# Bayesian Econometric Analysis (Mathematical Structure)

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September 25, 2020

Note : 1

$$\{\mathbf{A}|\mathbf{A} = \{a_1, a_2, \ldots, a_n\}\,,\ \mathbf{A} \in \mathbb{R}\}$$

$$\{\mathbf{B}|\mathbf{B} = \{b_1, b_2, \ldots, b_n\}\,, \mathbf{B}\ \in \mathbb{R}\}$$

$$\{(\mathbf{A} \times \mathbf{B})|\,\{a_1, a_2, \ldots, a_n\} \times \{b_1, b_2, \ldots, b_n\}\,, (\mathbf{A} \times \mathbf{B}) \to d(\mathbf{A} \times \mathbf{B})\}$$

$$\left\{f^0(x^0)|(\mathbf{A} \times \mathbf{B}) \to f^0(x^0), f^0(x^0) \Leftrightarrow k^0(f^0(x^0))\right\}$$

$$\left\{\theta_k^0|k^0(f^0(x^0)) \longrightarrow \theta_k^0/x_0\,, \theta_k^0 \in \mathbb{R}, x_0 \in \mathbb{R}\right\}$$

$$\{\mu^0|\mu^0 \in (x^0 \cap x), x^0\neg \perp x, (x^0 \cap x) \neq \{\}, x^0 \precsim x, \overline{x}^0 \sim \overline{x}, \mu^0 \succeq \bar{x}^0,$$

$$\mu^0 \succeq \bar{x}, \bar{x}^0 \in (\mu^0 \pm \sigma^0.r), \bar{x} \in (\mu^0 \pm \sigma.r), r \in \mathbb{R}^{\mathbb{L}}, x^0 \to \sigma^0, x \to \sigma\}$$

$$\left\{\boldsymbol{\beta}|\,\{\mathbf{Y}, \mathbf{X}\} \to \boldsymbol{\beta}, \mathbf{Y} \in \mathbb{R}^{\mathbb{L}}, \mathbf{X} \in \mathbb{R}^{\mathbb{L}}, \mathbf{Y} = \boldsymbol{\beta}\mathbf{X} + \mathbf{U}\right\}$$

$$\left\{\hat{\boldsymbol{\beta}}|f(\mathbf{Y}, \mathbf{X}) \to \hat{\boldsymbol{\beta}}, \mathbf{X} \cap \mathbf{U} = \{\}\right\}$$

$$\left\{\bar{\boldsymbol{\beta}}|\left\{\hat{\boldsymbol{\beta}}, \sigma^2, \underline{\boldsymbol{\beta}}, \mathbf{X}\right\} \to \bar{\boldsymbol{\beta}}, \sigma_0^2 \sim \mathbf{NG}(\ldots), or\, \sigma_0^2 \sim \mathbf{G}(\ldots), or\, \sigma_0^2 \sim \mathbf{N}(\ldots)\right\}$$

$$\mathbf{b} = \hat{\boldsymbol{\beta}} = (\mathbf{X}^{'}\mathbf{X})^{-1}\mathbf{X}^{'}\mathbf{Y}$$

$$\underline{\boldsymbol{V}} = \frac{v_0 \sigma_0^2 + \sum_{i=1}^{n} (x_i - \mu)^2}{v_0 + n}$$

$$\bar{\boldsymbol{\beta}} = \frac{\underline{\boldsymbol{V}}^{-1}}{\underline{\boldsymbol{V}}^{-1} + \sum x_i^2} . \underline{\boldsymbol{\beta}} + \hat{\boldsymbol{\beta}} . \frac{\sum x_i^2}{\underline{\boldsymbol{V}}^{-1} + \sum x_i^2}$$

$$Z_i, i = 1, 2, \ldots, N$$

$$\bar{m} = \frac{1}{N} \sum_{i=1}^{N} Z_i$$

$$v = N$$

$$m^* = \left( \frac{v}{v + T} \right) m + \left( \frac{T}{v + T} \right) \bar{y}$$

$$\bar{y} = \frac{1}{T} \sum_{i=1}^{T} y_t$$

$$\bar{\mu} = m^* + \mu^0 \times \left( \frac{\sigma^2}{v + T} \right)^{\frac{1}{2}}$$

$$\sigma_0^2 = \frac{v_0 \sigma_0^2 + \sum_{i=1}^{n} (x_i - \bar{x})^2 + \frac{n_0 . n}{n_0 + n} (\mu_0 - \bar{x})^2}{v_0 + n}$$

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$$y_t = x_t' \boldsymbol{\beta} + u_t$$

$$u_t \sim i.i.d.\, N(0, \sigma^2)$$

$$y_{T \times 1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}$$

$$x_{T\times 1} = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_T' \end{bmatrix}$$

$$f\left(\mathbf{y}|\boldsymbol{\beta},\mathbf{X};\sigma^2\right) = \prod_{t=1}^{T} \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](y_t - x_t'\boldsymbol{\beta})^2\right\}$$

$$f\left(\mathbf{y}|\boldsymbol{\beta},\mathbf{X};\sigma^2\right) = \frac{1}{(2\pi\sigma^2)^{\frac{T}{2}}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](y_t - x_t'\boldsymbol{\beta})'(y_t - x_t'\boldsymbol{\beta})\right\}$$

$$\boldsymbol{\beta} \sim N(\mathbf{b},\sigma^2\mathbf{M})$$

$$f\left(\boldsymbol{\beta};\sigma^2\right) = \frac{1}{(2\pi\sigma^2)^{\frac{k}{2}}} \mid \mathbf{M} \mid^{-\frac{1}{2}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\boldsymbol{\beta}-\mathbf{b})' \mid \mathbf{M} \mid^{-1} (\boldsymbol{\beta}-\mathbf{b})\right\}$$

$$f(\mathbf{y}|\mathbf{X};\sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{T}{2}}} \mid \mathbf{I}_T + \mathbf{XMX}' \mid^{-\frac{1}{2}} \times \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\mathbf{y}-\mathbf{Xb})' \mid \mathbf{I_T} + \mathbf{XMX}' \mid^{-1} (\mathbf{y}-\mathbf{Xb})\right\}$$

$$E(\boldsymbol{\beta}|\mathbf{y},\mathbf{X}) = \mathbf{b}^* = (\mathbf{M}^{-1} + \mathbf{X}'\mathbf{X})^{-1}(\mathbf{M^{-1}b} + \mathbf{X}'\mathbf{y})$$

$$\boldsymbol{\beta} \sim N(\mathbf{b}^*, \sigma^2(\mathbf{M}^{-1} + \mathbf{X}'\mathbf{X})^{-1})$$

$$\mathbf{y} \sim N(\mathbf{Xb}^*, \sigma^2(\mathbf{I}_T + \mathbf{XMX}')^{-1})$$

$$\mathbf{M^{-1}} = \lambda.\mathbf{I_k}$$

$$\lambda > 0$$

$$\boldsymbol{\beta} \sim N(\mathbf{b}^*, \sigma^2(\mathbf{M}^{-1} + \mathbf{X}'\mathbf{X})^{-1})$$

$$\mathbf{b}^* = (\lambda\mathbf{I_k} + \mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

$$f(\boldsymbol{\beta}|\sigma^2,\mathbf{y},\mathbf{X}) = \frac{1}{(2\pi\sigma^2)^{\frac{K}{2}}}|\mathbf{M}^*|^{-\frac{1}{2}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\mathbf{b}^*-\mathbf{b})^{'}(\mathbf{M}^*)^{-1}(\mathbf{b}^*-\mathbf{b})\right\}$$

$$f(\sigma^2|\mathbf{y},\mathbf{X}) = \frac{\sigma^{-2|(N^*/2)-1|}\,(\lambda^*/2)^{N^*/2}}{\Gamma(N^*/2)} \exp\left[-\lambda^*\sigma^{-2}/2\right]$$

$$N^* = N + T$$

$$\lambda^* = \lambda + (\mathbf{y}-\mathbf{Xb}^*)'(\mathbf{y}-\mathbf{Xb}^*) + (\mathbf{b}^*-\mathbf{b})'\mathbf{M}^{-1}(\mathbf{X}'\mathbf{X}+\mathbf{M}^{-1})^{-1}\mathbf{X}'\mathbf{X}(\mathbf{b}^*-\mathbf{b})$$

$$f(\mathbf{b}^*|\sigma^2,\mathbf{y},\mathbf{X}) = \frac{1}{(2\pi\sigma^2)^{\frac{K}{2}}}|\mathbf{M}^*|^{-\frac{1}{2}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\mathbf{b}^*-\mathbf{b})^{'}(\mathbf{M}^*)^{-1}(\mathbf{b}^*-\mathbf{b})\right\}$$

$$f(\sigma^2|\mathbf{y},\mathbf{X}) = \frac{\sigma^{-2|(N^*/2)-1|}(\lambda^*/2)^{N^*/2}}{\Gamma(N^*/2)} \exp\left[-\lambda^*\sigma^{-2}/2\right]$$

$$f(\mathbf{y}|\mathbf{X};\sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{T}{2}}} \mid \mathbf{I}_T{+}\mathbf{XMX'} \mid^{-\frac{1}{2}} \times \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\mathbf{y}-\mathbf{Xb}^*)' \mid \mathbf{I_T}+\mathbf{XMX'} \mid^{-1}(\mathbf{y}-\mathbf{Xb}^*)\right\}$$

$$f(\mathbf{y},\mathbf{b}^*,\sigma^2|\mathbf{X}) = \left\{f(\mathbf{b}^*|\sigma^2,\mathbf{y},\mathbf{X}).f(\sigma^2|\mathbf{y},\mathbf{X}).f(\mathbf{y}|\mathbf{X};\sigma^2)\right\}$$

$$f(\mathbf{y},\mathbf{b}^*,\sigma^2|\mathbf{X}) = \frac{1}{(2\pi\sigma^2)^{\frac{K}{2}}}|\mathbf{M}^*|^{-\frac{1}{2}} \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\boldsymbol{\beta}-\mathbf{b}^*)^{'}(\mathbf{M}^*)^{-1}(\boldsymbol{\beta}-\mathbf{b}^*)\right\}$$

$$\times\frac{\sigma^{-2|(N^*/2)-1|}(\lambda^*/2)^{N^*/2}}{\Gamma(N^*/2)} \exp\left[-\lambda^*\sigma^{-2}/2\right]$$

$$\times\frac{1}{(2\pi\sigma^2)^{\frac{T}{2}}} \mid \mathbf{I_T}{+}\mathbf{XMX'} \mid^{-\frac{1}{2}} \times \exp\left\{\left[-\frac{1}{2\sigma^2}\right](\mathbf{y}-\mathbf{Xb}^*)' \mid \mathbf{I_T}+\mathbf{XMX'} \mid^{-1}(\mathbf{y}-\mathbf{Xb}^*)\right\}$$

# References


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