Chapter-5
Simplification of Boolean Functions

5.2 Algebraic Simplification of Boolean Functions

Example 1:

$$F = ABC + AB'(A'C')'$$
$$= ABC + AB'(A+C)$$
$$= ABC + AB'.A + AB'C$$
$$= ABC + AB' + AB'C$$
$$= AC(B+B') + AB'$$
$$= AC + AB'$$

Example 2:

$$F = ABC + ABC' + AB'C$$
$$= ABC + ABC' + ABC + AB'C$$
$$= AB(C+C') + AC(B+B')$$
$$= AB + AC$$

$ABC + ABC' = AB$
$111 \quad 110 \quad 11-$

$ABC + AB'C = AC$
$111 \quad 101 \quad 1-1$

} 1-bit variation

To reduce circuit cost

- We need to reduce number of product terms (AND gates)
- We need to reduce inputs of AND gates (Literals)

5.4 Karnaugh Map Method for Simplification of Boolean Functions

5.4.1 Karnaugh Map (K-map)

This map provides a simple straightforward procedure for representation and simplification of Boolean functions.

It is generally used up to six variables.

Structure of K-map:

2-variable K-map:

A \ B	0	1
0	00	01
1	10	11

Cell

Adjacent cells vary in only 1-bit.

3-variable K-map:

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

0,1 | 1,3 | 3,2 Adjacent

4,5 | 5,7 | 7,6 Adjacent

0,4 | 1,5 | 3,7 | 2,6 Adjacent

0,2 Adjacent

4,6 Adjacent

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

Normal Sequence

00
01
10
11

Gray code Sequence

00
01
11
10

4-variable K-map:

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

0,1 | 1,3 | 3,2 Adjacent

4,5 | 5,7 | 7,6 Adjacent

12,13 | 13,15 | 15,14 Adjacent

8,9 | 9,11 | 11,10 Adjacent

0,4 | 4,12 | 12,8 Adjacent

1,5 | 5,13 | 13,9 Adjacent

3,7 | 7,15 | 15,11 Adjacent

2,6 | 6,14 | 14,10 Adjacent

0,2 Adjacent
4,6 Adjacent
12,14 Adjacent
8,10 Adjacent

0,8 Adjacent
1,9 Adjacent
3,11 Adjacent
2,10 Adjacent

5.4.2 Representation of Truth Table on a K-map

ABC	F
000	0
001	1
010	1
011	0
100	X
101	0
110	X
111	1

A \ BC	00	01	11	10
0	0	1	0	1
1	X	0	1	X

5.4.3 Representation of SOP Function on a K-map

$F = A'B'C + A'BC' + AB'C' + ABC$ [CSOP]

001 010 100 111

A \ BC	00	01	11	10
0		1		1
1	1		1	

SOP: $F = A'C + A'B + AB'C + BC$

5.4.2 Representation of Truth Table on a K-map

ABC	F
000	0
001	1
010	1
011	0
100	X
101	0
110	X
111	1

A \ BC	00	01	11	10
0	0	1	0	1
1	X	0	1	X

5.4.3 Representation of SOP Function on a K-map

$F = A'B'C + A'BC' + AB'C' + ABC$ [CSOP]

001 010 100 111

A \ BC	00	01	11	10
0		1		1
1	1		1	

SOP: $F = A'C + A'B + AB'C + BC$

0-1 01- 101 -11

001 010 011

011 011 111

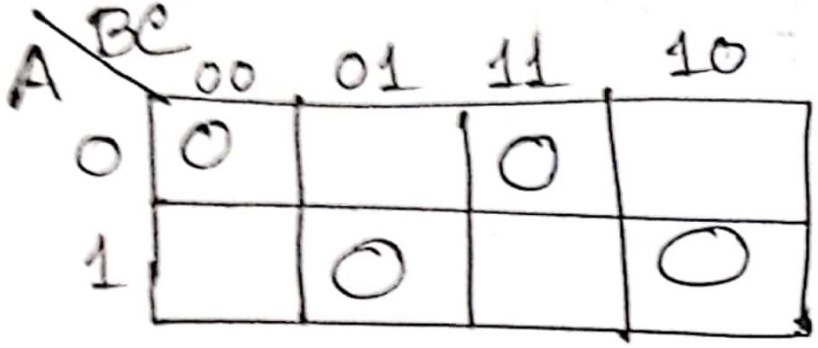
A \ BC	00	01	11	10
0		1	1	1
1		1	1	

5.4.4 Representation of POS Function on a k-map

CPOS:

$F = (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C)$

000 011 101 110



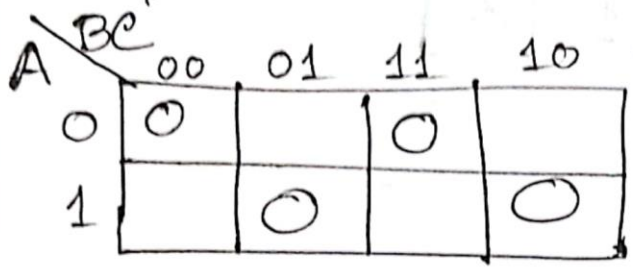
POS: $F = (B+C)(A'+B)(A+B'+C')$

5.1.4 Representation of POS function on a k-map

CPOS:

$F = (A+B+C)(A+B'+C')(A'+B+C')(A'+B'+C)$

000 011 101 110

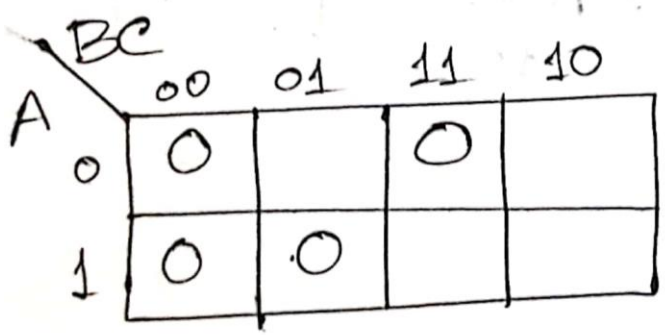


POS: $F = (B+C)(A'+B)(A+B'+C')$

_00 10_ 011

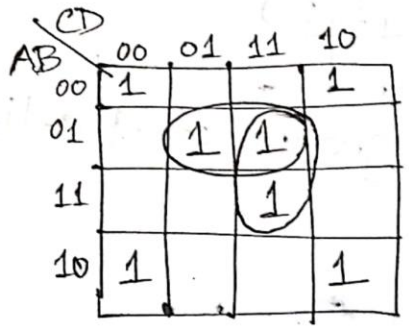
000 100

100 101



Example 1:

$F(A,B,C,D) = \sum(0, 2, 5, 7, 8, 10, 15)$



Principle:

* Largest Possible group

  Group size $= 2^m$

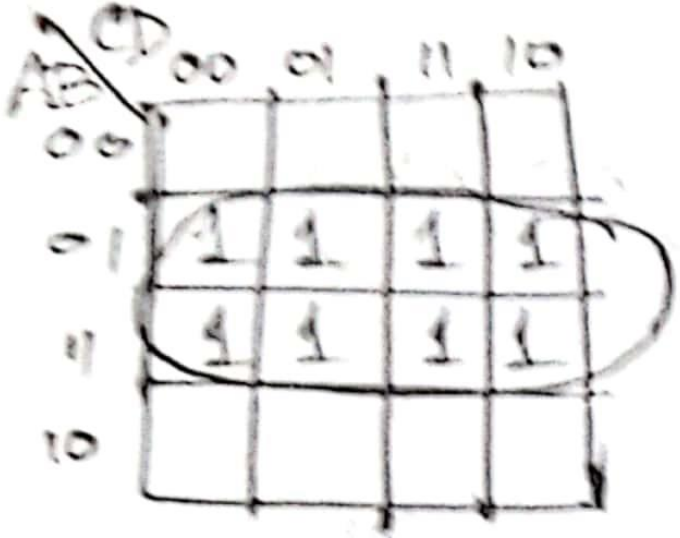
  $= 2^0, 2^1, 2^2, 2^3, 2^4$ $[m = 0, 1, 2, 3, 4]$

  $= 1, 2, 4, 8, 16$

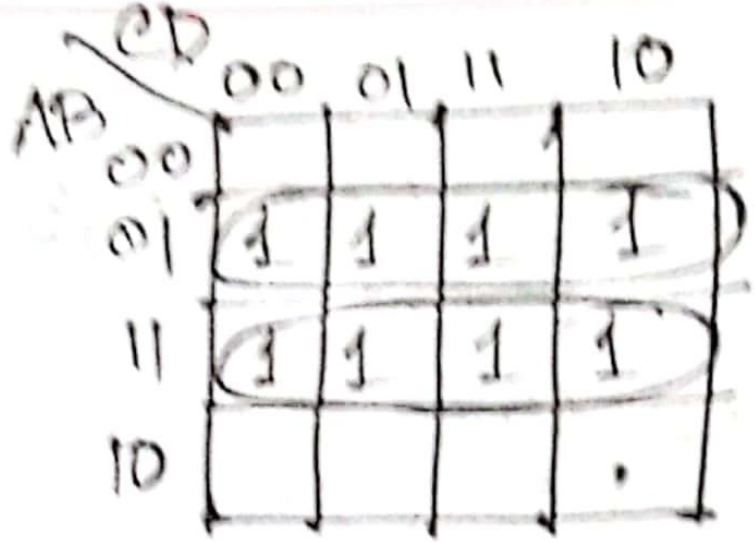
* Smallest Number of Groups

Procedure:

1. Find largest group.
2. Find next largest group. Include at least a new 1.
   Overlap can be done.

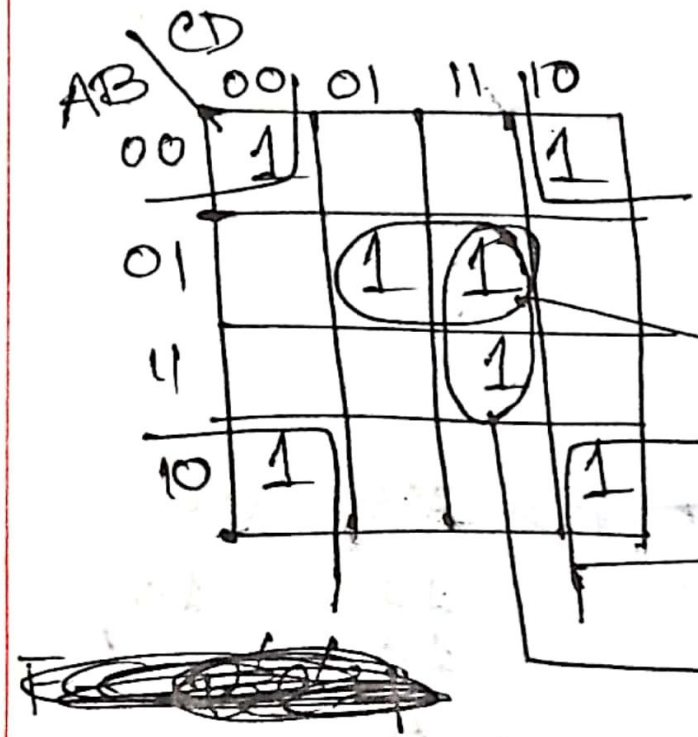


1 group of 8 cells.



2 groups of 4 cells

Example 1: $F(A,B,C,D) = \Sigma(0,2,5,7,8,10,15)$



AB	CD
00	00
10	10
B′	D′

$B'D'$

AB	CD
01	01
A′B	11
	D

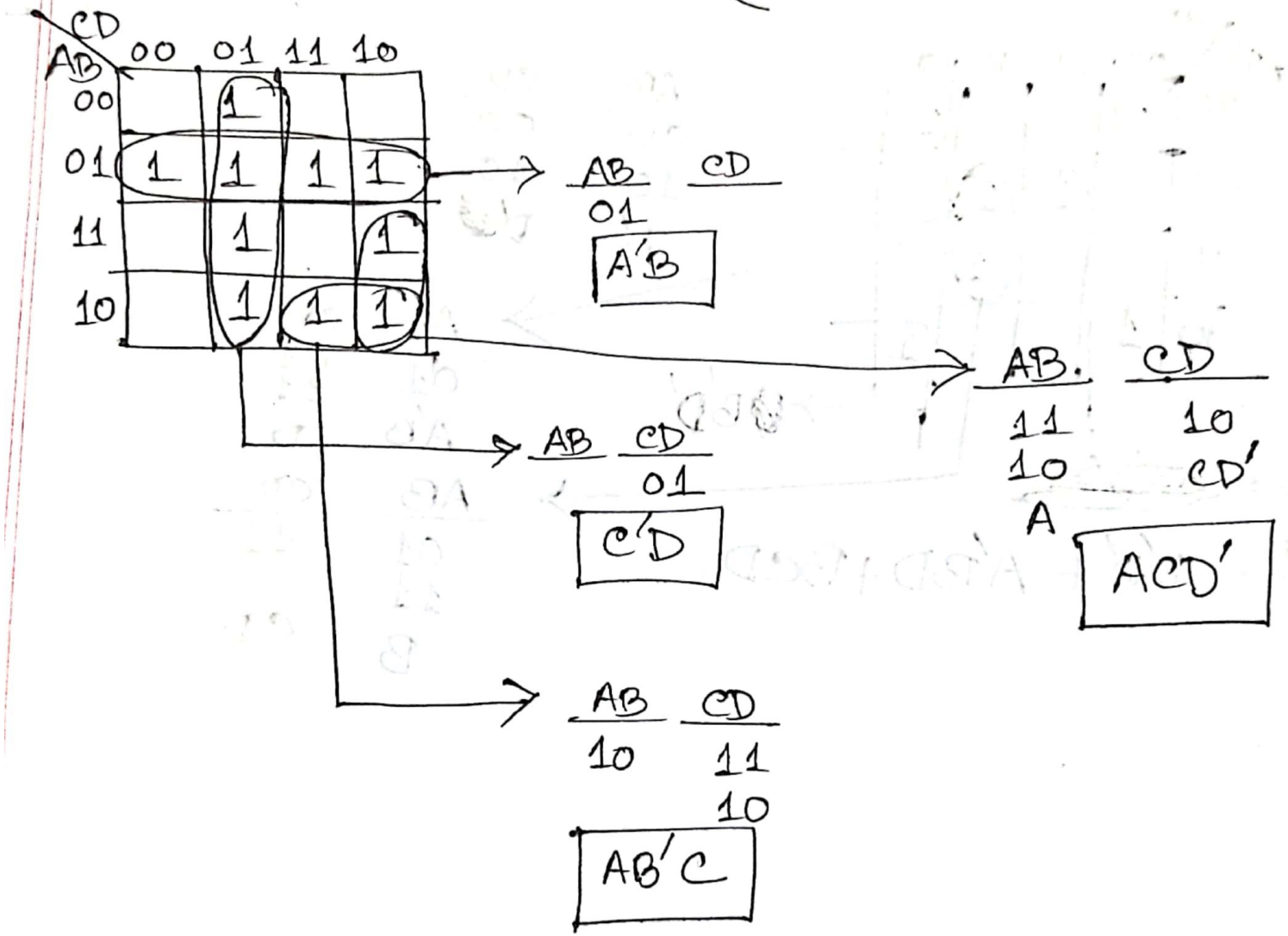
AB	CD
01	11
11	
B	CD

$$F = B'D' + A'BD + BCD$$

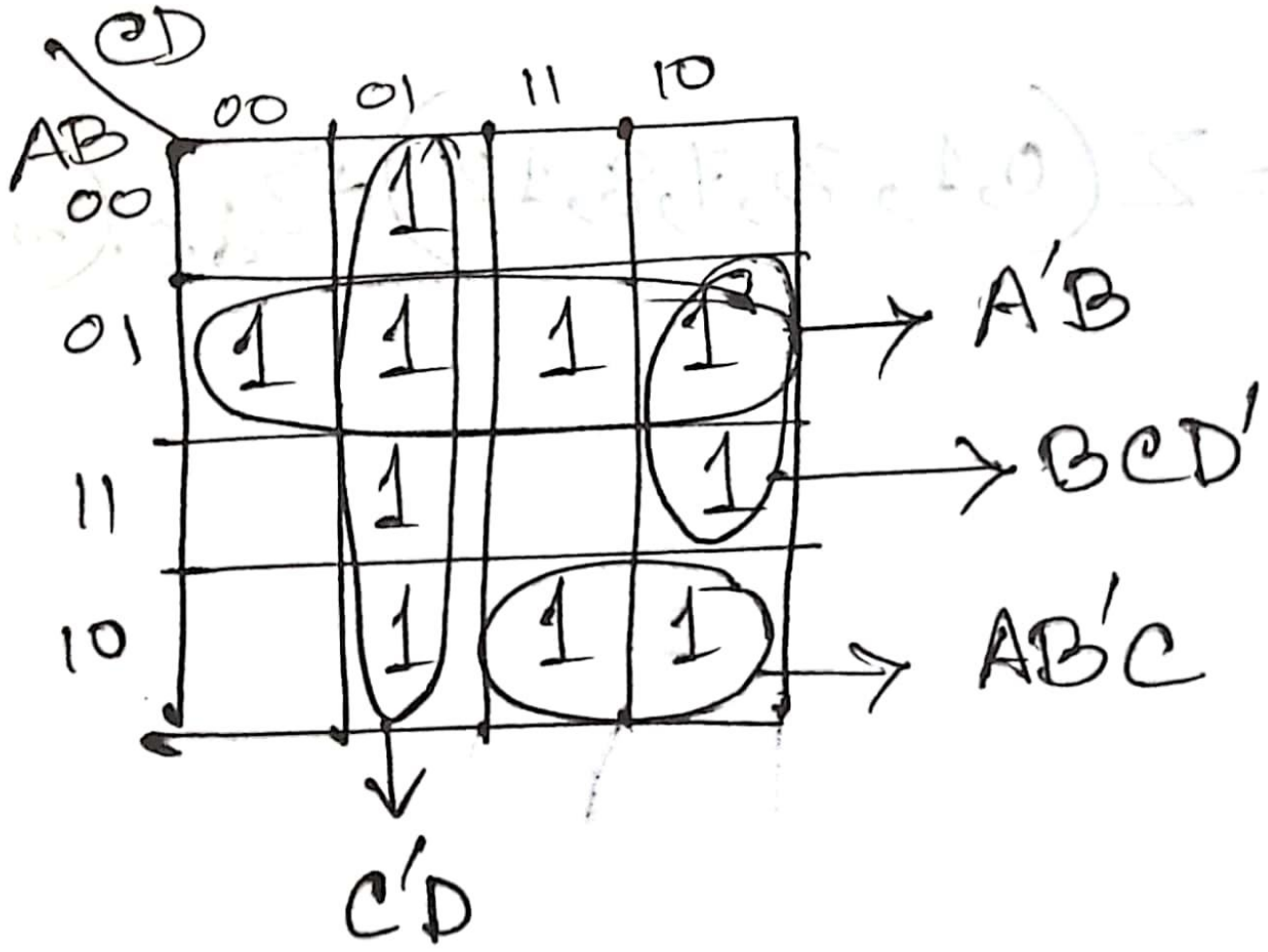
Example 2: $F(A,B,C,D)=\sum(1,4,5,6,7,9,10,11,13,14)$

AB \ CD	00	01	11	10
00		1		
01	1	1	1	1
11		1		1
10		1	1	1

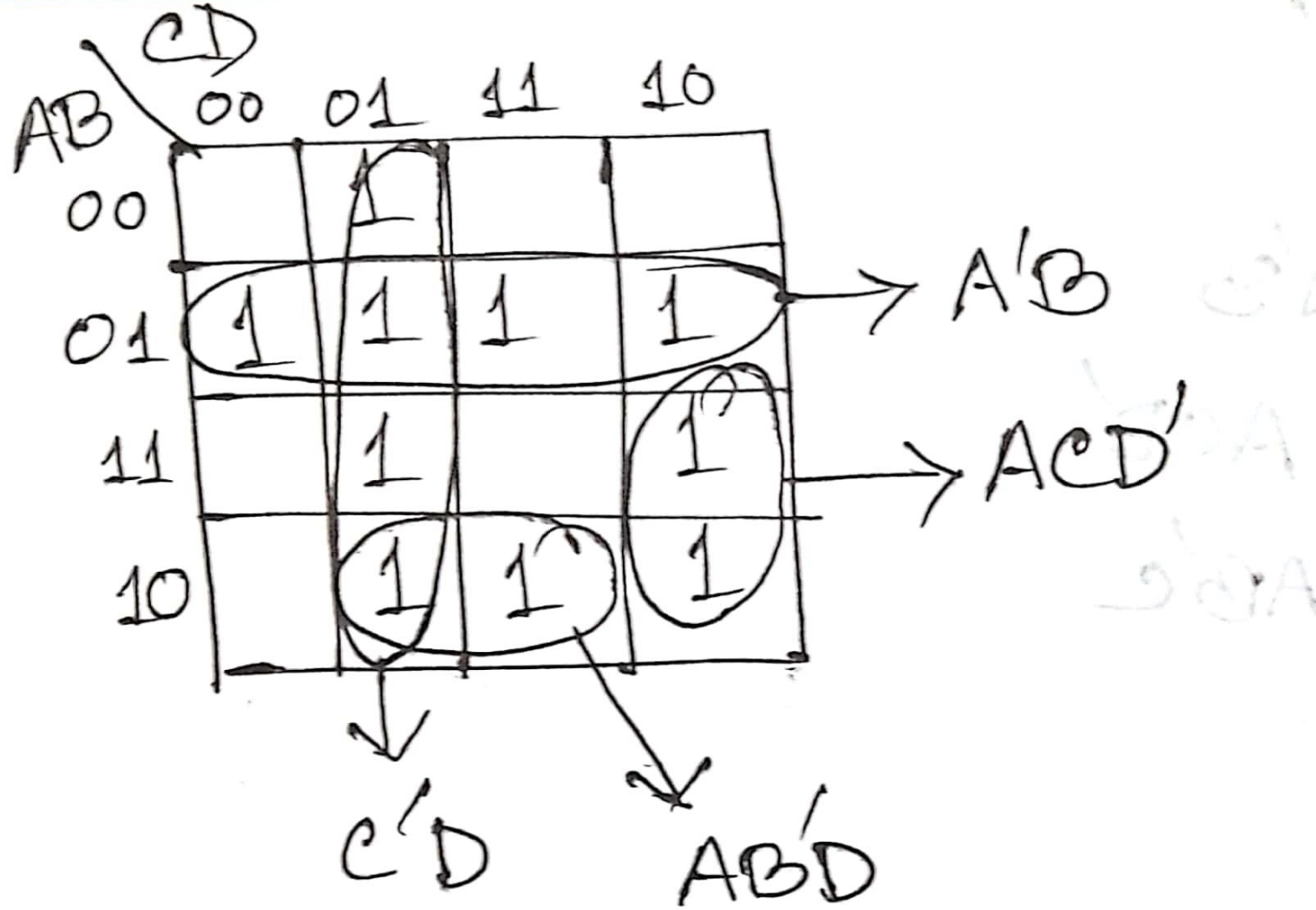
Example 2: $F(A,B,C,D) = \Sigma(1,4,5,6,7,9,10,11,13,14)$



$$F = A'B + C'D + ACD' + AB'C$$



$$F = A'B + C'D + BCD' + AB'C$$

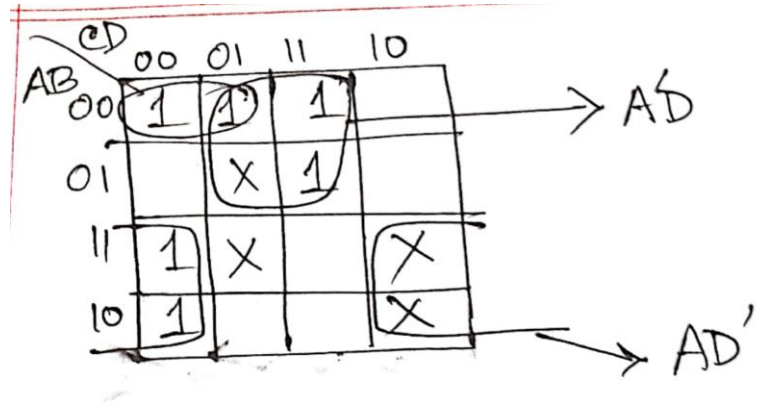


$$F = A'B + C'D + AB'D + ACD'$$

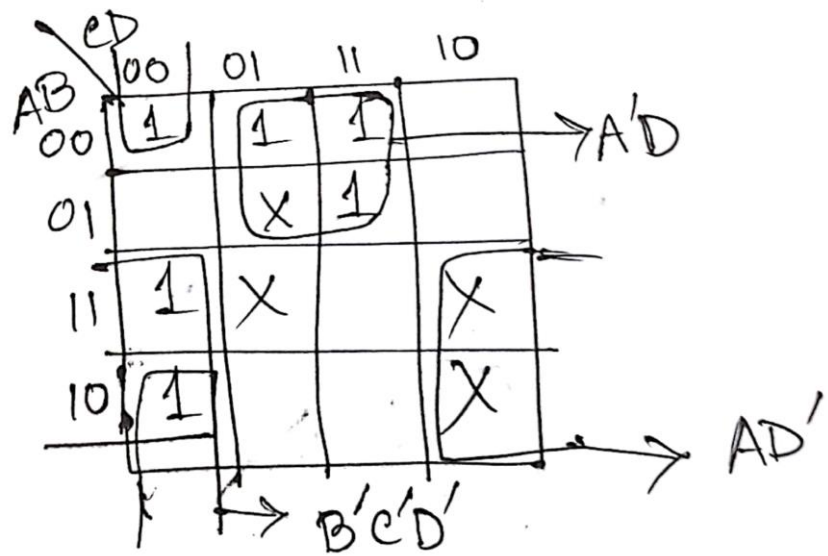
Example 3: $F(A,B,C,D) = \Sigma(0,1,3,7,8,12) + \Sigma_{d.c.}(5,10,13,14)$

AB \ CD	00	01	11	10
00	1	1	1	
01		X	1	
11	1	X		X
10	1			X

* Group with Don't Care (X) must have at least one 1 not covered by other group.
* Group of only don't care (X) is not required.



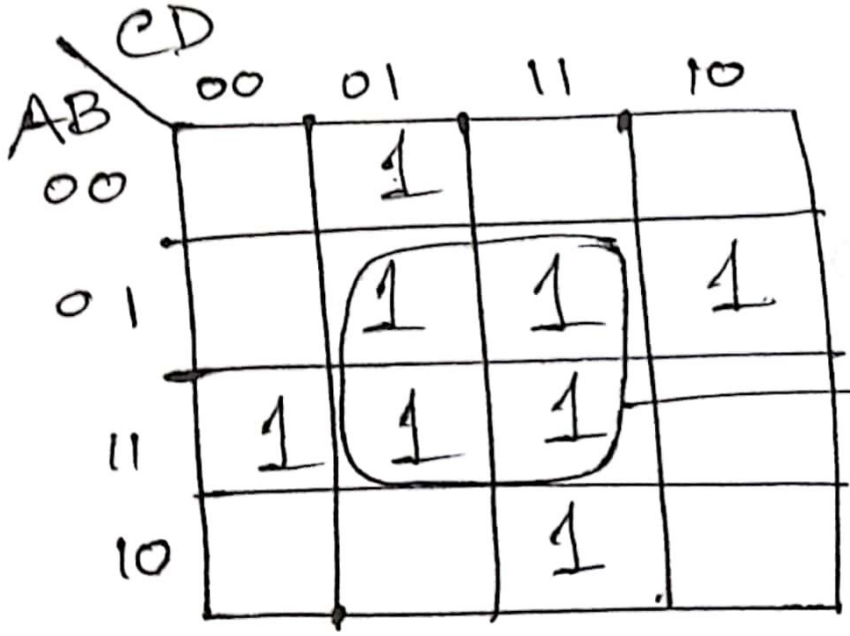
$F = A'D + AD' + A'B'C'$



$F = A'D + AD' + B'C'D'$

Example 2:

$$F(A,B,C,D) = \Sigma(1,5,6,7,11,12,13,15)$$

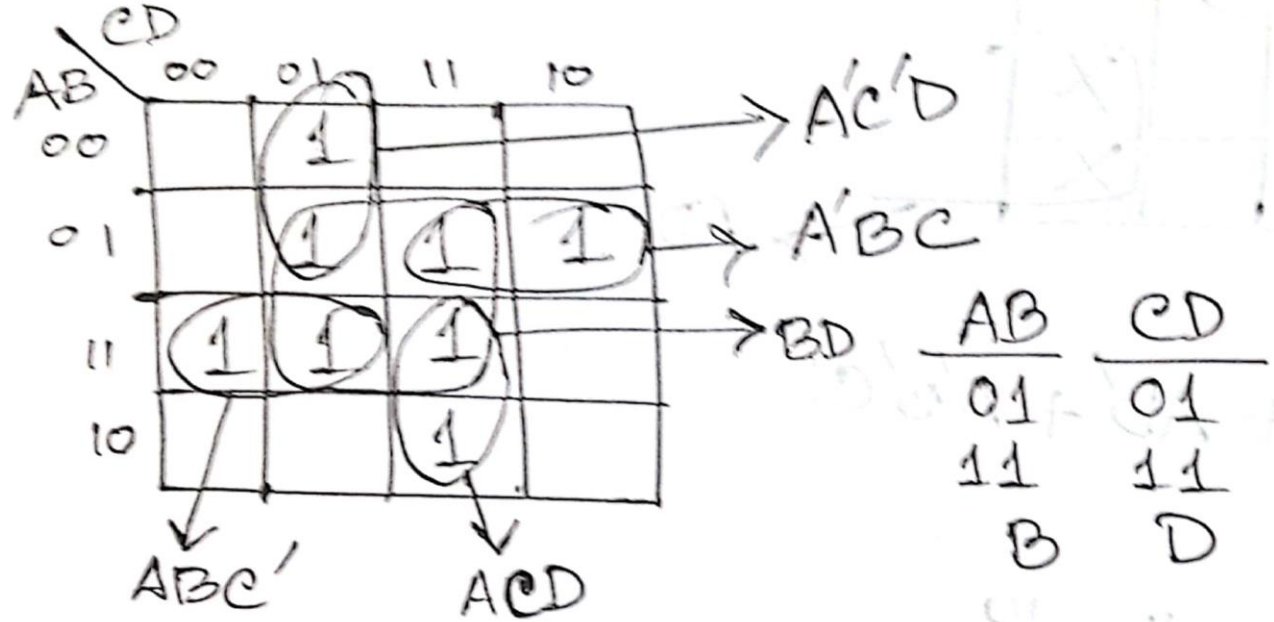


→ BD

AB	CD
01	01
11	11
B	D

Example 2:

$$F(A,B,C,D) = \Sigma(1,5,6,7,11,12,13,15)$$

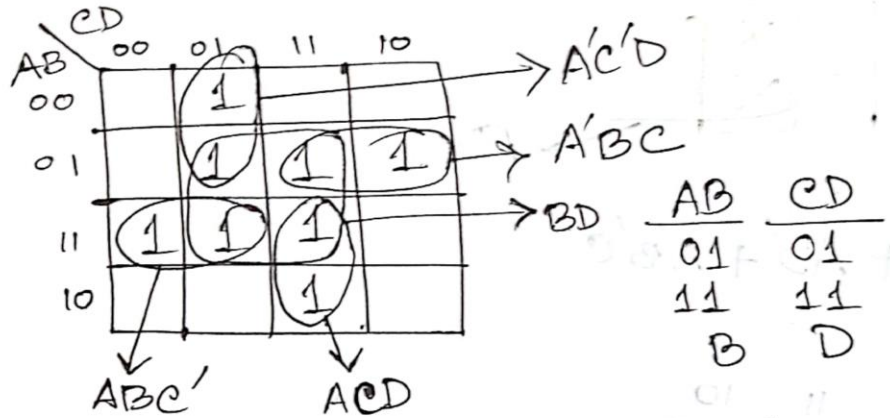


AB	CD
01	01
11	11
B	D

$$F = BD + A'C'D + ABC' + ACD + A'BC$$

Example 2:

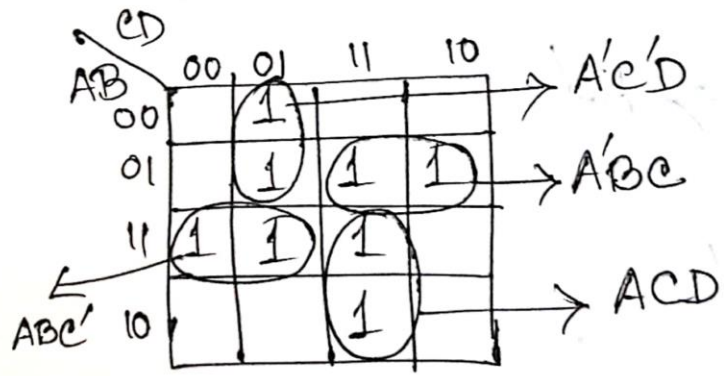
$F(A,B,C,D) = \Sigma(1,5,6,7,11,12,13,15)$



AB	CD
01	01
11	11
B	D

$$F = \underline{BD} + A'C'D + ABC' + ACD + A'BC$$

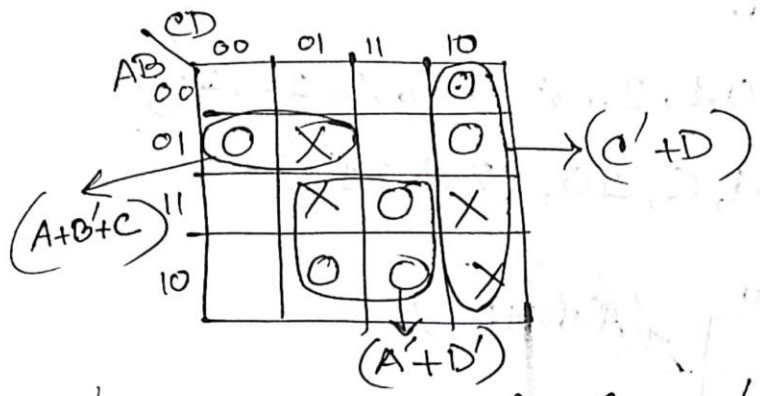
X



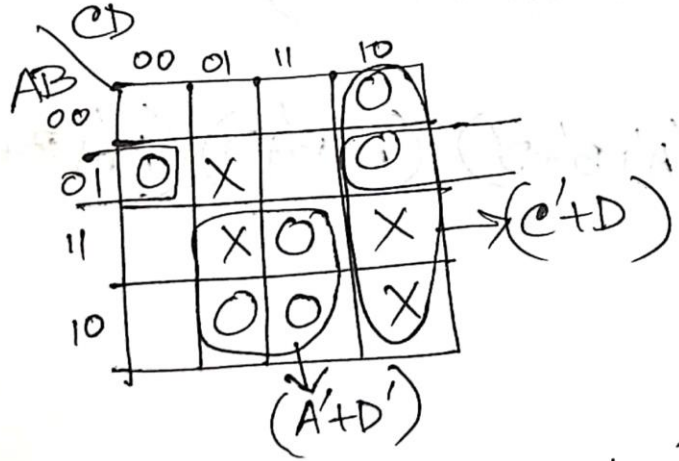
$$F = A'C'D + ABC' + ACD + A'BC$$

POS Simplification:

$F(A,B,C,D) = \Pi(2, 4, 6, 9, 11, 15) \cdot \Pi d.c.(5, 10, 13, 14)$



$F = (C'+D)(A'+D')(A+B'+C)$



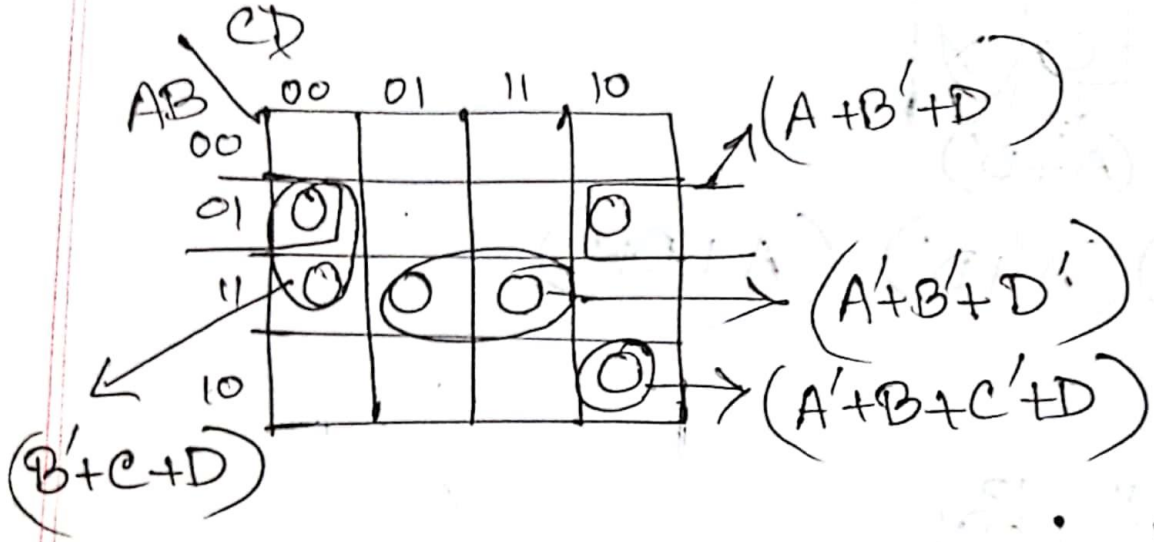
$F = (C'+D)(A'+D')(A+B'+D)$

$F(A,B,C,D) = \Sigma(0,1,2,3,5,7,8,9,11,14)$

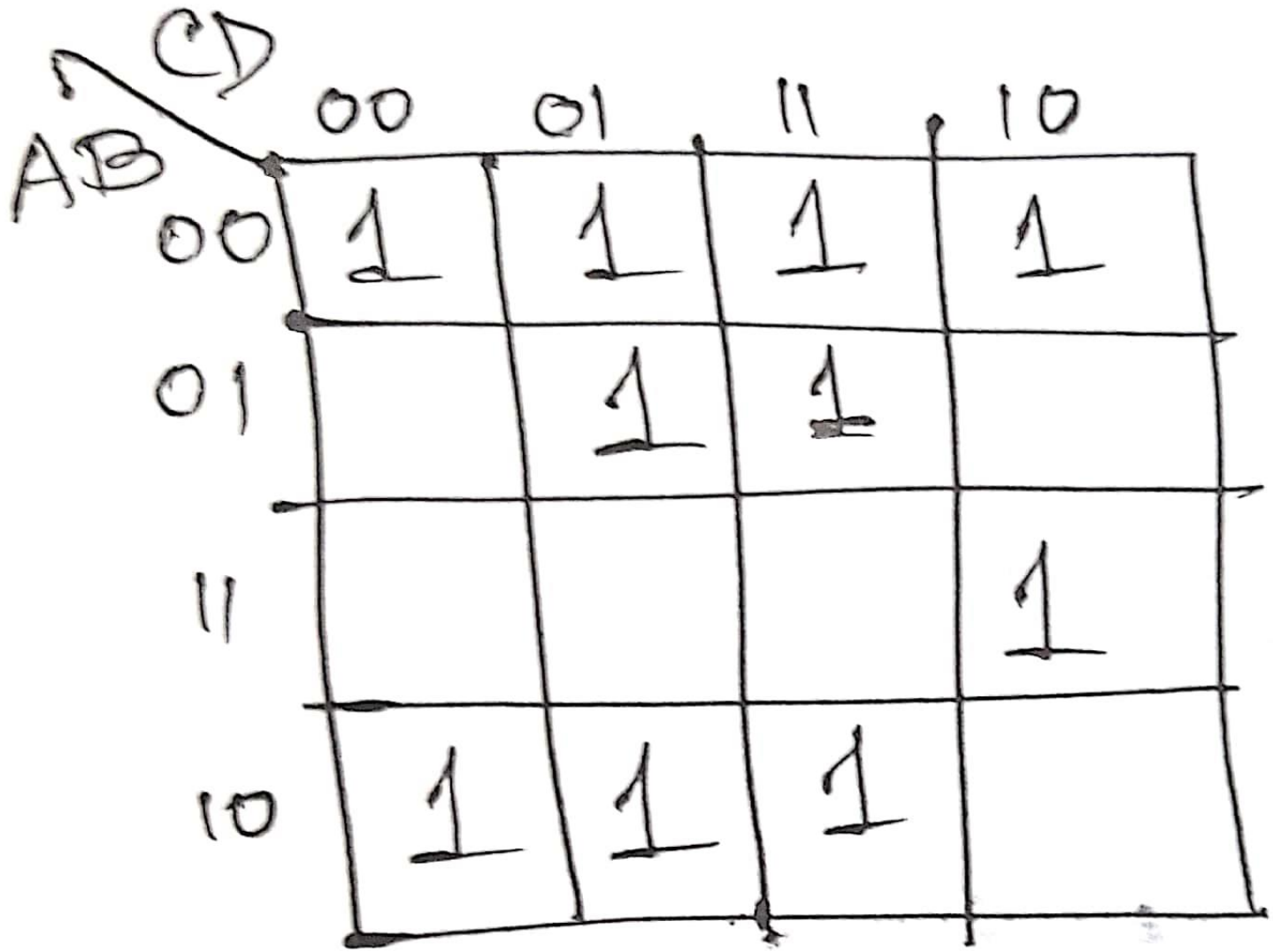
Find the simplified POS expression using K-map method.

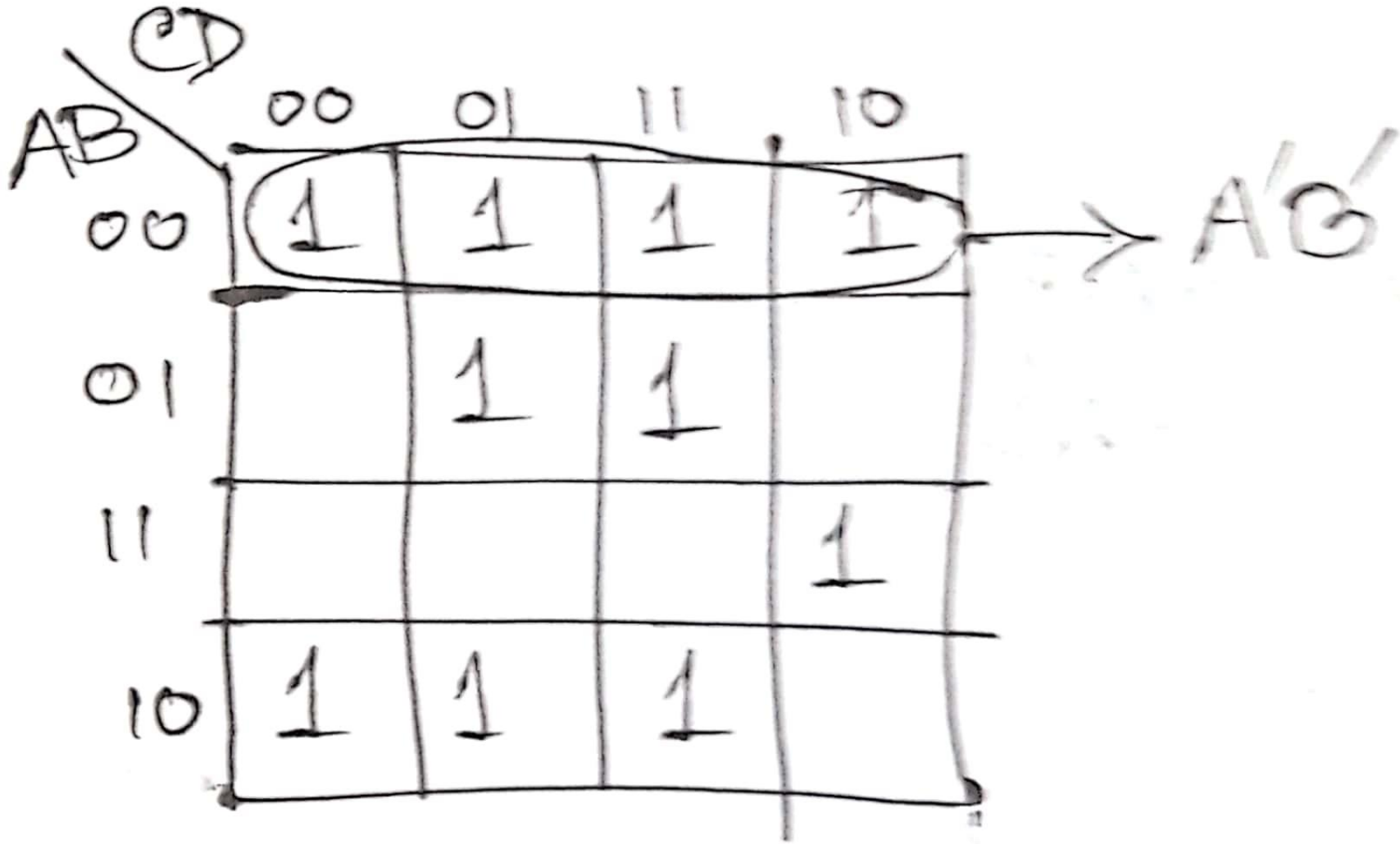
$F(A,B,C,D) = \Sigma(0,1,2,3,5,7,8,9,11,14)$

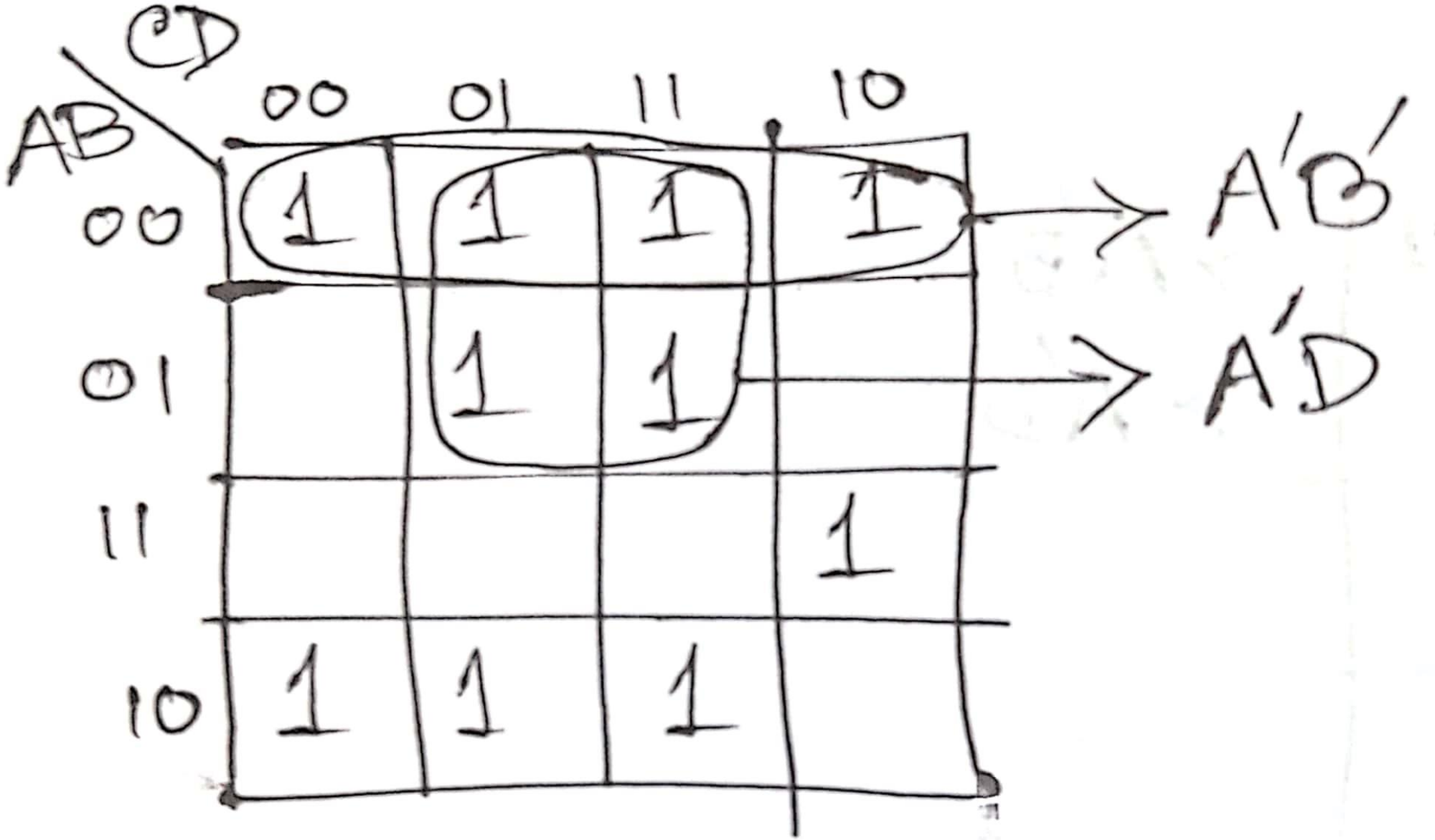
$= \Pi(4,6,10,12,13,15)$

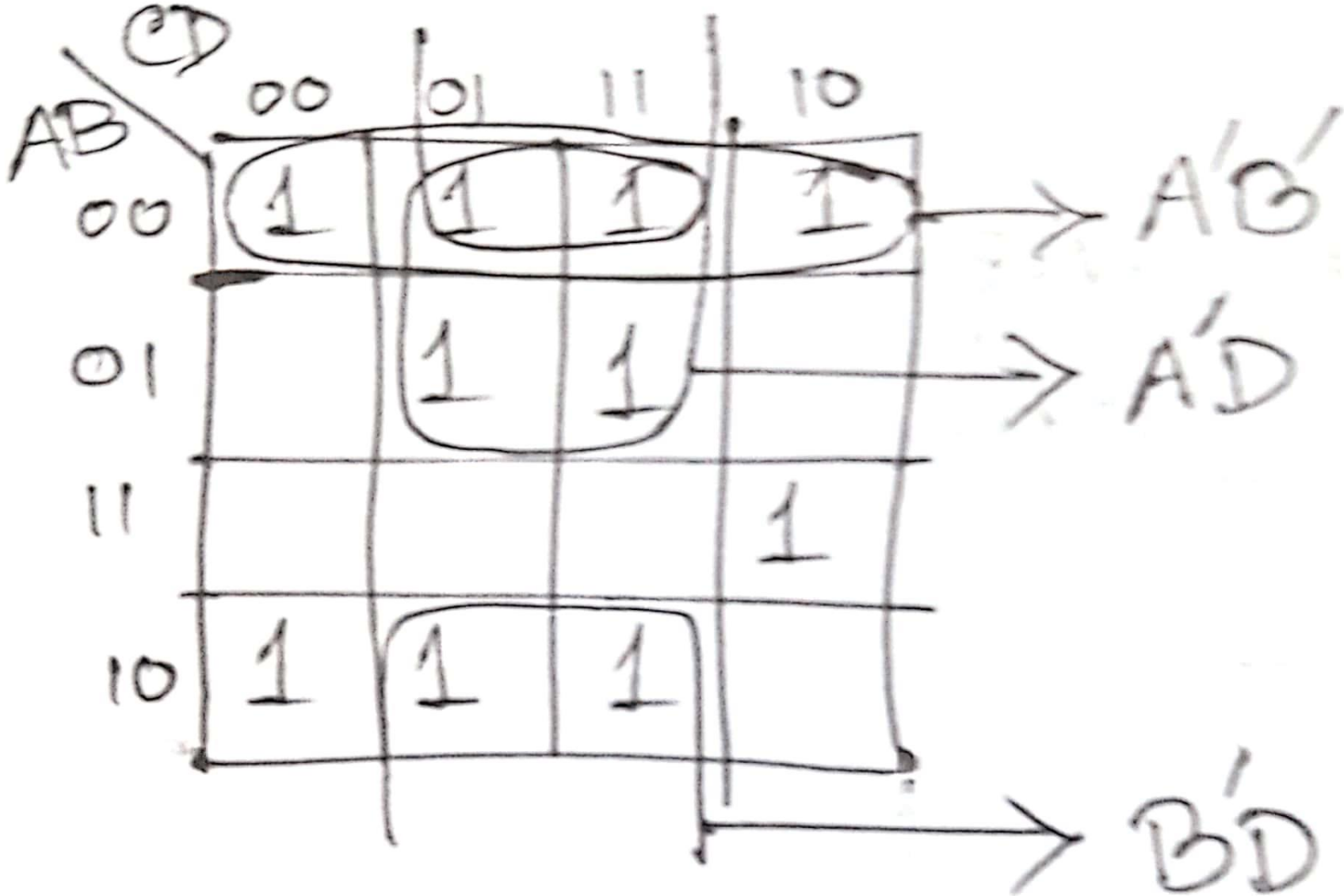


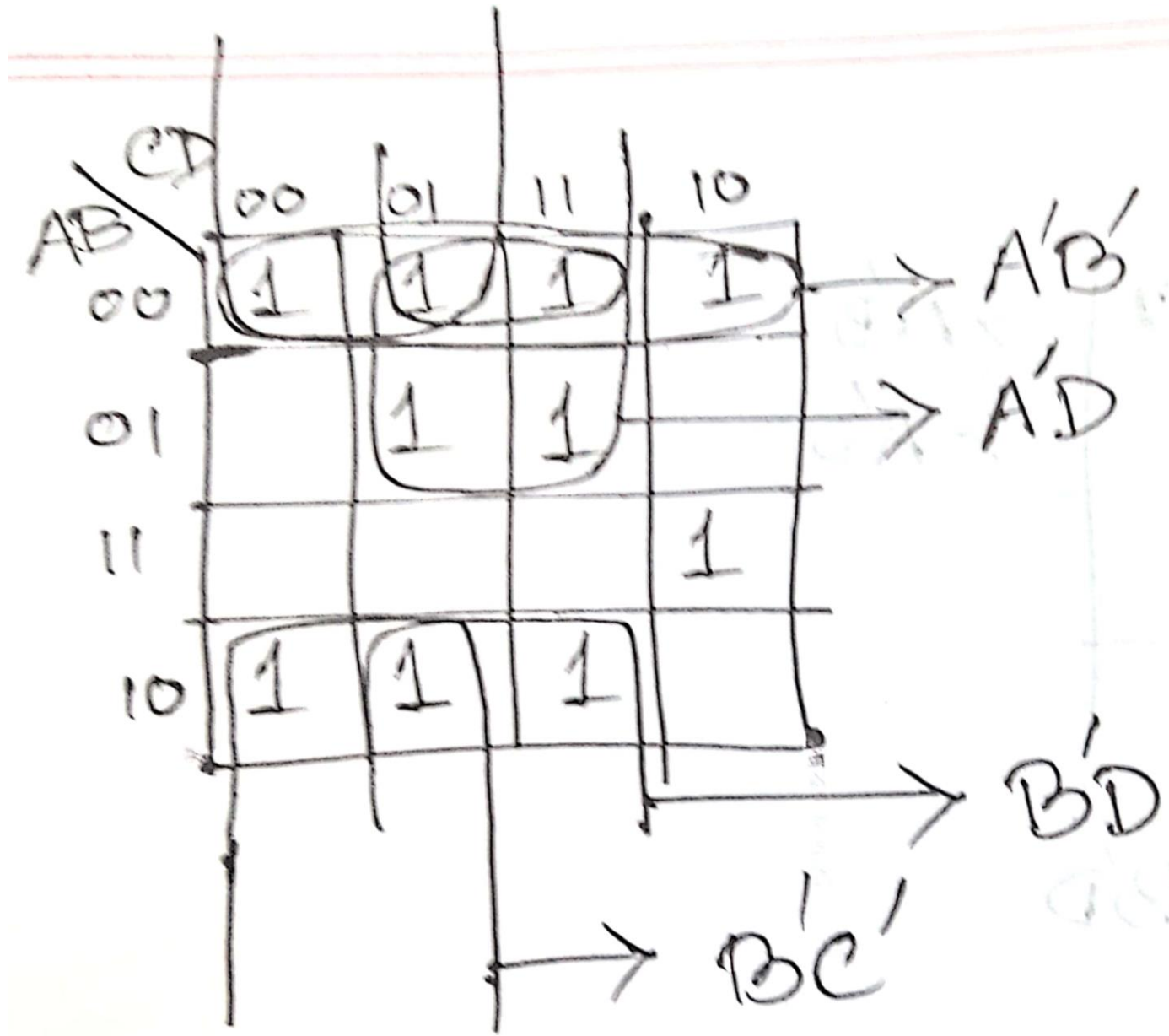
$F = (B'+C+D)(A'+B'+D')(A+B'+D)(A'+B+C'+D)$

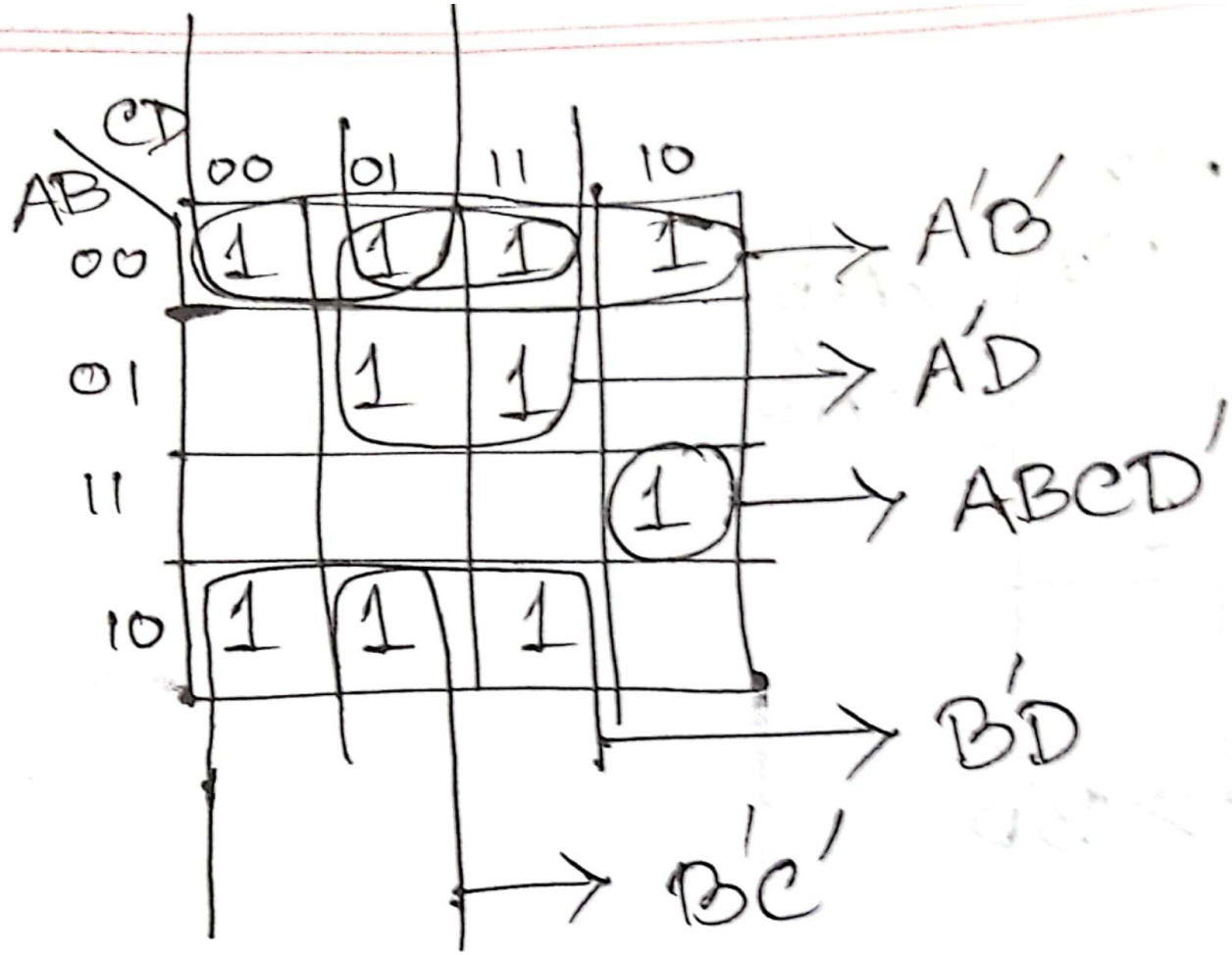

CD
AB
00
01
11
10
00
01
11
10
1
1
1
1
1
1
1
1
1
1
1
1
1
1


CD
AB
00
01
11
10
00
01
11
10
1
A'C'


CD
AB
00
01
11
10
00
01
11
10
1
A'B'
A'D


CD
AB
00
01
11
10
00
01
11
10
A'C'
A'D
B'D


CD
AB
00
01
11
10
00
01
11
10
A'B'
A'D
B'D
B'C'



$$F = A'B' + A'D + B'D + B'C' + ABCD'$$

5.3 Simplify the following sum of products functions using K-map method.

(a) $F(A,B,C,D) = \sum(0,1,2,3,7,8,9,10,11,12,13)$

(b) $F(A,B,C,D) = \sum(0,2,4,6,8) + \sum_{d.c.}(10,11,12,13,14,15)$

(c) $F(A,B,C,D) = \sum(0,2,4,9,12,15) + \sum_{d.c.}(1,5,7,10)$

(d) $F(A,B,C,D) = \sum(0,3,4,5,6,7,8,13,14)$

(e) $F(A,B,C,D) = \sum(4,6,7,9,10,11,12,14,15)$

(f) $F(A,B,C,D) = \sum(0,1,2,3,4,6,7,8,9,11,15)$

(g) $F(A,B,C,D) = \sum(1,3,4,5,7,8,9,11,14,15)$

(h) $F(A,B,C,D) = \sum(1,2,3,4,5,6,13,14,15)$

(i) $F(A,B,C,D) = ABC'D + A'BCD + A'B'C' + A'B'D' + AC' + AB'C + B'$

(j) $F(A,B,C,D) = \sum(1,3,7,11,15) + \sum_{d.c.}(0,2,5)$

(k) $f(x,y,z) = x'yz + x'yz' + xy'z' + xy'z$

(l) $F(A,B,C) = A'C + A'B + AB'C + BC$

(m) $F(A,B,C,D) = A'B'C' + B'CD' + A'BCD' + AB'C'$

(n) $F(A,B,C,D) = \sum(3,4,5,7,9,13,14,15)$

(o) $F(A,B,C,D) = \sum(0,1,2,3,5,7,8,9,11,14)$

5.4 Simplify the following product of sums functions using K-map method.

(a) $F(A,B,C,D) = \prod(1,4,5,6,11,12,13,14,15)$

(b) $F(A,B,C,D) = \prod(4,5,6,7,8,12) \cdot \prod_{d.c.}(1,2,3,9,11,14)$

(c) $F(A,B,C,D) = (A+B+C'+D')(A'+C+D')(A'+B+C'+D')$
$(B'+C)(B'+C')(A+B')(B'+D')$