Created by: shahrear
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source code: saddlediag.py
https://github.com/shahrear86/Dynamics
Reference:
Shone,Ronald,Economic Dynamics Phase Diagrams and Their Economic Application ,Second Edition,2002,Chambridge University Press,pp.171

$\dot{x} = x + y$

$\dot{y} = 4x + y$

$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

$\Rightarrow det(A) = -3$

$\Rightarrow A - \lambda.I = \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$

$\Rightarrow det(A - \lambda.I) = \lambda^2 - 2\lambda - 3$

$\Rightarrow \qquad = (\lambda - 3)(\lambda + 1)$

$\Rightarrow \qquad = 0$

Hence $\lambda = r = 3$ and $\lambda = s = -1$.

For $\lambda = r = 3$

$(A - \lambda.I)\,\nu^r = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \nu^r = 0$

$-2\nu_1^r + \nu_2^r = 0$

$4\nu_1^r - 2\nu_2^r = 0$

$\nu_1^r = 1$

$\nu_2^r = 2$

Hence one solution is

$u^1 = e^{rt} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

and

$\nu^r = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

For $\lambda = s = -1$

$(A - \lambda.I)\,\nu^s = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \nu^s = 0$

$2\nu_1^s + \nu_2^s = 0$

$4\nu_1^s + 2\nu_2^s = 0$

$\nu_1^s = 1$

$\nu_2^s = -2$

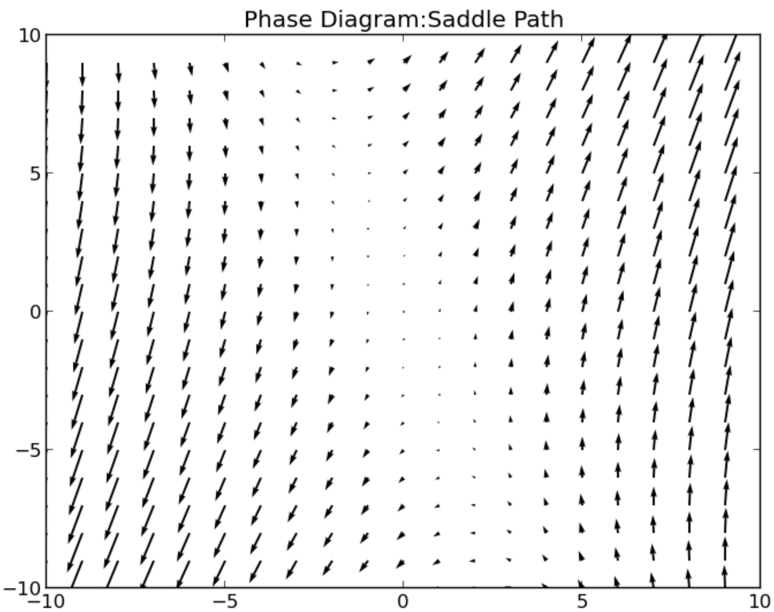
Hence, a second solution is

$u^2 = e^{st} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

and

$\nu^s = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

In phase diagram,
Unstable arm of the saddle is the line through the eigen vector ν^r
Stable arm of the saddle is the line through the eigen vector ν^s



Source Code:
```
# -*- coding: utf-8 -*- """ Created on Fri Apr 6 13:49:49 2018
@author: shahrear
reference books:
1.Economic Dynamics Phase Diagrams and Their Economic Application
Second Edition,
2002
by RONALD SHONE
University of Stirling
Chapter-4: Systems of first-order differential equations
Page-171
Example 4.13
2.User's Guide NumPy
User Guide Release 1.11.0
Written by the NumPy community
May 29, 2016
3.User's Guide Matplotlib
Release 2.1.0
by John Hunter, Darren Dale, Eric Firing, Michael Droettboom and the m
October 07, 2017
"""
```

```
import numpy as np
import matplotlib.pyplot as plt
X, Y = np.meshgrid(np.arange(-10, 10, 1), np.arange(-10, 10, 1))
Xdot=X+Y
Ydot=4*X+Y
plt.figure()
plt.title('Phase Diagram:Saddle Path')
Q=plt.quiver(X,Y,Xdot,Ydot,units='width')
plt.show()
```