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$\underline{\underline{1}}$ $f(x) = (2\sqrt{x}+1)\left(\frac{2-x}{x^2+3x}\right)$

$$\therefore f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx}(2\sqrt{x}+1)\left(\frac{2-x}{x^2+3x}\right)$$

$$= (2\sqrt{x}+1)\frac{d}{dx}\left(\frac{2-x}{x^2+3x}\right) + \left(\frac{2-x}{x^2+3x}\right)\frac{d}{dx}(2\sqrt{x}+1)$$

$$= (2\sqrt{x}+1)\left(\frac{(x^2+3x)\frac{d}{dx}(2-x) - (2-x)\frac{d}{dx}(x^2+3x)}{(x^2+3x)^2}\right) + \left(\frac{2-x}{x^2+3x}\right)\left(\frac{1}{\sqrt{x}}\right)$$

$$= (2\sqrt{x}+1)\left(\frac{(x^2+3x)(-1) - (2-x)(2x+3)}{(x^2+3x)^2}\right) + \left(\frac{2-x}{x^2+3x}\right)\left(\frac{1}{\sqrt{x}}\right)$$

$$= (2\sqrt{x}+1)\left(\frac{-x^2-3x-4x-6+2x^2+3x}{(x^2+3x)^2}\right) + \left(\frac{2-x}{x^2+3x}\right)\left(\frac{1}{\sqrt{x}}\right)$$

$$= (2\sqrt{x}+1)\left(\frac{x^2-4x-6}{(x^2+3x)^2}\right) + \left(\frac{2-x}{x^2+3x}\right)\left(\frac{1}{\sqrt{x}}\right)$$

$$= \frac{(2\sqrt{x}+1)(x^2-4x-6)}{(x^2+3x)^2} + \frac{(2-x)}{(x^2+3x)\times(\sqrt{x})}$$

2. $f(x) = \dfrac{4x+1}{x^2-5}$

$$\therefore f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx}\left(\frac{4x+1}{x^2-5}\right)$$

$$= \frac{(x^2-5)\frac{d}{dx}(4x+1) - (4x+1)\frac{d}{dx}(x^2-5)}{(x^2-5)^2}$$

$$= \frac{(x^2-5)\times 4 - (4x+1)\times 2x}{(x^2-5)^2}$$

$$= \frac{4x^2-20-8x^2-2x}{(x^2-5)^2}$$

$$= \frac{-4x^2-2x-20}{(x^2-5)^2}$$

$$= \frac{-2(2x^2+x+10)}{(x^2-5)^2}$$

(Ans)

3. $f(x) = \dfrac{x^2+1}{x-1}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x-1}\right)$$

$$= \frac{(x-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(2x) - (x^2+1)\times 1}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2}$$

$$= \frac{x^2 - 2x - 1}{(x-1)^2}$$

because of the horizontal line here, $\frac{dy}{dx} = 0$

$$\therefore \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x^2 - 2x - 1}{(x-1)^2} = 0$$

$$\Rightarrow x^2 - 2x - 1 = 0$$

$$\therefore x = \frac{-(-2) \pm \sqrt{(4+4)}}{2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{8}}{2}$$

$$\Rightarrow x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow x = 1 \pm \sqrt{2}$$

$$\therefore x = 2.414 \; ; \; -0.414$$

(Am)

4 given, $f(x) = \frac{x+3}{x+2}$; $y = x$ is prependicular

Here,

$\frac{dy}{dx} = -1$

$\Rightarrow \frac{d}{dx} f(x) = -1$

$\Rightarrow \frac{d}{dx}\left(\frac{x+3}{x+2}\right) = -1$

$\Rightarrow \frac{(x+2)\frac{d}{dx}(x+3) - (x+3)\frac{d}{dx}(x+2)}{(x+2)^2} = -1$

$\Rightarrow \frac{(x+2) - (x+3)}{(x+2)^2} = -1$

$\Rightarrow \frac{x+2-x-3}{(x+2)^2} = -1$

~~$\Rightarrow 5 = x^2+4x+4$~~

$\Rightarrow -(x^2+4x+4) = -1$

$\Rightarrow -x^2-4x-4+1 = 0$

$\Rightarrow -x^2-4x-3 = 0$

$\Rightarrow x^2+4x+3 = 0$

$\Rightarrow x^2+3x+x+3 = 0$

$\Rightarrow x(x+3)+1(x+3) = 0$

$\Rightarrow (x+3)(x+1) = 0$

Here, or,

$x+3 = 0$ $x+1 = 0$

$\therefore x = -3$ $\Rightarrow x = -1$

$\therefore$ the values of x is : $-3, -1$

5. $y = x^2 \cos x + 4 \sin x$

$$\therefore y'' = \frac{d}{dx}\left(\frac{d}{dx} y\right)$$

$$= \frac{d}{dx}\left\{\frac{d}{dx}\left(x^2 \cos x + 4 \sin x\right)\right\}$$

$$= \frac{d}{dx}\left(\frac{d}{dx} x^2 \cos x + \frac{d}{dx} 4 \sin x\right)$$

$$= \frac{d}{dx}\left\{\left(x^2 \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^2\right) + 4 \cos x\right\}$$

$$= \frac{d}{dx}\left\{\left(-\sin x \cdot x^2 + \cos x \cdot 2x\right) + 4 \cos x\right\}$$

$$= \frac{d}{dx}\left(2x \cos x + 4 \cos x - x^2 \sin x\right)$$

$$= \frac{d}{dx}(2x \cos x) + \frac{d}{dx}(4 \cos x) - \frac{d}{dx}(x^2 \sin x)$$

$$= \left(2x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} 2x\right) + (-4 \sin x) - \left(x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2\right)$$

$$= -2x \sin x + 2 \cos x - 4 \sin x - x^2 \cos x - 2x \sin x$$

$$= 2 \cos x - 4 \sin x - x^2 \cos x - 4x \sin x$$

$$= (2 - x^2) \cos x - 4 \sin x (x - 1)$$

(Ans)

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$y = \tan x$

$\therefore y'' = \frac{d}{dx}\left(\frac{d}{dx} y\right)$

$= \frac{d}{dx}\left(\frac{d}{dx} \tan x\right)$

$= \frac{d}{dx} \sec^2 x$

$= 2\sec x \cdot \frac{d}{dx}(\sec x)$

$= 2\sec x \cdot \sec x \cdot \tan x$

$= 2 \sec^2 x \tan x$

(Ans)

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We know,

tangent line Equation,

$y - y_1 = m(x - x_1)$

Here,

$m_{tan} = \frac{d}{dx} \sin x$

$= \cos x$

$= \cos \frac{\pi}{2}$ $[x = \pi/2]$

$= 0$

for the equation slope is 0,

$\therefore y - y_1 = 0(x - x_1)$

$\Rightarrow y - y_1 = 0$

$\therefore y - y_1 = 0$

(Ans)

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Here,

$$\sin\theta = \frac{3800}{s}$$

$$\Rightarrow s = \frac{3800}{\sin\theta}$$

$$\Rightarrow \frac{ds}{d\theta} = \frac{d}{d\theta}(3800 \operatorname{cosec}\theta)$$

$$\Rightarrow \frac{ds}{d\theta} = 3800(-\operatorname{cosec}\theta \cdot \cot\theta)$$

$$\Rightarrow \frac{ds}{d\theta} = -3800 \operatorname{cosec}\theta \cot\theta$$

if, $\theta = 30^\circ$

$$\therefore \frac{ds}{d\theta} = -3800 \times \operatorname{cosec}(30) \times \cot(30)$$

$$\Rightarrow \frac{ds}{d\theta} = -3800 \times 2 \times (\sqrt{3})$$

$$\therefore \frac{ds}{d\theta} = -7600\sqrt{3} \text{ ft/rad}$$

$$= -380\frac{\sqrt{3}\pi}{9}$$

$$= -229.75 \text{ ft/deg}$$

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Let,

$$y = \cos x$$

$$\Rightarrow \frac{dy}{dx} = -\sin x = \cos\left(\frac{\pi}{2} + x\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\cos x = \cos\left(\frac{2\pi}{2} + x\right)$$

$$\Rightarrow \frac{d^3y}{dx^3} = \sin x = \cos\left(3\frac{\pi}{2} + x\right)$$

$\vdots$

$$\Rightarrow \frac{d^ny}{dx^n} = \cos\left(n \cdot \frac{\pi}{2} + x\right)$$

we have to calculate for $n = 100$

$$\therefore \frac{d^{100}y}{dx^{100}}(\cos x) = \cos\left(100 \times \frac{\pi}{2} + x\right)$$

$$= \cos(50\pi + x)$$

$$= \cos(2\pi \times 25 + x)$$

$$= \cos x$$

(Ans)

10 Let $f(x) = \sin x$

$\therefore f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f''''(x) = \sin x$

$\therefore f^{(n)}(x) = \sin x$

So, for $n = 4k$, where k is constant.

$\therefore f^{(n)}(x) = \sin x$

(Ans)