting 4x1 vector of beam velocities can be linearly transformed to the instrument frame velocities.

$$ {}^{w}\dot{p}_d(t) = {}^{w}_{v}R(t) \; {}^{v}_{i}R \; {}^{i}\dot{p}_d(t) $$

- The world velocities are integrated, and the bottom track position determined using dead-reckoning:

$$ {}^{w}p_d(t) = \int_{t_0}^{t} {}^{w}\dot{p}_d(\tau)d\tau + {}^{w}p_d(t_0) $$

Courtesy of Ryan Eustice

Doppler Navigation Error Sources

3. Rotation from the vehicle frame to the world frame – from attitude sensor

1. DVL velocity in the instrument frame – from Doppler

$$ {}^{w}\dot{p}_d(t) = {}^{w}_{v}R(t)\; {}^{v}_{i}R\; {}^{i}\dot{p}_d(t) $$

2. Rotation from the instrument frame to the vehicle frame – estimated

Doppler velocity in the world frame

1. Doppler Velocity Sensing Accuracy
   - Sensor Accuracy
   - Frequency vs Range
   - Update Rate
   - Sound Velocity Calibration
2. Doppler to Attitude-Sensor Alignment
3. Attitude Sensor Accuracy

Courtesy of Ryan Eustice

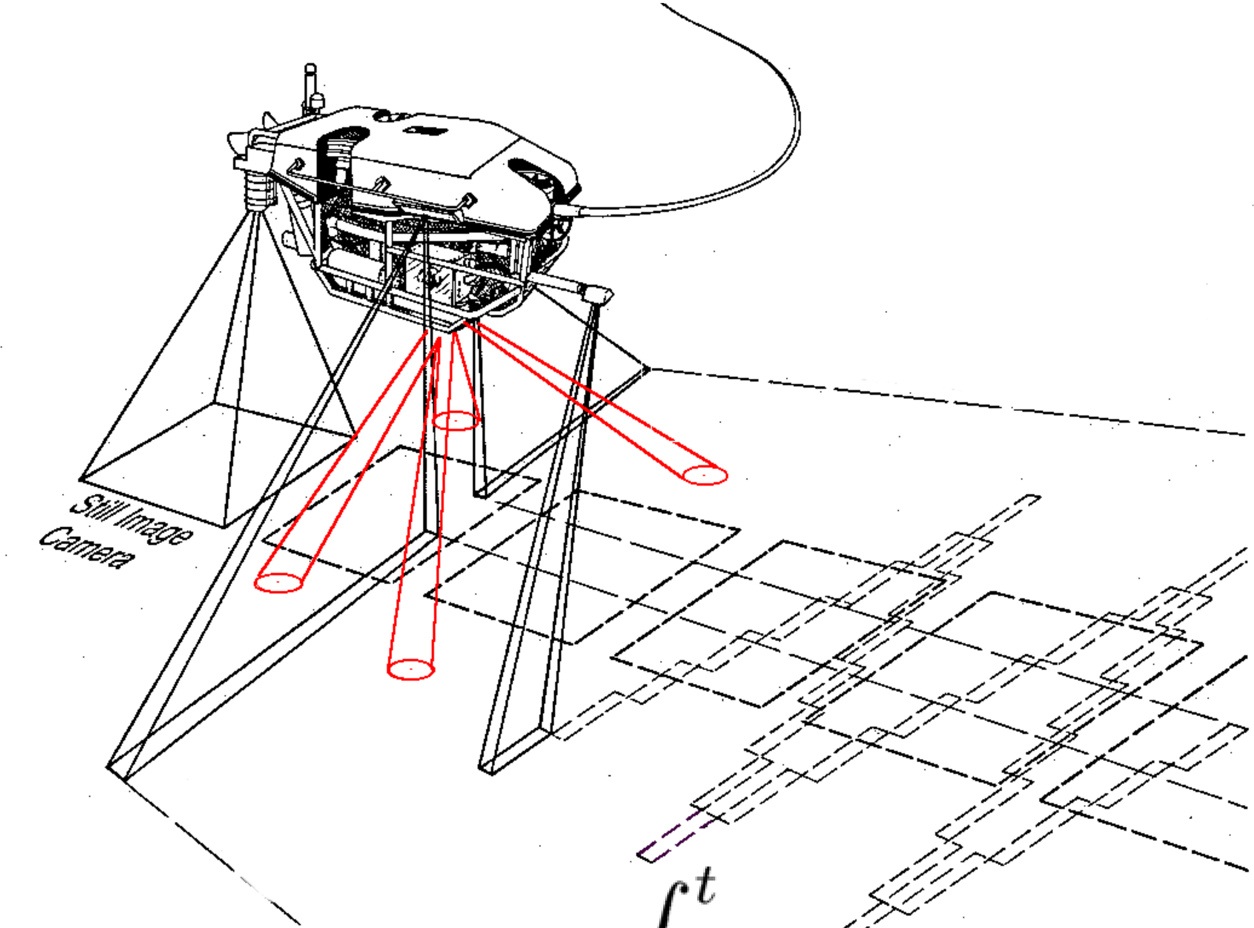
Doppler Navigation

Advantages

- Vehicle-mounted
- High update rate
- Easily deployed
- ADCP current profiles

Disadvantages

- Limited bottom-lock range:
  (25m @ 1,200 kHz, 800m @ 75 kHz)
- Requires precision attitude reference (North-seeking gyrocompass)
- Requires **calibration** of Doppler to attitude sensor.
- Accuracy ~0.1% - 5% of distance traveled
- Interference with other sonars
- Requires external reference (LBL, USBL, etc), for initialization and correction

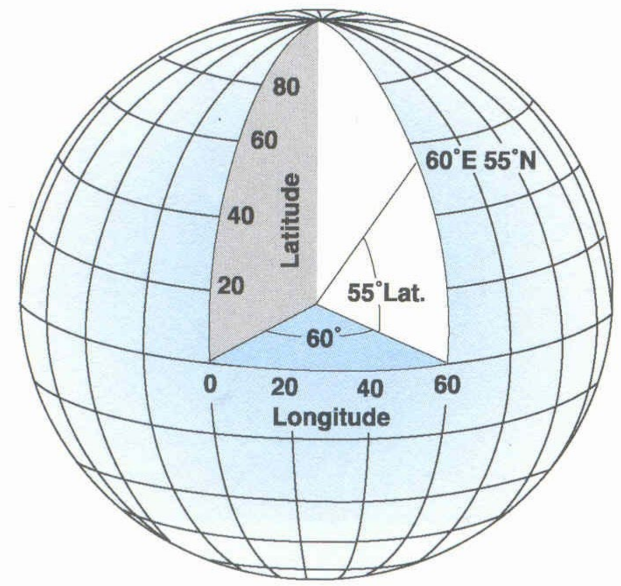


$$x(t) = x(t_0) + \int_{t_0}^{t} \dot{x}(\tau)d\tau$$

Courtesy of Ryan Eustice

Navigation: Latitude – Longitude (Y-X)

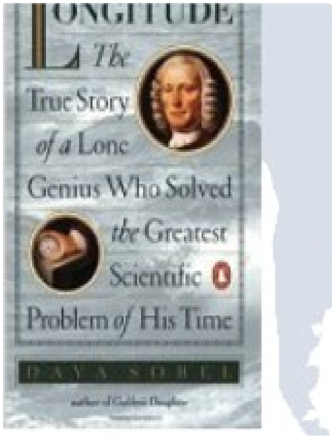
- Angular system of measurement of spherical earth
  - Reference to prime meridian – Greenwich England
  - Parallels
    - Lines of constant Latitude
    - 1 degree – 60 nm
  - Meridians
    - Lines of constant Longitude
    - 1 degree 60 nm at equator
    - 1 degree 42.5 nm at 45 N or S
  - Great Circle
    - Line created by surface intersecting plane that passes through center sphere



Courtesy of Ryan Eustice

Longitude = Time

- Longitude can be calculated using time difference between a reference location and you. (local apparent noon)
- The nautical mile (symbol M, NM, Nm or nmi) is a unit of length corresponding approximately to one minute of arc of latitude along any meridian.
- John Harrison – 1693-1776
  - H4 – lost 5.1 seconds on two month trip to Jamaica (1761)



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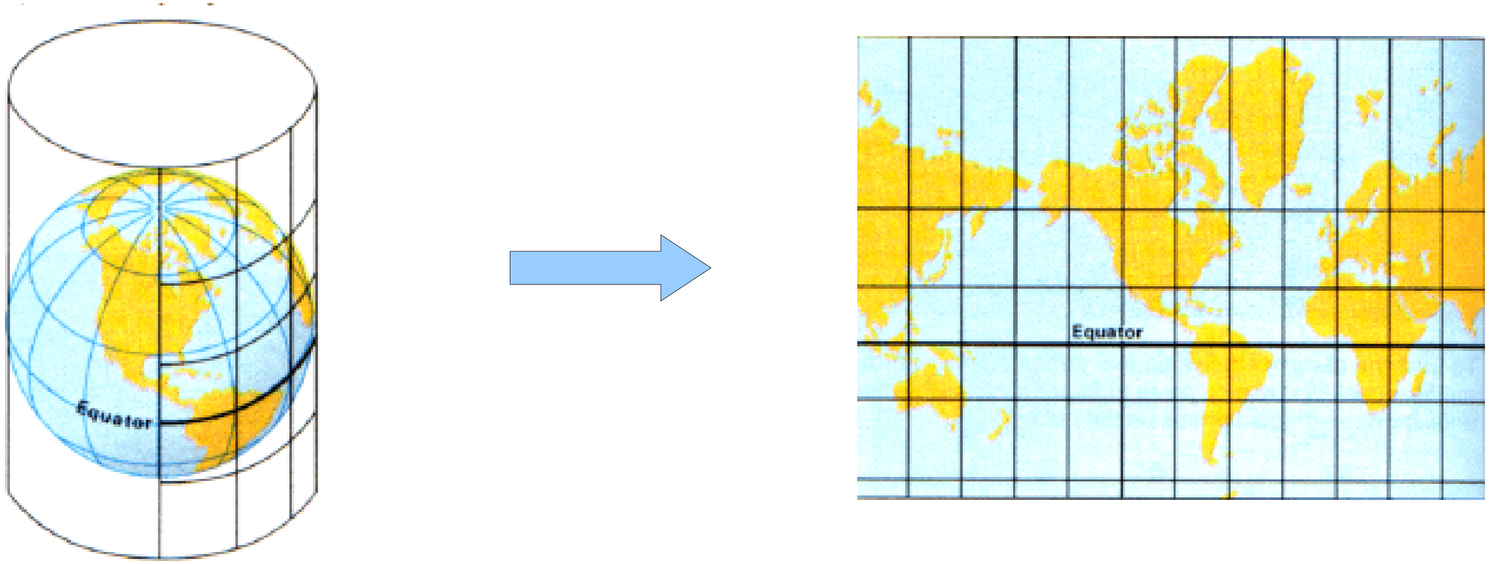
Map Projections

- Need a projection to move from 3D to 2D
- Now a Cartesian cord frame in (x,y), no longer angles



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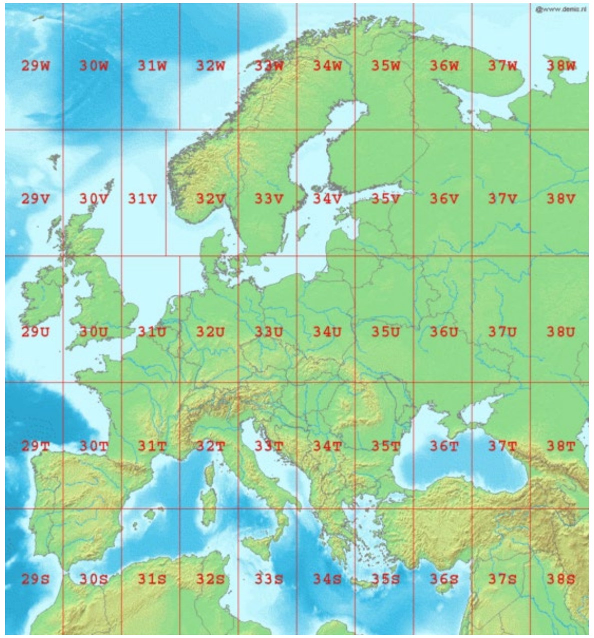
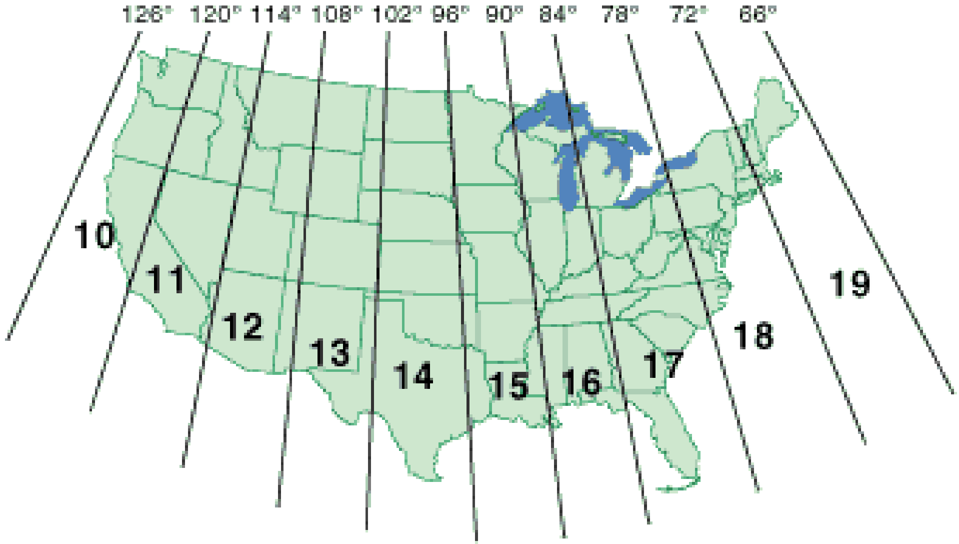
UTM (Universal Transverse Mercator)

- This is an ellipsoidal (WGS84) projection that divides the world into numbered zones in longitude, each 6 degrees wide
- Within each of the zones, the latitude and longitude difference from the central meridian is used to compute the UTM coordinates.
- Need a converter to switch Lat Lon ⟺ UTMs





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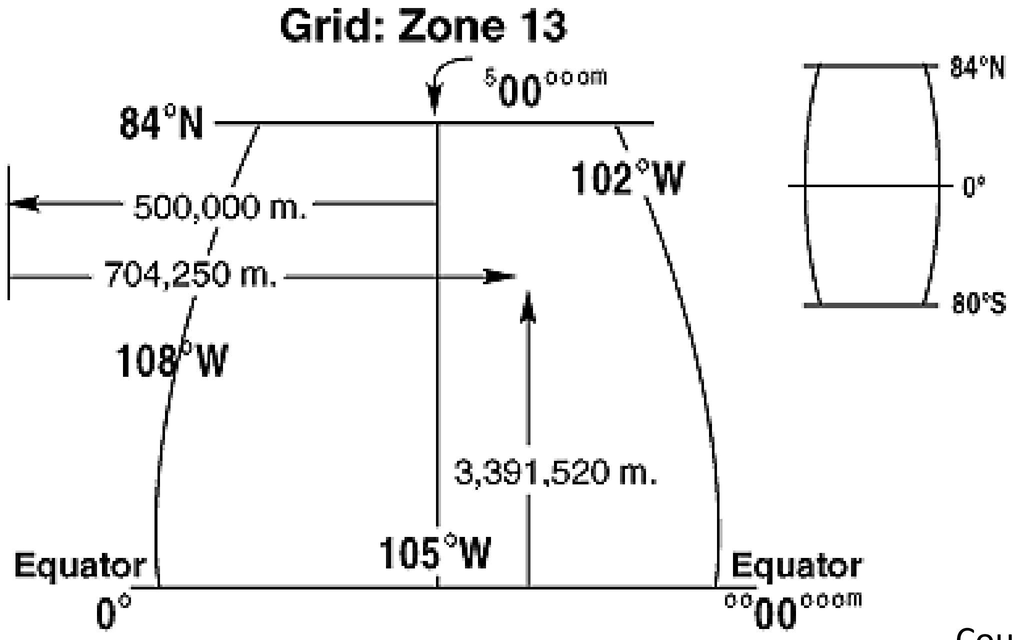
UTM Coordinates

- UTM coordinates are given as Northing and Easting. (The east coordinates have 500,000 added so that they are not negative west of the central meridian)
- UTM's are in meters
- Distortions inside one grid are 1 in 1000.

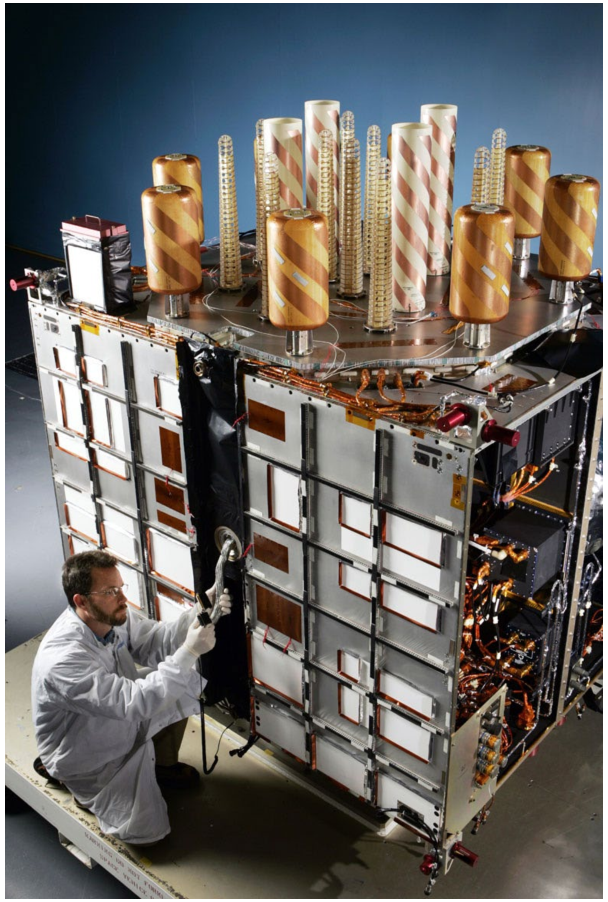


Courtesy of Ryan Eustice

GPS

- Launch some accurate clocks into orbit
  - Satellites broadcast the time and almanac of satellite positions
- Want to measure distance from receiver to satellites
  - dist = time * 299792458
- Assume receiver has atomic clock
  - Each satellite observation gives a sphere that the receiver must be on
  - Two spheres: circle
  - Three spheres: a point
- Receiver doesn’t have an atomic clock
  - Use fourth satellite, add local time as unknown variable.

- Tricky GPS characteristic
  - Error characteristic non-Gaussian!

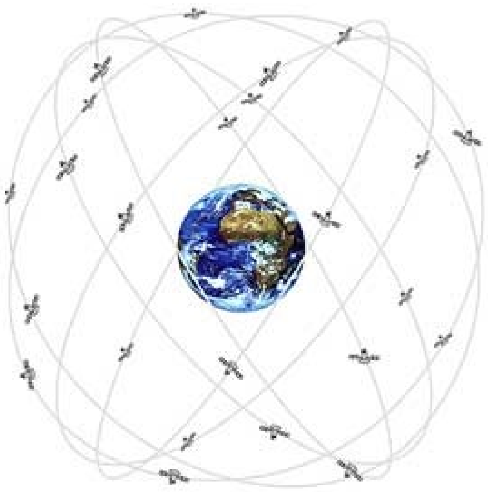


GPS IIR-15 (M), launched September 25, 2006

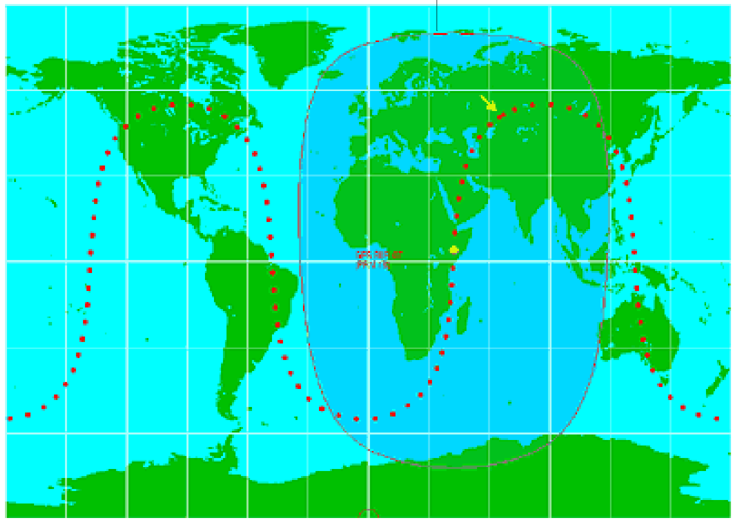
Courtesy of Ryan Eustice

GPS

- 24 satellites in 6 orbits
  - ~20,000 miles altitude
  - Orbit period ~ 12 hours
  - Each has multiple atomic clocks
  - Each broadcasts coded signals
    - L1 band – 1575.42 MHz - civilian
    - L2 band – 1272.60 MHz - military
  - (Fully) Operational in 1995
  - Positions relative to a WGS-84 ellipsoid (World Geodetic System)



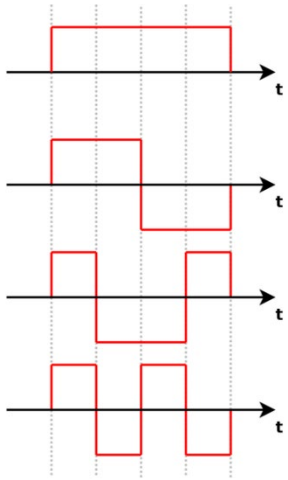
One satellite signal footprint

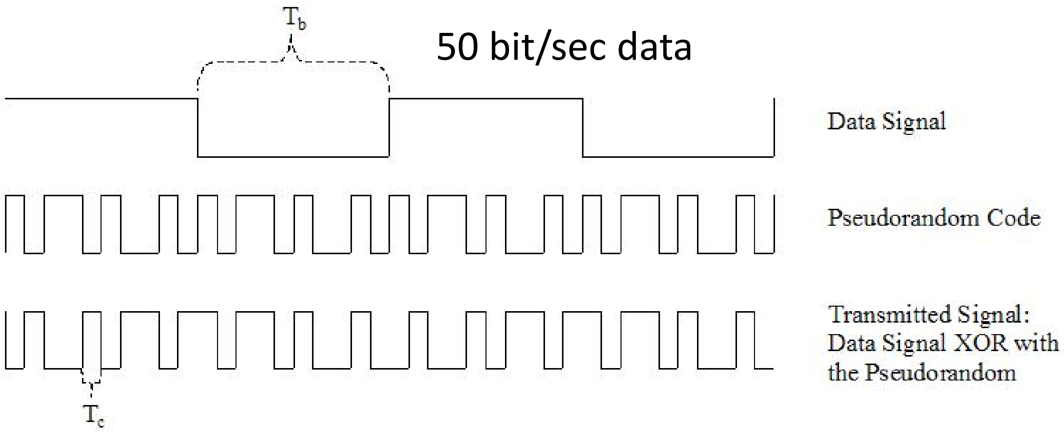


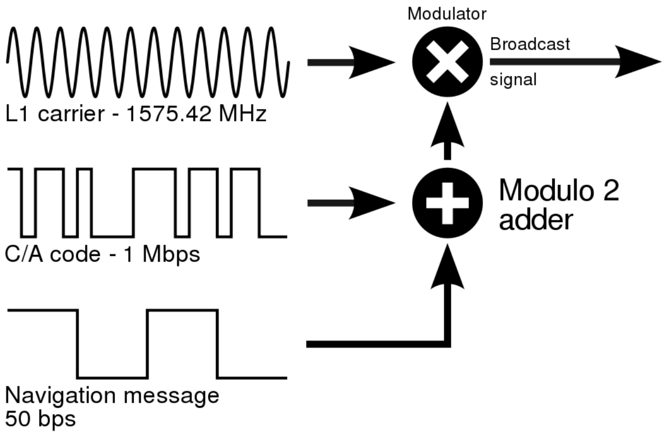
Courtesy of Ryan Eustice

GPS Signals

- Signals send information to receiver
  - Time, location of satellite, satellite ID
  - All broadcast on the same frequency, using different Pseudo Random Number (PRN) sequences
    - Code division multiple access (CDMA), spread spectrum
    - Civilian called C/A codes (Course Acquisition)



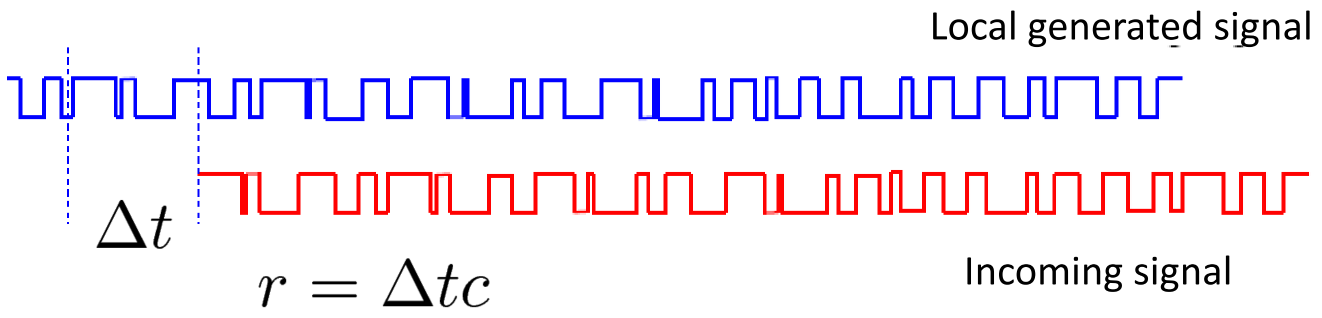




Courtesy of Ryan Eustice

GPS Receivers

- Need to determine time delay between sent and received signal
  - Use local time and generate a version of the PRN sequence
  - Correlate the incoming and local sequence to find delay
  - Sequences from other satellites do not correlate well
  - Want 4 signals – solve for positions and clock bias
    - Clock bias – all signals will be off by the same difference in the satellite clock and local clock
    - Solve for intersection of spheres
    - Typically ~15 meter errors



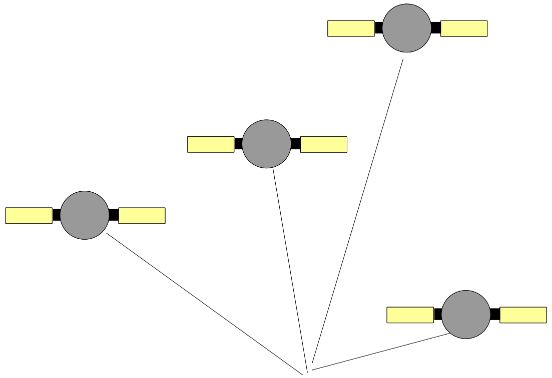
Courtesy of Ryan Eustice

GPS Errors

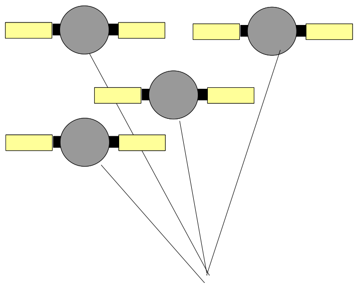
- Ephemeris (satellite position)
- Multipath
- Troposphere /Ionosphere
- Clock errors
- Satellite position (DOP)
  - Dilution of precision

Table 2 Standard error model - L1 C/A (no SA)

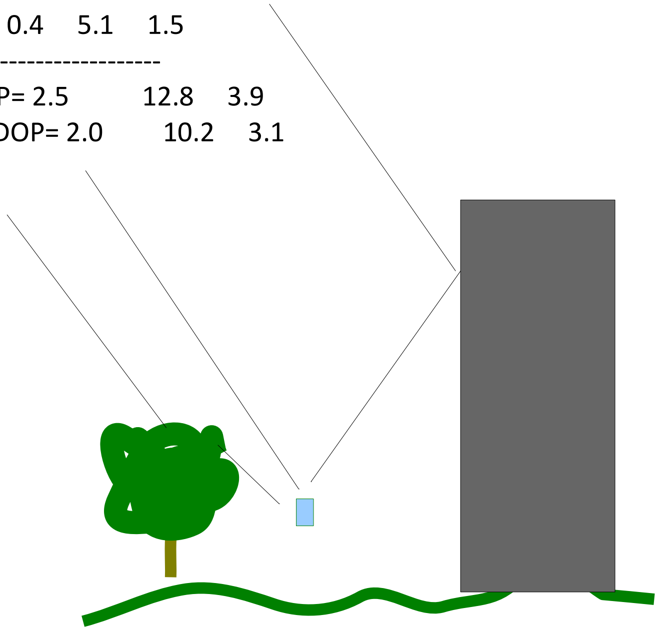
	One-sigma error, m			
Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock	2.0	0.7	2.1	0.0
Ionosphere	4.0	0.5	4.0	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1.0	1.0	1.4	1.4
Receiver measurement	0.5	0.2	0.5	0.5
User equivalent range error (UERE), rms*	5.1	1.4	5.3	1.6
Filtered UERE, rms	5.1	0.4	5.1	1.5
Vertical one-sigma errors--VDOP= 2.5			12.8	3.9
Horizontal one-sigma errors--HDOP= 2.0			10.2	3.1


Good

DOP < 4


Bad

DOP > 4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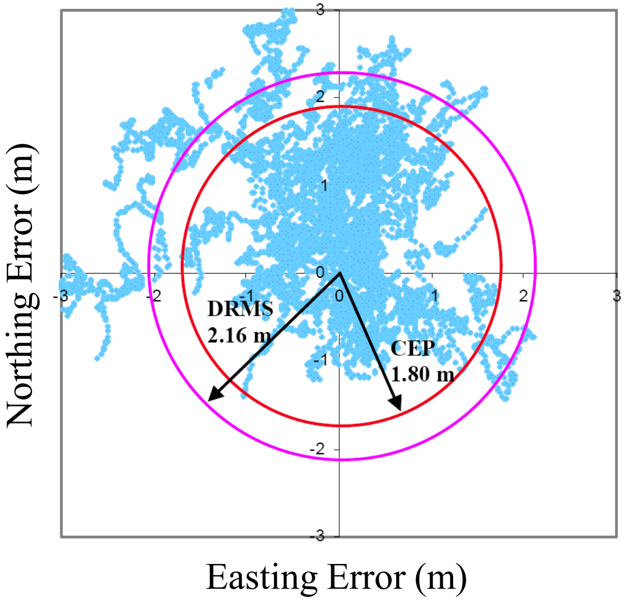
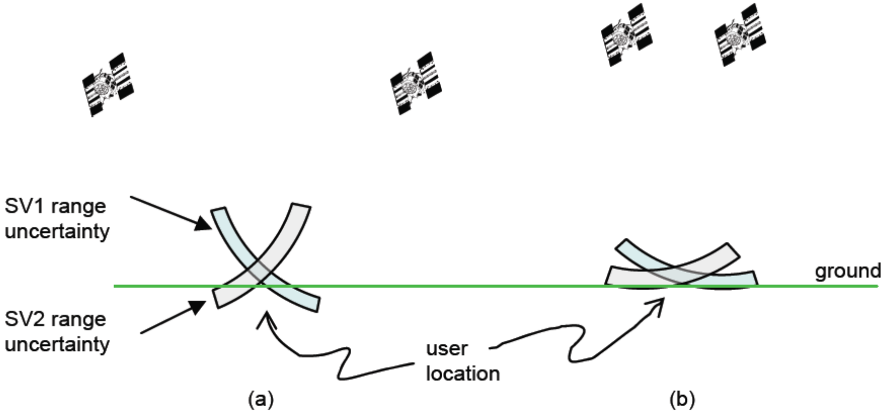
GPS Errors





- Single-point positions collected for 24 hours on a rooftop using an expensive NovAtel OEM4 receiver. One sigma uncertainties are on the order of 2m, and in most cases would be greater if the receiver was in motion.
- A minimum of 4 satellites is required to determine a 3-D position (*x,y,z,t*). Horizontal Dilution of Precision (HDOP) is a metric characterizing the quality of GPS satellite coverage. A smaller HDOP value represents a better solution. For a "reasonable" GPS solution, HDOP should be < 3.

Courtesy of Ryan Eustice

Improvements to GPS

- DGPS (Differential GPS)
  - Ground stations with known locations calculate corrections that can be applied to current signal
  - Corrections are sent out via ground based radio, need a special receiver to listen
  - Need to be within ~200 nm of station and be listening to the same set of satellites
  - Primarily atmospheric corrections

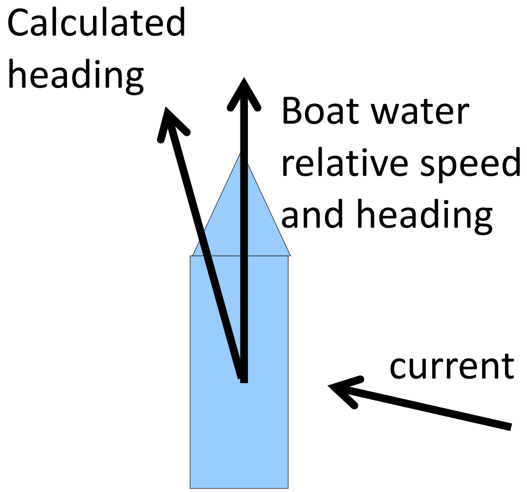
- WAAS (Wide Area Augmentation System)
  - Reduces error to ~3 meters, 95% of the time
  - Originally developed for FAA
  - Corrections broadcast via satellite to WAAS-enabled GPS receiver
    - (For the US only)
  - Euro Geostationary Navigation Overlay Service (EGNOS)

Courtesy of Ryan Eustice

Additional Measurements

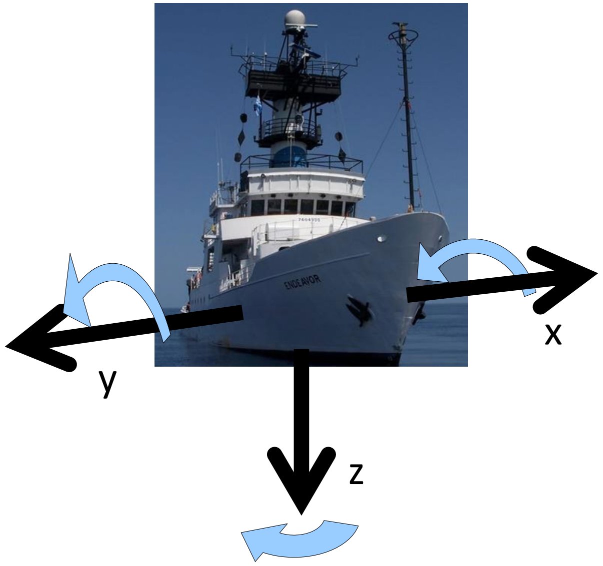
- Heading
  - Most GPS's will derive a heading from <u>motion</u>, use with caution (Course Over Ground)
  - Some will use two receivers to calculate a true heading,
    ~1 degree accurate (no motion required)

- Roll & Pitch
  - Using multiple receivers to directly measure
  - Very precise relative position (mm level) is determined between a pair of antennas (using differential carrier phase detection).
  - Two baselines composed of three antennas completely define the Euler angles associated with aircraft attitude and can be used to compute pitch, roll, and yaw angles.



Courtesy of Ryan Eustice

Angles



- Measure w.r.t. – Gravity vector and earths rotation axis
- Naval architecture convention for labeling axes of a ship or submarine
  - Yaw – rotation about z, + clockwise
  - Pitch – rotation about y, + bow up
  - Roll – rotation about x, + port up
- Use an angle sequence to describe angular position of a vessel
  - Yaw-pitch-roll is typical convention
  - **CAREFUL** – sequences do not permute
    - YPR ≠ RPY

Courtesy of Ryan Eustice

Magnetic Attitude Sensing Technology

3-Axis Magnetometer

- 1-5° Accuracy
- Low power < 1W
- Sensitive to heave, surge, sway.
- Cost $0.5K
- Power < 1W

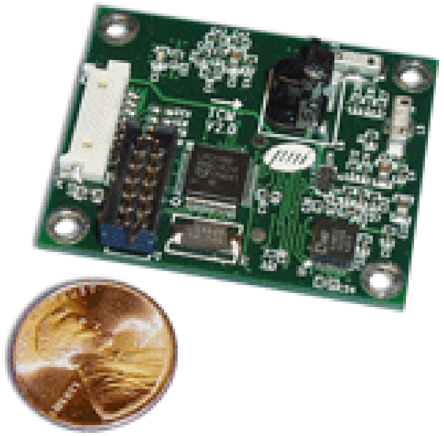
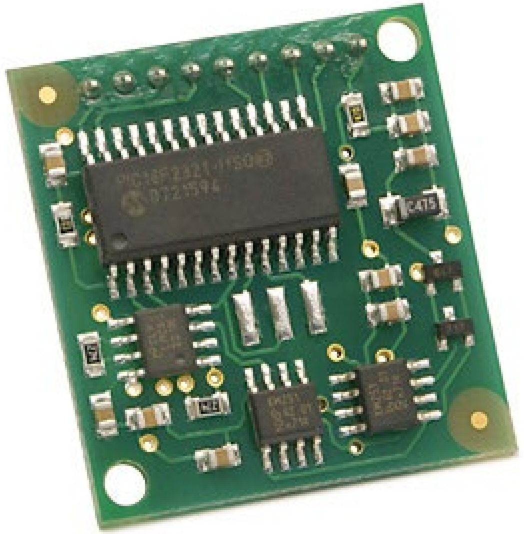


Image Credit: PNI Inc

Courtesy of Ryan Eustice

Magnetometer/Compass

- Always a popular idea
  - Error doesn't integrate over time.
  - Unfortunately, hard to make work reliably
- Many sources of interference
  - Robot itself
  - Buildings
    - In fact, some have built maps of environments by using the local magnetic flux as a landmark!

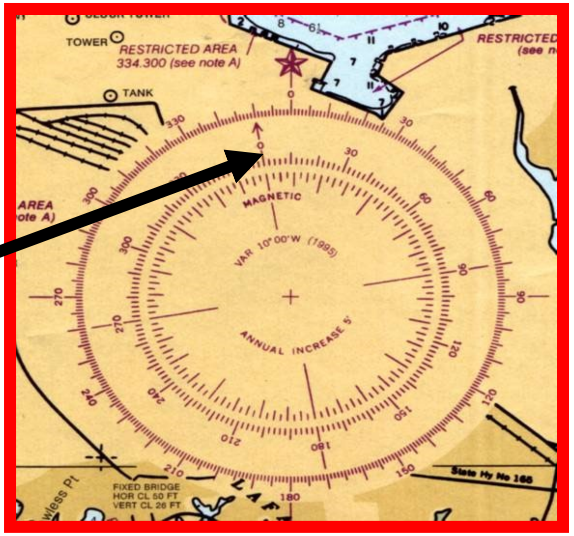


Courtesy of Ryan Eustice

Compass Errors

- Variation: the angle between a magnetic line of force and a geographic (true) meridian at any location on the earth.
- Causes:
  - Variation exists because the earth's magnetic and geographic poles are not co-located.
  - Magnetic anomalies in the earth's crust also contribute to variation.
  - Also called declination

This difference



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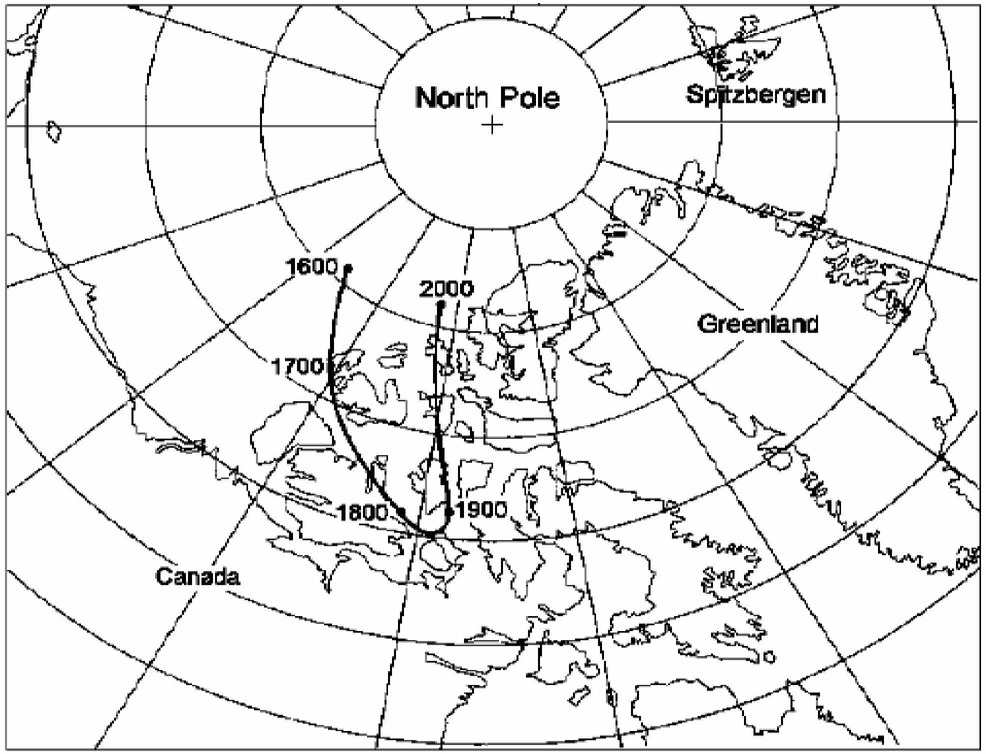
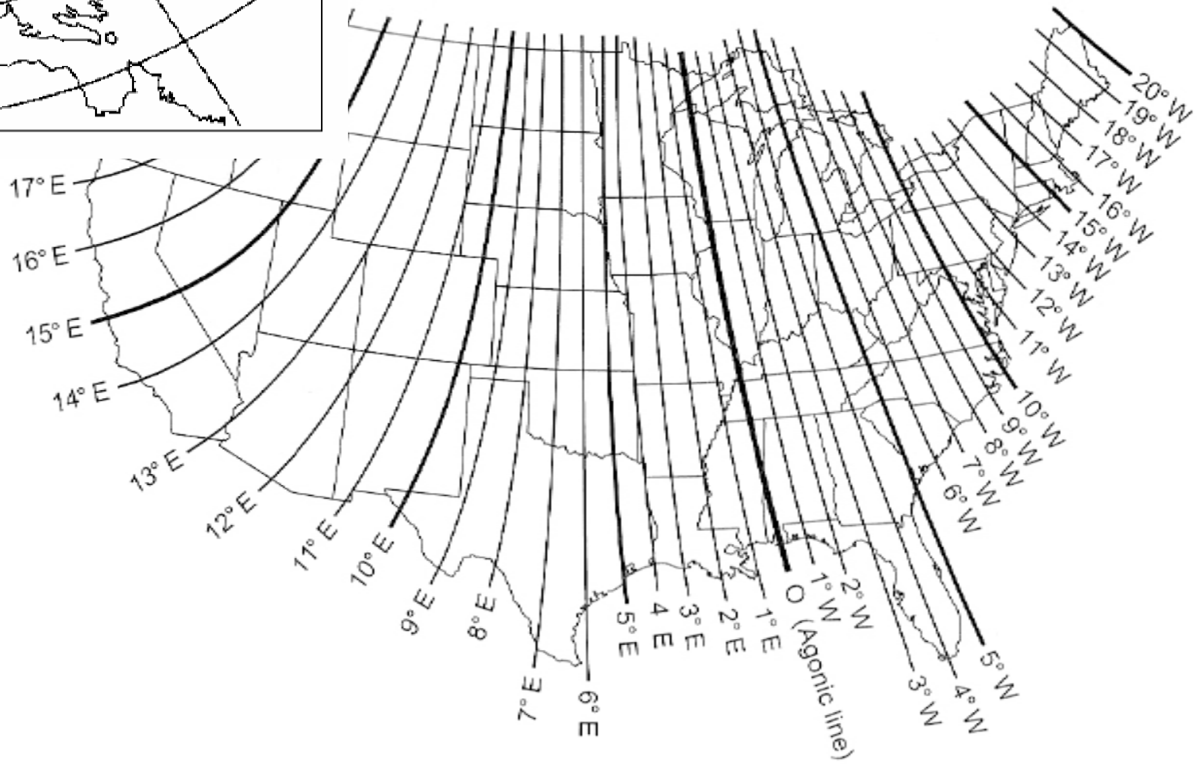




Courtesy of Ryan Eustice

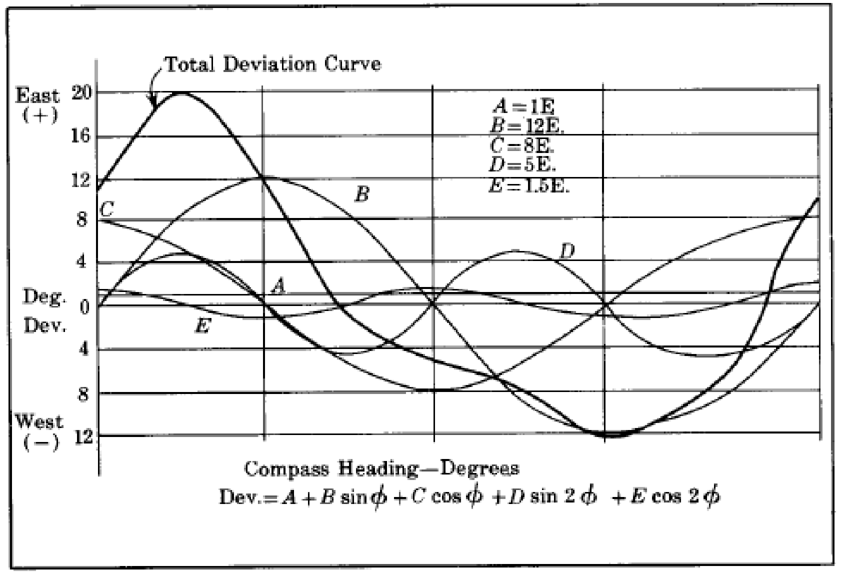
Compass Errors Con't

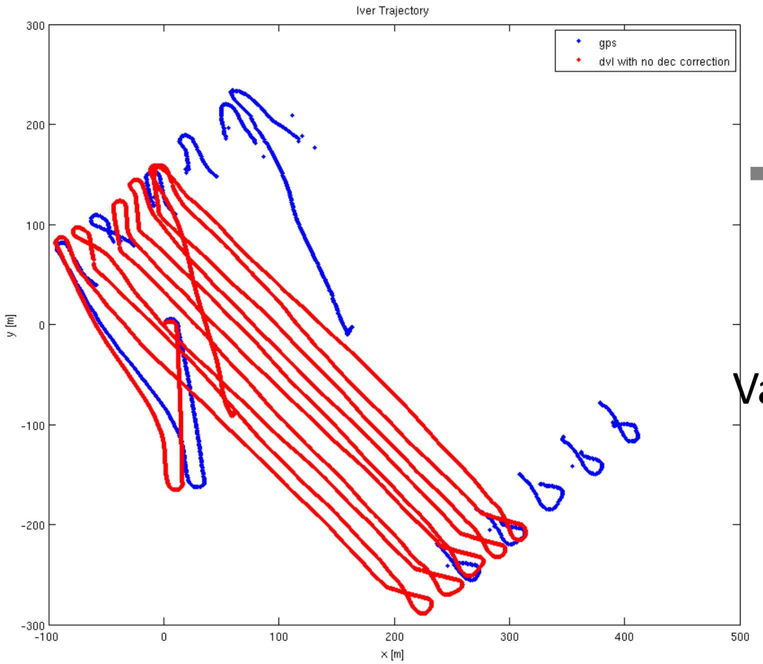
- <u>Deviation</u>: the angle between the actual magnetic meridian and the true meridian
- Causes:
  - Varies as a function of 0-360 angles on compass
  - Deviation is caused by the interaction with surrounding metallic structures and electrical systems with the earth's magnetic field

<u>Compass Calibrations</u>
Hard Iron – correct for distortions to the local magnetic field that are constant with orientation

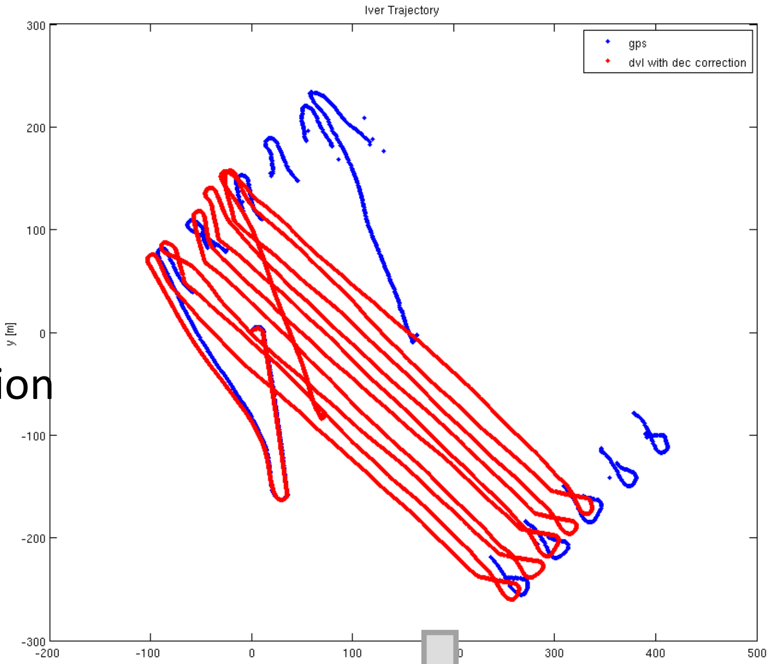
Soft Iron – distortions that vary with orientation

Courtesy of Ryan Eustice

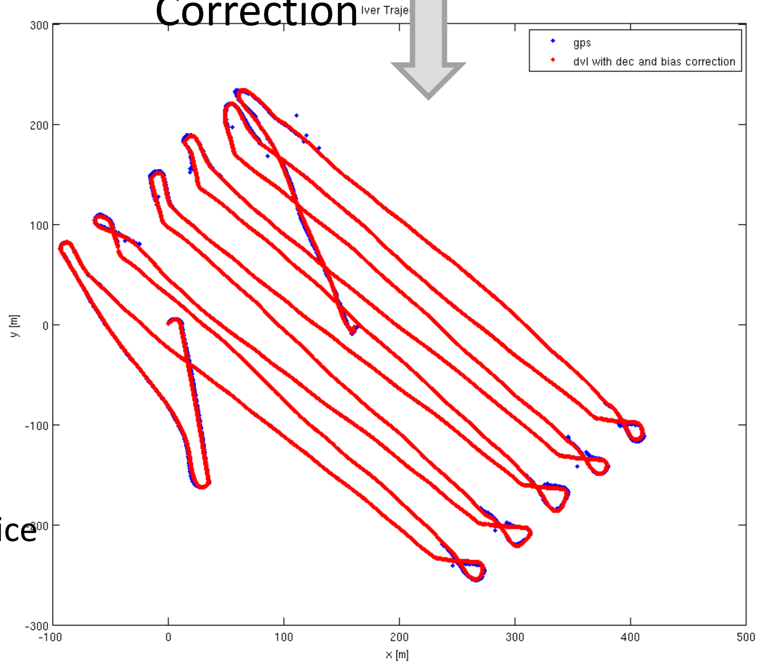




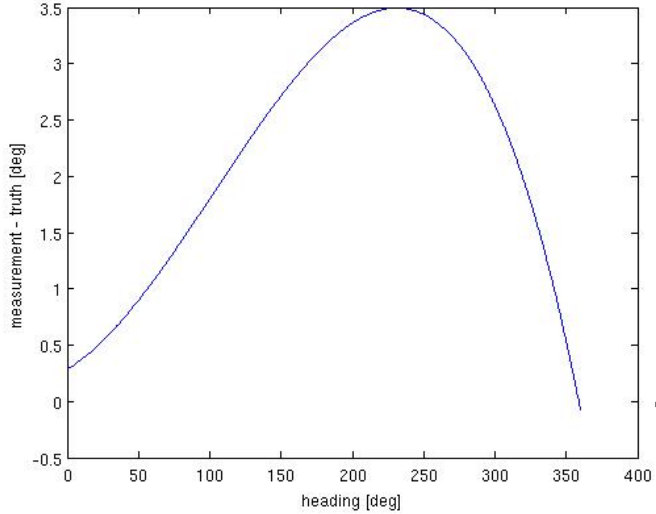
Variation Correction (~7˚)



Deviation Correction



Deviation correction as a function of measured heading (degrees)



Courtesy of Ryan Eustice

Inertial Navigation Systems

- Derived a position by integrating measurements of only **accelerations** and **angular rates**
  - Need to solve for position, velocity, orientation
  - Inertial implies you will measure the rotation of the earth as well. The inertial frame is fixed to the stars.
- Only get a fixed reference at the start
- Error will grow with time
  - Error dominated by gyro heading error
  - INS drift nm/hr = 60 * Gyro drift deg/hr
- Aided INS – uses additional measurements of Position, Speed or Attitude

Courtesy of Ryan Eustice