	COLLEGE OF COMPUTING AND INFORMATION SCIENCES
	Assignment-1 Fall 2020 Semester

Class Id	104964-65-66-67	**Course Title**	Numerical Computing
Program	BSCS	**Campus / Shift**	Main Campus / Morning
Submission Date	4^{th} October	**Total Marks**	100
Duration	07 Days	**Faculty Name**	Mr. Adnan Ullah Khan
Student Id	10761	**Student Name**	Shahreyar Asif

Instructions:

- Fill out your Student ID and Student Name in above header.
- A = Last three digits of S.I.D

[Problem - 1].

Where do the curves of y = cos(x) and y = x^3 – 1.A intersect? Use bisection.

Solution #01

$$y = \cos(x)$$

$$y = x^3 - 1.A$$

$$x^3 - 1.A = \cos(x)$$

$$x^3 - 1.A - \cos(x) = 0$$

$$x^3 - \cos(x) - 1.A = 0$$

$$x^3 - \cos(x) - 1.761 = 0$$

Iteration #01

$a = 1$ $\qquad$ $b = 1.5$

$$x_m = \frac{1+1.5}{2} = 1.25$$

$$f(1.25) = (1.25)^3 - \cos(1.25) - 1.761$$

$$= -0.123197$$

Iteration #2

$a = 1.25 \qquad b = 1.5$

$x_m = 1.375$

$f(1.375) = (1.375)^3 - \cos(1.375) - 1.761$
$= 0.644$

Iteration #3
$a = 1.25 \qquad b = 1.375$

$x_m = 1.3125$

$f(1.3125) = (1.3125)^3 - \cos(1.3125) - 1.761$
$= 0.24455$

Iteration #4
$a = 1.25 \qquad b = 1.3125$
$x_m = 1.28125$

$f(1.28125) = 0.05678$

Iteration #5

$a = 1.25 \qquad b = \cancel{1.3125}\ 1.28125$

$$x_m = \frac{1.25 + \cancel{1.3125}^{1.28125}}{2} = 1.265625$$

$f(x_m) = -0.0341700$

Iteration #6

$a = 1.265625 \qquad b = 1.28125$

~~f(~~ $x_m = 1.2734375$

$f(x_m) = 0.01106512$

Iteration #7

$a = 1.2734375 \qquad b = 1.28125$

$x_m = 1.27734375$

$f(x_m) = 0.03386$

The x is 1.27734375

The curve intersect at

$x = 1.27734375 \qquad y = 0.289258$

[Problem - 2].

Apply Newton's method to the equation $x^2 = A$ to derive this algorithm for getting the square root of N:

$$x_{n+1} = \frac{1}{2}(x_n + \frac{A}{x_n})$$

Solution #2

we have to drive.

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right)$$

the given eqn is

$$x^2 = A$$
$$x^2 - A = 0$$
$$f(x) = x^2 - A$$
$$f'(x) = 2x$$

The given formula of Newton Method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^2 - A)}{2x_n}$$

$$x_{n+1} = \frac{2x_n^2 - x_n^2 + A}{2x_n}$$

$$x_{n+1} = \frac{x_n^2 + A}{2x_n}$$

$$x_{n+1} = \frac{x_n^{\cancel{2}}}{2\cancel{x_n}} + \frac{A}{2x_n}$$

$$x_{n+1} = \frac{1}{2}\left(x_n - \frac{A}{x_n}\right)$$

$$x_{n+1} = \frac{1}{2}\left(x_n - \frac{761}{x_n}\right)$$

~~Iteration #1~~

$\cancel{x_1 = \frac{1}{2}(x}$

Iteration #1 $x_0 = 1$

$$x_1 = \frac{1}{2}\left(1 - \frac{761}{1}\right)$$

$$\boxed{x_1 = 380}$$

Iteration # 2 $x_n = 380$

$$x_R = \frac{1}{2}\left(380 - \frac{761}{380}\right)$$

$$\boxed{x_2 = 188.9986842}$$

Iteration # 3 ($x_n = 188.998684$)

$$x_3 = \frac{1}{2}\left(188.998684 - \frac{761}{188.998684}\right)$$

$$\boxed{x_3 = 92.486100}$$

Iteration #4 ($x_n = 92.486100$)

$$x_4 = \frac{1}{2}\left(92.486100 - \frac{761}{92.486100}\right)$$

$$\boxed{x_4 = 42.128918}$$

Iteration #5 ($x_n = 42.128918$)

$$x_5 = \frac{1}{2}\left(42.128918 - \frac{761}{42.128918}\right)$$

$$x_5 = 12.03265809$$

[Problem - 3].

Newton's method will find complex roots. Find the roots of these relation including the complex root

$$x^3 + 1.A = 0$$

Solution #03 (for Complex)

$$x^3 + 1.761 = 0$$

Iteration #01 (x_0)

$$x_0 = 1 + i$$

$$x_1 = 1 + i - \frac{f(1+i)}{f'(1+i)}$$

$$x_1 = 1 + i - \left(\frac{0.76 + i}{}\right)$$

$$x_1 = 1 + i - \left(\frac{-0.239 + 2i}{6i}\right)$$

$$x_1 = 0.6666 + 0.960166i$$

Iteration #2 ($x_1 = 0.666666 + 0.960166i$)

$$x_2 = (0.666666 + 0.960166i) - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.391855 + 1.563950i$$

Iteration #3 ($x_2 = 0.391855 + 1.563950i$)

$$x_3 = (0.391855 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.1905594 + 1.723660i$$

Iteration #04 ($x_3 = 0.190559 + 1.7236601i$)

$$x_4 = 0.190559 \overset{+1.72660i}{-} \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = -0.023725 + 1.787616i$$

Iteration #05 ($\cancel{-0.}\ x_4 = -0.023725 + 1.787616i$)

$$x_5 = (-0.023725 + 1.787616i) - \frac{f(x_4)}{f'(x_4)}$$

$$x_5 = -0.248333 + 1.780305i$$

Iteration #6 ($x_5 = -0.248333 + 1.780305i$)

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)}$$

$$x_6 = -0.486232 + 1.705733i$$

Iteration #7 (x_6)

$$x_7 = x_6 - \frac{f(x_6)}{f'(x_6)}$$

$$x_7 = -0.748416 + 1.558159i$$

Iteration # 8 (x_7)

$$x_8 = x_7 - \frac{f(x_7)}{f'(x_7)}$$

$$x_8 = -1.064217 + 1.328144i$$

Iteration # 9 (x_8)

$$x_9 = x_8 - \frac{f(x_8)}{f'(x_8)}$$

$$x_9 = -1.497952 + 1.031469i$$

Iteration # 10 (x_9)

$$x_{10} = x_9 - \frac{f(x_9)}{f'(x_9)}$$

$$x_{10} = -2.0929055 + 0.782791i$$

Iteration # 11 (x_{10})

$$x_{11} = x_{10} - \frac{f(x_{10})}{f'(x_{10})}$$

$$x_{11} = -2.725968 + 0.667065i$$

Iteration #12(x_{11})

$$x_{12} = x_{11} - \frac{f(x_{11})}{f'(x_{11})}$$

$$x_{12} = -3.325132 + 0.615441i$$

Iteration #13 (x_{12})

$$x_{13} = x_{12} - \frac{f(x_{12})}{f'(x_{12})}$$

$$x_{13} = -3.897030 + 0.587882i$$

Iteration #14 (x_{13})

$$x_{14} = x_{13} - \frac{f(x_{13})}{f'(x_{13})}$$

$$x_{14} = -4.451195 + 0.571160i$$

Iteration #15 (x_{14})

$$x_{15} = x_{14} - \frac{f(x_{14})}{f'(x_{14})}$$

$$x_{15} = -4.993499 + 0.560122i$$

Iteration #16 (x_{15})

$$x_{16} = x_{15} - \frac{f(x_{15})}{f'(x_{15})}$$

$$x_{16} = -5.527505 + 0.552395i$$

Iteration #17 (x_{16})

$$x_{17} = x_{16} - \frac{f(x_{16})}{f'(x_{16})}$$

$$x_{17} = -6.055474 + 0.546749i$$

Iteration # 18 (x_{17})

$$x_{18} = x_{17} - \frac{f(x_{17})}{f'(x_{17})}$$

$$x_{18} = -6.578907 + 0.542482i$$

Iteration #19 (x_{18})

$$x_{19} = x_{18} - \frac{f(x_{18})}{f'(x_{18})}$$

$$x_{19} = -7.098840 + 0.539173$$

Iteration #20 (x_{19})

$$x_{20} = x_{19} - \frac{f(x_{19})}{f'(x_{19})}$$

$$\boxed{x_{20} = -7.61601316 \pm 0.536549i}$$

Complex root =

FOR REAL ROOT:

Solution # 3 (Newtons Method)

$$x^3 + 1.761 = 0$$

formula

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Iteration #01 (x_0)

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1 - \frac{(1)^3 + 1.761}{3(1)^2}$$

$$x_1 = 0.07966$$

Iteration #02 (x_2)

$$x_1 = 0.0796666$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.079666 - \frac{f(0.079666)}{f'(0.079666)}$$

$$x_2 = 0.079666 - \frac{1.761505}{0.019040}$$

$$x_2 = -92.436352$$

Iteration #03 (x_3)

$$x_2 = -92.436352$$

$$x_3 = -92.436352 - \frac{f(-92.436352)}{f'(-92.436352)}$$

$$\left(\frac{-789818.7466}{25633.43751}\right)$$

$$x_3 = -61.624302$$

Iteration #04 (x_4)

$$x_3 = -61.624304$$

$$x_4 = -61.624304 - \frac{f(x_3)}{f'(x_3)}$$

$$x_4 = -61.624304 - \frac{(-234019.9131)}{11392.66453}$$

$$x_4 = -41.083023$$

Iteration #05 (x_5)

$$x_4 = -41.083023$$

$$x_5 = -41.083023 - \frac{f(x_4)}{f'(x_4)}$$

$$x_5 = -41.083023 - \left(\frac{-69338.77238}{5063.444336}\right)$$

$$x_5 = -27.389029$$

Iteration #06 (x_6)

$$x_5 = -27.389029$$

$$x_6 = -27.389029 - \frac{f(x_5)}{f'(x_5)}$$

$$x_6 = -27.389029 - \left(\frac{-20546.124413}{2250.476729}\right)$$

$$x_6 = -18.259352$$

Iteration #07 (x_7)

$$x_6 = -18.259352$$

$$x_7 = -18.259352 - \frac{f(x_6)}{f'(x_6)}$$

$$x_7 = -18.259352 - \left(\frac{-6085.978816}{1000.211806}\right)$$

$$x_7 = -12.174661$$

Iteration #08 (x_8)

$$x_7 = -12.174661$$

$$x_8 = -12.174661 - \frac{f(x_7)}{f'(x_7)}$$

$$x_8 = -12.174661 - \left(\frac{-1804.557113}{444.667111}\right)$$

$$x_8 = -8.116440$$

Iteration #09 (x_9)

$$x_8 = -8.116440$$

$$x_9 = -8.116440 - \frac{f(x_8)}{f'(x_8)}$$

$$x_9 = -8.116440 - \left(\frac{-534.683588}{197.629794}\right)$$

$$x_9 = -5.410959$$

Iteration #10 (x_{10})

$x_9 = -5.410959$

$$x_{10} = -5.410959 - \frac{f(x_9)}{f'(x_9)}$$

$$x_{10} = -5.410959 - \left(\frac{-158.424640}{29.278477}\right)$$

$$x_{10} = 4.681107 \times 10^{8}$$

Iteration # 11 (x_{11})

$x_{10} = 4.681107 \times 10^{-8}$

$$x_{11} = 4.681107 \times 10^{-8} - \frac{f(x_{10})}{f'(x_{10})}$$

$$x_{11} = 4.681107 \times 10^{-8} - \left(\frac{1.761}{6.573828 \times 10^{-15}}\right)$$

$$x_{11} = 4.681107 \times 10^{-8} - 2.678804 \times 10^{14}$$

$$x_{11} = -2.678804 \times 10^{14}$$

Iteration 12 (x_{12})

$$x_{11} = -2.678804 \times 10^{14}$$

$$x_{12} = -2.678804 \times 10^{14} - \frac{f(x_{11})}{f(x_{11})}$$

$$x_{12} = -2.678804 \times 10^{14} - \left(\frac{-2.678804 \times 10^{14}}{2.152797 \times 10^{29}}\right)$$

$$x_{12} = -0.3333333$$

Iteration #13 (x_{13})

$$x_{12} = -0.333333$$

$$x_{13} = -0.333333 - \frac{f(x_{12})}{f'(x_{12})}$$

$$x_{13} = -0.333333 - \left(\frac{1.723963}{0.333333}\right)$$

$$x_{13} = -5.5052323$$

Iteration #14 (x_{14})

$$x_{14} = -5.5052323 - \left(\frac{-165.089283}{90.922948}\right)$$

$$x_{14} = -3.320941$$

Iteration # 15 (x_{15})

$x_{14} = -3.320941$

$$x_{15} = -3.320941 - \frac{f(x_{14})}{f'(x_{14})}$$

$$x_{15} = -3.320941 - \left(\frac{-34.864514}{33.085947}\right)$$

$$x_{15} = -3.320941 - (-1.053755)$$

$$x_{15} = -2.267185$$

Iteration # 16 (x_{16})

$x_{15} = -2.267185$

$$x_{16} = -2.267185 - \frac{f(x_{15})}{f'(x_{15})}$$

$$x_{16} = -2.2 \quad - \left(\frac{-9.892621}{15.4203894}\right)$$

$$x_{16} = -1.625656$$

Iteration # 17 (x_{17})

$$x_{17} = -1.625656 - \left(\frac{f(x_{16})}{f'(x_{16})}\right)$$

$$x_{17} = -1.625656 - \left(\frac{-2.535215}{7.928272}\right)$$

$$\boxed{x_{17} = -1.305887}$$ ~~Ans~~

Iteration # 18

$x_{17} = {}^{-}1.305887$

$$x_{18} = -1.305887 - \frac{f(x_{17})}{f'(x_{17})}$$

$$x_{18} = -1.305887 - \left(\frac{-0.465982}{5.116022}\right)$$

$$\boxed{x_{18} = -1.214804}$$ Ans

[Problem - 4]. The sum of two numbers is A, the square root of their product is 2A. What are the numbers?

Solution #4

$$x + y = 761 \quad \text{— eq (i)}$$

$$\sqrt{xy} = 761 \times 2$$

$$\sqrt{xy} = 1522 \quad \text{— eq (ii)}$$

from eq (ii)

$$(\sqrt{xy})^2 = (1522)^2$$

$$xy = 2316484$$

$$y = \frac{2316484}{x}$$

Put in eq (i)

$$x + \frac{2316484}{x} = 761$$

$$\frac{x^2 + 2316484}{x} = 761$$

$$\boxed{x^2 - 761x + 2316484 = 0}$$

This is eqn have no roots

[Problem - 5]. DeSantis (1976) has derived are relationship for the compressibility factor of real gases of the form

$$z = \frac{1 + y + y^2 - y^3}{(1 + y)^3}$$

If z = 0.A, what is the value of y?

Solution #5

$$z = \frac{1+y+y^2-y^3}{(1+y)^3}$$

$\because z = 0.761$

$$0.761(1+y)^3 = 1+y+y^2-y^3$$

$$(0.761)\left((1)^3 + y^3 + 3(1)(y)(1+y)\right) = 1+y+y^2-y^3$$

$$0.761(1+y^3+3y+3y^2) = 1+y+y^2-y^3$$

$$0.761 + 0.761y^3 + 2.283y + 2.283y^2 = 1+y+y^2-y^3$$

$$0.761 + 0.761y^3 + 2.283y + 2.283y^2 = 1+y+y^2-y^3$$

$$1+y+y^2-y^3-0.761 - 0.761y^3 - 2.283y - 2.283y^2 = 0$$

$$0.239 - 1.283y - 1.283y^2 - 1.761y^3 = 0$$

$$1.761y^3 + 1.283y^2 + 1.283y - 0.239 = 0$$

By Using Bisection Method

Iteration #01

$a = 0 \quad b = 1$

$x_m = \frac{0+1}{2} = 0.5$

$f(x_m) = 0.846868$

Iteration #02

$a = 0 \quad b = 0.5$

$x_m = \frac{0.5}{2} = 0.25$

$f(x_m) = 0.189453$

Iteration #03

$a = 0 \quad b = 0.25$

$x_m = \frac{0.25}{2} = 0.125$

$f(x_m) = -0.055138$

Iteration #04

$a = 0.125 \quad b = 0.25$

$x_m = \frac{0.125 + 0.25}{2} = 0.1875$

$f(x_m) = 0.05827$

Iteration #05

$a = 0.125 \qquad b = 0.1875$

$$x_m = \frac{0.125 + 0.1875}{2} = 0.15625$$

$f(x_m) = -0.00049$

Iteration #06

$a = 0.15625 \qquad b = 0.1875$

$$x_m = \frac{0.15625 + 0.1875}{2}$$

$x_m = 0.171875$

$f(x_m) = 0.028357$

Iteration #07

$a = 0.15625 \qquad b = 0.171875$

$$x_m = \frac{0.15625 + 0.171875}{2} = 0.1640625$$

$f(x_m) = 0.088$

Iteration #08

$a = 0.15625 \qquad b = 0.1640625$

$$x_m = \frac{0.15625 + 0.1640625}{2}$$

$x_m = 0.16015625$

$f(x_m) = 0.006623$

Iteration #09

$a = 0.15625 \qquad b = 0.16015625$

$$x_m = \frac{0.15625 + 0.16015625}{2}$$

$x_m = 0.158203125$

$f(x_m) = 0.003058$

The root is 0.158203125