Bayesian Econometric Analysis (Mathematical Structure)

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Note : 2

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$v_0^{'} = v_0 + n$$

$$\Rightarrow v_0^{'} \sigma_0^{'2} = v_0 \sigma_0^2 + \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\Rightarrow \sigma_0^{'2} = \frac{v_0 \sigma_0^2 + \sum_{i=1}^{n} (x_i - \mu)^2}{v_0^{'}}$$

$$\Rightarrow \sigma_0^{'2} = \frac{v_0 \sigma_0^2 + \sum_{i=1}^{n} (x_i - \mu)^2}{v_0 + n}$$

$$S = \sum_{i=1}^{n} (x_i - \mu)^2$$

$$\Rightarrow p\left(\sigma^2 | v_0, \sigma_0^2\right) = \frac{\left(\sigma_0^2 \frac{v_0}{2}\right)^{\frac{v_0}{2}}}{\Gamma\left(\frac{v_0}{2}\right)} \frac{\exp\left[\frac{-v_0 \sigma_0^2}{2\sigma^2}\right]}{1 + \frac{v_0}{2}}$$

$$\Rightarrow p\left(\mathbf{X} | \mu, \sigma^2\right) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} \exp\left[-\frac{S}{2\sigma^2}\right]$$

$$p\left(\sigma^2, \mu | \mathbf{X}\right) = p\left(\mathbf{X} | \mu, \sigma^2\right) . p\left(\sigma^2 | v_0, \sigma_0^2\right)$$

$$\mu_0^{'} = \frac{n_0 \mu_0 + n\overline{x}}{n_0 + n}$$

$$p(\mu | \sigma^2, \mu_0, n_0) = \frac{1}{\sqrt{2\pi \frac{\sigma^2}{n_0}}} \exp\left(-\frac{n_0}{2\sigma^2} (\mu - \mu_0)^2\right)$$

$$p\left(\mathbf{X}|\mu,\sigma^2\right)=\left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}}\exp\left[-\frac{1}{2\sigma^2}\left(\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2+n\left(\overline{x}-\mu\right)^2\right)\right]$$

$$p\left(\mu,\sigma^2|\mathbf{X}\right).p(\mathbf{X})=p\left(\mathbf{X}|\mu,\sigma^2\right)p(\mu|\sigma^2,\mu_0,n_0)$$

$$p\left(\mu,\sigma^2|\mathbf{X}\right).p(\mathbf{X})=p\left(\mathbf{X}|\mu,\sigma^2\right).p(\mu|\sigma^2,\mu_0,n_0)$$

$$\mathbf{b}=\left(M^{-1}+\mathbf{X'X}\right)^{-1}\left(M^{-1}\overline{\mathbf{b}}+\mathbf{X'Y}\right)$$

$$\mathbf{b}-\boldsymbol{\beta}=\frac{\left(\frac{\sigma^2}{v+T}\right)^{-1}\left(\overline{\mathbf{b}}-\mathbf{b}\right)+\mathbf{X'e}}{\mathbf{X'X}}$$

$$\Rightarrow\mathbf{b}-\boldsymbol{\beta}=\frac{\frac{\left(\overline{\mathbf{b}}-\mathbf{b}\right)}{\left(\frac{\sigma^2}{v+T}\right)}+\mathbf{X'e}}{\mathbf{X'X}}$$

$$\Rightarrow\sqrt{T}(\mathbf{b}-\boldsymbol{\beta})=\frac{\frac{\left(\overline{\mathbf{b}}-\mathbf{b}\right)}{\left(\frac{\sigma^2}{v+T}\right)}+\frac{\mathbf{X'e}}{\sqrt{T}}}{\frac{\mathbf{X'X}}{T}}$$

$$\Rightarrow\sqrt{T}(\mathbf{b}-\boldsymbol{\beta})=\frac{\frac{\left(\overline{\mathbf{b}}-\mathbf{b}\right)}{\left(\frac{\sigma^2}{v+T}\right)}+\underset{T\to\infty}{lim}\frac{\mathbf{X'e}}{\sqrt{T}}}{\underset{T\to\infty}{lim}\frac{\mathbf{X'X}}{T}}$$

$$\mathbf{b}-\boldsymbol{\beta}=\left(\mathbf{X'X}\right)^{-1}\mathbf{X'e}$$

$$\Rightarrow\sqrt{T}(\mathbf{b}-\boldsymbol{\beta})=\left(\underset{T\to\infty}{lim}\frac{\mathbf{X'X}}{T}\right)^{-1}\underset{T\to\infty}{lim}\frac{\mathbf{X'e}}{\sqrt{T}}$$

References

[1] Time Series Analysis ; James D. Hamilton ; PRINCETON UNIVERSITY PRESS, PRINCETON, NEW JERSEY

[2] ECONOMETRIC ANALYSIS , 7th edition ; William H. Greene; New York University; Pearson Education Limited, England

[3] Bayesian Econometrics; Gary Koop; Department of Economics ; University of Glasgow ; John Wiley & Sons Ltd ; England

[4] EUCLIDEAN AND NON-EUCLIDEAN GEOMETRY; AN ANALYTIC APPROACH; Patrick J. Ryan ; CAMBRIDGE UNIVERSITY PRESS, USA