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Assignment-2

1. $f(x) = \begin{cases} x-1, & x \le 3 \\ 3x-7, & x > 3 \end{cases}$

(i) $\lim_{x \to 3^-} f(x) = 3 - 1 = 2$

(ii) $\lim_{x \to 3^+} f(x) = (3 \times 3) - 7 = 2$

(iii) $\lim_{x \to 3} f(x) = 2$; does exist

এখানে,

$\lim_{x \to 3^-} f(x) = 3 - 1$
$= 2$

আবার,

$\lim_{x \to 3^+} f(x) = (3 \times 3) - 7$
$= 2$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\therefore \lim_{x \to 3} f(x) = 2$; does exist.

(Ans)

2.

$$f(x) = \begin{cases} x-2, & x<0 \\ x^2, & 0 \le x \le 2 \\ 2x, & x>2 \end{cases}$$

(i) $\lim_{x \to 0} f(x)$

$\lim_{x \to 0^-} f(x) = 0-2$
$= -2$

আবার,
$\lim_{x \to 0^+} f(x) = 0^2$
$= 0$

$\therefore \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} f(x) =$ doesn't exist

(Ans)

(ii) $\lim_{x \to 1} f(x)$

এখানে,
$\lim_{x \to 1} f(x) = (1)^2$
$= 1$

$\therefore \lim_{x \to 1} f(x) =$ does exist

(Ans)

(iii) $\lim_{x\to 2} f(x)$

এখানে,

$$\lim_{x\to 2^-} f(x) = (2)^2 = 4$$

আবার,

$$\lim_{x\to 2^+} f(x) = 2\times 2 = 4$$

$$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\therefore \lim_{x\to 2} f(x) = 4 \text{ ; does exist}$$

(Ans)

3. $$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \times \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$= \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2}$$

$$= \frac{1}{\sqrt{0+4}+2}$$

$$= \frac{1}{2+2}$$

$$= \frac{1}{4}$$

(Ans)

4. $f(x) = \dfrac{x^3-1}{x-1}$

(i) $\lim_{x \to 1} f(x) = \lim_{x \to 1} \dfrac{x^3-1}{x-1}$

$= \lim_{x \to 1} \dfrac{(x-1)(x^2+1+x)}{(x-1)}$

$= \lim_{x \to 1} x^2+x+1$

$= (1)^2+1+1$

$= 3$

(Ans)

(ii) $f(1) =$ undefined

$f(x) = \dfrac{x^3-1}{x-1}$

এখানে, $x-1 \neq 0$

$\therefore x \neq 1$

$\therefore f(1) =$ undefined

(Ans)

5 $f(x) = \begin{cases} \frac{x^2-9}{x+3} & , x \neq 3 \\ k & , x = -3 \end{cases}$

$$L.H.L = \lim_{x \to 3^-} f(x)$$

$$= \lim_{x \to -3^-} \frac{x^2-9}{x+3}$$

$$= \lim_{x \to -3^-} \frac{(x-3)(x+3)}{(x+3)}$$

$$= \lim_{x \to -3^-} (x-3)$$

$$= -3 - 3$$

$$= -6$$

$$R.H.L = \lim_{x \to -3^+} f(x)$$

$$= k$$

According to the question,

$$f(-3) = \lim_{x \to -3} f(x)$$

[That is the function is continous]

if;∴ $L.H.L = R.H.L$

$$\Rightarrow \lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x)$$

$$\Rightarrow -6 = k$$

$$\therefore k = -6$$

(Ans)