

ISLR CH6 Exercises

```
library(tidyverse)

## -- Attaching packages -----
## v ggplot2 3.3.0      v purrr  0.3.3
## v tibble  3.0.1      v dplyr  0.8.5
## v tidyr   1.0.2      v stringr 1.4.0
## v readr   1.3.1      v forcats 0.5.0

## -- Conflicts -----
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()     masks stats::lag()

library(leaps)
hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# note that Salary is of type string and some of them are NA
sum(hitters.df$Salary=="NA")

## [1] 59

# first remove character NAs
hitters.df <- hitters.df[hitters.df$Salary != "NA",]

# now convert Salary into numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# Now convert it to tibble
hitters.df <- tibble(hitters.df)

# regsubsets() part of leaps library chooses best subset using RSS

regfit.full <- regsubsets(Salary ~ ., hitters.df, nvmax = 19)

#The summary shows the result of step 2 of algorithm 6.1 page 205 of the book
summary <- summary(regfit.full)

names(summary)

## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"
summary

## Subset selection object
## Call: regsubsets.formula(Salary ~ ., hitters.df, nvmax = 19)
## 19 Variables (and intercept)
##          Forced in Forced out
```

```

## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
##      AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1 ( 1 ) " " " " " " " " " " " " " " " " " "
## 2 ( 1 ) " " "*" " " " " " " " " " " " " " "
## 3 ( 1 ) " " "*" " " " " " " " " " " " " " "
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## 16 ( 1 ) "*" "*" "*" "*" "*" "*" " " "*" "*" " " " "*" "*"
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## 19 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" "*" "*" " " " "*" "*"
##      CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1 ( 1 ) " " " " " " " " " " " "
## 2 ( 1 ) " " " " " " " " " " " "
## 3 ( 1 ) " " " " " " "*" " " " " "
## 4 ( 1 ) " " " " "*" "*" " " " " " "
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## 9 ( 1 ) "*" " " "*" "*" " " " " " "
## 10 ( 1 ) "*" " " "*" "*" "*" " " " " "
## 11 ( 1 ) "*" "*" "*" "*" "*" " " " " "
## 12 ( 1 ) "*" "*" "*" "*" "*" "*" " " " "

```

```
## 13 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      " "
## 14 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      " "
## 15 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      " "
## 16 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      " "
## 17 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"
## 18 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"
## 19 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"

```

```
# coef(, n) returns coefficient estimates associated with best n variable model
coef(regfit.full,4)

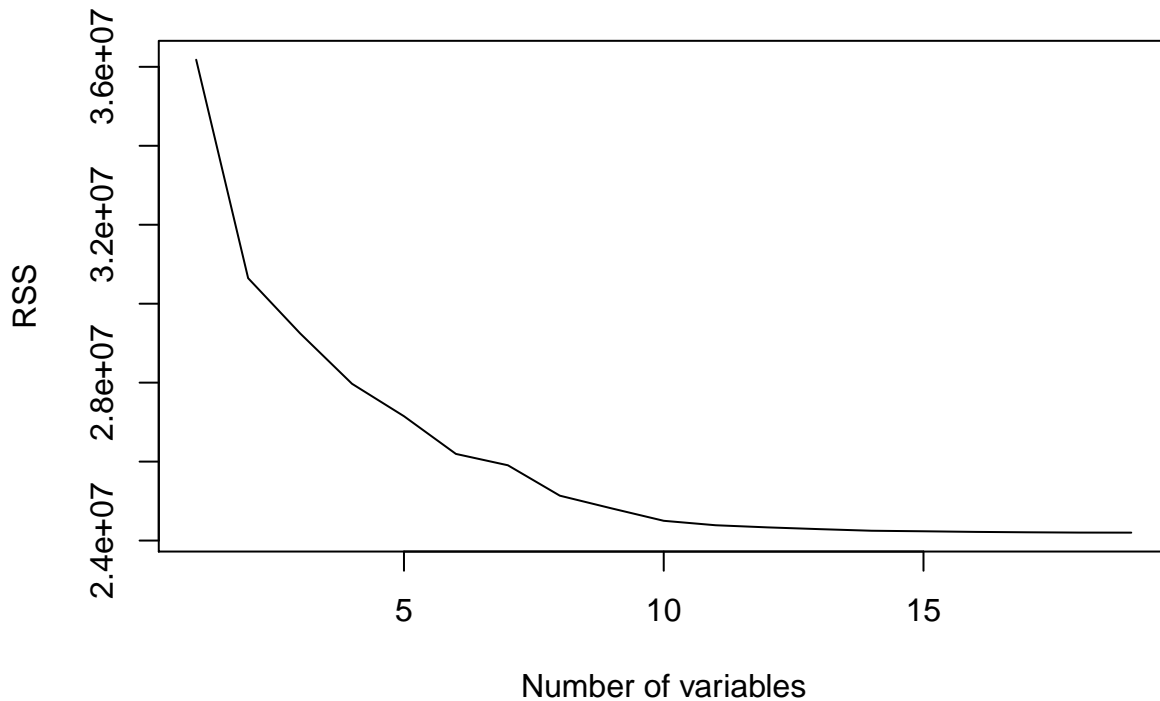
```

```
## (Intercept)      Hits      CRBI      DivisionW      PutOuts
## 13.9231044    2.6757978    0.6817790 -139.9538855    0.2735002

```

```
# plot Rsq , Cp and BIC
# par(mfrow=c(1,1))
plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")

```



```
plot(summary$adjr2,xlab = "Number of variables", ylab="Adjusted Rsquared",
      type = "l")

```

```
# which.max() returns location maximum point of the vector
(index <- which.max(summary$adjr2))

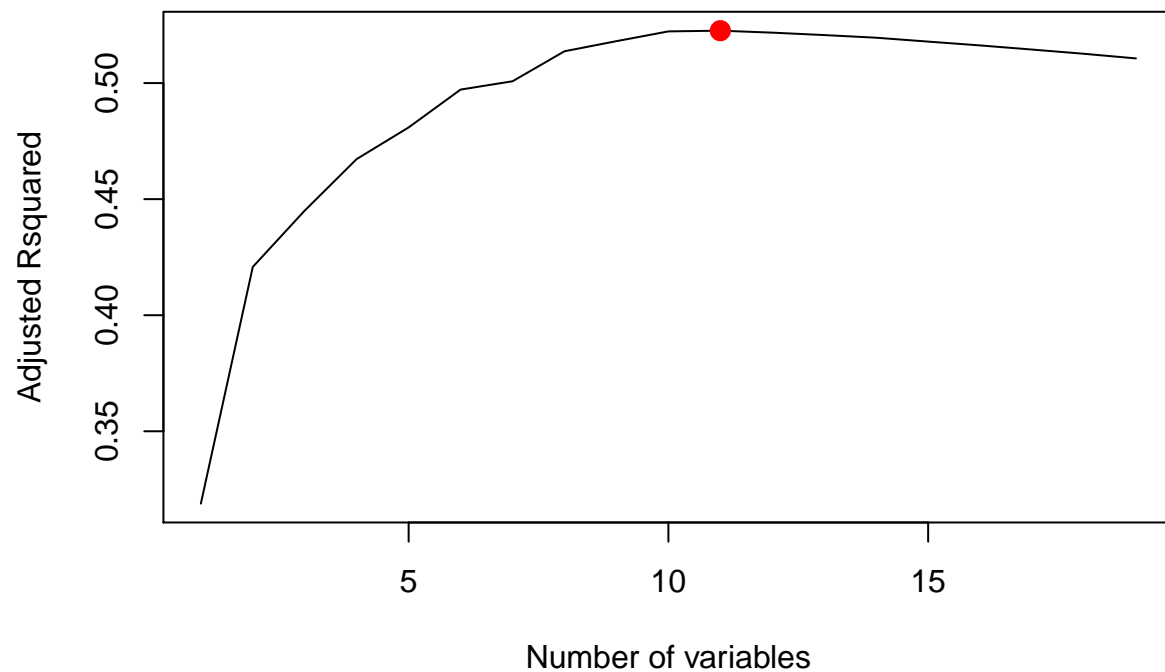
```

```
## [1] 11

```

```
points(index, summary$adjr2[index], col="red", cex=2, pch=20)

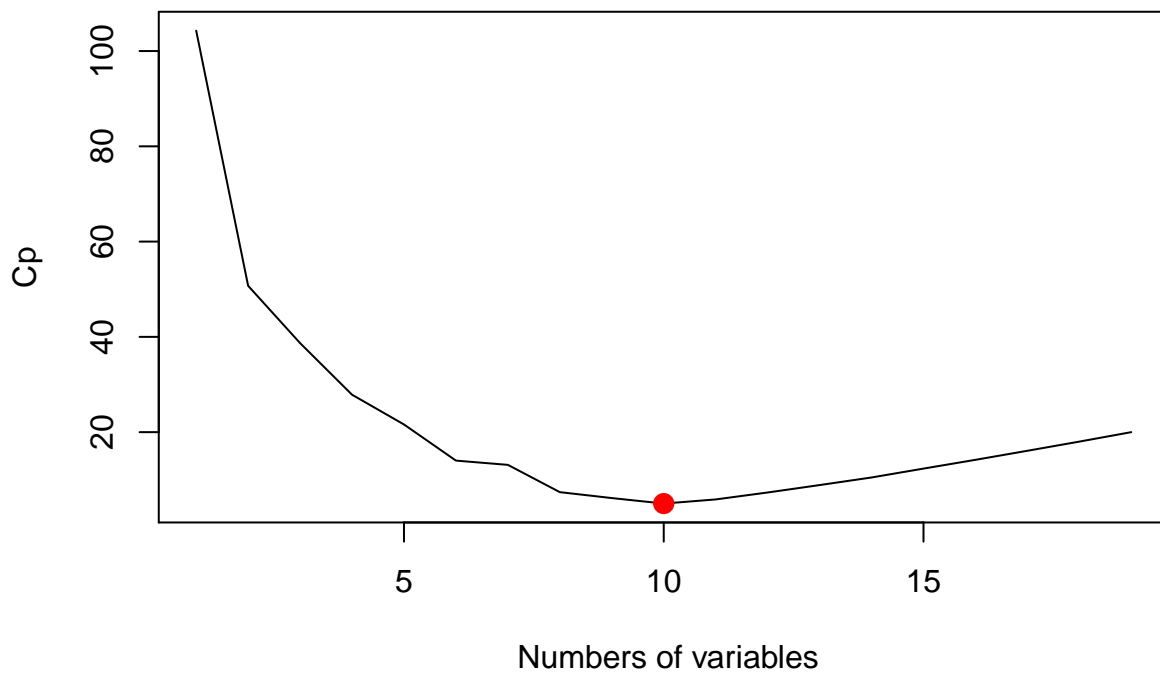
```



```
# which.min() returns location minimum point of the vector
plot(summary$cp, xlab = "Numbers of variables", ylab="Cp", type="l")
(index <- which.min(summary$cp))
```

```
## [1] 10
```

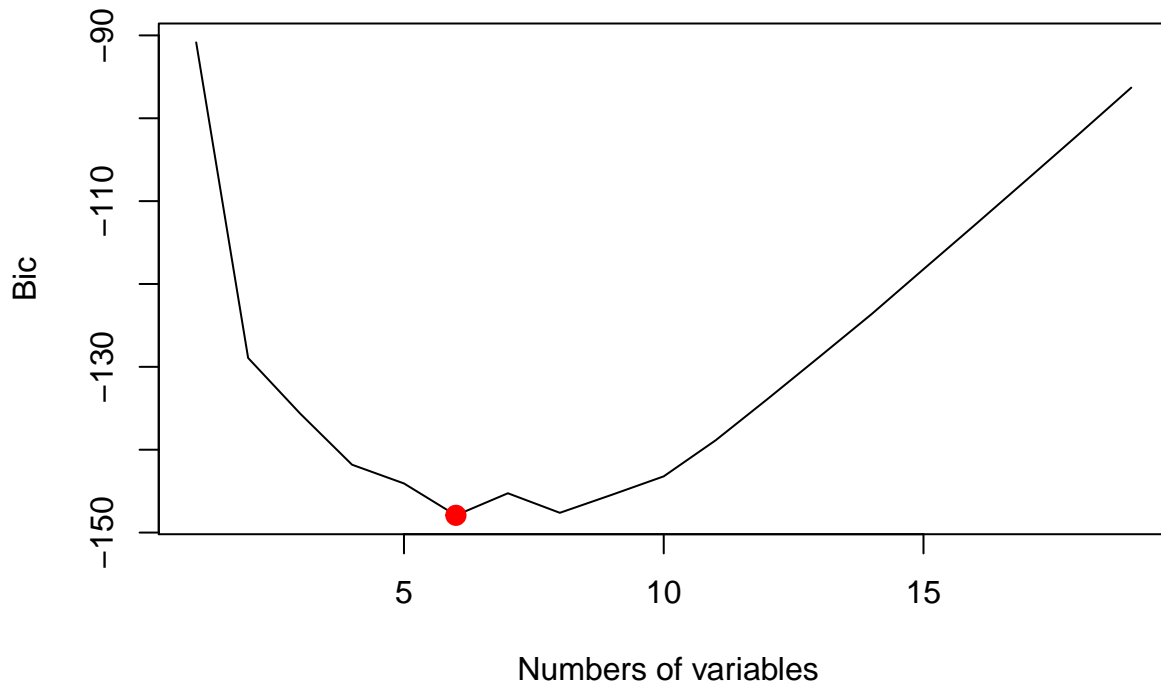
```
points(index, summary$cp[index], col="red", cex=2, pch=20)
```



```
# same for bic
plot(summary$bic, xlab = "Numbers of variables", ylab="Bic", type="l")
(index <- which.min(summary$bic))
```

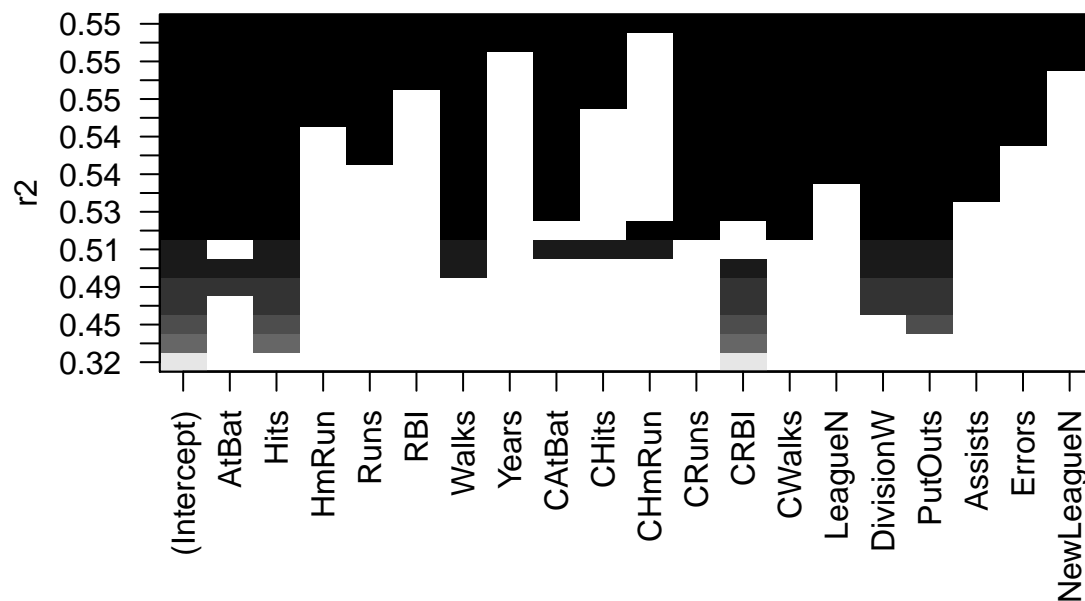
```
## [1] 6
```

```
points(index, summary$bic[index], col="red", cex=2, pch=20)
```

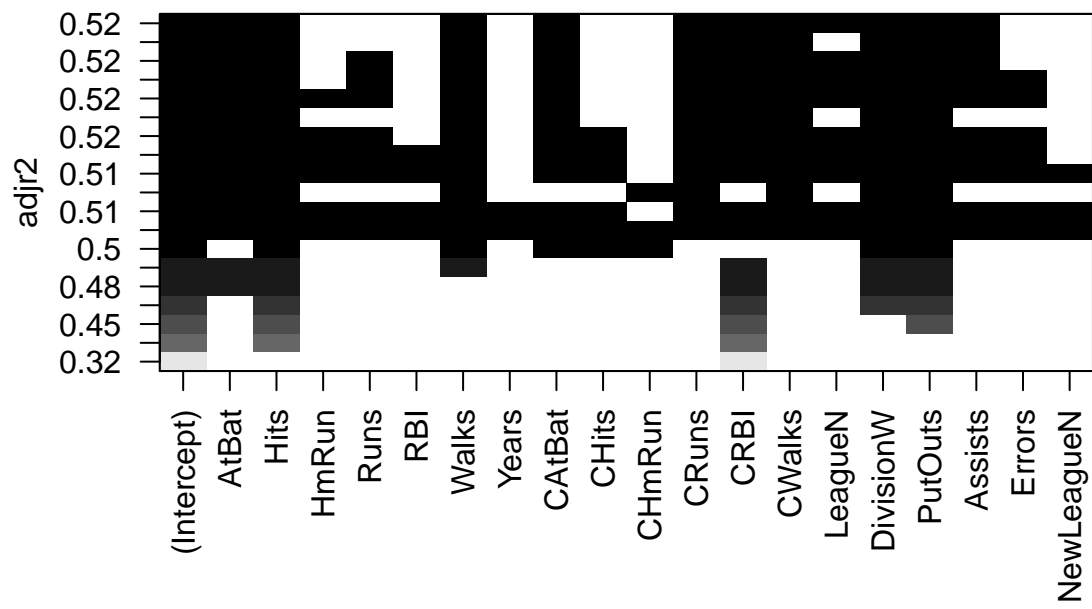


*# regsubsets() has builtin plot() command that displays selected variables for
best model with a given number of predictors*

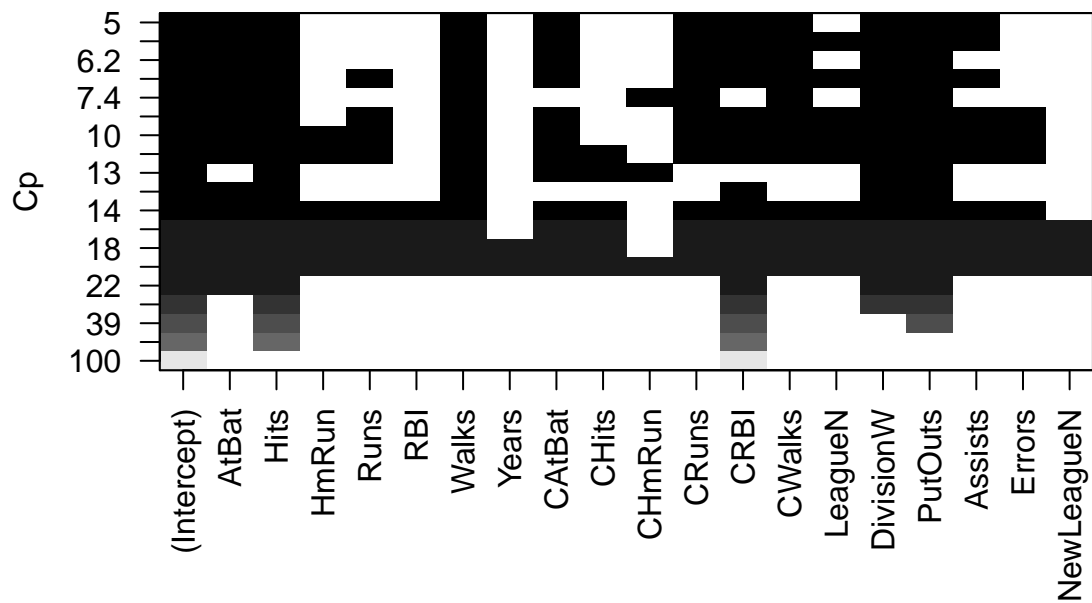
```
plot(regfit.full, scale="r2")
```



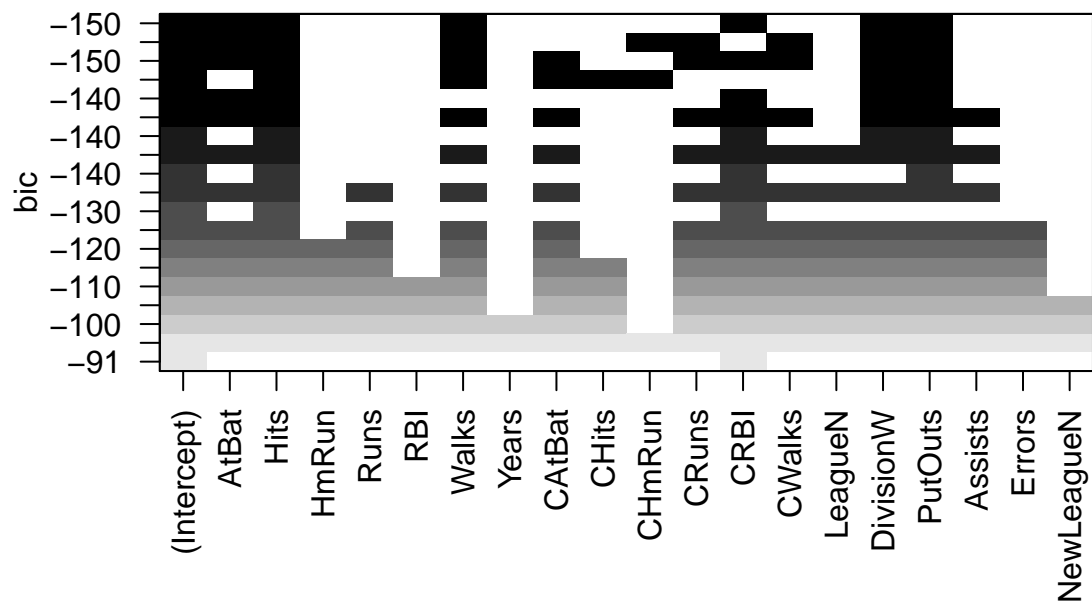
```
plot(regfit.full, scale="adjr2")
```



```
plot(regfit.full,scale="Cp")
```



```
plot(regfit.full,scale="bic")
```



```
library(tidyverse)
library(leaps)
hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# note that Salary is of type string and some of them are NA
sum(hitters.df$Salary=="NA")

## [1] 59

# first remove character NAs
hitters.df <- hitters.df[hitters.df$Salary != "NA",]

# now convert Salary into numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# Now convert it to tibble
hitters.df <- tibble(hitters.df)

# we can use regsubsets() to perform forward / backward stepwise selection
regfit.fwd <- regsubsets(Salary ~ ., data = hitters.df, nvmax=ncol(hitters.df),
  method = "forward")

summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = hitters.df, nvmax = ncol(hitters.df),
##   method = "forward")
## 19 Variables (and intercept)
##           Forced in Forced out
## AtBat      FALSE      FALSE
## Hits       FALSE      FALSE
## HmRun       FALSE      FALSE
## Runs       FALSE      FALSE
```

```

## RBI                FALSE      FALSE
## Walks              FALSE      FALSE
## Years              FALSE      FALSE
## CatBat             FALSE      FALSE
## CHits              FALSE      FALSE
## CHmRun             FALSE      FALSE
## CRuns              FALSE      FALSE
## CRBI               FALSE      FALSE
## CWalks             FALSE      FALSE
## LeagueN            FALSE      FALSE
## DivisionW          FALSE      FALSE
## PutOuts            FALSE      FALSE
## Assists            FALSE      FALSE
## Errors             FALSE      FALSE
## NewLeagueN         FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##      AtBat Hits HmRun Runs RBI Walks Years CatBat CHits CHmRun CRuns CRBI
## 1  ( 1 ) " " " " " " " " " " " " " " " " " "
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## 3  ( 1 ) " " "*" " " " " " " " " " " " " " "
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## 18 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" "*" " " " "*" "
## 19 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" "*" "*" " " " "*" "
##      CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) " " " " " " " " " "
## 2  ( 1 ) " " " " " " " " " "
## 3  ( 1 ) " " " " " " "*" " " "
## 4  ( 1 ) " " " " "*" "*" " " " "
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## 16 ( 1 ) "*" "*" "*" "*" "*" "*" " "

```



```
## 17 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"
## 18 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"
## 19 ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      "*"

# coefficient for the best model with 3 coefficients
(coefs <- coef(regfit.fwd,3))

## (Intercept)      Hits      CRBI      PutOuts
## -71.4592204    2.8038162    0.6825275    0.2735814

names(coefs)

## [1] "(Intercept)" "Hits"      "CRBI"      "PutOuts"

library(tidyverse)
library(leaps)
hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# note that Salary is of type string and some of them are NA
sum(hitters.df$Salary=="NA")

## [1] 59

# first remove character NAs
hitters.df <- hitters.df[hitters.df$Salary != "NA",]

# now convert Salary into numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# Now convert it to tibble
hitters.df <- tibble(hitters.df)
# first we create a vector that allocates each observation to one of K = 10 folds
k = 10
set.seed(1)

# create k folds
folds <- sample(1:k, nrow(hitters.df), replace = T)
# table(folds)
#  1  2  3  4  5  6  7  8  9 10
# 35 25 33 31 34 31 32 29 39 33

# number of features
noOfFeatures <- ncol(hitters.df) -1

# an empty accumulator to store MSE for each fold and each predictor
cv.errors <- matrix(NA, k, noOfFeatures,
  dimnames = list(NULL, paste(1:noOfFeatures)))

# perform a cross validation on a for loop
for (j in 1:k){
  # step# 2 of algorithm is evaluated on all folds except one of them each time
  # it chooses best models with number of features 1,2,..., noOfFeatures
  # on k-1 training folds
  best.fit <- regsubsets(Salary ~ ., data = hitters.df[folds != j, ],
    nvmax = noOfFeatures)
```

```

# Now build X matrix from test data
test.mat <- model.matrix(Salary ~ ., data = hitters.df[folds == j, ])

#model.matrix:
# -----
#(Intercept) AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns ...
#          1  202  53    4  31  26   27    9  1876  467    15  192 ...
#          1  239  60    0  30  11   22    6  1941  510    4  309 ...
#          1  472 116   16  60  62   74    6  1924  489   67  242 ...

# now compute CV test error for each of models that have best number of
# predictors on test fold # j
for(i in 1:noOfFeatures){
  # extract coefficients for model # i
  coefi <- coef(best.fit, id = i)

  # calculate cv test error for each row in test matrix()
  predicted_values <- test.mat[, names(coefi)] %*% coefi
  cv.errors[j,i] <- mean((predicted_values - hitters.df[folds ==j, ]$Salary)^2)
}
}

# finally calculate mean of CV MSE error for each model
cv.error.means <- rep(NA, ncol(cv.errors))
for (l in 1:ncol(cv.errors)){
  cv.error.means[l] <- mean(cv.errors[,l])
}

# find the minimum of all cv-MSE means and corresponding coefficients
print("No of selected Features is the one with minimum of all cv-MSE means: ")

## [1] "No of selected Features is the one with minimum of all cv-MSE means: "
(no.of.selected.features <- which.min(cv.error.means))

## [1] 10

# finally get the selected columns (remember it includes (Intercept) that has to be removed)
reg.best <- regsubsets(Salary ~ ., data = hitters.df,
                      nvmax = no.of.selected.features)
names(coef(best.fit, id = which.min(cv.error.means)))

## [1] "(Intercept)" "AtBat"      "Hits"      "Walks"      "CAtBat"
## [6] "CRuns"       "CRBI"       "CWalks"    "DivisionW"   "PutOuts"
## [11] "Assists"

library(tidyverse)
library(class)
library(boot)
library(leaps)

set.seed(17)
weekly.df = read.csv("/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Weekly.csv",
                    header=T, stringsAsFactors = F, na.strings = "?")
weekly.df = tibble(weekly.df)

```

```

#----- Some usual cleaning on character columns ----- #

# First remove all recods with spaces in character column Direction
weekly.df$Direction <- gsub('\\s+', '', weekly.df$Direction)

# Second remove all leading and trailing spaces from a character column "Direction"
weekly.df$Direction <- trimws(weekly.df$Direction, which = c("both"))

# Remove all records with "NA" or empty string in character column "Direction"
weekly.df <- weekly.df[!(tolower(weekly.df$Direction) == "na" |
                        weekly.df$Direction == ""), ]

# convert all character fields
weekly.df[sapply(weekly.df, is.character)] <-
  lapply(weekly.df[sapply(weekly.df, is.character)], as.factor)

#----- Find and remove NA in all columns ----- #

weekly.df <- na.omit(weekly.df)

# ----- stepwise forward feature selectin with CV -----#

# create k-fold
k <- 10
threshold <- 0.5

set.seed(1)

# create k folds
folds <- sample(1:k, nrow(weekly.df), replace = T)

# For folds with same size do:
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#                           size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

# number of features
noOfFeatures <- ncol(weekly.df) - 1

# an empty accumulator to store MSE for each fold and each predictor
cv.errors <- matrix(NA, k, noOfFeatures,
                    dimnames = list(NULL, paste(1:noOfFeatures)))

# perform a cross validation on a for loop
for (j in 1:k){
  # step# 2 of algorithm 6.3 page 209 is evaluated on all folds except one of
  # them each time it chooses best models with number of features
  # 1,2,..., noOfFeatures on k-1 training folds.
  best.fit <- regsubsets(Direction ~ ., data = weekly.df[folds != j, ],
                        nvmax = noOfFeatures, method = "backward")
}

```

```

# Compute CV test error for each of models that have best number of
# predictors on test fold # j
for(i in 1:noOfFeatures){

  # extract coefficients for model # i
  coefi <- coef(best.fit, id = i)

  # For classification we are interested in name
  # of features for model # i (not their coefficients)
  # except intercept
  predictorsOfModel <- names(coefi)[-1]
  # fit the model on k-1 training portion
  lda.fit <- MASS::lda(as.formula(paste("Direction~", paste(predictorsOfModel, collapse="+"))),
    data = weekly.df, family = binomial, subset = (folds != j))

  # predict on single validation fold
  lda.pred <- predict(lda.fit, weekly.df[folds == j, ], type = "response")
  # since contrasts(weekly.df$Direction) shows dummy variable 1 assigned to 'Up'
  # and since P(y=1|x) is glm.probs what we get is posterior of probability of 'Up' case
  stopifnot(length(lda.pred$class) == length(weekly.df[folds == j, ]$Direction))
  cv.errors[j,i] <- mean(lda.pred$class == weekly.df[folds == j, ]$Direction)
}
}

# finally calculate mean of CV MSE error for each model
cv.error.means <- rep(NA, ncol(cv.errors))
for (i in 1:ncol(cv.errors)){
  cv.error.means[i] <- mean(cv.errors[, i])
}

# find the minimum of all cv-MSE means and corresponding coefficients
print("minimum of all cv-MSE means and corresponding coefficients: ")

## [1] "minimum of all cv-MSE means and corresponding coefficients: "
(selected.no.of.features <- which.min(cv.error.means))

## [1] 4
lda.best <- regsubsets(Direction ~ ., data = weekly.df[folds != j, ],
  nvmax = selected.no.of.features, method = "backward")

# seems like model with following 5 predictors has least MSE error on test data
names(coef(lda.best, id = which.min(cv.error.means) ))

## [1] "(Intercept)" "Lag2" "Lag3" "Lag5" "Today"

library(tidyverse)
library(glmnet)

## Loading required package: Matrix

##
## Attaching package: 'Matrix'

## The following objects are masked from 'package:tidyr':
##

```

```
##      expand, pack, unpack
## Loaded glmnet 3.0-2
hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Salary
hitters.df$Salary <- gsub('\\s+', '', hitters.df$Salary)

# Second remove all leading and trailing spaces from a character column "Salary"
hitters.df$Salary <- trimws(hitters.df$Salary, which = c("both"))

# Remove all records with "NA" or empty string in character column "Salary"
hitters.df <- hitters.df[!(tolower(hitters.df$Salary) == "na" |
  hitters.df$Salary == ""), ]

# convert Salary column to numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# convert all character fields
hitters.df[sapply(hitters.df, is.character)] <-
  lapply(hitters.df[sapply(hitters.df, is.character)], as.factor)

# Find and remove NA in all columns
hitters.df <- na.omit(hitters.df)

# Use glmnet () for Ridge (glmnet only take numarical values)
# glmnet automatically standardize predictors unless we set standardize = F

# first create a matrix of all predictors
# model.matrix automatically transforms any qualitative variable to factor

x <- model.matrix(Salary~., hitters.df)[, -1]
y <- hitters.df$Salary

# apply Ridge
grid <- 10 ^ seq(10, -2, length = 100)
# alpha = 0 causes Ridge to be applied
ridge.model <- glmnet(x, y, alpha=0, lambda = grid)

# there are 100 values for lambda and associated to each we have
# number of ncol(hitters.df) coefficients

ridge.model$lambda[50]

## [1] 11497.57

# get the coefficients corresponding to 50th lambda:
rownames(coef(ridge.model))

## [1] "(Intercept)" "AtBat"      "Hits"      "HmRun"      "Runs"
## [6] "RBI"          "Walks"      "Years"      "CAatBat"    "CHits"
```

```
## [11] "CHmRun"      "CRuns"      "CRBI"      "CWalks"    "LeagueN"
## [16] "DivisionW"    "PutOuts"    "Assists"    "Errors"     "NewLeagueN"
```

```
coef(ridge.model)[,50]
```

```
##      (Intercept)      AtBat      Hits      HmRun      Runs
## 407.356050200    0.036957182    0.138180344    0.524629976    0.230701523
##           RBI           Walks           Years      CAtBat      CHits
##   0.239841459    0.289618741    1.107702929    0.003131815    0.011653637
##      CHmRun      CRuns      CRBI      CWalks      LeagueN
##   0.087545670    0.023379882    0.024138320    0.025015421    0.085028114
##   DivisionW    PutOuts      Assists      Errors    NewLeagueN
##  -6.215440973    0.016482577    0.002612988   -0.020502690    0.301433531
```

```
# for some reason there are two intercept coefficients at the begining
# we drop first one to calculate L2 norm of the coefficients
sqrt(sum(coef(ridge.model)[-1,50]^2))
```

```
## [1] 6.360612
```

```
# we use predict to get a new value for coefficients for any given value of lambda
predict(ridge.model, s = 51, type="coefficients")[1:20]
```

```
## [1] 4.784128e+01 -3.496519e-01 1.949106e+00 -1.267814e+00 1.147840e+00
## [6] 8.055626e-01 2.698472e+00 -6.123000e+00 5.606739e-03 1.056868e-01
## [11] 6.221438e-01 2.195339e-01 2.174176e-01 -1.464445e-01 4.567755e+01
## [16] -1.180038e+02 2.497163e-01 1.201684e-01 -3.262943e+00 -9.218087e+00
```

```
# now split the samples into test and training:
```

```
set.seed(10)
```

```
train <- sample(1:nrow(x), nrow(x)/2)
```

```
test <- (-train)
```

```
y.test <- y[test]
```

```
# Fit ridge regression model to trainig data
```

```
ridge.model <- glmnet(x[train, ], y[train], alpha=0,
                      lambda = ( 10 ^ seq(10, -2, length = 100)),
                      thresh = 1e-12)
```

```
# Evaluate MSE of the model on on the test set for lambda = 4
```

```
# to do the prediction we set newx argument to test set
```

```
ridge.predict <- predict(ridge.model, s=1000, newx = x[test,])
```

```
print ("now find the MSE corresponding to lambda = 4")
```

```
## [1] "now find the MSE corresponding to lambda = 4"
```

```
mean((ridge.predict-y.test)^2)
```

```
## [1] 139172.7
```

```
print ("just for comarison we use intercept to predict and calculate the MSE
      (lambda is set to a very large value)")
```

```
## [1] "just for comarison we use intercept to predict and calculate the MSE\n      (lambda is set to a
```

```
ridge.predict <- predict(ridge.model, s=1e+10, newx = x[test,])
```

```
mean((ridge.predict-y.test)^2)
```

```
## [1] 202640.1
print ("now compare Ridge with usuall regression (i.e when lambda = 0)")

## [1] "now compare Ridge with usuall regression (i.e when lambda = 0)"
# we set exact to T otherwise predict() function will
# interpolate over the grid of lambda values
ridge.predict <- predict(ridge.model, s=0, newx = x[test,],
                        exact = T, x = x[train,], y=y[train])

# calculate MSE
mean((ridge.predict - y.test)^2)

## [1] 145023.6
# if we want to fit a (unpenalized) least squares model, then we should use the
# lm() function, since that function provides more useful outputs, such as
# standard errors and p-values for the coefficients.

library(tidyverse)
library(glmnet)

hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Salary
hitters.df$Salary <- gsub('\\s+', '', hitters.df$Salary)

# Second remove all leading and trailing spaces from a character column "Salary"
hitters.df$Salary <- trimws(hitters.df$Salary, which = c("both"))

# Remove all records with "NA" or empty string in character column "Salary"
hitters.df <- hitters.df[!(tolower(hitters.df$Salary) == "na" |
  hitters.df$Salary == ""), ]

# convert Salary column to numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# convert all character fields to factor
hitters.df[sapply(hitters.df, is.character)] <-
  lapply(hitters.df[sapply(hitters.df, is.character)], as.factor)

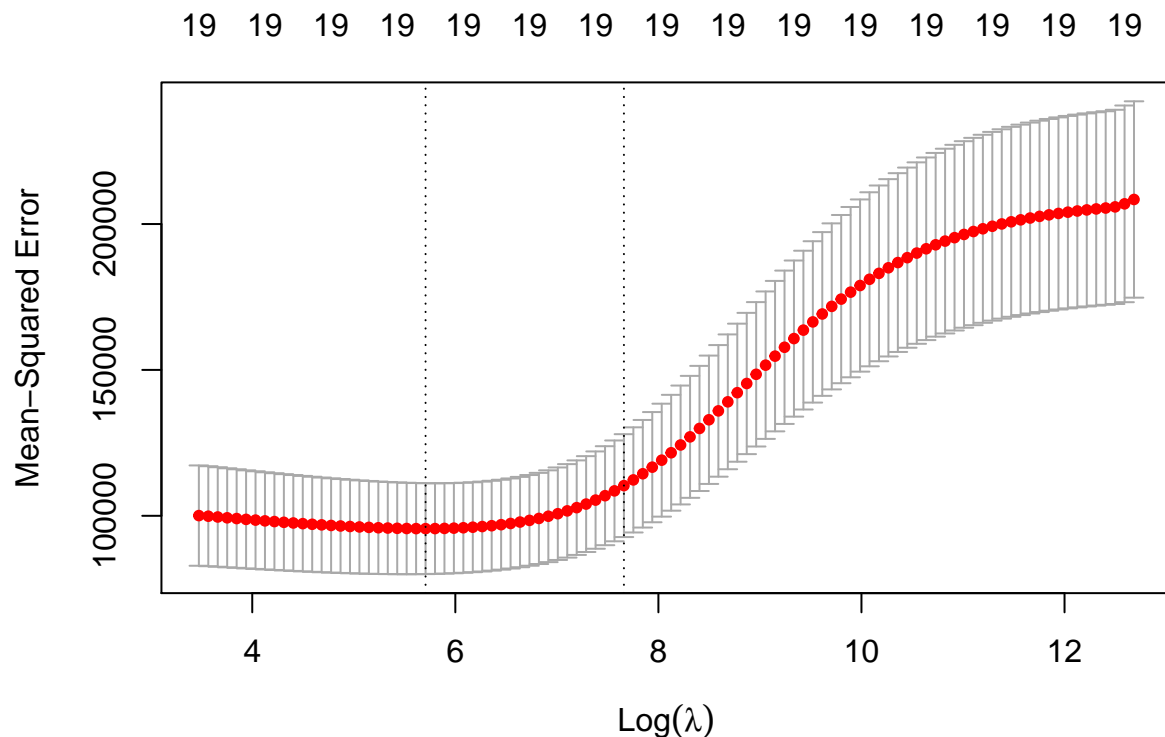
# Find and remove NA in all columns
hitters.df <- na.omit(hitters.df)

x <- model.matrix(Salary~., hitters.df)[, -1] # -1 is to drop the Intercept
y <- hitters.df$Salary

set.seed(10)
train <- sample(1:nrow(x), nrow(x)/2)
test <- (-train)
```

```
# Built-in cross-validation function, cv.glmnet().
# By default, the function performs ten-fold cross-validation,
# though this can be changed using the argument nfolds.
# we apply it on training portion of the data to find the lambda
# then we run the final model with the lambda on test data to get MSE
```

```
cv.out=cv.glmnet(x[train,],y[train],alpha=0)
plot(cv.out)
```



```
(bestlam=cv.out$lambda.min)
```

```
## [1] 300.8959
```

```
# What is the test MSE associated with this value of lambda?
```

```
# Fit ridge regression model to training data
```

```
ridge.model <- glmnet(x[train, ], y[train], alpha=0,
                      lambda = ( 10 ^ seq(10, -2, length = 100)),
                      thresh = 1e-12)
```

```
ridge.pred=predict(ridge.model,s=bestlam ,newx=x[test,])
```

```
mean((ridge.pred-y[test])^2)
```

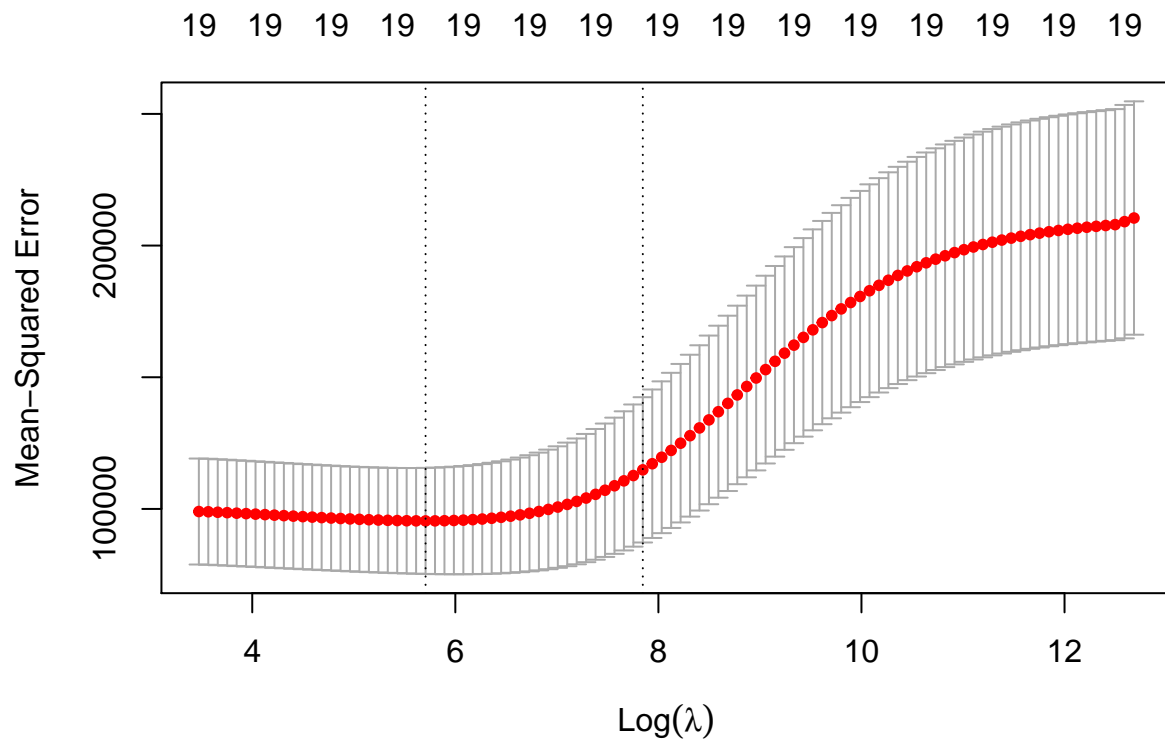
```
## [1] 143253.2
```

```
# now let's split the data into train / test and train
```

```
# the model on train data , get the best lambda and then run it on test portion
```

```
# with the best lambda we got to see the testMSE:
```

```
cv.out <- cv.glmnet(x[train, ], y[train], alpha=0)
plot(cv.out)
```

```
(best.lambda <- cv.out$lambda.min)
```

```
## [1] 300.8959
```

```
ridge.pred <- predict(cv.out, s=bestlam, newx = x[test, ])
```

```
sprintf("mean error on test data using %s is %s", bestlam, mean((ridge.pred - y[test])^ 2))
```

```
## [1] "mean error on test data using 300.895860794707 is 143257.455545951"
```

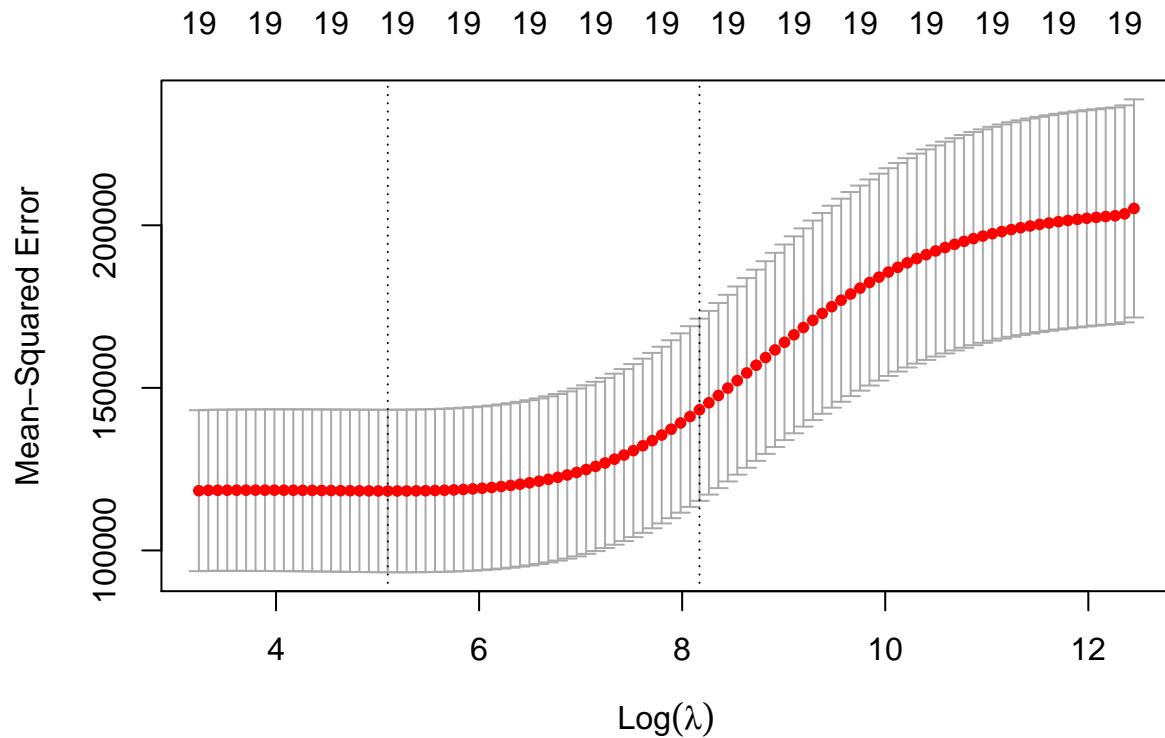
```
mean((ridge.pred - y[test])^ 2)
```

```
## [1] 143257.5
```

```
# finally we refit the model on the whole data and use the best lambda calculated  
# in CV
```

```
cv.out=cv.glmnet(x,y,alpha=0)
```

```
plot(cv.out)
```



```
predict(cv.out, type="coefficients" ,s=bestlam )
```

```
## 20 x 1 sparse Matrix of class "dgCMatrix"
```

```
##              1
## (Intercept) 13.60555790
## AtBat       0.07035200
## Hits        0.88449492
## HmRun        0.52082363
## Runs        1.07391429
## RBI          0.87905669
## Walks        1.65668620
## Years        1.15648560
## CAtBat       0.01133745
## CHits        0.05871840
## CHmRun       0.41453959
## CRuns        0.11702038
## CRBI         0.12398388
## CWalks       0.04940713
## LeagueN     23.01892904
## DivisionW   -81.49794763
## PutOuts      0.17107771
## Assists      0.03150566
## Errors      -1.44191373
## NewLeagueN   8.90014313
```

```
library(tidyverse)
```

```
library(glmnet)
```

```
weekly.df =
```

```
  read.csv("/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Weekly.csv",
           header=T, stringsAsFactors = F, na.strings = "?")
```

```

str(weekly.df)

## 'data.frame': 1089 obs. of 9 variables:
## $ Year : int 1990 1990 1990 1990 1990 1990 1990 1990 1990 1990 ...
## $ Lag1 : num 0.816 -0.27 -2.576 3.514 0.712 ...
## $ Lag2 : num 1.572 0.816 -0.27 -2.576 3.514 ...
## $ Lag3 : num -3.936 1.572 0.816 -0.27 -2.576 ...
## $ Lag4 : num -0.229 -3.936 1.572 0.816 -0.27 ...
## $ Lag5 : num -3.484 -0.229 -3.936 1.572 0.816 ...
## $ Volume : num 0.155 0.149 0.16 0.162 0.154 ...
## $ Today : num -0.27 -2.576 3.514 0.712 1.178 ...
## $ Direction: chr "Down" "Down" "Up" "Up" ...

# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Direction
weekly.df$Direction <- gsub('\\s+', '', weekly.df$Direction)

# Second remove all leading and trailing spaces from a character column "Direction"
weekly.df$Direction <- trimws(weekly.df$Direction, which = c("both"))

# Remove all records with "NA" or empty string in character column "Direction"
weekly.df <- weekly.df[!(tolower(weekly.df$Direction) == "na" |
                        weekly.df$Direction == ""), ]

# convert all character fields to factor
weekly.df[sapply(weekly.df, is.character)] <-
  lapply(weekly.df[sapply(weekly.df, is.character)], as.factor)

# Find and remove NA in all columns
weekly.df <- na.omit(weekly.df)

str(weekly.df)

## 'data.frame': 1089 obs. of 9 variables:
## $ Year : int 1990 1990 1990 1990 1990 1990 1990 1990 1990 1990 ...
## $ Lag1 : num 0.816 -0.27 -2.576 3.514 0.712 ...
## $ Lag2 : num 1.572 0.816 -0.27 -2.576 3.514 ...
## $ Lag3 : num -3.936 1.572 0.816 -0.27 -2.576 ...
## $ Lag4 : num -0.229 -3.936 1.572 0.816 -0.27 ...
## $ Lag5 : num -3.484 -0.229 -3.936 1.572 0.816 ...
## $ Volume : num 0.155 0.149 0.16 0.162 0.154 ...
## $ Today : num -0.27 -2.576 3.514 0.712 1.178 ...
## $ Direction: Factor w/ 2 levels "Down","Up": 1 1 2 2 2 1 2 2 2 1 ...

# contrasts(weekly.df$Direction)

# now we use cross validation to find the best lambda and corresponding coeffs

# First construct matrix from dataframe (and drop intercept column)
x <- model.matrix(Direction~., weekly.df)[,-1]
y <- ifelse(weekly.df$Direction == "Up", 1, 0)

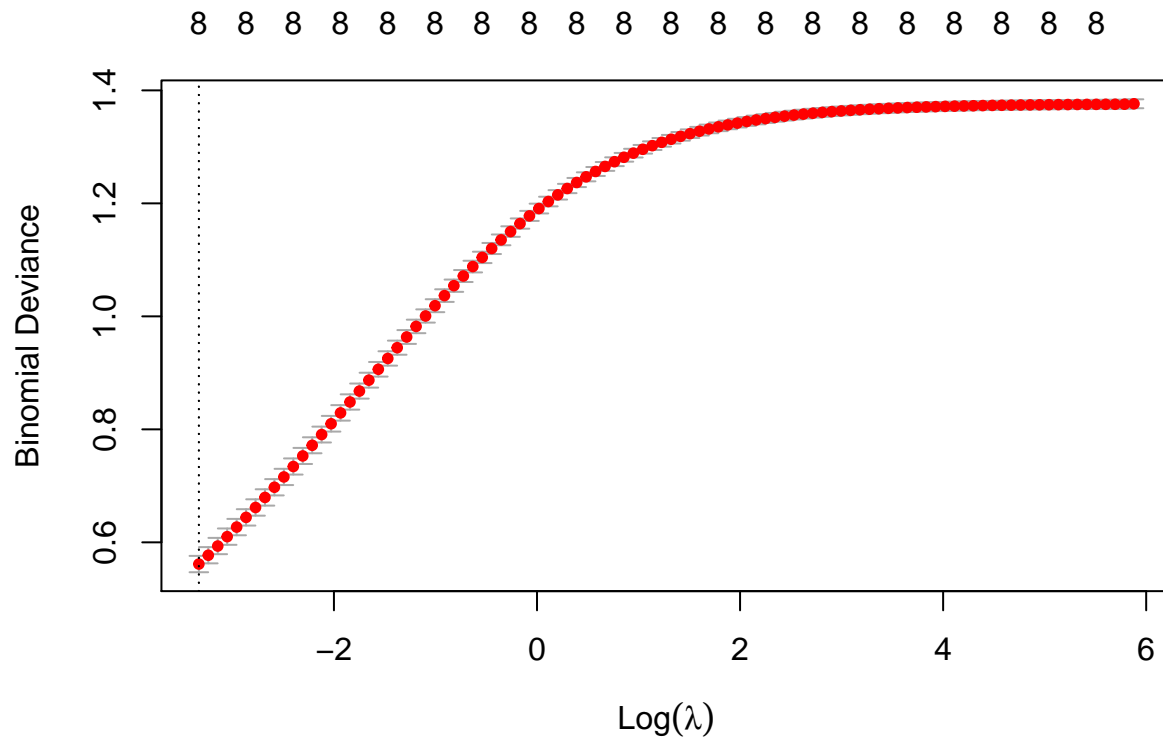
set.seed(10)

```

```
# We'll use the R function glmnet() [glmnet package] for computing penalized logistic regression.

# Built-in cross-validation function, cv.glmnet().
# By default, the function performs ten-fold cross-validation,
# though this can be changed using the argument nfolds.

cv.out=cv.glmnet(x, y, family = "binomial", alpha=0, lambda = NULL)
plot(cv.out)
```



```
(bestlam=cv.out$lambda.min)

## [1] 0.03577832
# get the coefficients:
predict(cv.out, type="coefficients" ,s=bestlam )

## 9 x 1 sparse Matrix of class "dgCMatrix"
##              1
## (Intercept)  9.238107856
## Year        -0.004539565
## Lag1        -0.011377731
## Lag2         0.036049566
## Lag3        -0.007947419
## Lag4        -0.021976657
## Lag5        -0.012921371
## Volume       0.012663021
## Today        0.966650674

print("Here is value of lambda for which the MSE is minimum")

## [1] "Here is value of lambda for which the MSE is minimum"
```

```

cv.out$lambda.min

## [1] 0.03577832
print("Here is one standard error value of lambda for which the MSE is minimum")

## [1] "Here is one standard error value of lambda for which the MSE is minimum"
cv.out$lambda.1se

## [1] 0.03577832
library(tidyverse)
library(glmnet)

weekly.df =
  read.csv("/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Weekly.csv",
           header=T, stringsAsFactors = F, na.strings = "?")

str(weekly.df)

## 'data.frame': 1089 obs. of 9 variables:
## $ Year : int 1990 1990 1990 1990 1990 1990 1990 1990 1990 1990 ...
## $ Lag1 : num 0.816 -0.27 -2.576 3.514 0.712 ...
## $ Lag2 : num 1.572 0.816 -0.27 -2.576 3.514 ...
## $ Lag3 : num -3.936 1.572 0.816 -0.27 -2.576 ...
## $ Lag4 : num -0.229 -3.936 1.572 0.816 -0.27 ...
## $ Lag5 : num -3.484 -0.229 -3.936 1.572 0.816 ...
## $ Volume : num 0.155 0.149 0.16 0.162 0.154 ...
## $ Today : num -0.27 -2.576 3.514 0.712 1.178 ...
## $ Direction: chr "Down" "Down" "Up" "Up" ...
# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Direction
weekly.df$Direction <- gsub('\\s+', '', weekly.df$Direction)

# Second remove all leading and trailing spaces from a character column "Direction"
weekly.df$Direction <- trimws(weekly.df$Direction, which = c("both"))

# Remove all records with "NA" or empty string in character column "Direction"
weekly.df <- weekly.df[!(tolower(weekly.df$Direction) == "na" |
                        weekly.df$Direction == ""), ]

# convert all character fields to factor
weekly.df[sapply(weekly.df, is.character)] <-
  lapply(weekly.df[sapply(weekly.df, is.character)], as.factor)

# Find and remove NA in all columns
weekly.df <- na.omit(weekly.df)

str(weekly.df)

## 'data.frame': 1089 obs. of 9 variables:
## $ Year : int 1990 1990 1990 1990 1990 1990 1990 1990 1990 1990 ...
## $ Lag1 : num 0.816 -0.27 -2.576 3.514 0.712 ...
## $ Lag2 : num 1.572 0.816 -0.27 -2.576 3.514 ...

```

```
## $ Lag3      : num  -3.936 1.572 0.816 -0.27 -2.576 ...
## $ Lag4      : num  -0.229 -3.936 1.572 0.816 -0.27 ...
## $ Lag5      : num  -3.484 -0.229 -3.936 1.572 0.816 ...
## $ Volume    : num   0.155 0.149 0.16 0.162 0.154 ...
## $ Today     : num   -0.27 -2.576 3.514 0.712 1.178 ...
## $ Direction: Factor w/ 2 levels "Down","Up": 1 1 2 2 2 1 2 2 2 1 ...
```

```
# contrasts(weekly.df$Direction)

# now we use cross validation to find the best lambda and corresponding coeffs

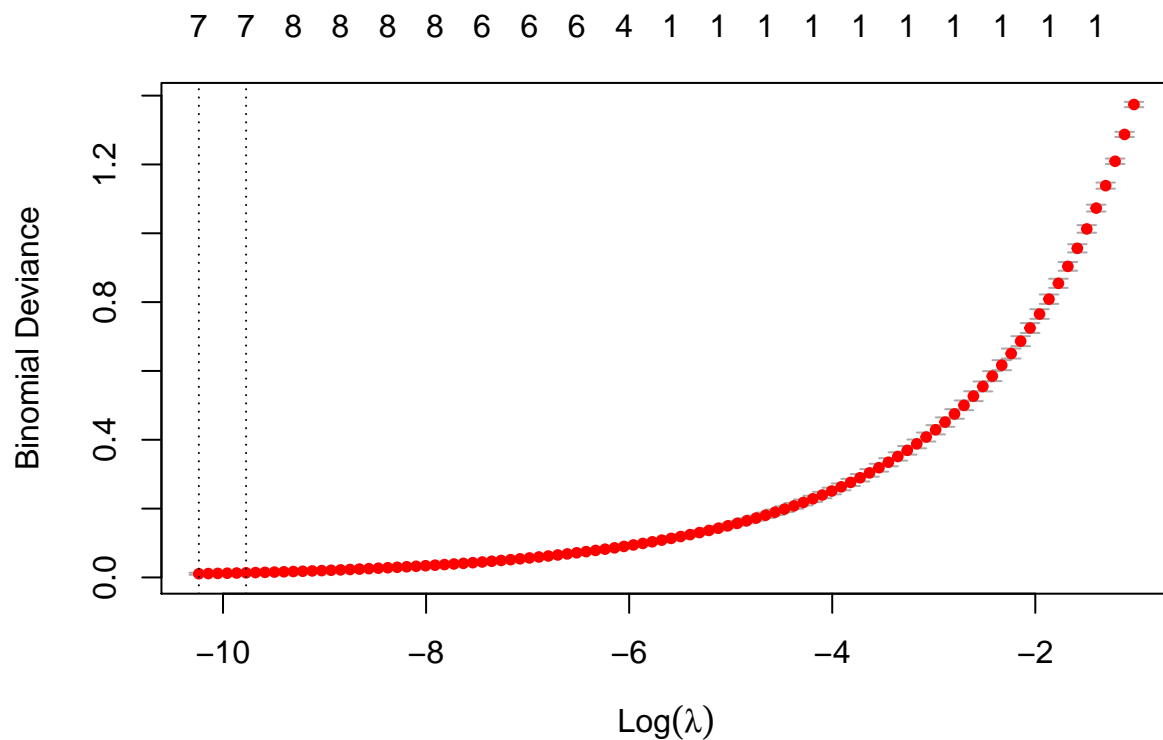
# First construct matrix from dataframe (and drop intercept column)
x <- model.matrix(Direction~., weekly.df)[-1]
y <- ifelse(weekly.df$Direction == "Up", 1, 0)

set.seed(10)

# We'll use the R function glmnet() [glmnet package]
# for computing penalized logistic regression.

# Built-in cross-validation function, cv.glmnet().
# By default, the function performs ten-fold cross-validation,
# though this can be changed using the argument nfolds.

cv.out=cv.glmnet(x, y, family = "binomial", alpha=1, lambda = NULL)
plot(cv.out)
```



```
(bestlam=cv.out$lambda.min)
```

```
## [1] 3.577832e-05
```

```

# gget the coefficients:
predict(cv.out, type="coefficients" ,s=bestlam )

## 9 x 1 sparse Matrix of class "dgCMatrix"
##              1
## (Intercept) -57.08275304
## Year        0.02827835
## Lag1        -0.61384069
## Lag2        0.06008024
## Lag3        0.28917265
## Lag4        .
## Lag5        0.47365956
## Volume      0.22529278
## Today       61.60602428

names(cv.out)

## [1] "lambda"      "cvm"          "cvsd"         "cvup"         "cvlo"
## [6] "nzero"       "call"         "name"         "glmnet.fit"   "lambda.min"
## [11] "lambda.1se"

print("Here is value of lambda for which the MSE is minimum")

## [1] "Here is value of lambda for which the MSE is minimum"
cv.out$lambda.min

## [1] 3.577832e-05

print("Here is one standard error value of lambda for which the MSE is minimum")

## [1] "Here is one standard error value of lambda for which the MSE is minimum"
cv.out$lambda.1se

## [1] 5.69692e-05

library(tidyverse)
library(pls)

##
## Attaching package: 'pls'
## The following object is masked from 'package:stats':
##
##      loadings

hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Salary
hitters.df$Salary <- gsub('\\s+', '', hitters.df$Salary)

# Second remove all leading and trailing spaces from a character column "Salary"
hitters.df$Salary <- trimws(hitters.df$Salary, which = c("both"))

```

```

# Remove all records with "NA" or empty string in character column "Salary"
hitters.df <- hitters.df[!(tolower(hitters.df$Salary) == "na" |
                           hitters.df$Salary == ""), ]

# convert Salary column to numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# convert all character fields
hitters.df[sapply(hitters.df, is.character)] <-
  lapply(hitters.df[sapply(hitters.df, is.character)], as.factor)

# Find and remove NA in all columns
hitters.df <- na.omit(hitters.df)

set.seed(2)

# fit the model using pcr
pcr.fit <- pcr(Salary ~ ., data = hitters.df, scale = T, validation="CV")

print ("Summary:")

## [1] "Summary:"
summary(pcr.fit)

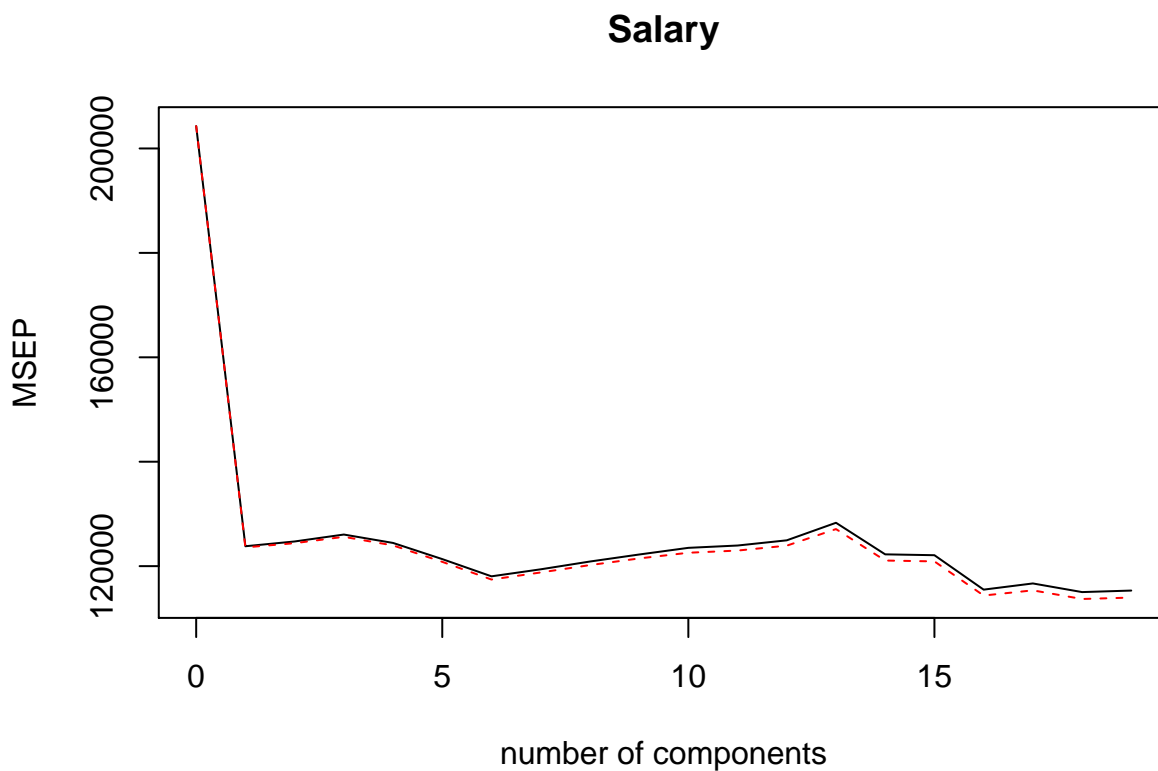
## Data:      X dimension: 263 19
## Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##      (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV              452    351.9   353.2   355.0   352.8   348.4   343.6
## adjCV           452    351.6   352.7   354.4   352.1   347.6   342.7
##      7 comps  8 comps  9 comps 10 comps 11 comps 12 comps 13 comps
## CV          345.5   347.7   349.6   351.4   352.1   353.5   358.2
## adjCV       344.7   346.7   348.5   350.1   350.7   352.0   356.5
##      14 comps 15 comps 16 comps 17 comps 18 comps 19 comps
## CV          349.7   349.4   339.9   341.6   339.2   339.6
## adjCV       348.0   347.7   338.2   339.7   337.2   337.6
##
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X          38.31   60.16   70.84   79.03   84.29   88.63   92.26   94.96
## Salary     40.63   41.58   42.17   43.22   44.90   46.48   46.69   46.75
##      9 comps 10 comps 11 comps 12 comps 13 comps 14 comps 15 comps
## X          96.28   97.26   97.98   98.65   99.15   99.47   99.75
## Salary     46.86   47.76   47.82   47.85   48.10   50.40   50.55
##      16 comps 17 comps 18 comps 19 comps
## X          99.89   99.97   99.99   100.00
## Salary     53.01   53.85   54.61   54.61

# Note that although the minimum value of RSME is associated with
# M = 16 (which is very close to 19) but for M = 7 we get a drastic

```



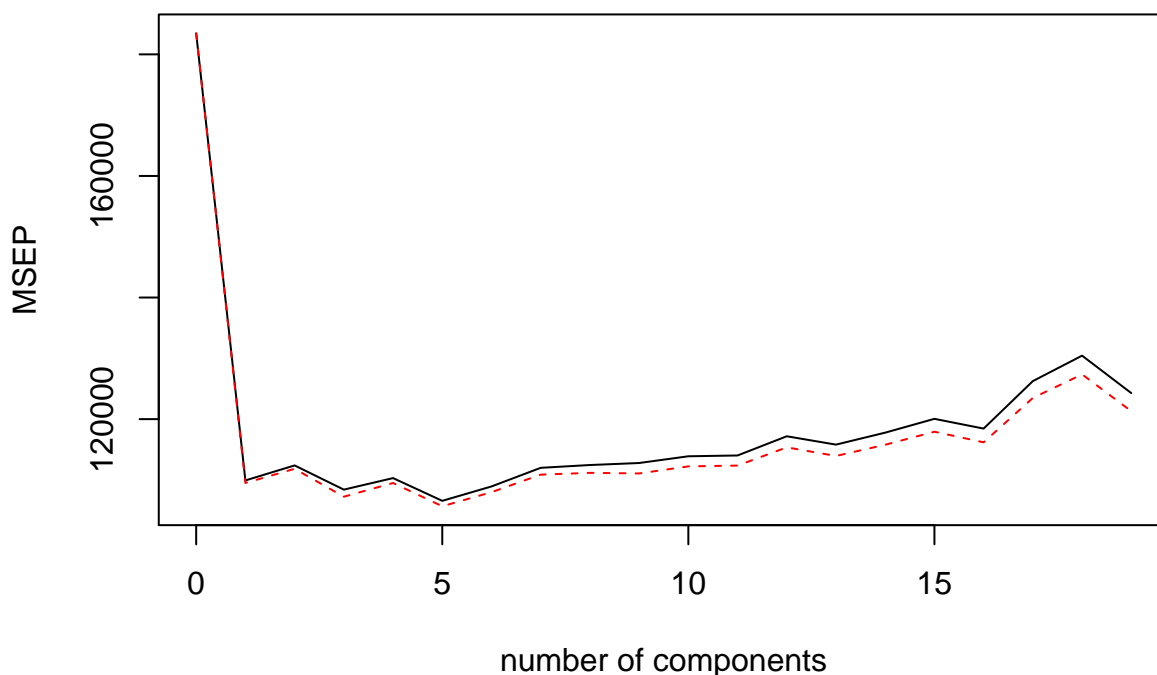
```
# decrease in RMSE which is very close to that of M = 17
# This suggests M=7 gives us good enough model
validationplot(pcr.fit, val.type = "MSEP")
```



```
# Now to see how model with 7 works on test data an compare it with
# model with M = 17 we split data into test and train and fit the model
# on train

set.seed(1)
train <- train <- sample(1:nrow(hitters.df), nrow(hitters.df)/2)
test <- (-train)
pcr.fit <- pcr(Salary ~ ., data=hitters.df, subset=train, scale=T, validation="CV")
validationplot(pcr.fit, val.type = "MSEP")
```

Salary



```
# now let's find the lowest cross validation error occurs M = 7 on the model
test.y <- hitters.df[test,]$Salary
pcr.pred <- predict(pcr.fit, hitters.df[test,], ncomp = 7)

sprintf("lowest MSE corresponding to M = 7 is %s (Ridge was 143257.45)", mean((pcr.pred-test.y)^2))

## [1] "lowest MSE corresponding to M = 7 is 140751.276313081 (Ridge was 143257.45)"
mean((pcr.pred-test.y)^2)

## [1] 140751.3

library(tidyverse)
library(pls)

weekly.df =
  read.csv("/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Weekly.csv",
           header=T, stringsAsFactors = F, na.strings = "?")
# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Direction
weekly.df$Direction <- gsub('\\s+', '', weekly.df$Direction)

# Second remove all leading and trailing spaces from a character column "Direction"
weekly.df$Direction <- trimws(weekly.df$Direction, which = c("both"))

# Remove all records with "NA" or empty string in character column "Direction"
weekly.df <- weekly.df[!(tolower(weekly.df$Direction) == "na" |
                        weekly.df$Direction == ""), ]

# convert all character fields to factor
```

```

weekly.df[sapply(weekly.df, is.character)] <-
  lapply(weekly.df[sapply(weekly.df, is.character)], as.factor)

# Find and remove NA in all columns
weekly.df <- na.omit(weekly.df)

# ----- outmost loop must be CV loop -----
# Note that as per "Tibshirani" cross validation must always be before
# dimension reduction or feature selection
# -----
set.seed(1)
k <- 10
threshold <- 0.5

folds <- sample(1:k, size = nrow(weekly.df), replace = T)
table(folds)

## folds
## 1 2 3 4 5 6 7 8 9 10
## 104 86 106 112 116 107 121 102 117 118

# folds with same size
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#   size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

results <- lapply(1:k, function(x) { # x is the index of test portion, the rest are for training

  #----- calculate all PCA scores on k-1 training fold -----
  # First create predictor matrix and response vector on test and train folds
  predictor.train.X <- model.matrix(Direction~., weekly.df[folds != x,])[ , -1]
  response.train.Y <- weekly.df[folds != x,]$Direction

  # use PCA to find principal components on k-1 training fold:
  pca.train <- princomp(predictor.train.X, cor=T) # PCA using correlation matrix

  # make a tibble from PCAs
  weekly.pca.train <- as_tibble(pca.train$scores[,]*-1) %>%
    add_column(Direction=response.train.Y)

  # Do exact same thing for test fold:
  predictor.test.X <- model.matrix(Direction~., weekly.df[folds == x,])[ , -1]
  response.test.Y <- weekly.df[folds == x,]$Direction

  # use PCA to find principal components on test fold:
  pca.test <- princomp(predictor.test.X, cor=T) # PCA using correlation matrix

  # make a tibble from PCAs
  weekly.pca.test <- as_tibble(pca.test$scores[,]*-1) %>%
    add_column(Direction=response.test.Y)

  # get list of all pca column names
  pca.cols <- colnames(weekly.pca.train[ , !(names(weekly.pca.train) %in% c("Direction"))])

```

```

# An empty tibble to collect all result of running LDA on given test fold
pca.results <- tibble (no.of.pcas = NULL,
                      posterior.up = NULL,
                      posterior.down = NULL,
                      predicted = NULL,
                      real = NULL)

for (pc.col.idx in 1:length(pca.cols)){

  pca.chosen.cols <- pca.cols[1:pc.col.idx]
  partial.weekly.pca.train <- weekly.pca.train[ ,pca.chosen.cols] %>%
    add_column(Direction=response.train.Y)

  # fit on train fold
  lda.fit <-
    MASS::lda(as.formula(paste("Direction~",
                                paste(pca.chosen.cols, collapse="+"))),
              data = partial.weekly.pca.train, family = binomial)

  # predict on test fold
  lda.pred <- predict(lda.fit, weekly.pca.test, type = "response")
  stopifnot(length( lda.pred$class) == length(weekly.pca.test$Direction))
  pca.results <- rbind (pca.results, tibble (no.of.pcas = pc.col.idx,
                                             posterior.up = lda.pred$posterior[, "Up"],
                                             posterior.down = lda.pred$posterior[, "Down"],
                                             predicted = lda.pred$class,
                                             real = weekly.pca.test$Direction))
}
return (pca.results)
})

# We have to find average of missclassification rate for each number of PCs
# cross all test folds

# first calculte missclassification rate per each number of PCAs

rates <- lapply(results, function (result) {
  return(result %>%
    group_by(no.of.pcas) %>%
    summarise(MSE = mean(predicted != real),
              FP_rates = table(predicted, real)[2,1]/(table(predicted, real)[2,1] + table(predicted, real)[2,2]),
              TP_rates = table(predicted, real)[2,2]/(table(predicted, real)[2,2] + table(predicted, real)[2,1]),
              precisions = table(predicted, real)[2,2]/(table(predicted, real)[2,2] + table(predicted, real)[2,1]),
              specificities = table(predicted, real)[2,1]/(table(predicted, real)[2,1] + table(predicted, real)[2,2]),
              nullClassifier = max( ( table(predicted, real)[1,1] + table(predicted, real)[2,1]) /
                                   (table(predicted, real)[1,1] + table(predicted, real)[2,1] +
                                    table(predicted, real)[1,2] + table(predicted, real)[2,2]),
                                   table(predicted, real)[1,2] + table(predicted, real)[2,2]) /
                                   (table(predicted, real)[1,1] + table(predicted, real)[2,1] +
                                    table(predicted, real)[1,2] + table(predicted, real)[2,2])
              ))
})

```

```

# Place rates for all folds in one df
(all.rates = do.call(rbind, rates))

## # A tibble: 80 x 7
##   no.of.pcas  MSE FP_rates TP_rates precisions specificities nullClassifier
##   <int> <dbl>   <dbl>   <dbl>   <dbl>       <dbl>       <dbl>
## 1         1 0.481     1     0.947     0.535         1         0.548
## 2         2 0.606     0.872   0.614     0.461       0.872       0.548
## 3         3 0.740     0.915   0.404     0.348       0.915       0.548
## 4         4 0.760     0.872   0.333     0.317       0.872       0.548
## 5         5 0.827     0.936   0.263     0.254       0.936       0.548
## 6         6 0.817     0.936   0.281     0.267       0.936       0.548
## 7         7 0.904     1       0.175     0.175         1         0.548
## 8         8 0.904     1       0.175     0.175         1         0.548
## 9         1 0.395     0.939   0.943     0.617       0.939       0.616
## 10        2 0.558     0.818   0.604     0.542       0.818       0.616
## # ... with 70 more rows

# get mean of rates cross all folds per each number of pcs (1, 2, ..., 8)
(all.rates %>%
  group_by(no.of.pcas) %>%
  summarise(MSE = mean(MSE),
            FP_rates = mean(FP_rates),
            TP_rates = mean(TP_rates),
            precisions = mean(precisions),
            specificities = mean(specificities),
            nullClassifier = mean(nullClassifier))
)

## # A tibble: 8 x 7
##   no.of.pcas  MSE FP_rates TP_rates precisions specificities nullClassifier
##   <int> <dbl>   <dbl>   <dbl>   <dbl>       <dbl>       <dbl>
## 1         1 0.454     0.968   0.956     0.554       0.968       0.558
## 2         2 0.503     0.740   0.688     0.542       0.740       0.558
## 3         3 0.513     0.648   0.597     0.538       0.648       0.558
## 4         4 0.505     0.611   0.582     0.547       0.611       0.558
## 5         5 0.504     0.614   0.587     0.546       0.614       0.558
## 6         6 0.517     0.617   0.564     0.535       0.617       0.558
## 7         7 0.530     0.622   0.543     0.523       0.622       0.558
## 8         8 0.530     0.623   0.545     0.523       0.623       0.558

# result shows 1 pca is the best , 2 or 3 number of PCAS are acceptable

library(tidyverse)
library(pls)

hitters.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/Hitters.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

# ----- Usual clean up first -----#

# First remove all recods with spaces in character column Salary
hitters.df$Salary <- gsub('\\s+', '', hitters.df$Salary)

```

```

# Second remove all leading and trailing spaces from a character column "Salary"
hitters.df$Salary <- trimws(hitters.df$Salary, which = c("both"))

# Remove all records with "NA" or empty string in character column "Salary"
hitters.df <- hitters.df[!(tolower(hitters.df$Salary) == "na" |
                           hitters.df$Salary == ""), ]

# convert Salary column to numeric
hitters.df$Salary <- as.numeric(as.character(hitters.df$Salary))

# convert all character fields
hitters.df[sapply(hitters.df, is.character)] <-
  lapply(hitters.df[sapply(hitters.df, is.character)], as.factor)

# Find and remove NA in all columns
hitters.df <- na.omit(hitters.df)

set.seed(2)

# fit the model using pls
pls.fit <- pls(Salary ~ ., data = hitters.df, scale = T, validation="CV")

print ("Summary:")

```

```
## [1] "Summary:"
```

```
summary(pcr.fit)
```

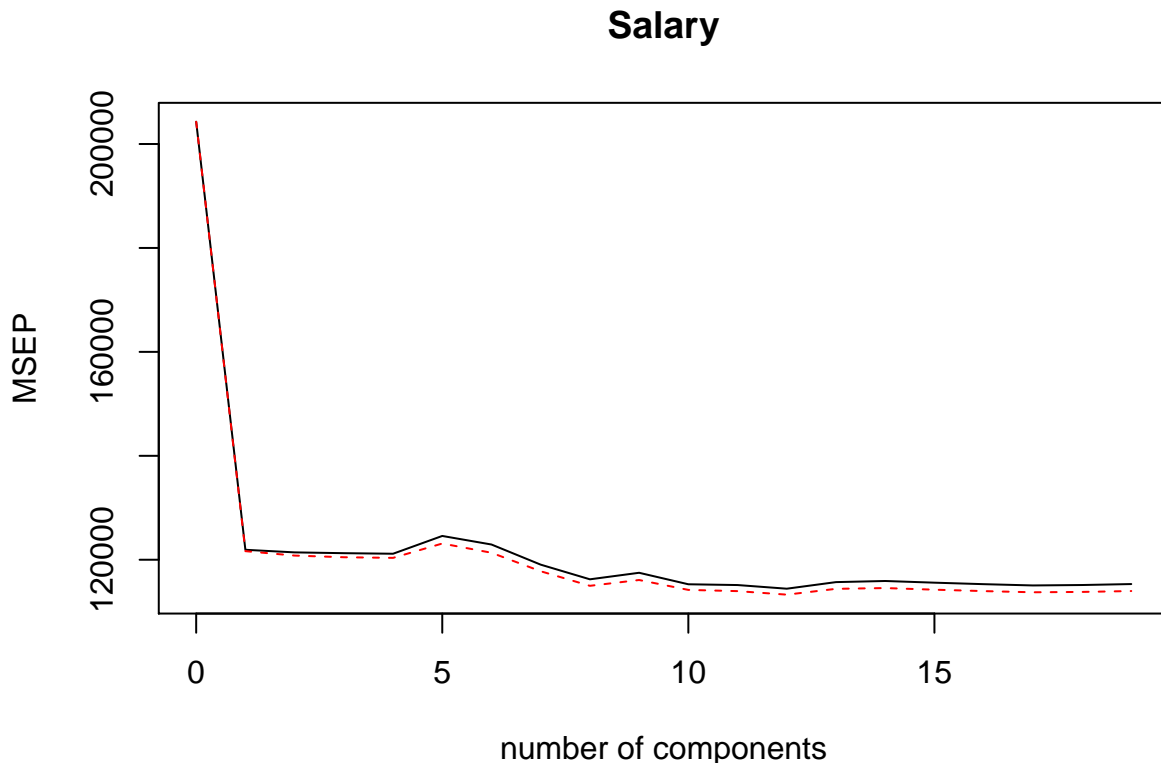
```

## Data:      X dimension: 131 19
## Y dimension: 131 1
## Fit method: svdpc
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##      (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           428.3   331.5   335.2   329.2   332.1   326.4   330.0
## adjCV        428.3   330.9   334.4   327.5   330.9   325.1   328.6
##      7 comps  8 comps  9 comps 10 comps 11 comps 12 comps 13 comps
## CV           334.6   335.3   335.8   337.5   337.7   342.3   340.3
## adjCV        333.0   333.4   333.3   335.0   335.2   339.6   337.6
##      14 comps 15 comps 16 comps 17 comps 18 comps 19 comps
## CV           343.2   346.5   344.2   355.3   361.2   352.5
## adjCV        340.3   343.4   340.9   351.4   356.9   348.3
##
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X           39.32   61.57   71.96   80.83   85.95   89.99   93.25   95.34
## Salary       43.87   43.93   47.36   47.37   49.52   49.55   49.63   50.98
##      9 comps 10 comps 11 comps 12 comps 13 comps 14 comps 15 comps
## X           96.55   97.61   98.28   98.85   99.22   99.53   99.79
## Salary       53.00   53.00   53.02   53.05   53.80   53.85   54.03
##      16 comps 17 comps 18 comps 19 comps
## X           99.91   99.97   99.99   100.00

```

```
## Salary      55.85      55.89      56.21      58.62
```

```
validationplot(pls.fit, val.type = "MSEP")
```



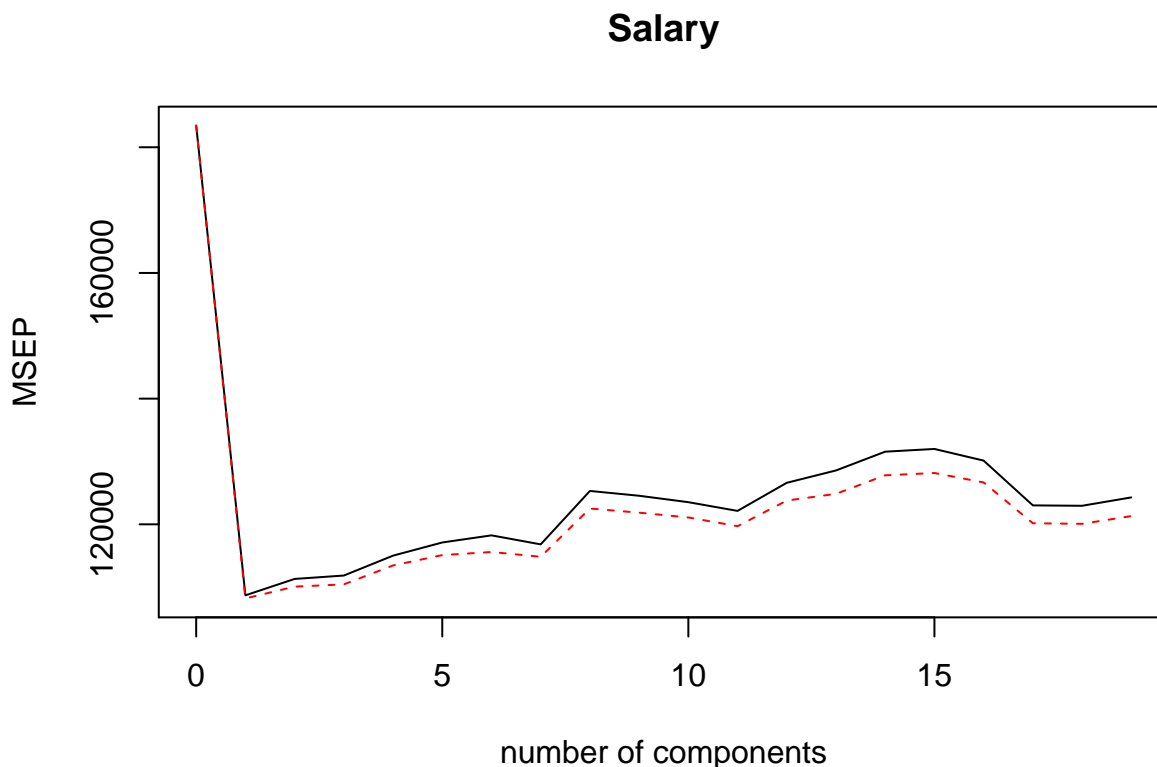
```
# Now to see how model with 7 works on test data an compare it with  
# model with M = 17 we split data into test and train and fit the model  
# on train
```

```
set.seed(1)  
train <- sample(1:nrow(hitters.df), nrow(hitters.df)/2)  
test <- (-train)  
pls.fit <- plsr(Salary ~ ., data=hitters.df, subset=train, scale=T, validation="CV")  
summary(pls.fit)
```

```
## Data:      X dimension: 131 19  
## Y dimension: 131 1  
## Fit method: kernelpls  
## Number of components considered: 19  
##  
## VALIDATION: RMSEP  
## Cross-validated using 10 random segments.  
##      (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  
## CV           428.3   329.7   333.6   334.4   339.1   342.2   343.9  
## adjCV        428.3   328.9   331.8   332.3   336.8   339.3   340.0  
##      7 comps  8 comps  9 comps 10 comps 11 comps 12 comps 13 comps  
## CV           341.8    354   352.9   351.5   349.5   355.8   358.6  
## adjCV        338.9    350   349.1   348.0   346.0   351.8   353.3  
##      14 comps 15 comps 16 comps 17 comps 18 comps 19 comps  
## CV           362.7   363.3   360.8   350.7   350.6   352.5  
## adjCV        357.5   358.0   355.9   346.7   346.5   348.3
```

```
##
## TRAINING: % variance explained
##      1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X      39.13   48.80   60.09   75.07   78.58   81.12   88.21   90.71
## Salary  46.36   50.72   52.23   53.03   54.07   54.77   55.05   55.66
##      9 comps 10 comps 11 comps 12 comps 13 comps 14 comps 15 comps
## X      93.17   96.05   97.08   97.61   97.97   98.70   99.12
## Salary  55.95   56.12   56.47   56.68   57.37   57.76   58.08
##     16 comps 17 comps 18 comps 19 comps
## X      99.61   99.70   99.95  100.00
## Salary  58.17   58.49   58.56   58.62
```

```
validationplot(pls.fit, val.type = "MSEP")
```



```
# now let's find the lowest cross validation error occurs M = 7 on the model
test.y <- hitters.df[test,]$Salary
pls.pred <- predict(pls.fit, hitters.df[test,] %>% select(-Salary), ncomp = 7)

sprintf("lowest MSE corresponding to M = 7 is %s (Ridge was 143257.45)", mean((pls.pred - test.y)^2))
```

```
## [1] "lowest MSE corresponding to M = 7 is 143971.58395204 (Ridge was 143257.45)"
```

```
# PLSR really does not add that much value to PCR
mean((pls.pred - test.y)^2)
```

```
## [1] 143971.6
```

```
library(tidyverse)
library(leaps)
```

```
X <- rnorm(100)
```



```
e <- rnorm(100)

Y <- .02 + 1.6*X -2.2 * X^2 + 5.9*X^3 + e

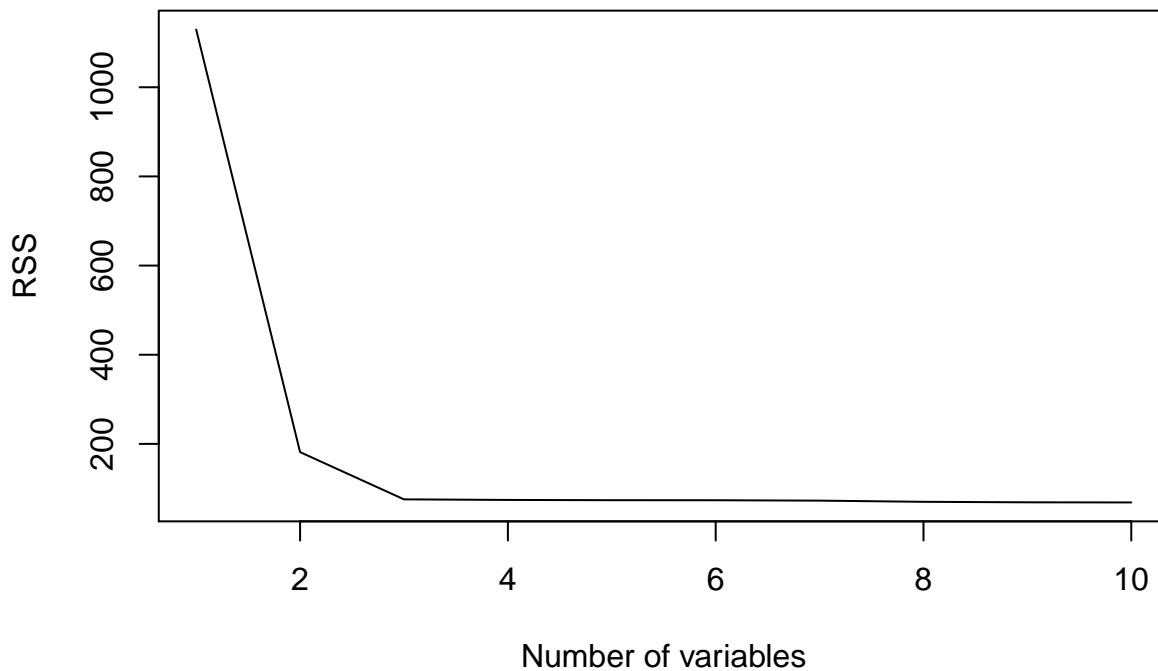
print("Part c:-----")

## [1] "Part c:-----"
df <- tibble(x1 = X, x2=X^2, x3=X^3, x4=X^4, x5=X^5, x6=X^6, x7=X^7,
             x8=X^8, x9=X^9, x10=X^10, y = Y)

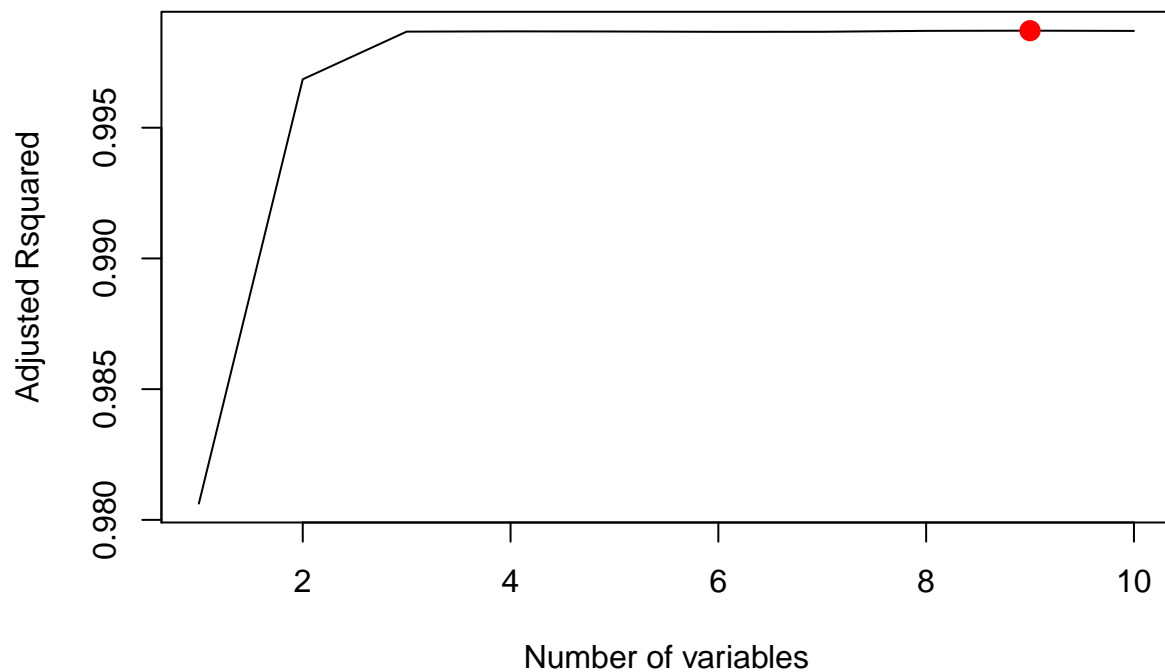
regfit.full <- regsubsets(y ~ ., df, nvmax = 10)

#The summary shows the result of step 2 of algorithm 6.1 page 205 of the book
summary <- summary(regfit.full)

plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")
```



```
# which.max() returns location maximum point of the vector
index <- which.max(summary$adjr2)
plot(summary$adjr2,xlab = "Number of variables", ylab="Adjusted Rsquared",
      type = "l")
points(index, summary$adjr2[index], col="red", cex=2, pch=20)
```

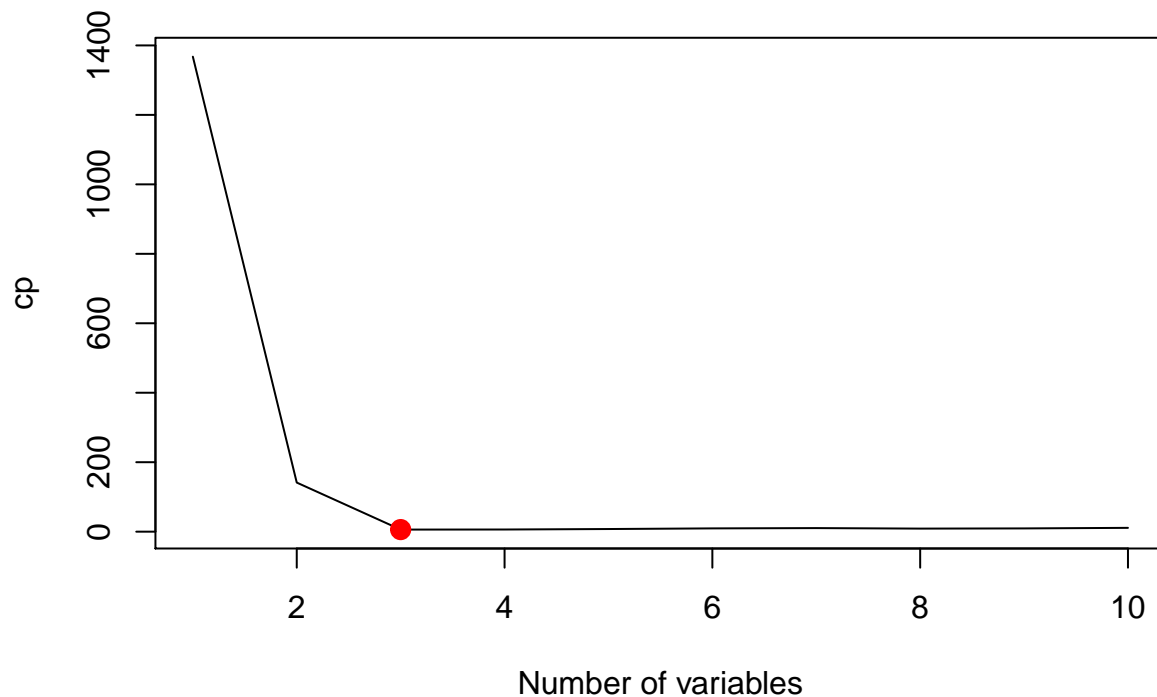


```
print("coefficients of the best model (adjr2) : ")

## [1] "coefficients of the best model (adjr2) : "
coef(regfit.full,index)

## (Intercept)          x1          x2          x3          x4          x5
##  0.25366462  2.03868008 -4.74449117  5.16091299  4.13326339  0.37779502
##          x6          x7          x8          x10
## -2.15279669 -0.05923650  0.42975731 -0.02806608

# which.min() returns location minimum point of the vector
index <- which.min(summary$cp)
plot(summary$cp,xlab = "Number of variables", ylab="cp", type = "l")
points(index, summary$cp[index], col="red", cex=2, pch=20)
```



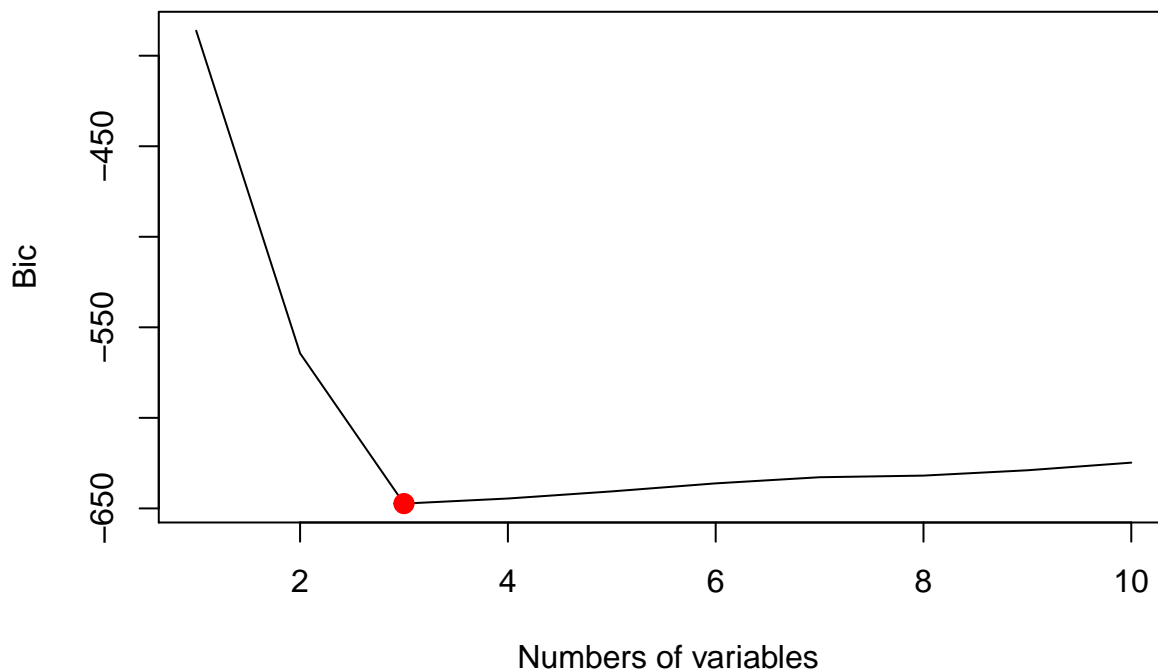
```
print("coefficients of the best model (cp) : ")

## [1] "coefficients of the best model (cp) : "
coef(regfit.full,index)

## (Intercept)          x1          x2          x3
##  0.1304897   1.6922530  -2.2822247   5.8583460

# same for bic
plot(summary$bic, xlab = "Numbers of variables", ylab="Bic", type="l")
(index <- which.min(summary$bic))

## [1] 3
points(index, summary$bic[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (bic) : ")

## [1] "coefficients of the best model (bic) : "
coef(regfit.full,index)

## (Intercept)          x1          x2          x3
##  0.1304897    1.6922530   -2.2822247    5.8583460
# coef(, n) returns coefficient estimates associated with best n variable model

print ("----- use CV with best subset selection -----" )

## [1] "----- use CV with best subset selection -----"

set.seed(1)
k <- 10

folds <- sample(1:k, size = nrow(df), replace = T)
table(folds)

## folds
##  1  2  3  4  5  6  7  8  9 10
##  9  7  7  6  9 14 14  9 11 14
# folds with same size
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#   size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

results <- lapply (1:k, function(j) { # x is the index of test portion, the rest are for training

  # this is to collect the MSEs for each test fold
```

```

msep <- tibble(no.of.coefs = NULL, MSE = NULL)
df.train <- df[folds != j, ]
df.test <- df[folds == j, ]
df.train.X <- df.train %>% select (-y)
df.test.X <- df.test %>% select (-y)
mat.test.X <- model.matrix(y~., data=df.test)
df.test.Y <- df.test$y

# step 2 of algorithm 6.1 page 205 of the book
regfit.full.train <- regsubsets(y ~ ., df.train, nvmax = ncol(df.train.X))

# apply the model with selected subsets on test set
# one at a time and calculate the MSE
for (i in 1:ncol(df.test.X)){
  (coefi <- coef(regfit.full.train, id = i))
  (pred <- mat.test.X[, names(coefi)] %*% coefi)
  (mse <- mean((pred - df.test.Y)^2))
  msep <- rbind (msep, tibble(no.of.coefs = i, MSE = mse))
}
return(msep)
})

allResults <- results[[1]]
for (i in 2 : length(results)){
  allResults <- rbind(allResults , results[[i]])
}

(allMse <- (allResults %>%
  group_by(no.of.coefs) %>%
  summarise(mse.mean = mean(MSE))) )

## # A tibble: 10 x 2
##   no.of.coefs  mse.mean
##   <int>      <dbl>
## 1         1    21.2
## 2         2     2.83
## 3         3     0.920
## 4         4     0.913
## 5         5     6.25
## 6         6    30.8
## 7         7    86.9
## 8         8  9814.
## 9         9 17003.
## 10        10 36230.

(noOfFeatures <- which.min(allMse$mse.mean))

## [1] 4

print ("The best subset of features selected corresponds to minimum CV_MSE")

## [1] "The best subset of features selected corresponds to minimum CV_MSE"
# train on the whole training set now

regfit.full.train <- regsubsets(y ~ ., df , nvmax = ncol(df %>% select (-y)))

```

```
coef(regfit.full.train, id = noOfFeatures)
```

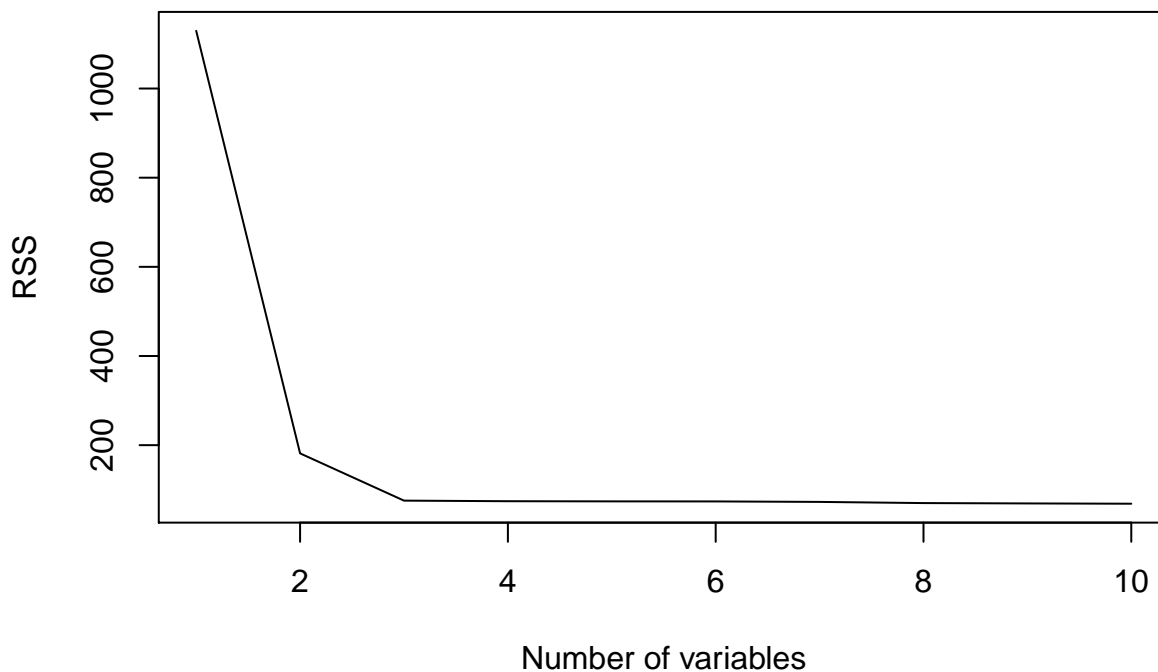
```
## (Intercept)          x1          x2          x3          x4
##  0.04468671  1.74391662 -2.10645119  5.83194832 -0.03328162
```

```
#- -----
print ("part d : Use forward stepwise selection to find the best selected subsets -----")
```

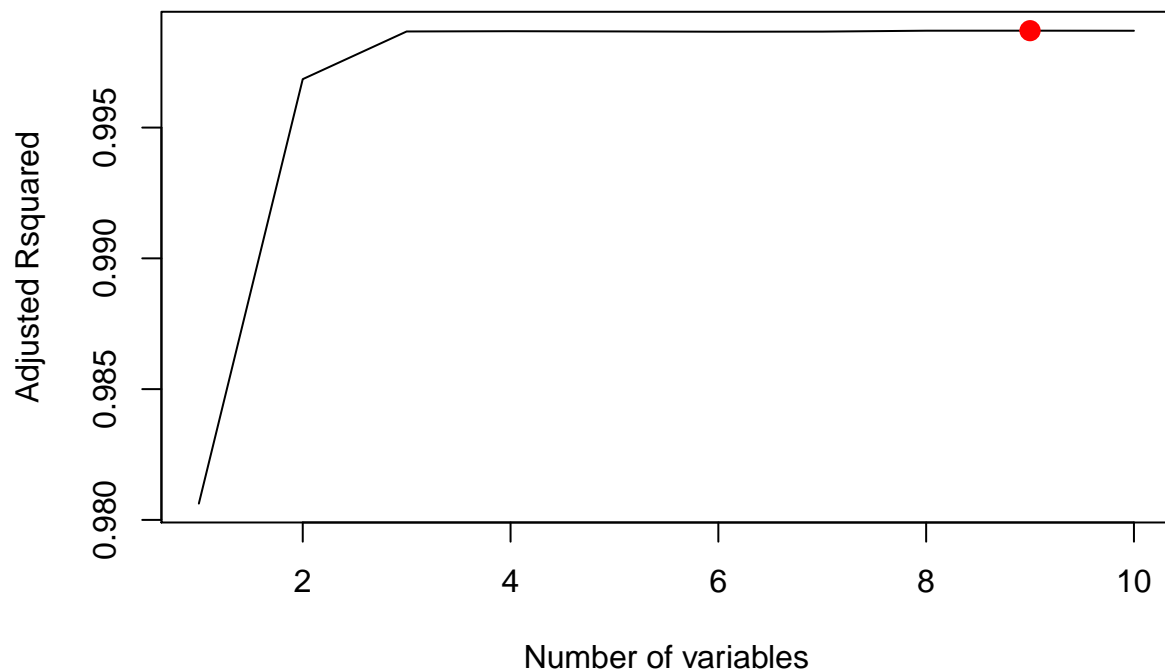
```
## [1] "part d : Use forward stepwise selection to find the best selected subsets -----"
regfit.fwd <- regsubsets(y ~ ., df, nvmax = 10, method="forward")
```

```
#The summary shows the result of step 2 of algorithm 6.2 page 207 of the book
summary <- summary(regfit.fwd)
```

```
plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")
```



```
# which.max() returns location maximum point of the vector
index <- which.max(summary$adjr2)
plot(summary$adjr2,xlab = "Number of variables", ylab="Adjusted Rsquared",
      type = "l")
points(index, summary$adjr2[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (adjr2) : ")
```

```
## [1] "coefficients of the best model (adjr2) : "
```

```
coef(regfit.fwd,index)
```

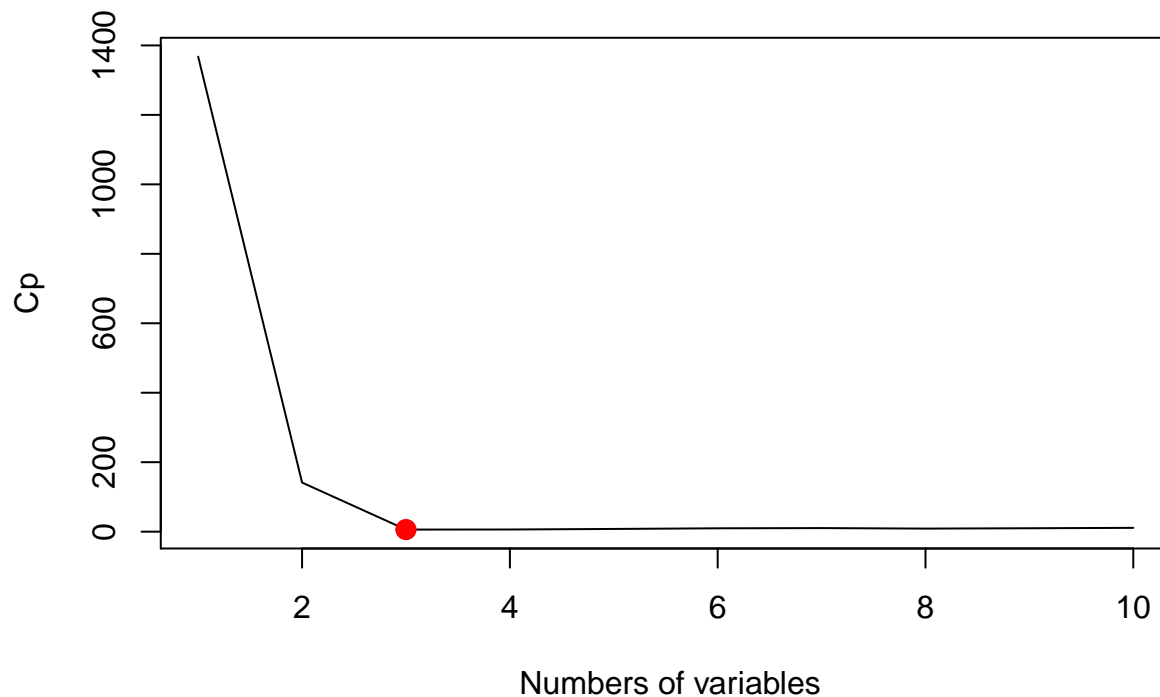
```
## (Intercept)      x1      x2      x3      x4      x5
## 0.241806111 1.900480339 -4.590763438 5.525996094 3.871308892 0.127746525
##      x6      x8      x9     x10
## -1.999182860 0.395212947 -0.003996494 -0.025692057
```

```
# which.min() returns location minimum point of the vector
```

```
index <- which.min(summary$cp)
```

```
plot(summary$cp, xlab = "Numbers of variables", ylab="Cp", type="l")
```

```
points(index, summary$cp[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (cp) : ")
```

```
## [1] "coefficients of the best model (cp) : "
```

```
coef(regfit.fwd,index)
```

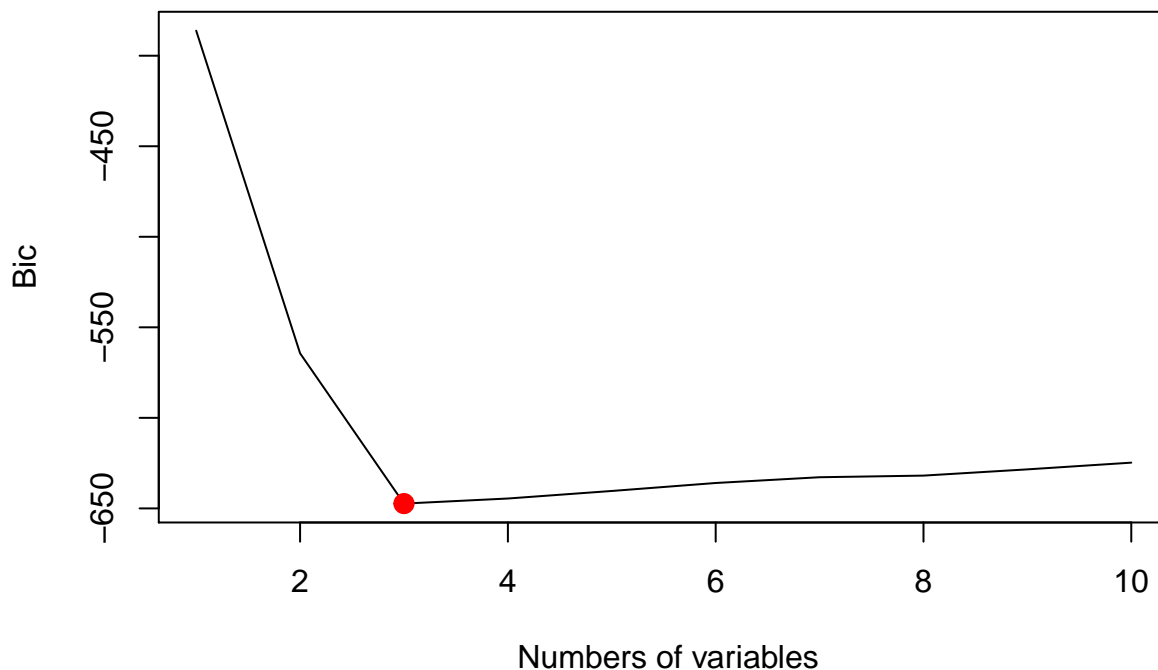
```
## (Intercept)          x1          x2          x3
##  0.1304897   1.6922530  -2.2822247   5.8583460
```

```
# same for bic
```

```
index <- which.min(summary$bic)
```

```
plot(summary$bic, xlab = "Numbers of variables", ylab="Bic", type="l")
```

```
points(index, summary$bic[index], col="red", cex=2, pch=20)
```

```
print("coefficients of the best model (bic) : ")

## [1] "coefficients of the best model (bic) : "
coef(regfit.fwd,index)

## (Intercept)          x1          x2          x3
##  0.1304897    1.6922530   -2.2822247    5.8583460

print ("----- use CV to find best forward stepwise selected model -----" )

## [1] "----- use CV to find best forward stepwise selected model -----"

set.seed(1)
k <- 10

folds <- sample(1:k, size = nrow(df), replace = T)
table(folds)

## folds
##  1  2  3  4  5  6  7  8  9 10
##  9  7  7  6  9 14 14  9 11 14

# folds with same size
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#   size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

results <- lapply (1:k, function(x) { # x is the index of test portion, the rest are for training

  # this is to collect the MSEs for each test fold
  mses <- tibble(no.of.coefs = NULL, MSE = NULL)

  df.train <- df[folds != x, ]
  df.test <- df[folds == x, ]
```

```

(df.train.X <- df.train %>% select (-y))
(df.test.X <- df.test %>% select (-y))
(mat.test.X <- model.matrix(y~., data=df.test))
(df.test.Y <- df.test$y)

# step 2 of algorithm 6.2 page 207 of the book
regfit.fwd.train <- regsubsets(y ~ ., df.train,
                             nvmax = ncol(df.train.X), method="forward")

# apply the model with selected subsets on test set
# one at a time and calculate the MSE
for (i in 1:ncol(df.test.X)){
  (coefi <- coef(regfit.fwd.train, id = i))
  (pred <- mat.test.X[, names(coefi)] %*% coefi)
  (mse <- mean((pred - df.test.Y)^2))
  mses <- rbind (mses, tibble(no.of.coefs = i, MSE = mse))
}
return(mses)
})

allResults <- results[[1]]
for (i in 2 : length(results)){
  allResults <- rbind(allResults , results[[i]])
}

(allMse <- (allResults %>%
  group_by(no.of.coefs) %>%
  summarise(mse.mean = mean(MSE))) )

## # A tibble: 10 x 2
##   no.of.coefs  mse.mean
##   <int>      <dbl>
## 1         1      21.2
## 2         2       2.83
## 3         3       0.920
## 4         4       0.913
## 5         5       1.50
## 6         6       8.00
## 7         7       7.05
## 8         8      96.1
## 9         9     1159.
## 10        10    36230.

(no.of.selected.features <- which.min(allMse$mse.mean))

## [1] 4

print ("The forward features selected correspond to minimum CV_MSE")

## [1] "The forward features selected correspond to minimum CV_MSE"

# train on the whole training set now
regfit.fwd.train <- regsubsets(y ~ ., df , nvmax = ncol(df %>% select (-y)),
                             method = "forward")
coef(regfit.fwd.train, id = no.of.selected.features)

```

```
## (Intercept)          x1          x2          x3          x4
## 0.04468671  1.74391662 -2.10645119  5.83194832 -0.03328162

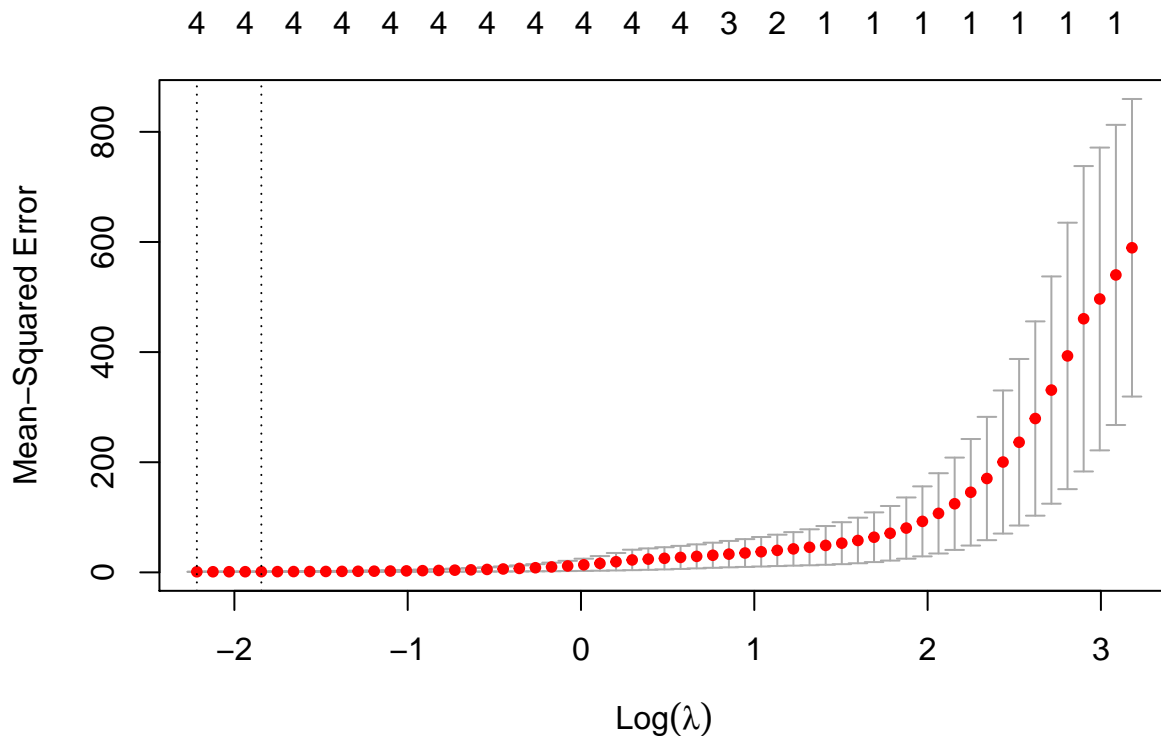
#-----
print("Part e: Fit lasso model and use cv to find the best value for lamda -----")

## [1] "Part e: Fit lasso model and use cv to find the best value for lamda -----"
#-----

library(glmnet)
set.seed(1)

# First construct matrix from dataframe (and drop intercept column)
x <- model.matrix(y~., df)[-1]
y <- df$y

cv.out=cv.glmnet(x, y, alpha=1, lambda = NULL)
plot(cv.out)
```



```
print("Here is value of lambda for which the MSE is minimum")

## [1] "Here is value of lambda for which the MSE is minimum"
(bestlam=cv.out$lambda.min)

## [1] 0.108985
print("Here are the coefficients corresponding to best value of lambda:")

## [1] "Here are the coefficients corresponding to best value of lambda:"
predict(cv.out, type="coefficients" ,s=bestlam )

## 11 x 1 sparse Matrix of class "dgCMatrix"
```

```

##              1
## (Intercept) -0.05681726
## x1          1.65284155
## x2          -1.99192279
## x3           5.82137607
## x4          -0.03838223
## x5           .
## x6           .
## x7           .
## x8           .
## x9           .
## x10          .

print ( "Lasso coefficients are not as close as that of all feature selections !!")

## [1] "Lasso coefficients are not as close as that of all feature selections !!"
print("Here is one standard error value of lambda for which the MSE is minimum")

## [1] "Here is one standard error value of lambda for which the MSE is minimum"
one.SE.lam <- cv.out$lambda.1se

print("Here are the coefficients corresponding to one standard error value of lambda:")

## [1] "Here are the coefficients corresponding to one standard error value of lambda:"
predict(cv.out, type="coefficients" ,s=one.SE.lam )

## 11 x 1 sparse Matrix of class "dgCMatrix"
##              1
## (Intercept) -0.09769231
## x1          1.61472612
## x2          -1.94823793
## x3           5.81658758
## x4          -0.03952492
## x5           .
## x6           .
## x7           .
## x8           .
## x9           .
## x10          .

#-----
print("Part f: Perform Best subset selection and lasso on new data -----")

## [1] "Part f: Perform Best subset selection and lasso on new data -----"
#-----

Y <- 12 - 45.3 * X^7
df <- tibble(x1 = X, x2=X^2, x3=X^3, x4=X^4, x5=X^5, x6=X^6, x7=X^7,
             x8=X^8, x9=X^9, x10=X^10, y = Y)
library(glmnet)
set.seed(1)

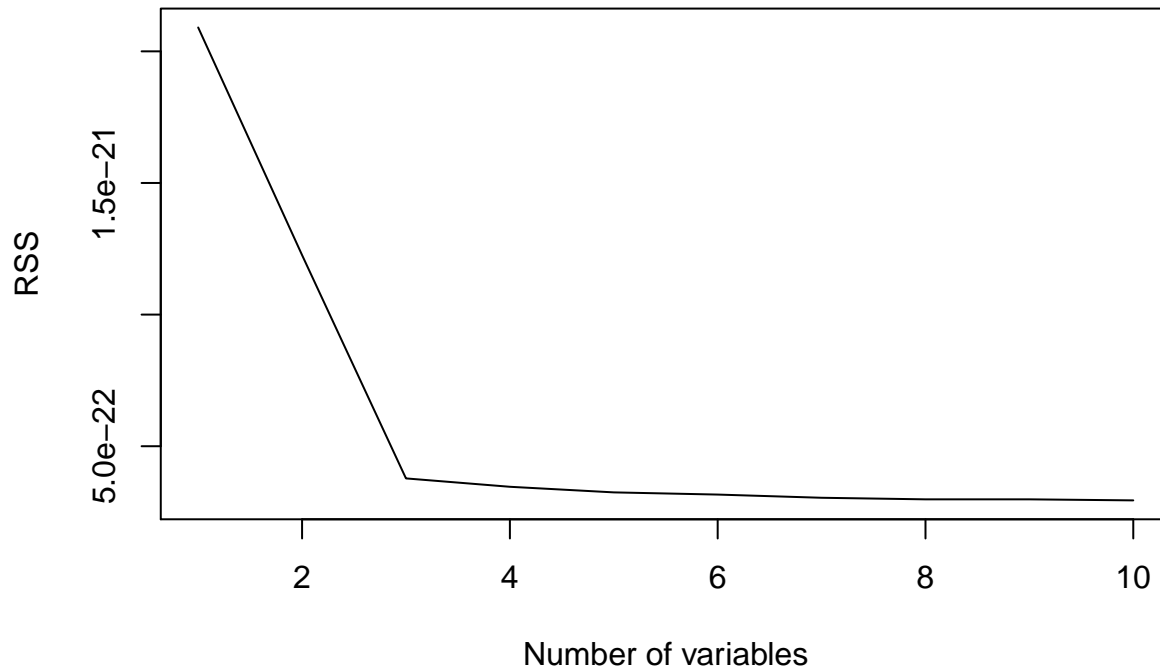
print ("Perform best subset selection: ")

```

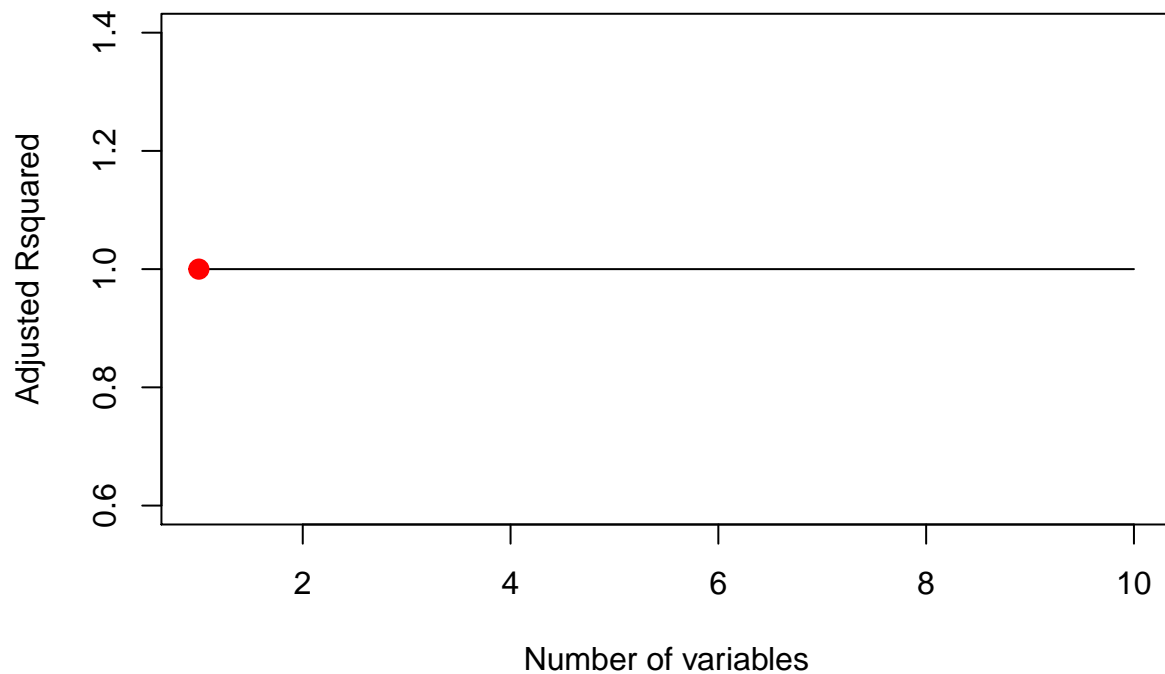
```
## [1] "Perform best subset selection: "
regfit.full <- regsubsets(y ~ ., df, nvmax = 10)

#The summary shows the result of step 2 of algorithm 6.1 page 205 of the book
summary <- summary(regfit.full)

plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")
```



```
# which.max() returns location maximum point of the vector
index <- which.max(summary$adjr2)
plot(summary$adjr2,xlab = "Number of variables", ylab="Adjusted Rsquared",
      type = "l")
points(index, summary$adjr2[index], col="red", cex=2, pch=20)
```

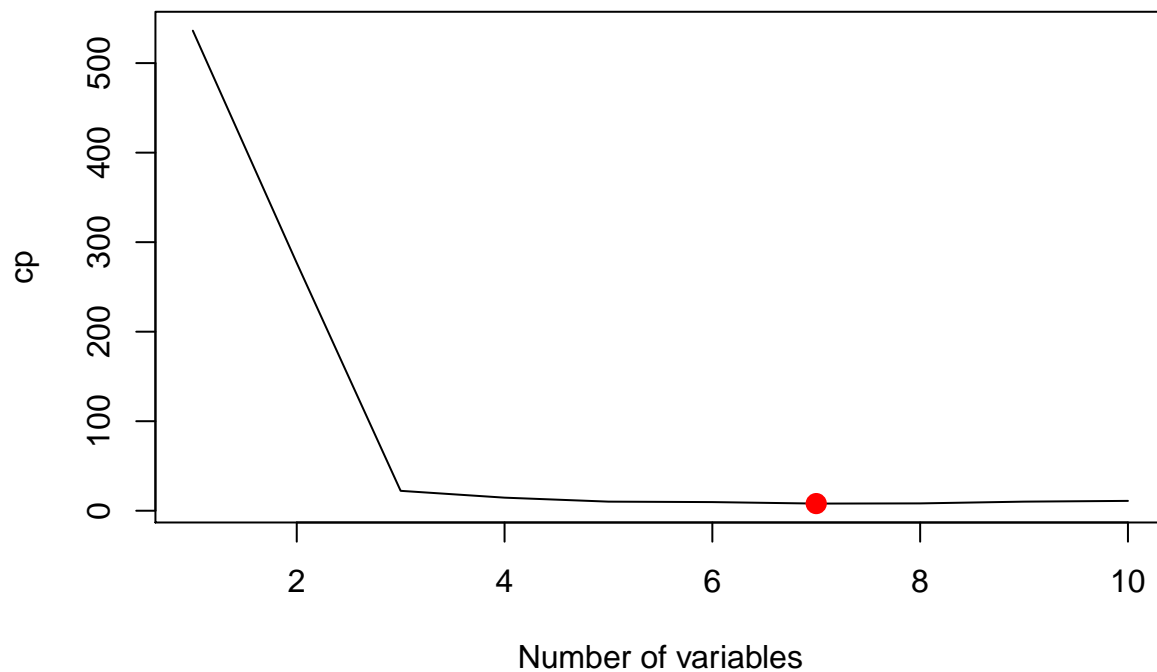


```
print("coefficients of the best model (adjr2) : ")

## [1] "coefficients of the best model (adjr2) : "
coef(regfit.full,index)

## (Intercept)          x7
##      12.0         -45.3

# which.min() returns location minimum point of the vector
index <- which.min(summary$cp)
plot(summary$cp,xlab = "Number of variables", ylab="cp", type = "l")
points(index, summary$cp[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (cp) : ")

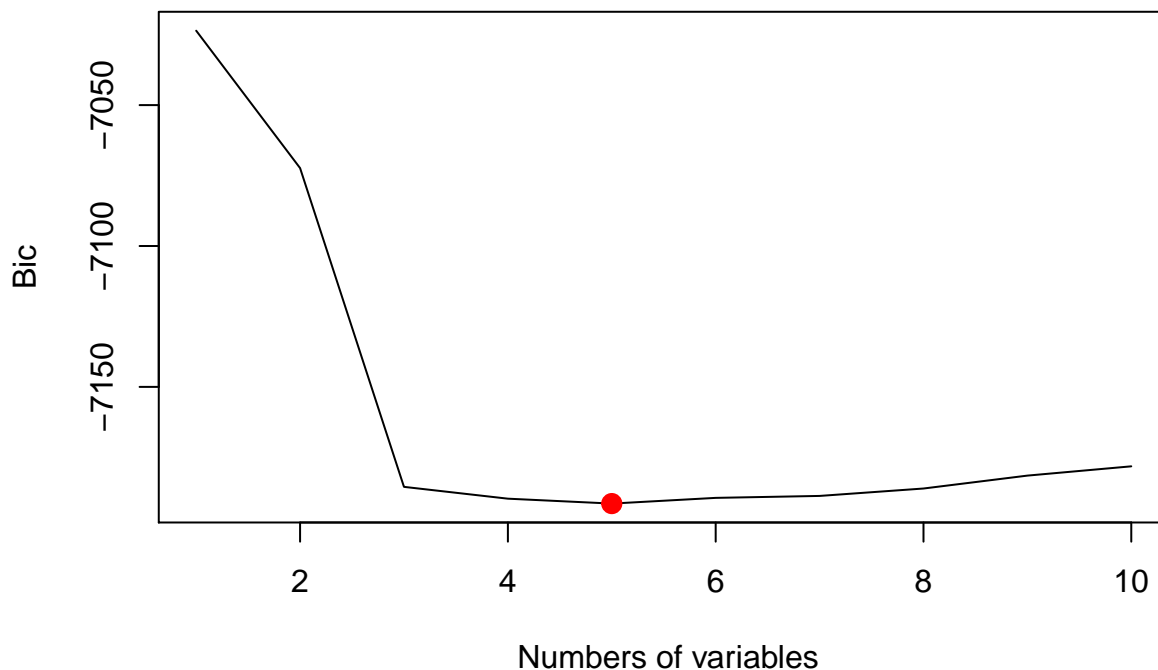
## [1] "coefficients of the best model (cp) : "
coef(regfit.full,index)

##      (Intercept)          x1          x2          x3          x5
##  1.200000e+01  1.949149e-12  7.458195e-13  5.604306e-12 -3.916634e-12
##           x7          x9          x10
## -4.530000e+01 -6.896989e-14 -2.863481e-15

# same for bic
plot(summary$bic, xlab = "Numbers of variables", ylab="Bic", type="l")
(index <- which.min(summary$bic))

## [1] 5

points(index, summary$bic[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (bic) : ")

## [1] "coefficients of the best model (bic) : "
coef(regfit.full,index)

##      (Intercept)          x1          x2          x5          x7
##  1.200000e+01  4.010558e-12  5.727175e-13 -4.867231e-14 -4.530000e+01
##           x10
## -1.519573e-15

# coef(, n) returns coefficient estimates associated with best n variable model

print ("----- use CV with best subset selection for new data -----" )

## [1] "----- use CV with best subset selection for new data -----"

set.seed(1)
k <- 10

folds <- sample(1:k, size = nrow(df), replace = T)
table(folds)

## folds
##  1  2  3  4  5  6  7  8  9 10
##  9  7  7  6  9 14 14  9 11 14

# folds with same size
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#   size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

results <- lapply (1:k, function(x) { # x is the index of test portion, the rest are for training
```



```

# this is to collect the MSEs for each test fold
mses <- tibble(no.of.coefs = NULL, MSE = NULL)
df.train <- df[folds != x, ]
df.test <- df[folds == x, ]
df.train.X <- df.train %>% select (-y)
df.test.X <- df.test %>% select (-y)
(mat.test.X <- model.matrix(y~., data=df.test))
(df.test.Y <- df[folds == x, ]$y)

# step 2 of algorithm 6.1 page 205 of the book
regfit.full.train <- regsubsets(y ~ ., df.train, nvmax = ncol(df.train.X))

# apply the model with selected subsets on test set
# one at a time and calculate the MSE
for (i in 1:ncol(df.test.X)){
  (coefi <- coef(regfit.full.train, id = i))
  (pred <- mat.test.X[, names(coefi)] %*% coefi)
  (mse <- mean((pred - df.test.Y)^2))
  mses <- rbind (mses, tibble(no.of.coefs = i, MSE = mse))
}
return(mses)
})

allResults <- results[[1]]
for (i in 2 : length(results)){
  allResults <- rbind(allResults , results[[i]])
}

(allMse <- (allResults %>%
  group_by(no.of.coefs) %>%
  summarise(mse.mean = mean(MSE))) )

## # A tibble: 10 x 2
##   no.of.coefs mse.mean
##   <int>     <dbl>
## 1         1 1.22e-23
## 2         2 2.64e-23
## 3         3 1.96e-23
## 4         4 4.62e-22
## 5         5 1.69e-22
## 6         6 2.23e-21
## 7         7 9.39e-21
## 8         8 1.13e-20
## 9         9 2.67e-20
## 10        10 2.61e-20

(no.of.selected.features <- which.min(allMse$mse.mean))

## [1] 1
print ("The best subset of features selected correspond to minimum CV_MSE")

## [1] "The best subset of features selected correspond to minimum CV_MSE"

```

```

# train on the whole training set now
regfit.full.train <- regsubsets(y ~ ., df, nvmax = ncol(df %>% select (-y)))
coef(regfit.full.train, id = no.of.selected.features)

## (Intercept)          x7
##          12.0        -45.3

print(" Clearly applying CV on Best subset selection provides nonsensical result.")

## [1] " Clearly applying CV on Best subset selection provides nonsensical result."
print ("Apply Lasso  cross validation to find the best lambda and corresponding coeffs")

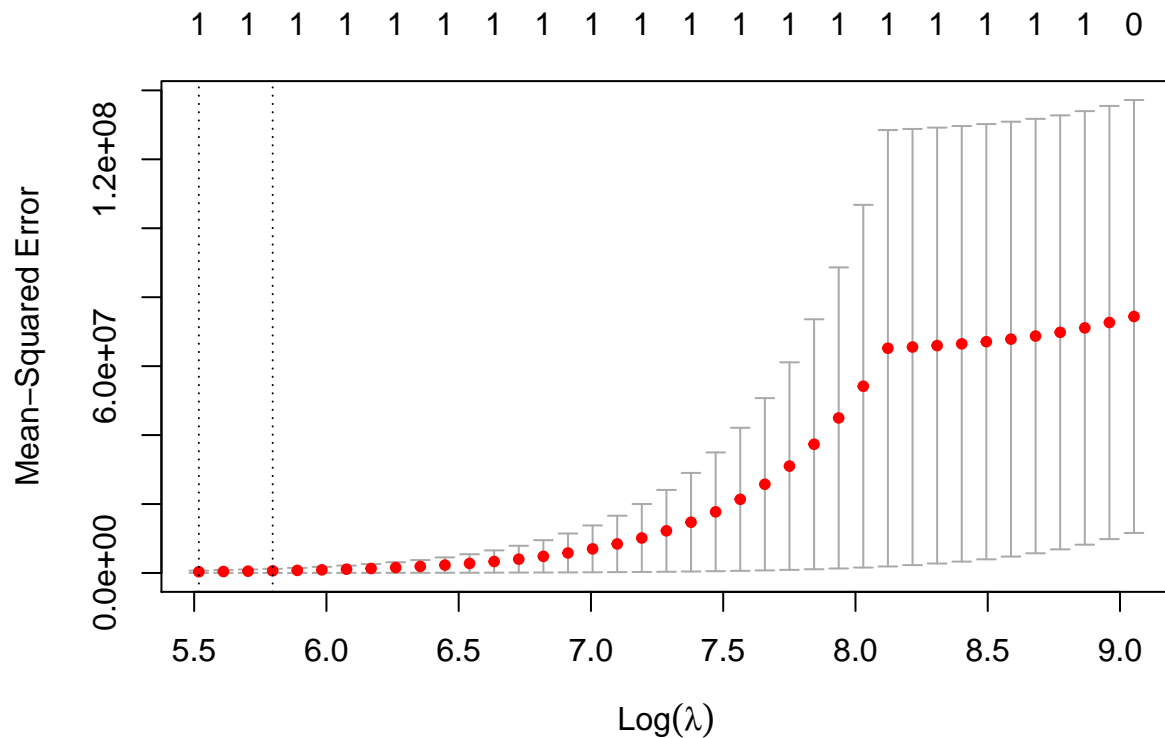
## [1] "Apply Lasso  cross validation to find the best lambda and corresponding coeffs"

library(glmnet)
set.seed(1)

# First construct matrix from dataframe (and drop intercept column)
x <- model.matrix(y~., df)[-1]
y <- df$y

cv.out=cv.glmnet(x, y, alpha=1, lambda = NULL)
plot(cv.out)

```



```

print("Here is value of lambda for which the MSE is minimum")

## [1] "Here is value of lambda for which the MSE is minimum"

(bestlam=cv.out$lambda.min)

## [1] 249.0647

```

```

print("Here are the coefficients corresponding to best value of lambda:")

## [1] "Here are the coefficients corresponding to best value of lambda:"
predict(cv.out, type="coefficients" ,s=bestlam )

## 11 x 1 sparse Matrix of class "dgCMatrix"
##              1
## (Intercept) 19.32170
## x1          .
## x2          .
## x3          .
## x4          .
## x5          .
## x6          .
## x7         -43.97948
## x8          .
## x9          .
## x10         .

print("Here is one standard error value of lambda for which the MSE is minimum")

## [1] "Here is one standard error value of lambda for which the MSE is minimum"
one.SE.lam <- cv.out$lambda.1se

print("Here are the coefficients corresponding to one standard error value of lambda:")

## [1] "Here are the coefficients corresponding to one standard error value of lambda:"
predict(cv.out, type="coefficients" ,s=one.SE.lam )

## 11 x 1 sparse Matrix of class "dgCMatrix"
##              1
## (Intercept) 21.67886
## x1          .
## x2          .
## x3          .
## x4          .
## x5          .
## x6          .
## x7         -43.55435
## x8          .
## x9          .
## x10         .

library(tidyverse)
library(glmnet)
library(pls)
library(leaps)

set.seed(1)
college.df = read.csv("/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/College.csv",
                      header=T, stringsAsFactors = F, na.strings = "?")
college.df = tibble(college.df)

# str(college.df)

```

```

#----- Some usual cleaning on character columns ----- #

# First remove all recods with spaces in character column Private
college.df$Private <- gsub('\\s+', '', college.df$Private)

# Second remove all leading and trailing spaces from a character column "Private"
college.df$Private <- trimws(college.df$Private, which = c("both"))

# Remove all records with "NA" or empty string in character column "Private"
college.df <- college.df[!(tolower(college.df$Private) == "na" |
                           college.df$Private == ""), ]

# convert all character fields
college.df[sapply(college.df, is.character)] <-
  lapply(college.df[sapply(college.df, is.character)], as.factor)

#----- Find and remove NA in all columns ----- #
college.df <- na.omit(college.df)

# str(college.df)
set.seed(1)
print("a: Split into train / test data sets -----")

## [1] "a: Split into train / test data sets -----"

train <- sample(1:nrow(college.df), nrow(college.df)/2)
test <- (-train)

train.df <- college.df[train, ]
test.df <- college.df[test, ]
y.train <- train.df$Apps
y.test <- test.df$Apps

print("b: fit a linear model -----")

## [1] "b: fit a linear model -----"

# fit a model
df.lm <- lm (Apps ~ ., data = train.df)

# Now predict Apps for test data
pred.lm <- predict(df.lm, test.df)

print("Linear model test MSE:")

## [1] "Linear model test MSE:"
(lm.MSE <- mean( (pred.lm - y.test)^2))

## [1] 1135758

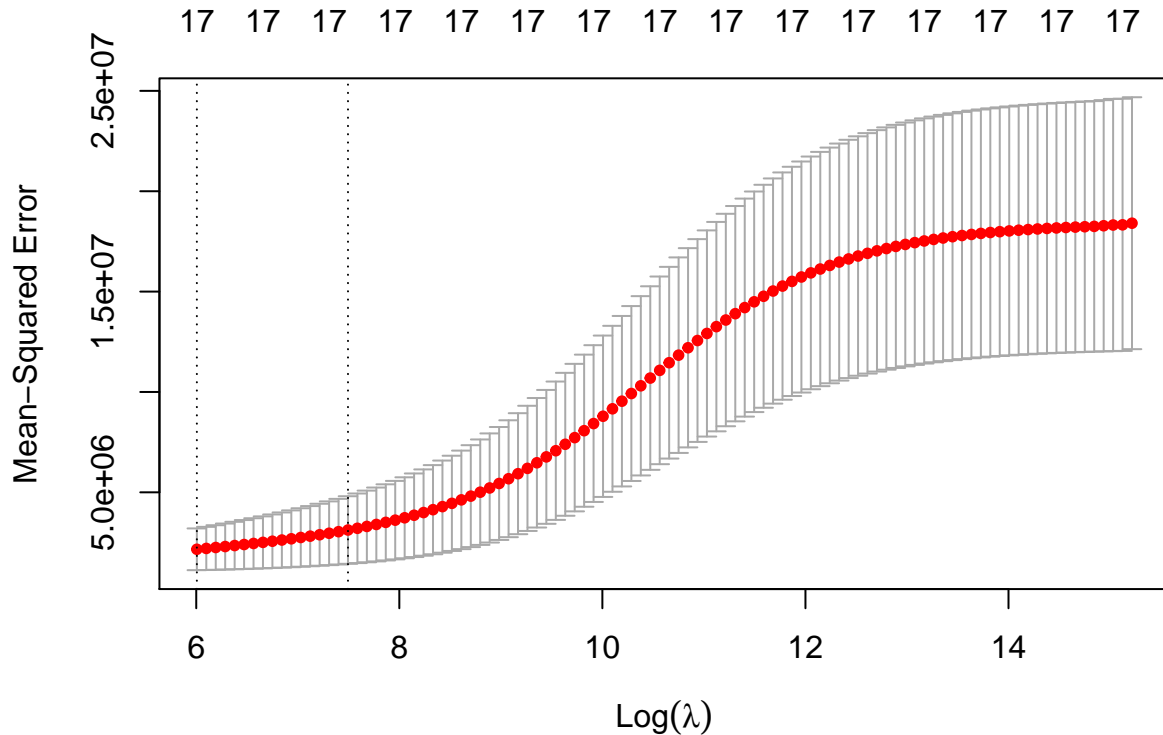
print("c: fit Ridge regression model on training set and get test.mse-----")

## [1] "c: fit Ridge regression model on training set and get test.mse-----"
# use magic model.matrix to convert dataframe into a matrix for Ridge and Lasso
x.train <- model.matrix(Apps~., train.df)[, -1]

```

```
x.test <- model.matrix(Apps~., test.df)[, -1]
```

```
cv.out <- cv.glmnet(x.train, y.train, alpha=0) # Ridge
plot(cv.out)
```



```
best.lambda <- cv.out$lambda.min
```

```
# predict the model on test
```

```
pred.ridge <- predict(cv.out, s=best.lambda, newx=x.test)
```

```
sprintf("Ridge test mse for best lambda: %s", best.lambda)
```

```
## [1] "Ridge test mse for best lambda: 405.840359582873"
```

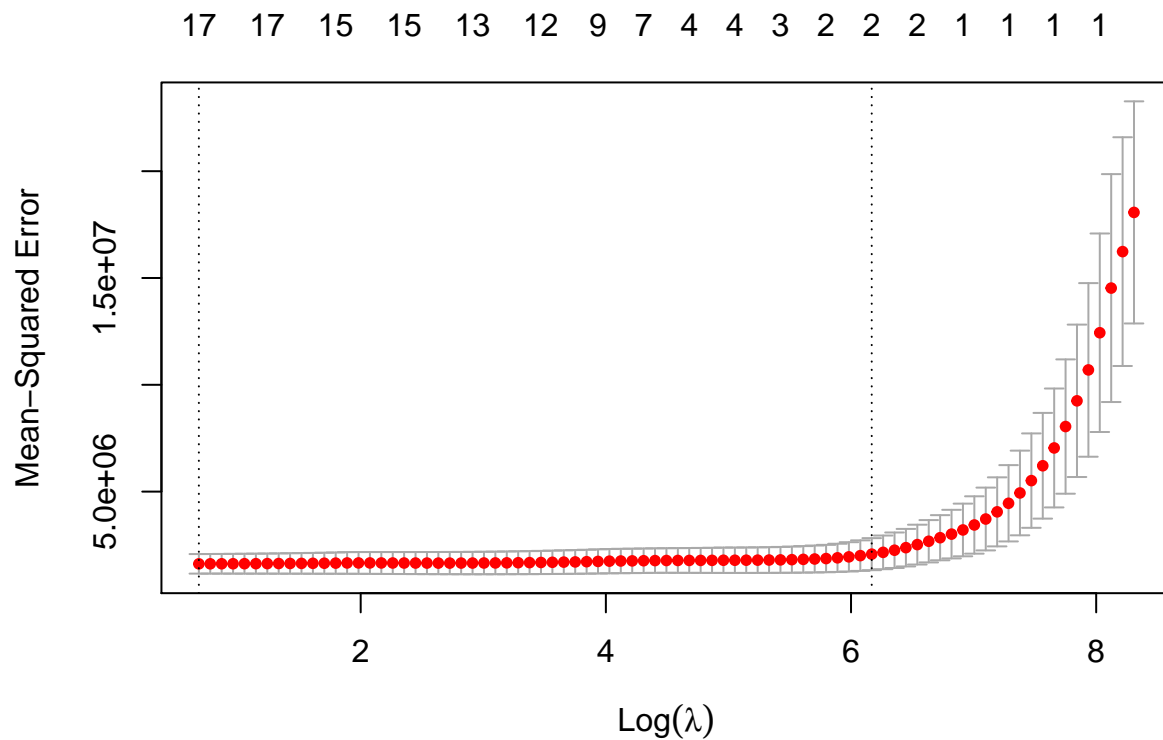
```
(ridge.mse <- mean((pred.ridge - y.test)^2))
```

```
## [1] 976261.5
```

```
print("d: fit Lasso regression model on training set and get test.mse and coeffs")
```

```
## [1] "d: fit Lasso regression model on training set and get test.mse and coeffs"
```

```
cv.out <- cv.glmnet(x.train, y.train, alpha=1) # Lasso
plot(cv.out)
```



```
best.lambda <- cv.out$lambda.min

# predict the model on test
pred.lasso <- predict(cv.out, s=best.lambda, newx=x.test)

sprintf("Lasso test mse for best lambda: %s", best.lambda)

## [1] "Lasso test mse for best lambda: 1.97343997085518"

(lasso.mse <- mean((pred.lasso - y.test)^2))

## [1] 1115901

print("Coeffs for Lasso: ")

## [1] "Coeffs for Lasso: "

(coefs.ridge <- predict (cv.out, type = "coefficients", s = best.lambda))

## 18 x 1 sparse Matrix of class "dgCMatrix"
##           1
## (Intercept) -7.688896e+02
## PrivateYes  -3.127034e+02
## Accept      1.762718e+00
## Enroll     -1.318195e+00
## Top10perc   6.482356e+01
## Top25perc  -2.081406e+01
## F.Undergrad 7.119149e-02
## P.Undergrad 1.246161e-02
## Outstate   -1.049091e-01
## Room.Board  2.088305e-01
## Books       2.926466e-01
## Personal    3.955068e-03
```

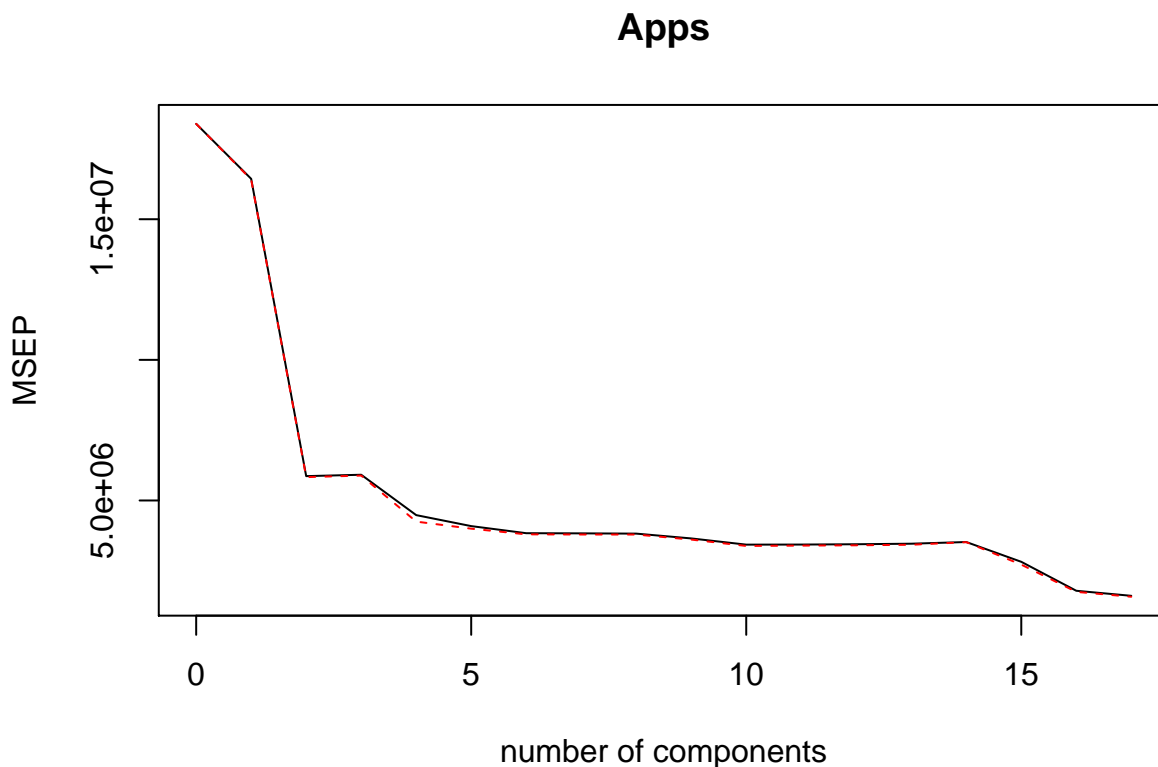
```
## PhD          -1.455463e+01
## Terminal     5.395858e+00
## S.F.Ratio    2.171398e+01
## perc.alumni  5.088260e-01
## Expend       4.824455e-02
## Grad.Rate    7.036148e+00
```

```
print("e: Fit a PCR model on training set with M chosen by CS and get test.mse")
```

```
## [1] "e: Fit a PCR model on training set with M chosen by CS and get test.mse"
```

```
pcr.fit <- pcr (Apps~., data=train.df, scale=T, validation="CV")
```

```
validationplot(pcr.fit, val.type="MSEP")
```



```
print("Minimum CV Root MSE is for M=17 components which is 100 so CV MSE is 10000")
```

```
## [1] "Minimum CV Root MSE is for M=17 components which is 100 so CV MSE is 10000"
```

```
print("Looking at validation plot we see that M = 15 or 16 should suffice")
```

```
## [1] "Looking at validation plot we see that M = 15 or 16 should suffice"
```

```
# Now apply model with M=17 on test data and calculate MSE'
```

```
M = 17
```

```
pcr.pred <- predict(pcr.fit, x.test, ncomp = M)
```

```
sprintf("pcr test cv_mse for when best number of component is: %s", M)
```

```
## [1] "pcr test cv_mse for when best number of component is: 17"
```

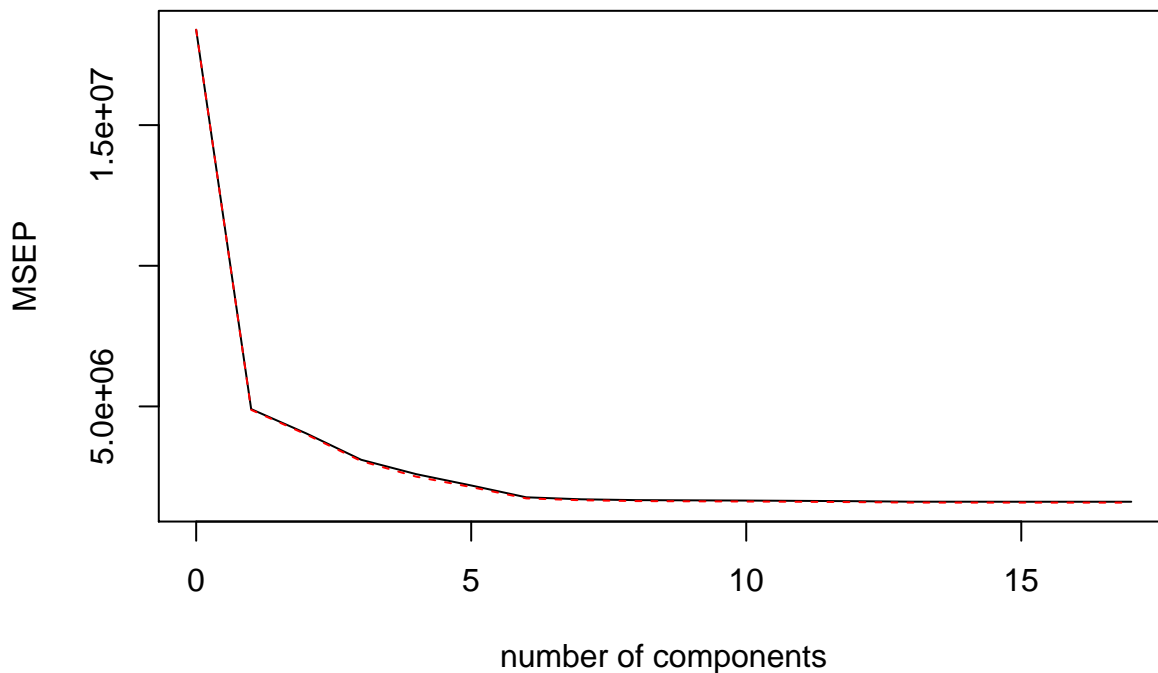
```
(pcr.mse <- mean((pcr.pred - y.test)^2))
```

```
## [1] 1135758
sprintf("pcr test mse for best number of component: %s is %s:", M, pcr.mse)

## [1] "pcr test mse for best number of component: 17 is 1135758.31783053:"
print("f: Fit a PLS model on training set with M chosen by CS and get test.mse")

## [1] "f: Fit a PLS model on training set with M chosen by CS and get test.mse"
pls.fit <- plsr(Apps ~ ., data = college.df, subset = train, scale=T, validation = "CV")
validationplot(pls.fit, val.type="MSEP")
```

Apps



```
print("Minimum CV Root MSE is for M=13 components which is 1118")

## [1] "Minimum CV Root MSE is for M=13 components which is 1118"
# Now apply model with M=13 on test data and calculate MSE'
M = 13
pls.pred <- predict(pls.fit, x.test, ncomp = M)

(pls.mse <- mean((pls.pred - y.test)^2))

## [1] 1140255
sprintf("pls test mse for for best number of component: %s is %s:", M, pls.mse)

## [1] "pls test mse for for best number of component: 13 is 1140255.01397807:"
library(tidyverse)
library(glmnet)
library(pls)
library(leaps)
```



```

print("a) Generate a data set with p = 20 features, n = 1,000")

## [1] "a) Generate a data set with p = 20 features, n = 1,000"

set.seed(10)
X <- matrix(rep(NA, 1000 * 20), c(1000,20))
for (i in 1: 20)
  X[, i] <- rnorm(1000)

e <- rnorm(1000)

beta <- c(12,0,2.6, -123,0,11.2,56,-7,0,0,0,13,-41,2.2,0,8.7, -18,0,19,0.03)

# name the features
feature.names <- c("x1", "x2", "x3", "x4", "x5", "x6", "x7", "x8", "x9", "x10",
  "x11", "x12", "x13", "x14", "x15", "x16", "x17", "x18", "x19", "x20" )
colnames(X) <- feature.names
names(beta) <- feature.names
beta

##      x1      x2      x3      x4      x5      x6      x7      x8      x9      x10
##  12.00    0.00    2.60 -123.00    0.00   11.20   56.00   -7.00    0.00    0.00
##   x11    x12    x13    x14    x15    x16    x17    x18    x19    x20
##   0.00   13.00 -41.00    2.20    0.00    8.70 -18.00    0.00   19.00    0.03

Y <- rep(NA, 1000)

for (i in 1 : 1000)
  Y[[i]] <- beta %*% X[i, ] + e[[i]]

# now split the samples into test and training:
print("b) Split the data into train setcontaining 100 and test set containing 900")

## [1] "b) Split the data into train setcontaining 100 and test set containing 900"

train <- sample(1:nrow(X), nrow(X)/10)
test <- (-train)

train.x <- X[train, ]
train.y <- Y[train]

test.x <- X[test, ]
test.y <- Y[test]

df.train.X <- as_tibble(train.x)
df.train <- df.train.X %>% add_column(y = train.y, .before = "x1")
df.test.X <- as_tibble(test.x)
df.test <- df.test.X %>% add_column(y = test.y, .before = "x1")

print("c) Perform best subset selection on training set:")

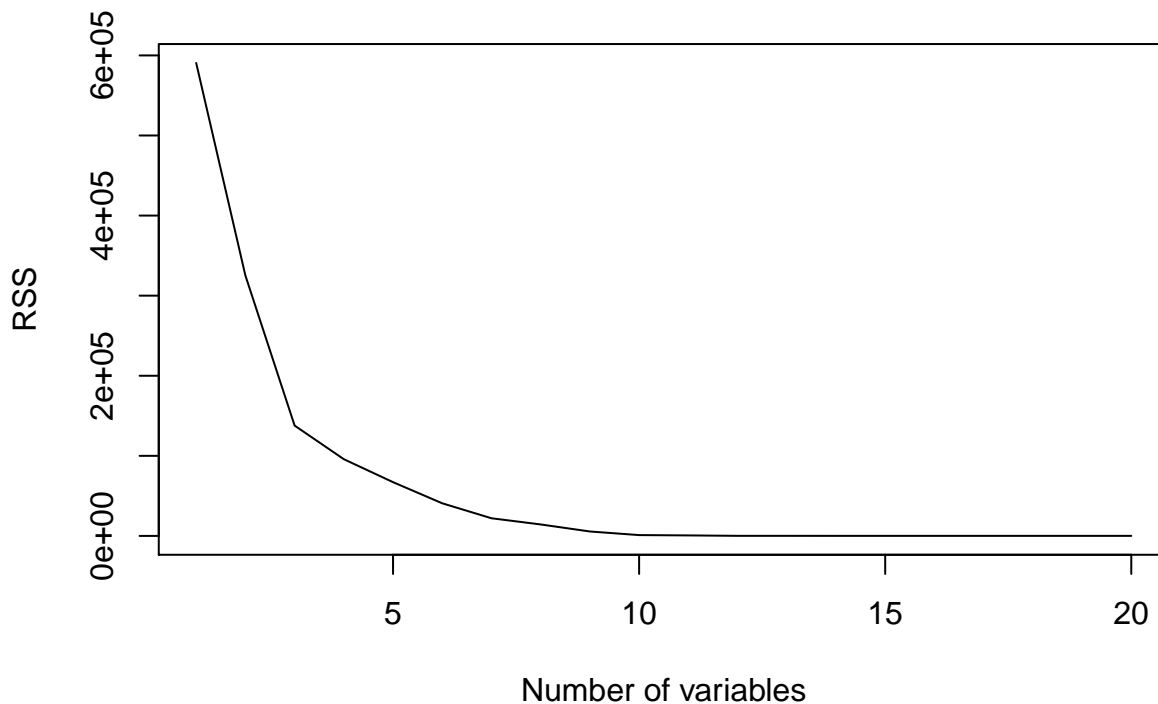
## [1] "c) Perform best subset selection on training set:"
regfit.full <- regsubsets(y ~ ., df.train, nvmax = 20)

#The summary shows the result of step 2 of algorithm 6.1 page 205 of the book

```

```
summary <- summary(regfit.full)
```

```
plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")
```



```
print("d) Plot test MSE for model of each size")
```

```
## [1] "d) Plot test MSE for model of each size"
```

```
# First prepare test data as matrix, this is only for convinience
```

```
# when using dot product %*%: coefficients %*% test.X rows
```

```
df.test.mat <- model.matrix(y~., data = df.test)
```

```
# loop through all models , get the coefficients and calculate MSE
```

```
mse <- rep(NA, 20)
```

```
for (i in 1:20){
```

```
  coeffs <- coef(regfit.full, i)
```

```
  yhat <- df.test.mat[, names(coeffs)] %*% coeffs
```

```
  mse[i] <- mean((test.y - yhat)^2)
```

```
}
```

```
(mse_data <- tibble(comps = 1:20, mse=mse))
```

```
## # A tibble: 20 x 2
```

```
##   comps     mse
```

```
##   <int>   <dbl>
```

```
## 1     1 6116.
```

```
## 2     2 2904.
```

```
## 3     3 1276.
```

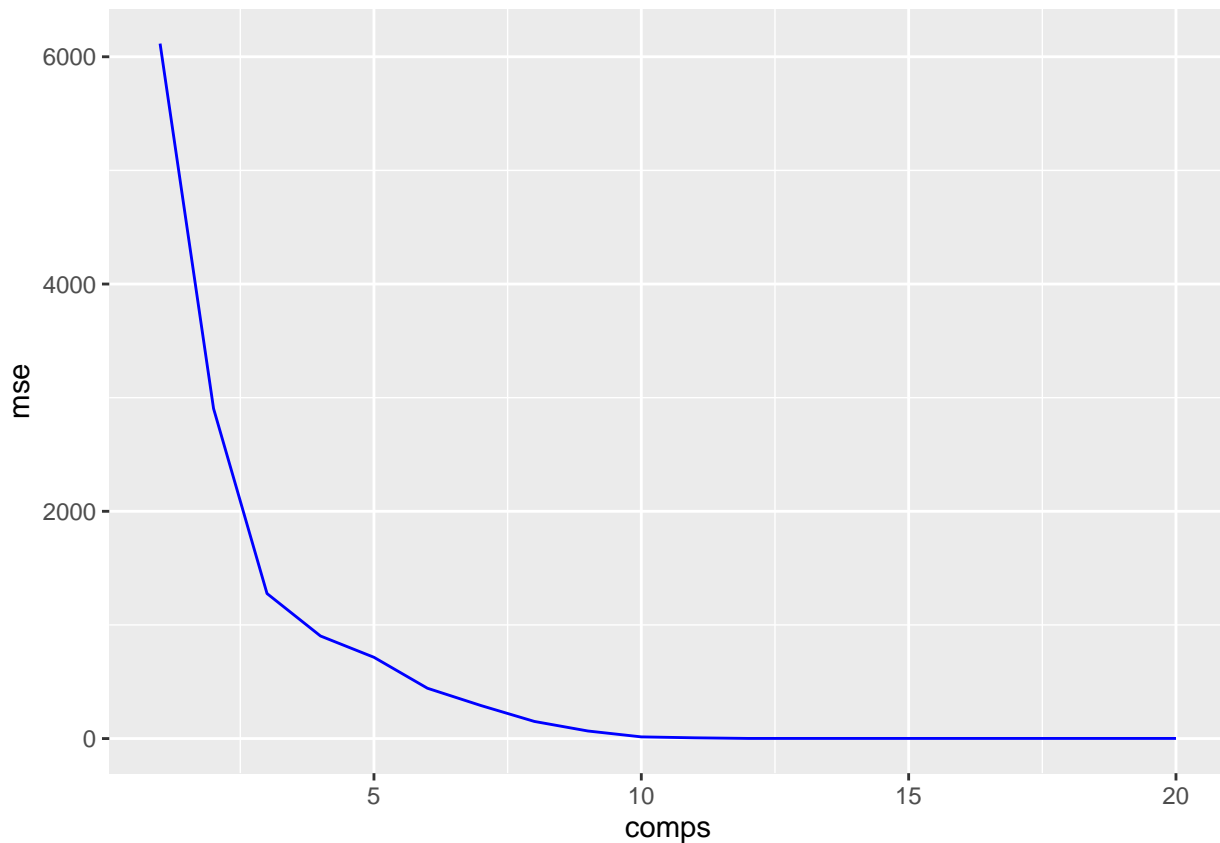
```
## 4     4  903.
```

```
## 5     5  715.
```

```
## 6     6  443.
```

```
## 7      7 291.
## 8      8 151.
## 9      9 66.7
## 10     10 14.9
## 11     11 6.75
## 12     12 1.23
## 13     13 1.28
## 14     14 1.30
## 15     15 1.32
## 16     16 1.36
## 17     17 1.34
## 18     18 1.34
## 19     19 1.35
## 20     20 1.36
```

```
ggplot(mse_data, aes(comps, mse))+
  geom_line(color="blue")
```



```
print("e) For model size with 12 components MSE is minimized: 12 --> 1.234475")
```

```
## [1] "e) For model size with 12 components MSE is minimized: 12 --> 1.234475"
```

```
print("f) How does the model at which the test MSE is minimized? compare the coeffs")
```

```
## [1] "f) How does the model at which the test MSE is minimized? compare the coeffs"
```

```
(coeffs <- coef(regfit.full, 12))
```

```
## (Intercept)          x1          x3          x4          x6          x7
```

```
##      0.1211867    11.9871248     2.6070986 -123.0928886    10.9639610    56.0180802
##           x8           x12           x13           x14           x16           x17
##     -6.9507468    13.1406636   -40.8673157     2.3919952     8.6684323   -17.9409391
##           x19
##     18.9294841
```

```
beta
```

```
##      x1      x2      x3      x4      x5      x6      x7      x8      x9      x10
##    12.00     0.00     2.60 -123.00     0.00    11.20    56.00    -7.00     0.00     0.00
##     x11     x12     x13     x14     x15     x16     x17     x18     x19     x20
##     0.00    13.00   -41.00     2.20     0.00     8.70   -18.00     0.00    19.00     0.03
```

```
print("comparision shows all zero coefficients successfully predicted and non zero
      coefficients are close approximation to real ones, only the 20th
      coefficient is missing ")
```

```
## [1] "comparision shows all zero coefficients successfully predicted and non zero\n"
```

```
coefficients
```

```
print("
(Intercept)      x1      x3      x4
0.1211867  11.9871248    2.6070986 -123.0928886
           12.00         2.60        -123.00

           x6           x7           x8
10.9639610    56.0180802   -6.9507468
11.20         56.00         -7.00

           x12          x13          x14
13.1406636  -40.8673157    2.3919952
13.00        -41.00         2.20

           x16          x17          x19
8.6684323  -17.9409391   18.9294841
8.70        -18.00         19.00         0.03

")
```

```
## [1] "\n (Intercept)      x1      x3      x4      \n 0.1211867  11.9871248    2.6070986
```

```
print("g) create a plot displaying ...")
```

```
## [1] "g) create a plot displaying ..."
```

```
# loop over all models
```

```
distance.squared <- rep(0, 20)
```

```
for (i in 1:20){
  (coeffs <- coef(regfit.full, i)[-1]) # drop the intercept
  for (j in 1:i)
    (distance.squared[i] <- distance.squared[i] + (coeffs [j] - beta[names(coeffs [j])])^2 )
}
dist <- sqrt(distance.squared)
(distance.vs.model <- tibble(model = 1:20, distance = dist))
```

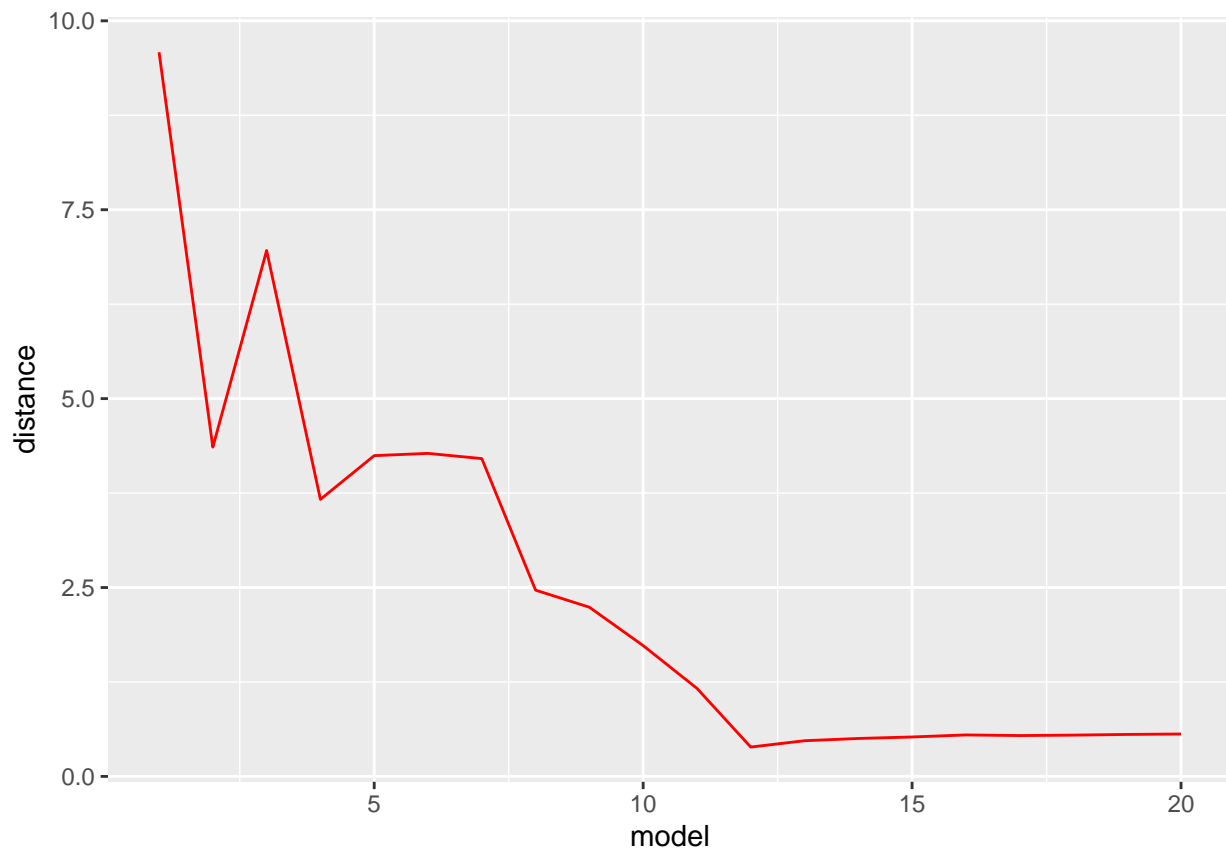
```
## # A tibble: 20 x 2
```

```
##   model distance
```

```
##   <int>     <dbl>
```

```
## 1      1      9.58
## 2      2      4.36
## 3      3      6.96
## 4      4      3.67
## 5      5      4.25
## 6      6      4.28
## 7      7      4.21
## 8      8      2.47
## 9      9      2.24
## 10     10     1.73
## 11     11     1.17
## 12     12     0.389
## 13     13     0.472
## 14     14     0.502
## 15     15     0.522
## 16     16     0.549
## 17     17     0.541
## 18     18     0.547
## 19     19     0.556
## 20     20     0.561
```

```
ggplot(distance.vs.model, aes(model, distance))+
  geom_line(color="red")
```



```
print("As graph shows on model with 12 components (the model we found in part d)
the distance between two sets of coefficients are minimum")
```

```
## [1] "As graph shows on model with 12 components (the model we found in part d) \nthe distance between"
```

```

library(tidyverse)
library(glmnet)
library(pls)
library(leaps)

boston.df = read.csv(
  "/Users/shahrdadshadab/env/my-R-project/ISLR/Data/datasets/BostonHousing.csv",
  header=T, stringsAsFactors = F, na.strings = "?")

str(boston.df)

## 'data.frame': 506 obs. of 14 variables:
## $ crim : num 0.00632 0.02731 0.02729 0.03237 0.06905 ...
## $ zn : num 18 0 0 0 0 12.5 12.5 12.5 12.5 ...
## $ indus : num 2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 ...
## $ chas : int 0 0 0 0 0 0 0 0 0 ...
## $ nox : num 0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 ...
## $ rm : num 6.58 6.42 7.18 7 7.15 ...
## $ age : num 65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
## $ dis : num 4.09 4.97 4.97 6.06 6.06 ...
## $ rad : int 1 2 2 3 3 3 5 5 5 ...
## $ tax : int 296 242 242 222 222 222 311 311 311 311 ...
## $ ptratio: num 15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
## $ b : num 397 397 393 395 397 ...
## $ lstat : num 4.98 9.14 4.03 2.94 5.33 ...
## $ medv : num 24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...

# Find and remove NA in all columns
boston.df <- na.omit(boston.df)
set.seed(1)
print("Split into train / test data sets -----")

## [1] "Split into train / test data sets -----"

train <- sample(1:nrow(boston.df), nrow(boston.df)/2)
test <- (-train)

df.train <- boston.df[train, ]
df.test <- boston.df[test, ]
train.y <- df.train$crim
test.y <- df.test$crim

print ("Perform best subset selection on train data: ")

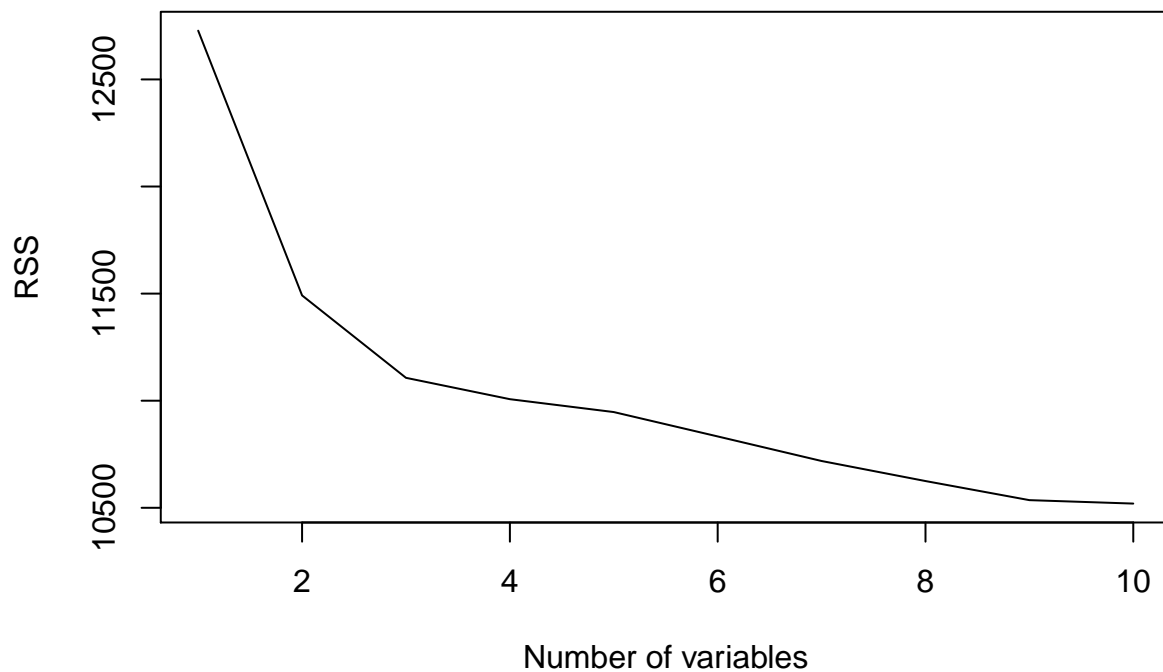
## [1] "Perform best subset selection on train data: "

regfit.full <- regsubsets(crim ~ ., df.train, nvmax = 10)

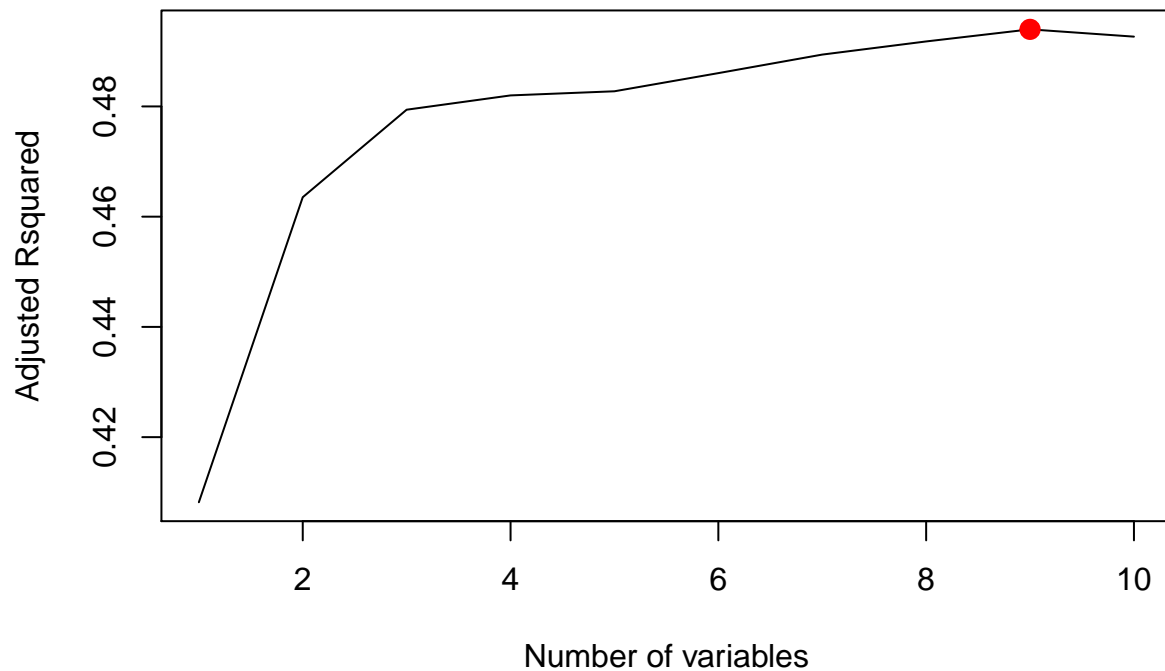
#The summary shows the result of step 2 of algorithm 6.1 page 205 of the book
summary <- summary(regfit.full)

plot(summary$rss, xlab = "Number of variables", ylab="RSS",type = "l")

```



```
# which.max() returns location maximum point of the vector
index <- which.max(summary$adjr2)
plot(summary$adjr2,xlab = "Number of variables", ylab="Adjusted Rsquared",
      type = "l")
points(index, summary$adjr2[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (adjr2) : ")
```

```
## [1] "coefficients of the best model (adjr2) : "
```

```
coef(regfit.full,index)
```

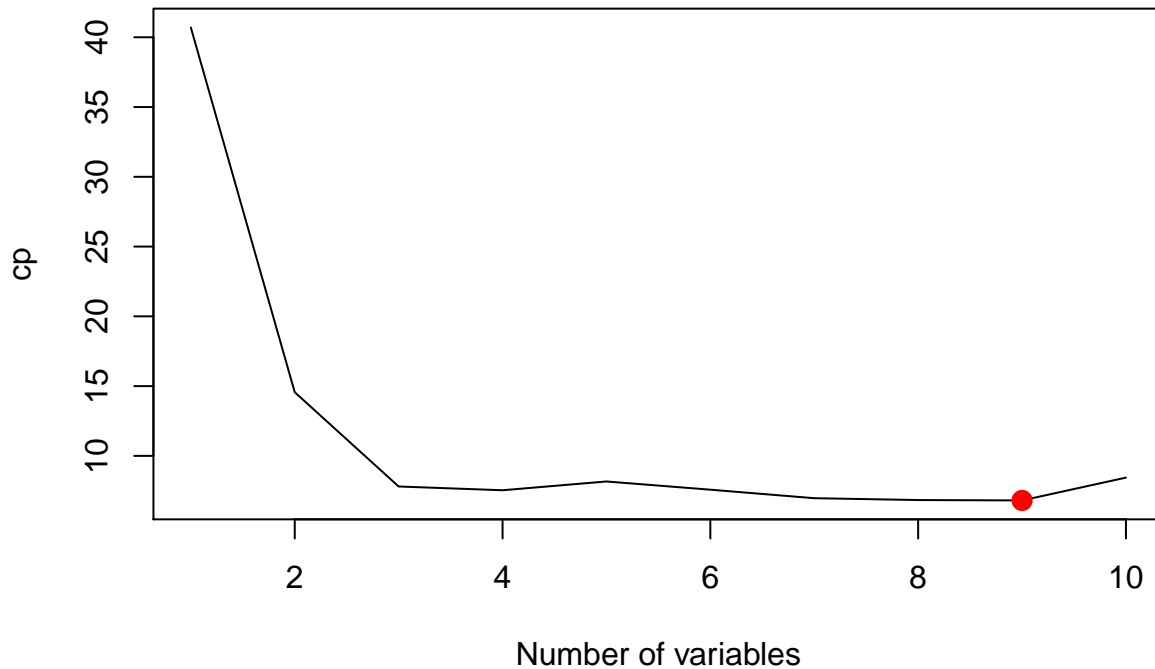
```
## (Intercept)          zn          indus          nox          dis          rad
## 24.83928145  0.04131571 -0.15408884 -12.05853934 -1.05123802  0.55802734
##      ptratio          b          lstat          medv
## -0.47520557 -0.01309050  0.25525000 -0.18608243
```

```
# which.min() returns location minimum point of the vector
```

```
index <- which.min(summary$cp)
```

```
plot(summary$cp,xlab = "Number of variables", ylab="cp", type = "l")
```

```
points(index, summary$cp[index], col="red", cex=2, pch=20)
```



```
print("coefficients of the best model (cp) : ")
```

```
## [1] "coefficients of the best model (cp) : "
```

```
coef(regfit.full,index)
```

```
## (Intercept)          zn          indus          nox          dis          rad
## 24.83928145  0.04131571 -0.15408884 -12.05853934 -1.05123802  0.55802734
##      ptratio          b          lstat          medv
## -0.47520557 -0.01309050  0.25525000 -0.18608243
```

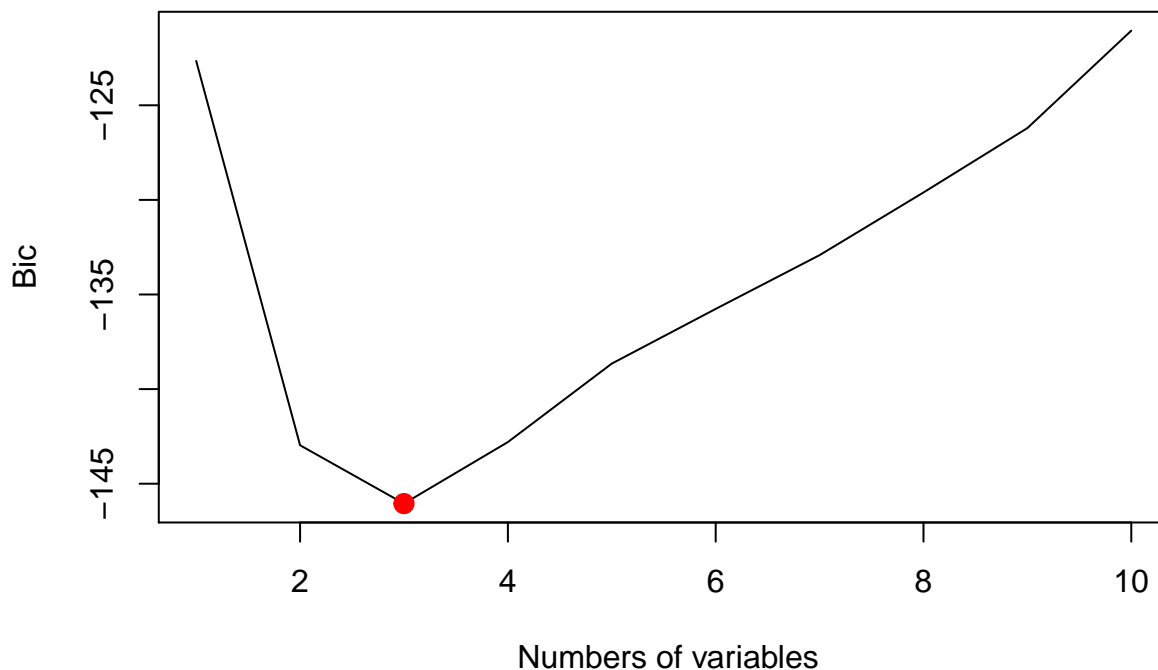
```
# same for bic
```

```
plot(summary$bic, xlab = "Numbers of variables", ylab="Bic", type="l")
```

```
(index <- which.min(summary$bic))
```

```
## [1] 3
```

```
points(index, summary$bic[index], col="red", cex=2, pch=20)
```

```
print("coefficients of the best model (bic) : ")

## [1] "coefficients of the best model (bic) : "
coef(regfit.full,index)

## (Intercept)      rad      b      lstat
## 0.72163825 0.46965678 -0.01530067 0.31662147

print("looks a subset of three variables provides a model with acceptable performance")

## [1] "looks a subset of three variables provides a model with acceptable performance"
print ("----- use CV with best subset selection -----" )

## [1] "----- use CV with best subset selection -----"

k <- 10

folds <- sample(1:k, size = nrow(df), replace = T)
table(folds)

## folds
##  1  2  3  4  5  6  7  8  9 10
## 13  8 14 11  9  7 13  7 11  7

# folds with same size
# sameSizefolds <- sample(rep(1:k, length.out = nrow(weekly.df)),
#   size = nrow(weekly.df), replace = F)
# table(sameSizefolds)

results <- lapply (1:k, function(x) { # x is the index of test portion, the rest are for training

  # this is to collect the MSEs for each test fold
  mses <- tibble(no.of.coefs = NULL, MSE = NULL)
  df.train.cv <- df.train[folds != x, ]
```

```

df.test.cv <- df.train[folds == x, ]
df.train.cv.X <- df.train.cv %>% select (-crim)
df.test.cv.X <- df.test.cv %>% select (-crim)
(mat.test.cv.X <- model.matrix(crim~., data=df.test.cv))
(df.test.cv.Y <- df.test.cv$crim)

# step 2 of algorithm 6.1 page 205 of the book
regfit.full.train <- regsubsets(crim ~ ., df.train.cv,
                               nvmax = ncol(df.train.cv.X))

# apply the model with selected subsets on test set
# one at a time and calculate the MSE on test fold
for (i in 1:ncol(df.test.cv.X)){
  (coefi <- coef(regfit.full.train, id = i))
  (pred <- mat.test.cv.X[, names(coefi)] %*% coefi)
  (mse <- mean((pred - df.test.cv.Y)^2))
  mses <- rbind (mses, tibble(no.of.coefs = i, MSE = mse))
}
return(mses)
})

allResults <- results[[1]]
for (i in 2 : length(results)){
  allResults <- rbind(allResults , results[[i]])
}

(allMse <- (allResults %>%
  group_by(no.of.coefs) %>%
  summarise(mse.mean = mean(MSE))) )

```

```

## # A tibble: 13 x 2
##   no.of.coefs mse.mean
##   <int>     <dbl>
## 1         1     51.0
## 2         2     49.2
## 3         3     47.9
## 4         4     47.8
## 5         5     48.4
## 6         6     47.5
## 7         7     47.2
## 8         8     46.6
## 9         9     45.9
## 10        10     46.0
## 11        11     45.6
## 12        12     45.6
## 13        13     45.5

```

```
print ("The best subset of features selected corresponds to minimum CV_MSE:")
```

```
## [1] "The best subset of features selected corresponds to minimum CV_MSE:"
```

```
(idx <- which.min(allMse$mse.mean))
```

```
## [1] 13
```

```
print("However looking at the data shows subset of features with 9 components
      is very close to the minimum we found so we use it")
```

```
## [1] "However looking at the data shows subset of features with 9 components \n      is very close to
M=9"
```

```
regfit.full.train <- regsubsets(crim ~ ., df.train,
                               nvmax = ncol(df.train) - 1)
```

```
sprintf(" Now apply the the model with %s No. of components to test data:", idx)
```

```
## [1] " Now apply the the model with 13 No. of components to test data:"
```

```
(coefi <- coef(regfit.full.train, id = M))
```

```
## (Intercept)      zn      indus      nox      dis      rad
## 24.83928145  0.04131571 -0.15408884 -12.05853934 -1.05123802  0.55802734
##      ptratio      b      lstat      medv
## -0.47520557 -0.01309050  0.25525000 -0.18608243
```

```
(mat.df.test <- model.matrix(crim~., df.test))
```

```
##      (Intercept)      zn indus chas      nox      rm      age      dis rad tax ptratio
## 2              1  0.0  7.07    0 0.4690 6.421  78.9  4.9671  2 242    17.8
## 3              1  0.0  7.07    0 0.4690 7.185  61.1  4.9671  2 242    17.8
## 4              1  0.0  2.18    0 0.4580 6.998  45.8  6.0622  3 222    18.7
## 5              1  0.0  2.18    0 0.4580 7.147  54.2  6.0622  3 222    18.7
## 6              1  0.0  2.18    0 0.4580 6.430  58.7  6.0622  3 222    18.7
## 7              1 12.5  7.87    0 0.5240 6.012  66.6  5.5605  5 311    15.2
## 8              1 12.5  7.87    0 0.5240 6.172  96.1  5.9505  5 311    15.2
## 9              1 12.5  7.87    0 0.5240 5.631 100.0  6.0821  5 311    15.2
## 10             1 12.5  7.87    0 0.5240 6.004  85.9  6.5921  5 311    15.2
## 11             1 12.5  7.87    0 0.5240 6.377  94.3  6.3467  5 311    15.2
## 12             1 12.5  7.87    0 0.5240 6.009  82.9  6.2267  5 311    15.2
## 17             1  0.0  8.14    0 0.5380 5.935  29.3  4.4986  4 307    21.0
## 18             1  0.0  8.14    0 0.5380 5.990  81.7  4.2579  4 307    21.0
## 21             1  0.0  8.14    0 0.5380 5.570  98.1  3.7979  4 307    21.0
## 23             1  0.0  8.14    0 0.5380 6.142  91.7  3.9769  4 307    21.0
## 26             1  0.0  8.14    0 0.5380 5.599  85.7  4.4546  4 307    21.0
## 30             1  0.0  8.14    0 0.5380 6.674  87.3  4.2390  4 307    21.0
## 32             1  0.0  8.14    0 0.5380 6.072 100.0  4.1750  4 307    21.0
## 34             1  0.0  8.14    0 0.5380 5.701  95.0  3.7872  4 307    21.0
## 38             1  0.0  5.96    0 0.4990 5.850  41.5  3.9342  5 279    19.2
## 46             1  0.0  6.91    0 0.4480 5.682  33.8  5.1004  3 233    17.9
## 47             1  0.0  6.91    0 0.4480 5.786  33.3  5.1004  3 233    17.9
## 52             1 21.0  5.64    0 0.4390 6.115  63.0  6.8147  4 243    16.8
## 54             1 21.0  5.64    0 0.4390 5.998  21.4  6.8147  4 243    16.8
## 55             1 75.0  4.00    0 0.4100 5.888  47.6  7.3197  3 469    21.1
## 56             1 90.0  1.22    0 0.4030 7.249  21.9  8.6966  5 226    17.9
## 57             1 85.0  0.74    0 0.4100 6.383  35.7  9.1876  2 313    17.3
## 58             1 100.0 1.32    0 0.4110 6.816  40.5  8.3248  5 256    15.1
## 59             1 25.0  5.13    0 0.4530 6.145  29.2  7.8148  8 284    19.7
## 63             1 25.0  5.13    0 0.4530 6.456  67.8  7.2255  8 284    19.7
## 66             1 80.0  3.37    0 0.3980 6.290  17.8  6.6115  4 337    16.1
## 67             1 80.0  3.37    0 0.3980 5.787  31.1  6.6115  4 337    16.1
```

## 68	1	12.5	6.07	0	0.4090	5.878	21.4	6.4980	4	345	18.9
## 69	1	12.5	6.07	0	0.4090	5.594	36.8	6.4980	4	345	18.9
## 71	1	0.0	10.81	0	0.4130	6.417	6.6	5.2873	4	305	19.2
## 74	1	0.0	10.81	0	0.4130	6.245	6.2	5.2873	4	305	19.2
## 76	1	0.0	12.83	0	0.4370	6.286	45.0	4.5026	5	398	18.7
## 80	1	0.0	12.83	0	0.4370	5.874	36.6	4.5026	5	398	18.7
## 81	1	25.0	4.86	0	0.4260	6.727	33.5	5.4007	4	281	19.0
## 82	1	25.0	4.86	0	0.4260	6.619	70.4	5.4007	4	281	19.0
## 83	1	25.0	4.86	0	0.4260	6.302	32.2	5.4007	4	281	19.0
## 87	1	0.0	4.49	0	0.4490	6.015	45.1	4.4272	3	247	18.5
## 88	1	0.0	4.49	0	0.4490	6.121	56.8	3.7476	3	247	18.5
## 90	1	0.0	3.41	0	0.4890	7.079	63.1	3.4145	2	270	17.8
## 91	1	0.0	3.41	0	0.4890	6.417	66.1	3.0923	2	270	17.8
## 94	1	28.0	15.04	0	0.4640	6.211	28.9	3.6659	4	270	18.2
## 95	1	28.0	15.04	0	0.4640	6.249	77.3	3.6150	4	270	18.2
## 96	1	0.0	2.89	0	0.4450	6.625	57.8	3.4952	2	276	18.0
## 97	1	0.0	2.89	0	0.4450	6.163	69.6	3.4952	2	276	18.0
## 99	1	0.0	2.89	0	0.4450	7.820	36.9	3.4952	2	276	18.0
## 100	1	0.0	2.89	0	0.4450	7.416	62.5	3.4952	2	276	18.0
## 101	1	0.0	8.56	0	0.5200	6.727	79.9	2.7778	5	384	20.9
## 106	1	0.0	8.56	0	0.5200	5.851	96.7	2.1069	5	384	20.9
## 109	1	0.0	8.56	0	0.5200	6.474	97.1	2.4329	5	384	20.9
## 112	1	0.0	10.01	0	0.5470	6.715	81.6	2.6775	6	432	17.8
## 114	1	0.0	10.01	0	0.5470	6.092	95.4	2.5480	6	432	17.8
## 115	1	0.0	10.01	0	0.5470	6.254	84.2	2.2565	6	432	17.8
## 119	1	0.0	10.01	0	0.5470	5.872	73.1	2.4775	6	432	17.8
## 120	1	0.0	10.01	0	0.5470	5.731	65.2	2.7592	6	432	17.8
## 123	1	0.0	25.65	0	0.5810	5.961	92.9	2.0869	2	188	19.1
## 125	1	0.0	25.65	0	0.5810	5.879	95.8	2.0063	2	188	19.1
## 128	1	0.0	21.89	0	0.6240	5.693	96.0	1.7883	4	437	21.2
## 131	1	0.0	21.89	0	0.6240	6.458	98.9	2.1185	4	437	21.2
## 134	1	0.0	21.89	0	0.6240	5.822	95.4	2.4699	4	437	21.2
## 136	1	0.0	21.89	0	0.6240	6.335	98.2	2.1107	4	437	21.2
## 137	1	0.0	21.89	0	0.6240	5.942	93.5	1.9669	4	437	21.2
## 139	1	0.0	21.89	0	0.6240	5.857	98.2	1.6686	4	437	21.2
## 142	1	0.0	21.89	0	0.6240	5.019	100.0	1.4394	4	437	21.2
## 144	1	0.0	19.58	0	0.8710	5.468	100.0	1.4118	5	403	14.7
## 146	1	0.0	19.58	0	0.8710	6.130	100.0	1.4191	5	403	14.7
## 147	1	0.0	19.58	0	0.8710	5.628	100.0	1.5166	5	403	14.7
## 150	1	0.0	19.58	0	0.8710	5.597	94.9	1.5257	5	403	14.7
## 151	1	0.0	19.58	0	0.8710	6.122	97.3	1.6180	5	403	14.7
## 154	1	0.0	19.58	0	0.8710	5.709	98.5	1.6232	5	403	14.7
## 155	1	0.0	19.58	1	0.8710	6.129	96.0	1.7494	5	403	14.7
## 156	1	0.0	19.58	1	0.8710	6.152	82.6	1.7455	5	403	14.7
## 157	1	0.0	19.58	0	0.8710	5.272	94.0	1.7364	5	403	14.7
## 158	1	0.0	19.58	0	0.6050	6.943	97.4	1.8773	5	403	14.7
## 159	1	0.0	19.58	0	0.6050	6.066	100.0	1.7573	5	403	14.7
## 161	1	0.0	19.58	1	0.6050	6.250	92.6	1.7984	5	403	14.7
## 162	1	0.0	19.58	0	0.6050	7.489	90.8	1.9709	5	403	14.7
## 164	1	0.0	19.58	1	0.6050	8.375	93.9	2.1620	5	403	14.7
## 166	1	0.0	19.58	0	0.6050	6.101	93.0	2.2834	5	403	14.7
## 169	1	0.0	19.58	0	0.6050	6.319	96.1	2.1000	5	403	14.7
## 171	1	0.0	19.58	0	0.6050	5.875	94.6	2.4259	5	403	14.7
## 173	1	0.0	4.05	0	0.5100	5.572	88.5	2.5961	5	296	16.6

## 175	1	0.0	4.05	0	0.5100	5.859	68.7	2.7019	5	296	16.6
## 177	1	0.0	4.05	0	0.5100	6.020	47.2	3.5549	5	296	16.6
## 178	1	0.0	4.05	0	0.5100	6.315	73.4	3.3175	5	296	16.6
## 179	1	0.0	4.05	0	0.5100	6.860	74.4	2.9153	5	296	16.6
## 180	1	0.0	2.46	0	0.4880	6.980	58.4	2.8290	3	193	17.8
## 182	1	0.0	2.46	0	0.4880	6.144	62.2	2.5979	3	193	17.8
## 183	1	0.0	2.46	0	0.4880	7.155	92.2	2.7006	3	193	17.8
## 184	1	0.0	2.46	0	0.4880	6.563	95.6	2.8470	3	193	17.8
## 186	1	0.0	2.46	0	0.4880	6.153	68.8	3.2797	3	193	17.8
## 188	1	45.0	3.44	0	0.4370	6.782	41.1	3.7886	5	398	15.2
## 189	1	45.0	3.44	0	0.4370	6.556	29.1	4.5667	5	398	15.2
## 190	1	45.0	3.44	0	0.4370	7.185	38.9	4.5667	5	398	15.2
## 191	1	45.0	3.44	0	0.4370	6.951	21.5	6.4798	5	398	15.2
## 192	1	45.0	3.44	0	0.4370	6.739	30.8	6.4798	5	398	15.2
## 195	1	60.0	2.93	0	0.4010	6.604	18.8	6.2196	1	265	15.6
## 196	1	80.0	0.46	0	0.4220	7.875	32.0	5.6484	4	255	14.4
## 197	1	80.0	1.52	0	0.4040	7.287	34.1	7.3090	2	329	12.6
## 199	1	80.0	1.52	0	0.4040	7.274	38.3	7.3090	2	329	12.6
## 200	1	95.0	1.47	0	0.4030	6.975	15.3	7.6534	3	402	17.0
## 201	1	95.0	1.47	0	0.4030	7.135	13.9	7.6534	3	402	17.0
## 203	1	82.5	2.03	0	0.4150	7.610	15.7	6.2700	2	348	14.7
## 205	1	95.0	2.68	0	0.4161	8.034	31.9	5.1180	4	224	14.7
## 208	1	0.0	10.59	0	0.4890	5.783	72.7	4.3549	4	277	18.6
## 209	1	0.0	10.59	1	0.4890	6.064	59.1	4.2392	4	277	18.6
## 210	1	0.0	10.59	1	0.4890	5.344	100.0	3.8750	4	277	18.6
## 211	1	0.0	10.59	1	0.4890	5.960	92.1	3.8771	4	277	18.6
## 215	1	0.0	10.59	0	0.4890	5.412	9.8	3.5875	4	277	18.6
## 216	1	0.0	10.59	0	0.4890	6.182	42.4	3.9454	4	277	18.6
## 220	1	0.0	13.89	1	0.5500	6.373	92.4	3.3633	5	276	16.4
## 222	1	0.0	6.20	1	0.5070	6.164	91.3	3.0480	8	307	17.4
## 223	1	0.0	6.20	1	0.5070	6.879	77.7	3.2721	8	307	17.4
## 226	1	0.0	6.20	0	0.5040	8.725	83.0	2.8944	8	307	17.4
## 227	1	0.0	6.20	0	0.5040	8.040	86.5	3.2157	8	307	17.4
## 228	1	0.0	6.20	0	0.5040	7.163	79.9	3.2157	8	307	17.4
## 232	1	0.0	6.20	0	0.5040	7.412	76.9	3.6715	8	307	17.4
## 235	1	0.0	6.20	1	0.5070	6.726	66.5	3.6519	8	307	17.4
## 236	1	0.0	6.20	0	0.5070	6.086	61.5	3.6519	8	307	17.4
## 238	1	0.0	6.20	0	0.5070	7.358	71.6	4.1480	8	307	17.4
## 240	1	30.0	4.93	0	0.4280	6.606	42.2	6.1899	6	300	16.6
## 243	1	30.0	4.93	0	0.4280	6.358	52.9	7.0355	6	300	16.6
## 244	1	30.0	4.93	0	0.4280	6.393	7.8	7.0355	6	300	16.6
## 245	1	22.0	5.86	0	0.4310	5.593	76.5	7.9549	7	330	19.1
## 250	1	22.0	5.86	0	0.4310	6.718	17.5	7.8265	7	330	19.1
## 251	1	22.0	5.86	0	0.4310	6.487	13.0	7.3967	7	330	19.1
## 253	1	22.0	5.86	0	0.4310	6.957	6.8	8.9067	7	330	19.1
## 256	1	80.0	3.64	0	0.3920	5.876	19.1	9.2203	1	315	16.4
## 257	1	90.0	3.75	0	0.3940	7.454	34.2	6.3361	3	244	15.9
## 258	1	20.0	3.97	0	0.6470	8.704	86.9	1.8010	5	264	13.0
## 259	1	20.0	3.97	0	0.6470	7.333	100.0	1.8946	5	264	13.0
## 260	1	20.0	3.97	0	0.6470	6.842	100.0	2.0107	5	264	13.0
## 261	1	20.0	3.97	0	0.6470	7.203	81.8	2.1121	5	264	13.0
## 262	1	20.0	3.97	0	0.6470	7.520	89.4	2.1398	5	264	13.0
## 264	1	20.0	3.97	0	0.6470	7.327	94.5	2.0788	5	264	13.0
## 266	1	20.0	3.97	0	0.6470	5.560	62.8	1.9865	5	264	13.0

## 267	1	20.0	3.97	0	0.6470	7.014	84.6	2.1329	5	264	13.0
## 268	1	20.0	3.97	0	0.5750	8.297	67.0	2.4216	5	264	13.0
## 269	1	20.0	3.97	0	0.5750	7.470	52.6	2.8720	5	264	13.0
## 272	1	20.0	6.96	0	0.4640	6.240	16.3	4.4290	3	223	18.6
## 276	1	40.0	6.41	0	0.4470	6.854	42.8	4.2673	4	254	17.6
## 278	1	40.0	6.41	1	0.4470	6.826	27.6	4.8628	4	254	17.6
## 283	1	20.0	3.33	1	0.4429	7.645	49.7	5.2119	5	216	14.9
## 288	1	52.5	5.32	0	0.4050	6.209	31.3	7.3172	6	293	16.6
## 291	1	80.0	4.95	0	0.4110	6.861	27.9	5.1167	4	245	19.2
## 292	1	80.0	4.95	0	0.4110	7.148	27.7	5.1167	4	245	19.2
## 294	1	0.0	13.92	0	0.4370	6.127	18.4	5.5027	4	289	16.0
## 301	1	70.0	2.24	0	0.4000	6.871	47.4	7.8278	5	358	14.8
## 302	1	34.0	6.09	0	0.4330	6.590	40.4	5.4917	7	329	16.1
## 303	1	34.0	6.09	0	0.4330	6.495	18.4	5.4917	7	329	16.1
## 308	1	33.0	2.18	0	0.4720	6.849	70.3	3.1827	7	222	18.4
## 309	1	0.0	9.90	0	0.5440	6.635	82.5	3.3175	4	304	18.4
## 310	1	0.0	9.90	0	0.5440	5.972	76.7	3.1025	4	304	18.4
## 311	1	0.0	9.90	0	0.5440	4.973	37.8	2.5194	4	304	18.4
## 312	1	0.0	9.90	0	0.5440	6.122	52.8	2.6403	4	304	18.4
## 314	1	0.0	9.90	0	0.5440	6.266	82.8	3.2628	4	304	18.4
## 315	1	0.0	9.90	0	0.5440	6.567	87.3	3.6023	4	304	18.4
## 317	1	0.0	9.90	0	0.5440	5.914	83.2	3.9986	4	304	18.4
## 318	1	0.0	9.90	0	0.5440	5.782	71.7	4.0317	4	304	18.4
## 319	1	0.0	9.90	0	0.5440	6.382	67.2	3.5325	4	304	18.4
## 320	1	0.0	9.90	0	0.5440	6.113	58.8	4.0019	4	304	18.4
## 321	1	0.0	7.38	0	0.4930	6.426	52.3	4.5404	5	287	19.6
## 322	1	0.0	7.38	0	0.4930	6.376	54.3	4.5404	5	287	19.6
## 323	1	0.0	7.38	0	0.4930	6.041	49.9	4.7211	5	287	19.6
## 334	1	0.0	5.19	0	0.5150	6.316	38.1	6.4584	5	224	20.2
## 335	1	0.0	5.19	0	0.5150	6.310	38.5	6.4584	5	224	20.2
## 337	1	0.0	5.19	0	0.5150	5.869	46.3	5.2311	5	224	20.2
## 341	1	0.0	5.19	0	0.5150	5.968	58.5	4.8122	5	224	20.2
## 347	1	0.0	4.39	0	0.4420	5.898	52.3	8.0136	3	352	18.8
## 348	1	85.0	4.15	0	0.4290	6.516	27.7	8.5353	4	351	17.9
## 349	1	80.0	2.01	0	0.4350	6.635	29.7	8.3440	4	280	17.0
## 350	1	40.0	1.25	0	0.4290	6.939	34.5	8.7921	1	335	19.7
## 351	1	40.0	1.25	0	0.4290	6.490	44.4	8.7921	1	335	19.7
## 352	1	60.0	1.69	0	0.4110	6.579	35.9	10.7103	4	411	18.3
## 354	1	90.0	2.02	0	0.4100	6.728	36.1	12.1265	5	187	17.0
## 356	1	80.0	1.91	0	0.4130	5.936	19.5	10.5857	4	334	22.0
## 357	1	0.0	18.10	1	0.7700	6.212	97.4	2.1222	24	666	20.2
## 358	1	0.0	18.10	1	0.7700	6.395	91.0	2.5052	24	666	20.2
## 361	1	0.0	18.10	0	0.7700	6.398	88.0	2.5182	24	666	20.2
## 363	1	0.0	18.10	0	0.7700	5.362	96.2	2.1036	24	666	20.2
## 365	1	0.0	18.10	1	0.7180	8.780	82.9	1.9047	24	666	20.2
## 367	1	0.0	18.10	0	0.7180	4.963	91.4	1.7523	24	666	20.2
## 369	1	0.0	18.10	0	0.6310	4.970	100.0	1.3325	24	666	20.2
## 370	1	0.0	18.10	1	0.6310	6.683	96.8	1.3567	24	666	20.2
## 372	1	0.0	18.10	0	0.6310	6.216	100.0	1.1691	24	666	20.2
## 373	1	0.0	18.10	1	0.6680	5.875	89.6	1.1296	24	666	20.2
## 374	1	0.0	18.10	0	0.6680	4.906	100.0	1.1742	24	666	20.2
## 376	1	0.0	18.10	0	0.6710	7.313	97.9	1.3163	24	666	20.2
## 379	1	0.0	18.10	0	0.6710	6.380	96.2	1.3861	24	666	20.2
## 380	1	0.0	18.10	0	0.6710	6.223	100.0	1.3861	24	666	20.2

## 381	1	0.0	18.10	0	0.6710	6.968	91.9	1.4165	24	666	20.2
## 383	1	0.0	18.10	0	0.7000	5.536	100.0	1.5804	24	666	20.2
## 384	1	0.0	18.10	0	0.7000	5.520	100.0	1.5331	24	666	20.2
## 385	1	0.0	18.10	0	0.7000	4.368	91.2	1.4395	24	666	20.2
## 386	1	0.0	18.10	0	0.7000	5.277	98.1	1.4261	24	666	20.2
## 387	1	0.0	18.10	0	0.7000	4.652	100.0	1.4672	24	666	20.2
## 388	1	0.0	18.10	0	0.7000	5.000	89.5	1.5184	24	666	20.2
## 393	1	0.0	18.10	0	0.7000	5.036	97.0	1.7700	24	666	20.2
## 394	1	0.0	18.10	0	0.6930	6.193	92.6	1.7912	24	666	20.2
## 397	1	0.0	18.10	0	0.6930	6.405	96.0	1.6768	24	666	20.2
## 398	1	0.0	18.10	0	0.6930	5.747	98.9	1.6334	24	666	20.2
## 400	1	0.0	18.10	0	0.6930	5.852	77.8	1.5004	24	666	20.2
## 403	1	0.0	18.10	0	0.6930	6.404	100.0	1.6390	24	666	20.2
## 407	1	0.0	18.10	0	0.6590	4.138	100.0	1.1781	24	666	20.2
## 409	1	0.0	18.10	0	0.5970	5.617	97.9	1.4547	24	666	20.2
## 410	1	0.0	18.10	0	0.5970	6.852	100.0	1.4655	24	666	20.2
## 411	1	0.0	18.10	0	0.5970	5.757	100.0	1.4130	24	666	20.2
## 417	1	0.0	18.10	0	0.6790	6.782	90.8	1.8195	24	666	20.2
## 424	1	0.0	18.10	0	0.6140	6.103	85.1	2.0218	24	666	20.2
## 425	1	0.0	18.10	0	0.5840	5.565	70.6	2.0635	24	666	20.2
## 426	1	0.0	18.10	0	0.6790	5.896	95.4	1.9096	24	666	20.2
## 429	1	0.0	18.10	0	0.6790	6.193	78.1	1.9356	24	666	20.2
## 430	1	0.0	18.10	0	0.6790	6.380	95.6	1.9682	24	666	20.2
## 432	1	0.0	18.10	0	0.5840	6.833	94.3	2.0882	24	666	20.2
## 439	1	0.0	18.10	0	0.7400	5.935	87.9	1.8206	24	666	20.2
## 440	1	0.0	18.10	0	0.7400	5.627	93.9	1.8172	24	666	20.2
## 441	1	0.0	18.10	0	0.7400	5.818	92.4	1.8662	24	666	20.2
## 447	1	0.0	18.10	0	0.7400	6.341	96.4	2.0720	24	666	20.2
## 448	1	0.0	18.10	0	0.7400	6.251	96.6	2.1980	24	666	20.2
## 451	1	0.0	18.10	0	0.7130	6.749	92.6	2.3236	24	666	20.2
## 452	1	0.0	18.10	0	0.7130	6.655	98.2	2.3552	24	666	20.2
## 453	1	0.0	18.10	0	0.7130	6.297	91.8	2.3682	24	666	20.2
## 454	1	0.0	18.10	0	0.7130	7.393	99.3	2.4527	24	666	20.2
## 455	1	0.0	18.10	0	0.7130	6.728	94.1	2.4961	24	666	20.2
## 456	1	0.0	18.10	0	0.7130	6.525	86.5	2.4358	24	666	20.2
## 457	1	0.0	18.10	0	0.7130	5.976	87.9	2.5806	24	666	20.2
## 460	1	0.0	18.10	0	0.7130	6.081	84.4	2.7175	24	666	20.2
## 462	1	0.0	18.10	0	0.7130	6.376	88.4	2.5671	24	666	20.2
## 468	1	0.0	18.10	0	0.5840	6.003	94.5	2.5403	24	666	20.2
## 469	1	0.0	18.10	0	0.5800	5.926	71.0	2.9084	24	666	20.2
## 470	1	0.0	18.10	0	0.5800	5.713	56.7	2.8237	24	666	20.2
## 472	1	0.0	18.10	0	0.5320	6.229	90.7	3.0993	24	666	20.2
## 474	1	0.0	18.10	0	0.6140	6.980	67.6	2.5329	24	666	20.2
## 475	1	0.0	18.10	0	0.5840	5.427	95.4	2.4298	24	666	20.2
## 477	1	0.0	18.10	0	0.6140	6.484	93.6	2.3053	24	666	20.2
## 479	1	0.0	18.10	0	0.6140	6.185	96.7	2.1705	24	666	20.2
## 482	1	0.0	18.10	0	0.5320	6.750	74.9	3.3317	24	666	20.2
## 486	1	0.0	18.10	0	0.5830	6.312	51.9	3.9917	24	666	20.2
## 487	1	0.0	18.10	0	0.5830	6.114	79.8	3.5459	24	666	20.2
## 489	1	0.0	27.74	0	0.6090	5.454	92.7	1.8209	4	711	20.1
## 491	1	0.0	27.74	0	0.6090	5.093	98.0	1.8226	4	711	20.1
## 493	1	0.0	27.74	0	0.6090	5.983	83.5	2.1099	4	711	20.1
## 494	1	0.0	9.69	0	0.5850	5.707	54.0	2.3817	6	391	19.2
## 495	1	0.0	9.69	0	0.5850	5.926	42.6	2.3817	6	391	19.2

## 497	1	0.0	9.69	0	0.5850	5.390	72.9	2.7986	6	391	19.2
## 499	1	0.0	9.69	0	0.5850	6.019	65.3	2.4091	6	391	19.2
## 502	1	0.0	11.93	0	0.5730	6.593	69.1	2.4786	1	273	21.0
## 504	1	0.0	11.93	0	0.5730	6.976	91.0	2.1675	1	273	21.0
## 506	1	0.0	11.93	0	0.5730	6.030	80.8	2.5050	1	273	21.0
##	b lstat medv										
## 2	396.90	9.14	21.6								
## 3	392.83	4.03	34.7								
## 4	394.63	2.94	33.4								
## 5	396.90	5.33	36.2								
## 6	394.12	5.21	28.7								
## 7	395.60	12.43	22.9								
## 8	396.90	19.15	27.1								
## 9	386.63	29.93	16.5								
## 10	386.71	17.10	18.9								
## 11	392.52	20.45	15.0								
## 12	396.90	13.27	18.9								
## 17	386.85	6.58	23.1								
## 18	386.75	14.67	17.5								
## 21	376.57	21.02	13.6								
## 23	396.90	18.72	15.2								
## 26	303.42	16.51	13.9								
## 30	380.23	11.98	21.0								
## 32	376.73	13.04	14.5								
## 34	358.77	18.35	13.1								
## 38	396.90	8.77	21.0								
## 46	396.90	10.21	19.3								
## 47	396.90	14.15	20.0								
## 52	393.97	9.43	20.5								
## 54	396.90	8.43	23.4								
## 55	396.90	14.80	18.9								
## 56	395.93	4.81	35.4								
## 57	396.90	5.77	24.7								
## 58	392.90	3.95	31.6								
## 59	390.68	6.86	23.3								
## 63	396.90	6.73	22.2								
## 66	396.90	4.67	23.5								
## 67	396.90	10.24	19.4								
## 68	396.21	8.10	22.0								
## 69	396.90	13.09	17.4								
## 71	383.73	6.72	24.2								
## 74	377.17	7.54	23.4								
## 76	383.23	8.94	21.4								
## 80	396.06	9.10	20.3								
## 81	396.90	5.29	28.0								
## 82	395.63	7.22	23.9								
## 83	396.90	6.72	24.8								
## 87	395.99	12.86	22.5								
## 88	395.15	8.44	22.2								
## 90	396.06	5.70	28.7								
## 91	392.18	8.81	22.6								
## 94	396.33	6.21	25.0								
## 95	396.90	10.59	20.6								
## 96	357.98	6.65	28.4								

97 391.83 11.34 21.4
99 393.53 3.57 43.8
100 396.90 6.19 33.2
101 394.76 9.42 27.5
106 394.05 16.47 19.5
109 395.24 12.27 19.8
112 395.59 10.16 22.8
114 396.90 17.09 18.7
115 388.74 10.45 18.5
119 338.63 15.37 20.4
120 391.50 13.61 19.3
123 378.09 17.93 20.5
125 379.38 17.58 18.8
128 392.11 17.19 16.2
131 395.04 12.60 19.2
134 388.69 15.03 18.4
136 394.67 16.96 18.1
137 378.25 16.90 17.4
139 392.04 21.32 13.3
142 396.90 34.41 14.4
144 396.90 26.42 15.6
146 172.91 27.80 13.8
147 169.27 16.65 15.6
150 351.85 21.45 15.4
151 372.80 14.10 21.5
154 261.95 15.79 19.4
155 321.02 15.12 17.0
156 88.01 15.02 15.6
157 88.63 16.14 13.1
158 363.43 4.59 41.3
159 353.89 6.43 24.3
161 338.92 5.50 27.0
162 374.43 1.73 50.0
164 388.45 3.32 50.0
166 240.16 9.81 25.0
169 297.09 11.10 23.8
171 292.29 14.43 17.4
173 396.90 14.69 23.1
175 393.23 9.64 22.6
177 393.23 10.11 23.2
178 395.60 6.29 24.6
179 391.27 6.92 29.9
180 396.90 5.04 37.2
182 396.90 9.45 36.2
183 394.12 4.82 37.9
184 396.90 5.68 32.5
186 387.11 13.15 29.6
188 393.87 6.68 32.0
189 382.84 4.56 29.8
190 396.90 5.39 34.9
191 377.68 5.10 37.0
192 389.71 4.69 30.5
195 376.70 4.38 29.1
196 394.23 2.97 50.0

```

## 197 396.90 4.08 33.3
## 199 392.20 6.62 34.6
## 200 396.90 4.56 34.9
## 201 384.30 4.45 32.9
## 203 395.38 3.11 42.3
## 205 390.55 2.88 50.0
## 208 389.43 18.06 22.5
## 209 381.32 14.66 24.4
## 210 396.90 23.09 20.0
## 211 393.25 17.27 21.7
## 215 348.93 29.55 23.7
## 216 393.63 9.47 25.0
## 220 393.74 10.50 23.0
## 222 395.24 21.46 21.7
## 223 390.39 9.93 27.5
## 226 382.00 4.63 50.0
## 227 387.38 3.13 37.6
## 228 372.08 6.36 31.6
## 232 376.14 5.25 31.7
## 235 360.20 8.05 29.0
## 236 376.75 10.88 24.0
## 238 390.07 4.73 31.5
## 240 383.78 7.37 23.3
## 243 372.75 11.22 22.2
## 244 374.71 5.19 23.7
## 245 372.49 12.50 17.6
## 250 393.74 6.56 26.2
## 251 396.28 5.90 24.4
## 253 386.09 3.53 29.6
## 256 395.18 9.25 20.9
## 257 386.34 3.11 44.0
## 258 389.70 5.12 50.0
## 259 383.29 7.79 36.0
## 260 391.93 6.90 30.1
## 261 392.80 9.59 33.8
## 262 388.37 7.26 43.1
## 264 393.42 11.25 31.0
## 266 392.40 10.45 22.8
## 267 384.07 14.79 30.7
## 268 384.54 7.44 50.0
## 269 390.30 3.16 43.5
## 272 396.90 6.59 25.2
## 276 396.90 2.98 32.0
## 278 393.45 4.16 33.1
## 283 377.07 3.01 46.0
## 288 396.90 7.14 23.2
## 291 396.90 3.33 28.5
## 292 396.90 3.56 37.3
## 294 396.90 8.58 23.9
## 301 390.86 6.07 24.8
## 302 395.75 9.50 22.0
## 303 383.61 8.67 26.4
## 308 396.90 7.53 28.2
## 309 396.90 4.54 22.8

```

310 396.24 9.97 20.3
311 350.45 12.64 16.1
312 396.90 5.98 22.1
314 393.39 7.90 21.6
315 395.69 9.28 23.8
317 390.70 18.33 17.8
318 396.90 15.94 19.8
319 395.21 10.36 23.1
320 396.23 12.73 21.0
321 396.90 7.20 23.8
322 396.90 6.87 23.1
323 396.90 7.70 20.4
334 389.71 5.68 22.2
335 389.40 6.75 20.7
337 396.90 9.80 19.5
341 396.90 9.29 18.7
347 364.61 12.67 17.2
348 392.43 6.36 23.1
349 390.94 5.99 24.5
350 389.85 5.89 26.6
351 396.90 5.98 22.9
352 370.78 5.49 24.1
354 384.46 4.50 30.1
356 376.04 5.57 20.6
357 377.73 17.60 17.8
358 391.34 13.27 21.7
361 374.56 7.79 25.0
363 380.79 10.19 20.8
365 354.55 5.29 21.9
367 316.03 14.00 21.9
369 375.52 3.26 50.0
370 375.33 3.73 50.0
372 366.15 9.53 50.0
373 347.88 8.88 50.0
374 396.90 34.77 13.8
376 396.90 13.44 15.0
379 396.90 23.69 13.1
380 393.74 21.78 10.2
381 396.90 17.21 10.4
383 396.90 23.60 11.3
384 396.90 24.56 12.3
385 285.83 30.63 8.8
386 396.90 30.81 7.2
387 396.90 28.28 10.5
388 396.90 31.99 7.4
393 396.90 25.68 9.7
394 396.90 15.17 13.8
397 396.90 19.37 12.5
398 393.10 19.92 8.5
400 338.16 29.97 6.3
403 376.11 20.31 12.1
407 370.22 23.34 11.9
409 314.64 26.40 17.2
410 179.36 19.78 27.5

```

## 411 2.60 10.11 15.0
## 417 21.57 25.79 7.5
## 424 2.52 23.29 13.4
## 425 3.65 17.16 11.7
## 426 7.68 24.39 8.3
## 429 96.73 21.52 11.0
## 430 60.72 24.08 9.5
## 432 81.33 19.69 14.1
## 439 68.95 34.02 8.4
## 440 396.90 22.88 12.8
## 441 391.45 22.11 10.5
## 447 318.01 17.79 14.9
## 448 388.52 16.44 12.6
## 451 0.32 17.44 13.4
## 452 355.29 17.73 15.2
## 453 385.09 17.27 16.1
## 454 375.87 16.74 17.8
## 455 6.68 18.71 14.9
## 456 50.92 18.13 14.1
## 457 10.48 19.01 12.7
## 460 396.90 14.70 20.0
## 462 391.43 14.65 17.7
## 468 331.29 21.32 19.1
## 469 368.74 18.13 19.1
## 470 396.90 14.76 20.1
## 472 395.33 12.87 19.6
## 474 374.68 11.66 29.8
## 475 352.58 18.14 13.8
## 477 396.21 18.68 16.7
## 479 379.70 18.03 14.6
## 482 393.07 7.74 23.7
## 486 388.62 10.58 21.2
## 487 392.68 14.98 19.1
## 489 395.09 18.06 15.2
## 491 318.43 29.68 8.1
## 493 396.90 13.35 20.1
## 494 396.90 12.01 21.8
## 495 396.90 13.59 24.5
## 497 396.90 21.14 19.7
## 499 396.90 12.92 21.2
## 502 391.99 9.67 22.4
## 504 396.90 5.64 23.9
## 506 396.90 7.88 11.9
## attr("assign")
## [1] 0 1 2 3 4 5 6 7 8 9 10 11 12 13

yhat <- mat.df.test[, names(coefi)] %*% coefi

mse.best.subset <- mean ((test.y - yhat)^2)
sprintf( "----- MSE on test data for best model with 8 component: %s",
        mse.best.subset)

## [1] "----- MSE on test data for best model with 8 component: 41.4927142070165"

```

```
print("c: fit Ridge regression model on training set and get test.mse-----")
```

```
## [1] "c: fit Ridge regression model on training set and get test.mse-----"
```

```
# use magic model.matrix to convert dataframe into a matrix for Ridge and Lasso
```

```
x.train <- model.matrix(crim~., df.train)[, -1]
```

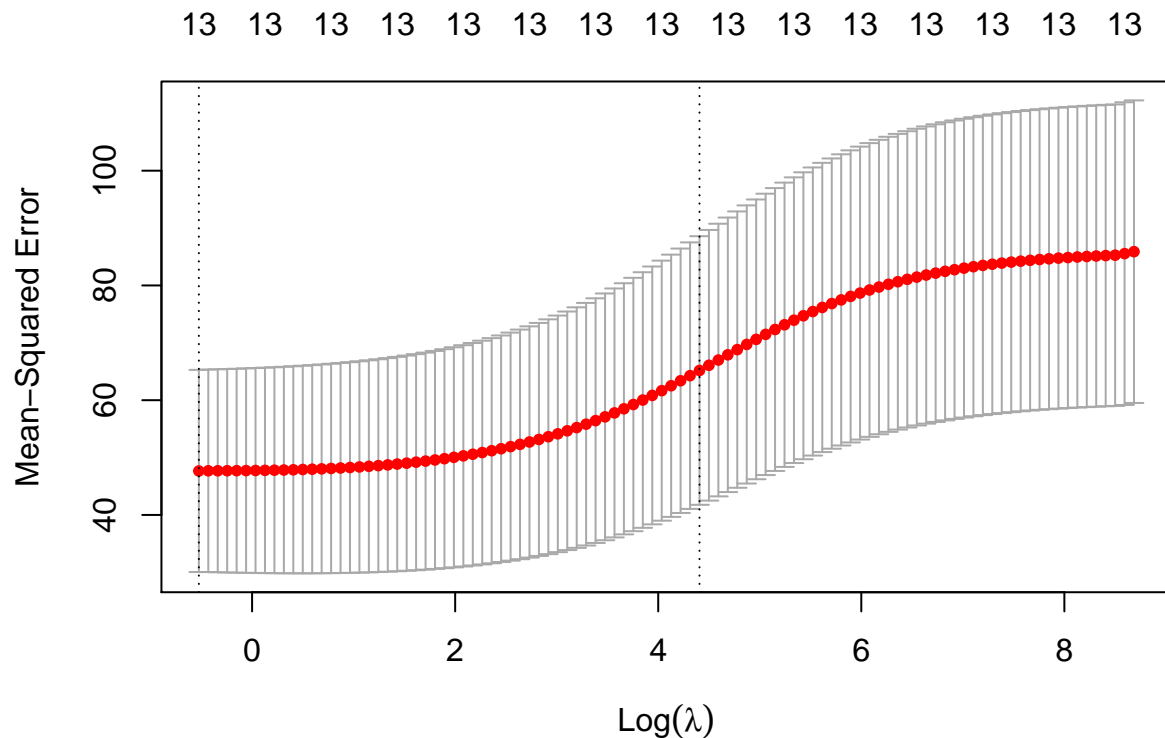
```
y.train <- (df.train)$crim
```

```
x.test <- model.matrix(crim~., df.test)[, -1]
```

```
y.test <- df.test$crim
```

```
cv.out <- cv.glmnet(x.train, y.train, alpha=0) # Ridge
```

```
plot(cv.out)
```



```
best.lambda <- cv.out$lambda.min
```

```
# predict the model on test
```

```
pred.ridge <- predict(cv.out, s=best.lambda, newx=x.test)
```

```
sprintf("Ridge test cv_mse for best lambda: %s", best.lambda)
```

```
## [1] "Ridge test cv_mse for best lambda: 0.591915933409596"
```

```
ridge.mse <- mean((pred.ridge - y.test)^2)
```

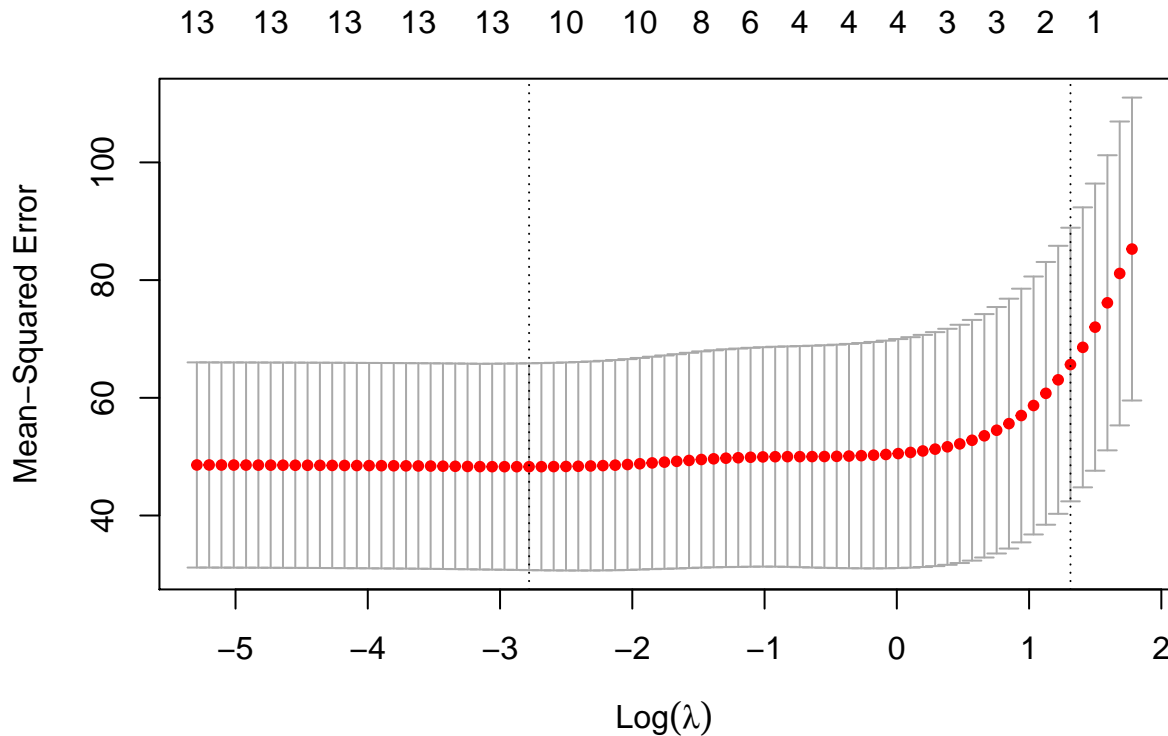
```
sprintf("-----Ridge MSE on test data for model with best lambda %s is %s",  
        best.lambda, ridge.mse)
```

```
## [1] "-----Ridge MSE on test data for model with best lambda 0.591915933409596 is 40.9277664623"
```

```
print("fit Lasso regression model on training set and get test.mse ")
```

```
## [1] "fit Lasso regression model on training set and get test.mse "
```

```
cv.out <- cv.glmnet(x.train, y.train, alpha=1) # Lasso
plot(cv.out)
```



```
best.lambda <- cv.out$lambda.min
```

```
# predict the model on test
```

```
pred.lasso <- predict(cv.out, s=best.lambda, newx=x.test)
```

```
sprintf("Lasso test cv_mse for best lambda: %s", best.lambda)
```

```
## [1] "Lasso test cv_mse for best lambda: 0.0620100453480678"
```

```
lasso.mse <- mean((pred.lasso - y.test)^2)
```

```
sprintf( "-----Lasso MSE on test data for model with best lambda %s is %s",
         best.lambda, lasso.mse)
```

```
## [1] "-----Lasso MSE on test data for model with best lambda 0.0620100453480678 is 40.931404807"
```

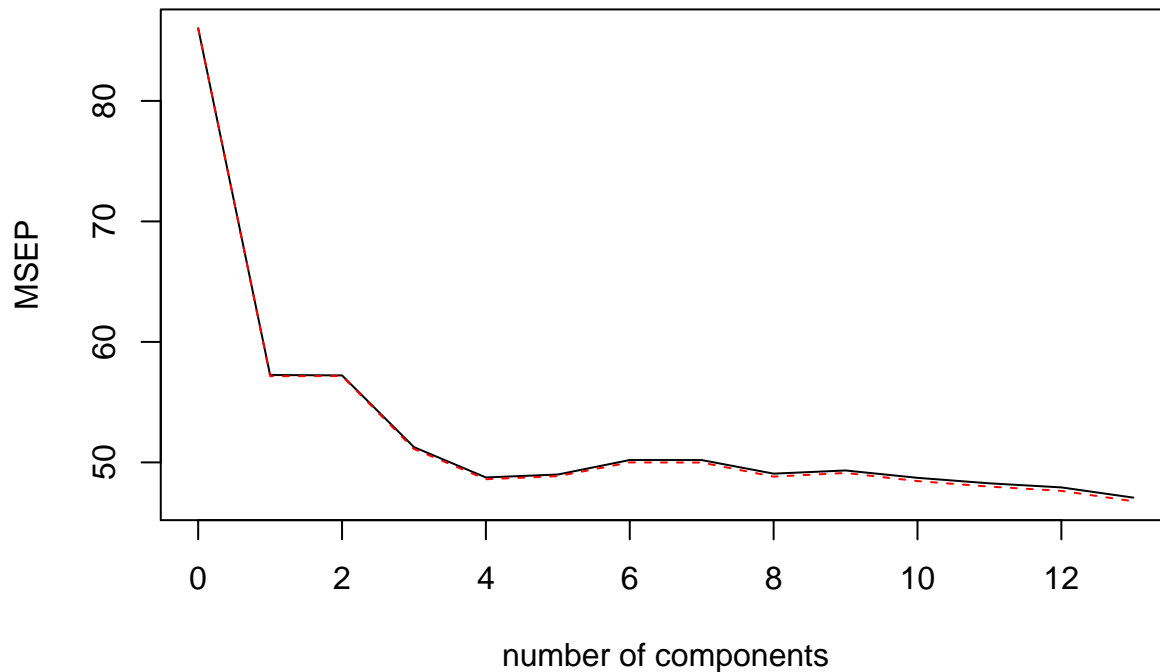
```
print("Fit a PCR model on training set with M chosen by CS and get test.mse")
```

```
## [1] "Fit a PCR model on training set with M chosen by CS and get test.mse"
```

```
pcr.fit <- pcr (crim~., data=df.train, scale=T, validation="CV")
```

```
validationplot(pcr.fit, val.type="MSEP")
```

crim



```
print("Minimum CV Root MSE is for M=4 components which is 50 so CV MSE is 2500")
```

```
## [1] "Minimum CV Root MSE is for M=4 components which is 50 so CV MSE is 2500"
```

```
# Now apply model with M=17 on test data and calculate MSE'
```

```
M = 4
```

```
pcr.pred <- predict(pcr.fit, x.test, ncomp = M)
```

```
pcr.mse <- mean((pcr.pred - y.test)^2)
```

```
sprintf("pcr test mse for best number of component: %s is %s:", M, pcr.mse)
```

```
## [1] "pcr test mse for best number of component: 4 is 43.9110729287546:"
```

```
sprintf("b) best subset selection has the best test MSE: %s
        comparing with Ridge test MSE %s, Lasso test MSE %s
        and PCR test %s MSEtherefore we prefer it to other three",
        mse.best.subset, ridge.mse, lasso.mse, pcr.mse)
```

```
## [1] "b) best subset selection has the best test MSE: 41.4927142070165 \n
```

comparing with Ridge

```
sprintf("c) Best subset selection only includes %s number of components
        probably because other features add noise",M)
```

```
## [1] "c) Best subset selection only includes 4 number of components\n
```

probably because other

```
# createDataPartition(
```

```
# y,
```

```
# times = 1,
```

```
# p = 0.5,
```

```
# list = TRUE,
```

```
# groups = min(5, length(y))
```

```
# )
```

```
# createFolds(y, k = 10, list = TRUE, returnTrain = FALSE)
```

```

#
# createMultiFolds(y, k = 10, times = 5)
#
# createTimeSlices(y, initialWindow, horizon = 1, fixedWindow = TRUE, skip = 0)
#
# groupKfold(group, k = length(unique(group)))
#
# createResample(y, times = 10, list = TRUE)
# =====
# Arguments
# y: a vector of outcomes. For createTimeSlices, these should be in
#    chronological order.
# times: the number of partitions to create
# p : the percentage of data that goes to training
# list : logical - should the results be in a list (TRUE) or a matrix with the
#        number of rows equal to floor(p * length(y)) and times columns.
# groups: for numeric y, the number of breaks in the quantiles (see below)
# k: an integer for the number of folds.
# returnTrain : a logical. When true, the values returned are the sample
#                positions corresponding to the data used during training.
#                This argument only works in conjunction with list = TRUE
# initialWindow: Initial number of consecutive values in each training set sample
# horizon: Number of consecutive values in test set sample
# fixedWindow: logical, if FALSE, all training samples start at 1
# skip: integer, how many (if any) resamples to skip to thin the total amount
# group: a vector of groups whose length matches the number of rows in the
#        overall data set.

```

```

library(tidyverse)
library(caret)

```

```

## Loading required package: lattice

##
## Attaching package: 'lattice'

## The following object is masked from 'package:boot':
##
##     melanoma

##
## Attaching package: 'caret'

## The following object is masked from 'package:pls':
##
##     R2

## The following object is masked from 'package:purrr':
##
##     lift

```

```

library(glmnet)

```

```

data(oil)
createDataPartition(oilType, 2)

```



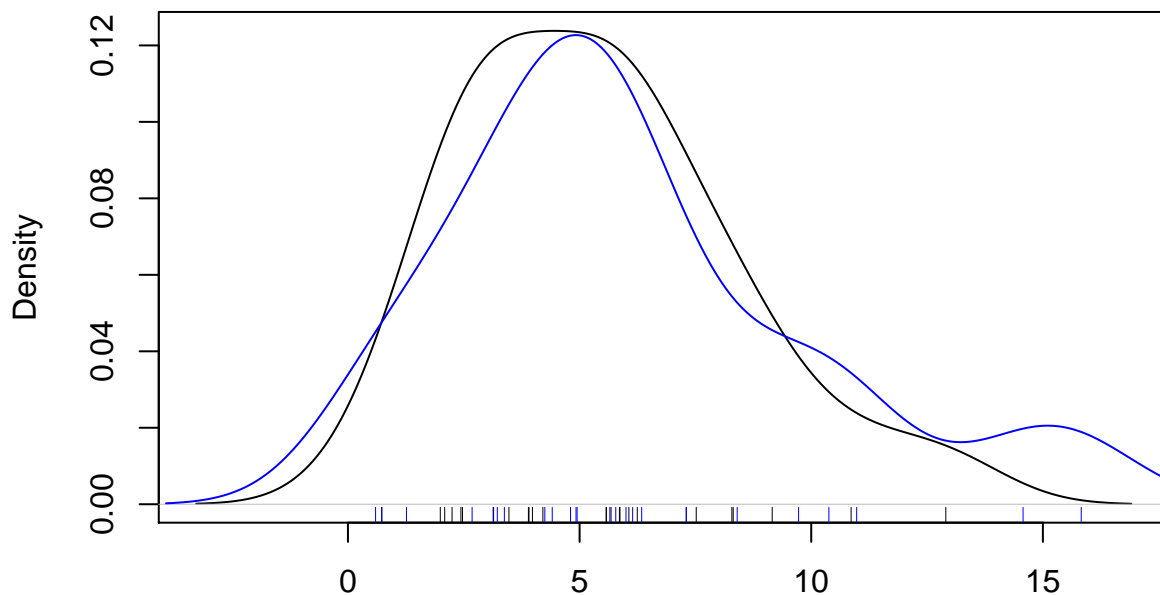
```
## $Resample1
## [1] 3 8 9 10 11 13 14 15 16 21 22 23 25 26 27 28 29 31 34 36 40 41 42 43 45
## [26] 49 51 52 54 58 59 62 63 64 67 69 71 74 75 76 77 78 81 82 83 86 88 90 95 96
##
## $Resample2
## [1] 5 7 9 10 11 12 14 15 17 19 21 22 24 25 29 30 35 38 40 41 42 43 46 47 48
## [26] 51 53 55 57 59 60 61 67 68 69 70 73 74 75 76 78 79 82 86 87 88 92 93 95 96

x <- rgamma(50, 3, .5)
inA <- createDataPartition(x, list = FALSE)

plot(density(x[inA]))
rug(x[inA])

points(density(x[-inA]), type = "l", col = 4)
rug(x[-inA], col = 4)
```

density.default(x = x[inA])



N = 26 Bandwidth = 1.335

```
createResample(oilType, 2)
```

```
## $Resample1
## [1] 2 2 5 5 6 7 8 9 9 10 10 11 13 15 15 16 16 17 20 21 22 22 25 25 27
## [26] 30 33 34 34 36 36 37 38 39 39 40 40 41 42 43 43 43 44 45 46 46 46 47 47 47
## [51] 49 50 50 54 55 59 59 60 60 62 62 63 64 64 66 67 67 68 68 70 72 75 77 78 79
## [76] 79 79 80 80 82 85 85 86 87 87 87 88 88 89 90 90 91 92 95 95 96
##
## $Resample2
## [1] 2 3 4 4 4 5 5 8 9 11 12 14 15 15 17 18 18 18 18 19 19 21 24 24 24
## [26] 25 26 27 27 29 29 30 34 34 35 38 38 41 41 41 42 45 46 46 47 47 47 47 50 51
## [51] 52 52 53 57 58 59 59 59 60 62 63 63 67 68 68 70 72 73 74 74 75 77 78 79 79
```

```
## [76] 80 80 80 82 83 84 85 85 85 86 87 88 89 89 90 90 92 94 94 96 96
```

```
createFolds(oilType, 10)
```

```
## $Fold01
## [1] 3 15 20 23 49 53 61 72 74 84 94
##
## $Fold02
## [1] 7 10 21 27 46 54 60 69 75 78 86
##
## $Fold03
## [1] 2 5 16 17 22 30 45 57 77
##
## $Fold04
## [1] 14 37 38 43 51 64 81 82 90 92
##
## $Fold05
## [1] 4 29 50 56 65 66 68 76 83 89 91
##
## $Fold06
## [1] 6 12 31 33 39 40 42 47 59
##
## $Fold07
## [1] 1 8 13 28 44 52 85 87
##
## $Fold08
## [1] 19 34 41 55 63 67 73 80 93 96
##
## $Fold09
## [1] 9 11 24 26 32 70 79 95
##
## $Fold10
## [1] 18 25 35 36 48 58 62 71 88
```

```
createFolds(oilType, 5, FALSE)
```

```
## [1] 3 3 2 2 1 5 4 2 3 5 5 3 4 3 4 1 4 5 5 1 2 4 3 5 2 4 3 4 3 4 3 5 1 4 5 1 5 1
## [39] 2 1 4 4 1 2 1 4 1 3 1 5 3 3 1 4 5 2 3 4 3 1 4 2 2 4 2 3 4 5 1 3 5 3 2 4 1 2
## [77] 3 3 1 3 1 2 2 4 2 3 4 5 1 5 2 5 5 1 2 5
```

```
createFolds(rnorm(21))
```

```
## $Fold01
## [1] 2 21
##
## $Fold02
## [1] 6 11
##
## $Fold03
## [1] 1 5 8
##
## $Fold04
## [1] 17 19
##
## $Fold05
## [1] 7 10
```

```
##
## $Fold06
## [1] 3 16
##
## $Fold07
## [1] 15 20
##
## $Fold08
## [1] 4 12
##
## $Fold09
## [1] 14 18
##
## $Fold10
## [1] 9 13

createTimeSlices(1:9, 5, 1, fixedWindow = FALSE)
```

```
## $train
## $train$Training5
## [1] 1 2 3 4 5
##
## $train$Training6
## [1] 1 2 3 4 5 6
##
## $train$Training7
## [1] 1 2 3 4 5 6 7
##
## $train$Training8
## [1] 1 2 3 4 5 6 7 8
##
##
## $test
## $test$Testing5
## [1] 6
##
## $test$Testing6
## [1] 7
##
## $test$Testing7
## [1] 8
##
## $test$Testing8
## [1] 9
```

```
createTimeSlices(1:9, 5, 1, fixedWindow = TRUE)
```

```
## $train
## $train$Training5
## [1] 1 2 3 4 5
##
## $train$Training6
## [1] 2 3 4 5 6
##
## $train$Training7
```

```
## [1] 3 4 5 6 7
##
## $train$Training8
## [1] 4 5 6 7 8
##
##
## $test
## $test$Testing5
## [1] 6
##
## $test$Testing6
## [1] 7
##
## $test$Testing7
## [1] 8
##
## $test$Testing8
## [1] 9
```

```
createTimeSlices(1:9, 5, 3, fixedWindow = TRUE)
```

```
## $train
## $train$Training5
## [1] 1 2 3 4 5
##
## $train$Training6
## [1] 2 3 4 5 6
##
##
## $test
## $test$Testing5
## [1] 6 7 8
##
## $test$Testing6
## [1] 7 8 9
```

```
createTimeSlices(1:9, 5, 3, fixedWindow = FALSE)
```

```
## $train
## $train$Training5
## [1] 1 2 3 4 5
##
## $train$Training6
## [1] 1 2 3 4 5 6
##
##
## $test
## $test$Testing5
## [1] 6 7 8
##
## $test$Testing6
## [1] 7 8 9
```

```
createTimeSlices(1:15, 5, 3)
```

```
## $train
```

```

## $train$Training05
## [1] 1 2 3 4 5
##
## $train$Training06
## [1] 2 3 4 5 6
##
## $train$Training07
## [1] 3 4 5 6 7
##
## $train$Training08
## [1] 4 5 6 7 8
##
## $train$Training09
## [1] 5 6 7 8 9
##
## $train$Training10
## [1] 6 7 8 9 10
##
## $train$Training11
## [1] 7 8 9 10 11
##
## $train$Training12
## [1] 8 9 10 11 12
##
##
## $test
## $test$Testing05
## [1] 6 7 8
##
## $test$Testing06
## [1] 7 8 9
##
## $test$Testing07
## [1] 8 9 10
##
## $test$Testing08
## [1] 9 10 11
##
## $test$Testing09
## [1] 10 11 12
##
## $test$Testing10
## [1] 11 12 13
##
## $test$Testing11
## [1] 12 13 14
##
## $test$Testing12
## [1] 13 14 15

createTimeSlices(1:15, 5, 3, skip = 2)

## $train
## $train$Training05
## [1] 1 2 3 4 5

```

```

##
## $train$Training08
## [1] 4 5 6 7 8
##
## $train$Training11
## [1] 7 8 9 10 11
##
##
## $test
## $test$Testing05
## [1] 6 7 8
##
## $test$Testing08
## [1] 9 10 11
##
## $test$Testing11
## [1] 12 13 14

createTimeSlices(1:15, 5, 3, skip = 3)

## $train
## $train$Training5
## [1] 1 2 3 4 5
##
## $train$Training9
## [1] 5 6 7 8 9
##
##
## $test
## $test$Testing5
## [1] 6 7 8
##
## $test$Testing9
## [1] 10 11 12

set.seed(131)
groups <- sort(sample(letters[1:4], size = 20, replace = TRUE))
table(groups)

## groups
## a b c d
## 6 5 4 5

folds <- groupKfold(groups)
lapply(folds, function(x, y) table(y[x]), y = groups)

## $Fold1
##
## b c d
## 5 4 5
##
## $Fold2
##
## a c
## 6 4
##

```

```
## $Fold3
##
## a b d
## 6 5 5
```