EAST WEST UNIVERSITY

Mid-II Examination, Fall-2021
Department of Mathematics and Physical Sciences
Course Code: MAT 205 (Linear Algebra and Complex Variables)
Section: 7, Time: 90 minutes, Full Marks: 40
Course Instructor: Dr. Nepal Chandra Roy (DNCR)

N.B.: Answer all the questions. Figure in the right margin indicate full marks.

1. (a) Find the eigenvalues and eigenvectors of the matrix [6]

$$A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}.$$

(b) Find the matrix P that diagonalizes A in 1(a) and determine $P^{-1}AP$. [4]

2. (a) Find the characteristic equation of the matrix [5]

$$A=\begin{bmatrix}1 & 2 & 2\\ 3 & 1 & 0\\ 1 & 1 & 1\end{bmatrix}.$$

(b) Verify Cayley-Hamilton theorem for A in 2(a). [5]

3. Consider the vector space $\mathbb{R}^3$ with the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis $u_1=(1,1,1), u_2=(1,0,1)$ and $u_3=(0,0,1)$. [10]

4. (a) If a and b are complex numbers, prove that $|a+b|\le|a|+|b|$. [5]

(b) For the function, $f(z)=u+iv$ defined by [5]

$$f(z)=\begin{cases}\dfrac{(\bar{z})^2}{z}, & z\neq 0\\ 0, & z=0\end{cases}$$

show that $\dfrac{\partial u}{\partial x}=\dfrac{\partial v}{\partial y}$ is satisfied at $z = 0$.