

A Technical Note for The Computable General Equilibrium (CGE) modeling: The Shoven-Whalley framework for the system of a Power Generator

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Let us consider a system, from which we require the minimum profit. Price for per unit can be measured in terms of the value in the market. Does that system give us the required utility? Here, the utility is also measured in terms of the value in that market.

Here, I have considered a Power Generating Process of a generator, perhaps a Geo-Thermal power generator for example.

I have considered the hypothetical data from a complete one cycle, in the form of a Micro Consistent Matrix (MCM). Therefore the price has it's own dynamic nature. The whole procedure follows the Computable General Equilibrium (CGE) in the form of The Shoven-Whalley framework.

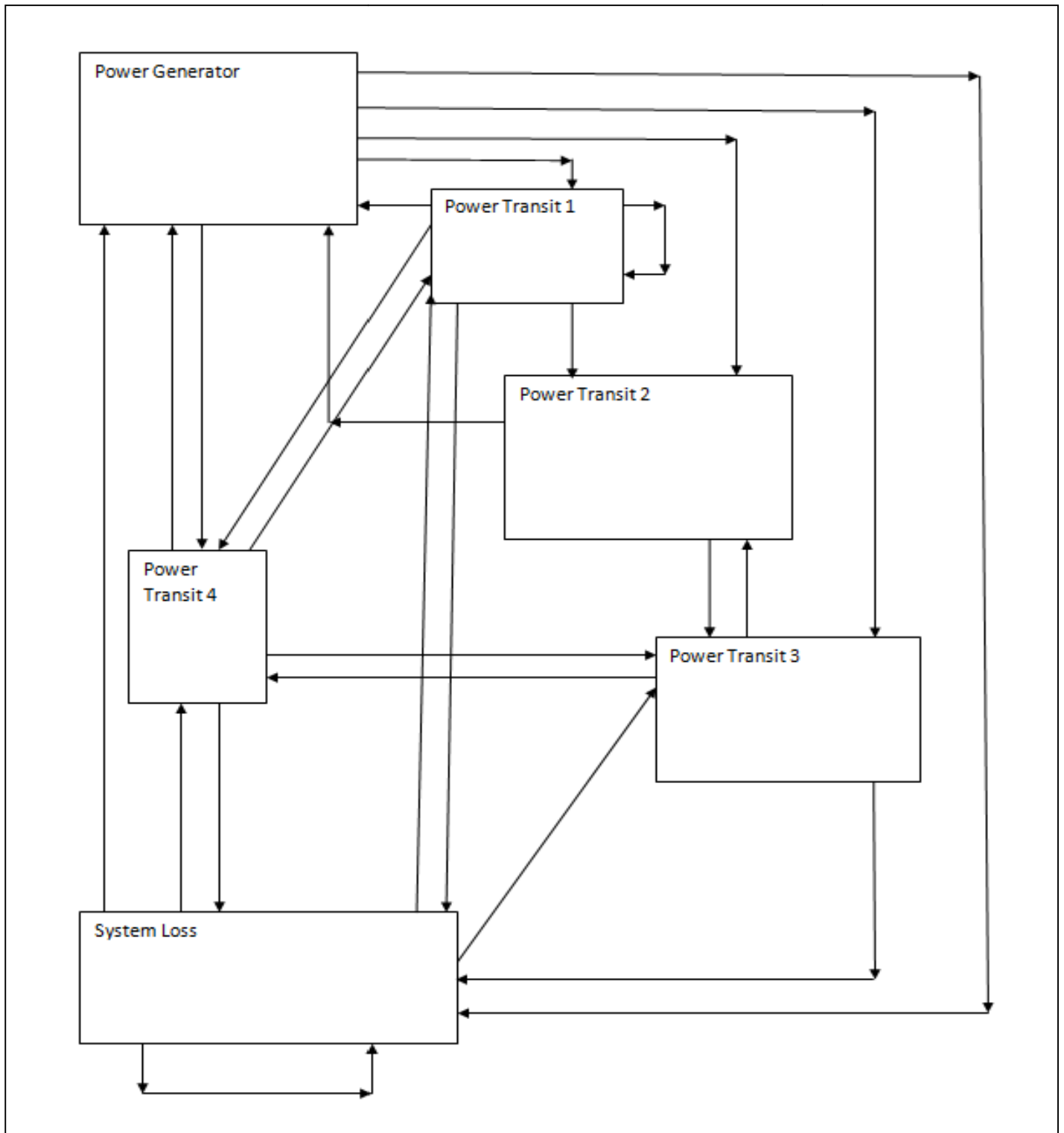


Diagram of the hypothetical system of a Power Generator

Use Matrix	Power Transit 1	Power Transit 2	Power Transit 3	Power Transit 4	Power Generat or	System Loss	Total
Power Transit 1	0	0	0	0	10	6	16
Power Transit 2	0	0	0	0	10	7	17
Power Transit 3	0	2	0	0	0	30	32
Power Transit 4	10	0	15	0	0	0	25
Power Generator	0	0	0	8	0	17	25
Prime Source	0	0	0	0	50	0	50
System Loss	2	0	1	0.5	2	3	8.5
Total	12	2	16	8.5	72	63	173.5

Make Matrix	Power Transit 1	Power Transit 2	Power Transit 3	Power Transit 4	Power Generato r	System Loss	Total
Power Transit 1	1	0	0	0	15	0	16
Power Transit 2	2	0	0	0	15	0	17
Power Transit 3	0	2	0	0	30	0	32
Power Transit 4	1	0	11	0	2	11	25
Power Generator	10	0	0	15	0	0	25
Prime Source	0	0	0	0	5	45	50
System Loss	2	0	1	0.5	5	0	8.5
Total	16	2	12	15.5	72	56	173.5

Let, The price vector \mathbf{p} for the relevant input and output

The quantity of the use vector \mathbf{k} for any input

The quantity of the make vector \mathbf{q} for any output

I have separated the system loss and add an ultimate source of the power, called 'Prime Source' in order to balance the transformation of the power, which keeps the total amount of the power fixed.

The quantity of the use vector $\underline{\mathbf{k}}$ for the system loss

The quantity of the make vector $\underline{\mathbf{q}}$ for the system loss

The total use of any input θ

The total make of any output H

The total use of any extra input $\underline{\theta}$

The total make of any extra output \underline{h}

Profit from the system:

$$\left\{ \underline{\pi} \mid \mathbf{p} \in R_{++}^n, \mathbf{q} \in R_+^n, \mathbf{k} \in R_+^n, \underline{\pi} = \sum \{ (\mathbf{p} \cdot \mathbf{q}) - (\mathbf{p} \cdot \mathbf{k}) \} \geq 0 \right\}$$

Profit from the system loss:

$$\left\{ \underline{\pi} \mid \mathbf{p} \in R_{++}^n, \underline{\mathbf{q}} \in R_+^n, \underline{\mathbf{k}} \in R_+^n, \underline{\pi} = \sum \{ (\mathbf{p} \cdot \underline{\mathbf{q}}) - (\mathbf{p} \cdot \underline{\mathbf{k}}) \} \geq 0 \right\}$$

System loss adjustment:

$$\begin{aligned} & \{ \theta > H \mid \theta \in R_+, H \in R_+, \underline{\theta} \in R_+, \underline{h} \in R_+, k \in \theta, q \in H, q \in (H + \underline{h}), k \in (\theta - \underline{\theta}) \} \\ & \{ \theta < H \mid \theta \in R_+, H \in R_+, \underline{\theta} \in R_+, \underline{h} \in R_+, k \in \theta, q \in H, q \in (H - \underline{h}), k \in (\theta + \underline{\theta}) \} \\ & \{ \theta = H \mid \theta \in R_+, H \in R_+, \underline{\theta} \in R_+, \underline{h} \in R_+, k \in \theta, q \in H \} \end{aligned}$$

Utility maximization:

$$\left\{ \max \mathbf{U} \mid x_i \in R_+^n, \rho < 1, U = \left(\sum_{i=1}^n x_i^\rho \right)^{\frac{1}{\rho}} \rightarrow R \right\}$$

$$\{x^i(\mathbf{p}, y) \mid (\mathbf{p}, y) \in R_{++}^n \times R_+^n\}$$

$$\{x_i(\mathbf{p}, y) \mid x_i \in R_+^n, u(x_0) \geq u(x_1) \Leftrightarrow x_0 \succeq x_1, i = 0, 1, \dots, n\}$$

Optimum supply (activities) from the system:

$$\hat{x}_i(\hat{\mathbf{p}}, y) = \frac{\hat{p}_i^{r-1} \cdot y}{\sum_{i=1}^N \hat{p}_i^r} \dots\dots\dots (1)$$

Indirect utility:

$$\left\{ V \mid y \in R_+, \hat{\mathbf{p}} \in R_{++}^n, \rho < 1, r \equiv \frac{\rho}{\rho - 1}, V(\hat{\mathbf{p}}, y) \in R \right\}$$

$$V(\hat{\mathbf{p}}, y) = y \left[\frac{\sum_{i=1}^n \hat{p}_i^r}{(\sum_{i=1}^N \hat{p}_i^r)^\rho} \right]^{\frac{1}{\rho}} \dots\dots\dots (2)$$

The hypothetical data came from one cycle of the power generating process of the dynamic system of the Generator. Therefore to solve the system of the

equations, the coefficients we will need, could be a question, from where those coefficients will come.

One possible solution of this problem, we can use the technical coefficients.

The following two technical coefficient matrixes have one same base line, the total input intake for each part of the system.

Technical Coefficient of Use Matrix						
0.00000	0.00000	0.00000	0.00000	0.13889	0.09524	0.09222
0.00000	0.00000	0.00000	0.00000	0.13889	0.11111	0.09798
0.00000	1.00000	0.00000	0.00000	0.00000	0.47619	0.18444
0.83333	0.00000	0.93750	0.00000	0.00000	0.00000	0.14409
0.00000	0.00000	0.00000	0.94118	0.00000	0.26984	0.14409
0.00000	0.00000	0.00000	0.00000	0.69444	0.00000	0.28818
0.16667	0.00000	0.06250	0.05882	0.02778	0.04762	0.04899
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

Technical Coefficient of Make Matrix to Use Matrix						
0.08333	0.00000	0.00000	0.00000	0.20833	0.00000	0.09222
0.16667	0.00000	0.00000	0.00000	0.20833	0.00000	0.09798
0.00000	1.00000	0.00000	0.00000	0.41667	0.00000	0.18444
0.08333	0.00000	0.68750	0.00000	0.02778	0.17460	0.14409
0.83333	0.00000	0.00000	1.76471	0.00000	0.00000	0.14409
0.00000	0.00000	0.00000	0.00000	0.06944	0.71429	0.28818
0.16667	0.00000	0.06250	0.05882	0.06944	0.00000	0.04899
1.33333	1.00000	0.75000	1.82353	1.00000	0.88889	1.00000

To find out the optimum price level, I have used the solver, `[x, fval, info] = fsolve (@f, [...])` to find out the optimum price level from a system of equations.

<p>Optimum price level :</p> <p>-3.6458e-001 \approx 0 1.8229e-001 \approx 0 5.9778e+000 -7.2790e-010 \approx 0 -2.2882e-010 \approx 0 4.1008e+000 2.0373e+000</p>	<p>$\rho=0.2$ (given)</p> <p>Optimum activity level :</p> <p>-0.5603374 + 0.0032408i 0.94782 + 0.93692i 0.012078 + 0.011939i -4.1986e+010 + 2.4283e+008i -1.7837e+011 + 1.0316e+009i 0.019346 + 0.019124i 0.046383 + 0.045849i</p>
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Indirect Utility:

-3.0460e+012 - 7.0480e+010i

References:

1. Thomas F. Rutherford, Calibrated CES Utility Functions: A Worked Example, Zürich, Switzerland, March 24, 2008.
2. Ronald E. Miller, Peter D. Blair, **Input-Output Analysis** Foundations and Extensions, Second Edition, CAMBRIDGE UNIVERSITY PRESS, 2009.
3. Geoffrey A. Jehle and Philip J. Reny, Advanced Microeconomic Theory, Pearson Education Limited, 2011.

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