Chapter 2
Number Systems and Codes

Binary Number System

2.4 Binary Number System

| 3 | 2 | 1 | 0 | Positional Weight

2^3 2^2 2^1 2^0

8   4   2   1

Binary to Decimal

$(1101)_2$
8 1 2 1

$8 + 4 + 1 = (13)_{10}$

$(100)_2 = (4)_{10}$
1 2 1

Decimal to Binary

(8 1 2 1)
$(9)_{10} = (1001)_2$

$(7)_{10} = (111)_2 = (0111)_2$

Most Significant Bit (MSB)   Least Significant Bit (LSB)
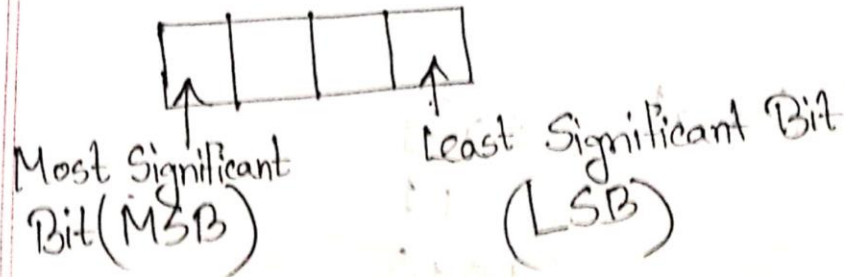
$n = 4 \implies 2^4 = 16$ Combinations

Binary 8 4 2 1	Decimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

MSB   LSB

Binary Addition:

4+8

$4 \rightarrow 0100$

$+ 8 \rightarrow 1000$

$12 \rightarrow 1100$

5+12

$5 \rightarrow 0101$

$+12 \rightarrow 1100$

$17 \rightarrow 10001$

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1's Complement

$(10110)_2 \xrightarrow{\text{1's complement}} (01001)_2$

$(1010)_2 \xrightarrow{\text{1's complement}} (0101)_2$

2's Complement

$(0110)_2 \xrightarrow{\text{1's complement}}$
$$\begin{array}{r} 1 \\ 1001 \\ +1 \\ \hline (1010)_2 \end{array}$$

Another Way: $(0110)_2 \xrightarrow{\text{2's complement}} (1010)_2$

$(0011)_2 \xrightarrow{\text{1's complement}}$
$$\begin{array}{r} 1100 \\ +1 \\ \hline (1101)_2 \end{array}$$

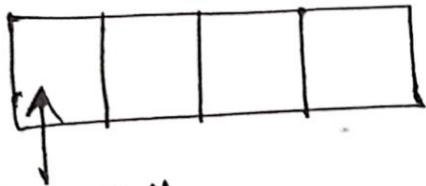Another Way: $(0011)_2 \xrightarrow{\text{2's complement}} (1101)_2$

2's complement Signed Number System

Range: $\qquad -2^{n-1}$ to $+\left(2^{n-1}-1\right)$

$n=4 \implies -2^{4-1}$ to $+\left(2^{4-1}-1\right)$

$\qquad\qquad = -2^{3}$ to $+\left(2^{3}-1\right)$

$\qquad\qquad = -8$ to $+7$

Sign Bit

Sign Bit $= 0 \implies$ Positive Number

Sign Bit $= 1 \implies$ Negative Number

Binary $-8\ 4\ 2\ 1$	Decimal
0 0 0 0	+0
0 0 0 1	+1
0 0 1 0	+2
0 0 1 1	+3
0 1 0 0	+4
0 1 0 1	+5
0 1 1 0	+6
0 1 1 1	+7
1 0 0 0	-8
1 0 0 1	-7
1 0 1 0	-6
1 0 1 1	-5
1 1 0 0	-4
1 1 0 1	-3
1 1 1 0	-2
1 1 1 1	-1

Algebraic Addition

$x + y$	x	y
	+	+
	+	-
	-	+
	-	-

$\boxed{++}$ $x = 3, y = 2$

$3 \rightarrow \quad 00\overset{1}{1}1$

$2 \rightarrow \quad 0010$

$5 \rightarrow \quad 0101$

$x = 4, y = 5$

$4 \rightarrow \quad 0\overset{1}{1}00$

$5 \rightarrow \quad 0101$

$9 \rightarrow \quad 1001 \text{(Overflow)}$

$\boxed{+ \; -}$ $x = 3, \; y = -2$

$3 \rightarrow 0011$

$-2 \rightarrow 1110$

$(2)_{10} = (0010)_2 \xrightarrow{\;2's\;} (1110)_2 = (-2)_{10}$

$3 \rightarrow \overset{1 \; 1}{0011}$

$-2 \rightarrow 1110$

$1 \rightarrow 10001$

↓

Discard

$x = 3, \ y = -4$

$(4)_{10} = (0100)_2 \xrightarrow{\text{2's}} (1100)_2 = (-4)_{10}$

$3 \rightarrow 0011$

$-4 \rightarrow 1100$

$-1 \rightarrow 1111$

$$\boxed{\; - \quad + \;} \qquad x = -3, \; y = 4$$

$$(3)_{10} = (0011)_2 \xrightarrow{\;2's\;} (1101)_2 = (-3)_{10}$$

$$\begin{array}{r} 1 \\ -3 \to \quad 1101 \\ 4 \to \quad 0100 \\ \hline 1 \to \lfloor 1 \rfloor 0001 \end{array}$$

$$\downarrow$$

Discard

$$\boxed{- \; -} \quad x = -3, \; y = -4$$

$$(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2 = (-3)_{10}$$

$$(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$$

$$-3 \rightarrow \overset{1}{1}101$$

$$-4 \rightarrow 1100$$

$$\rule{6cm}{0.4pt}$$

$$-7 \rightarrow \underset{\downarrow}{\boxed{1}}\, 10\,01$$

$$\downarrow$$

Discard

$$x = -4, \quad y = -5$$

$$(4)_{10} = (0100)_2 \xrightarrow{\text{2's}} (1100)_2 = (-4)_{10}$$

$$(5)_{10} = (0101)_2 \xrightarrow{\text{2's}} (1011)_2 = (-5)_{10}$$

$$
\begin{array}{rcl}
-4 & \rightarrow & 1100 \\
-5 & \rightarrow & 1011 \\
\hline
-9 & \rightarrow & \boxed{1}\,0111 \quad \text{(Underflow)}
\end{array}
$$

Discard

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Subtraction:

$x - y = x + (-y)$ where $-y = $ 2's complement of y

$\boxed{+\ +}$ $x = 4, y = 2$

$x - y = 4 - 2 = 4 + (-2)$ $(2)_{10} = (0010)_2 \xrightarrow{2's} (1110)_2$

$\overset{1}{4} \rightarrow 0100$ $= (-2)_{10}$

$-2 \rightarrow 1110$

$2 \rightarrow \underline{1}0010$

$\downarrow$

Discard

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$x = 2, y = 4$

$x - y = 2 - 4 = 2 + (-4)$ | $(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$

$2 \rightarrow \quad 0010$

$-4 \rightarrow \quad 1100$

_____

$-2 \rightarrow \quad 1110$

$\boxed{+ \ -}$ $x = 3, \ y = -2$

$x - y = 3 - (-2) = 3 + 2$

$\overset{1}{3 \to 0011}$

$2 \to 0010$

$5 \to 0101$

$x = 4, \ y = -5$

$x - y = 4 - (-5) = 4 + 5$

$\overset{1}{4 \to 0100}$

$5 \to 0101$

$9 \to 1001 \, (\text{overflow})$

$$\boxed{-7}$$

$$x = -3, \quad y = 2$$

$$x - y = (-3) - (2) = (-3) + (-2)$$

$$\begin{array}{r} -3 \to \overset{1}{1}101 \\ -2 \to 1110 \\ \hline -5 \to 1\,1011 \end{array}$$

$$\downarrow$$

Discard

$$(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2$$
$$= (-3)_{10}$$

$$(2)_{10} = (0010)_2 \xrightarrow{2's} (1110)_2$$
$$= (-2)_{10}$$

$x = -5, y = 4$

$x - y = (-5) - 4 = (-5) + (-4)$

$-5 \rightarrow 1011$

$-4 \rightarrow 1100$

$-9 \rightarrow 10111$ (Underflow)

$\downarrow$

Discard

$(5)_{10} = (0101)_2 \xrightarrow{2's} (1011)_2$

$= (-5)_{10}$

$(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2$

$= (-4)_{10}$

$\boxed{\text{--}}$ $x = -3, \; y = -2$

$x - y = (-3) - (-2) = (-3) + (2)$ $\Big|$ $(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2$

$-3 \rightarrow 1101$

$2 \rightarrow 0010$

$\hspace{3em} = (-3)_{10}$

_____

$-1 \rightarrow 1111$

$x = -3, \; y = -4$

$x - y = (-3) - (-4) = (-3) + (4)$

$\overset{1}{-3} \rightarrow 1101$

$4 \rightarrow 0100$

_____

$1 \rightarrow 1,0001$

$\downarrow$

Discard

Examples

Addition in 2's Complement Signed Number System

Example 1: $7 \longrightarrow$ 0111 Minuend

 $- 5 \longrightarrow (+)$ 1011 2's complement of subtrahend

\---   \------

 $+ 2$ (1)0010

$\hookrightarrow$ Discard end carry

The answer is 0010, which is equivalent to $(+2)_{10}$.

Example 2: $5 \longrightarrow$ 0101 Minuend

 $- 7 \longrightarrow (+)$ 1001 2's complement of subtrahend

\---   \------

 $- 2$ 1110

Examples

Subtraction in 2's Complement Signed Number System

Example 1: $48 - 23 = 48 + (- 23) = + 25$

$$
\begin{array}{rl}
48 \longrightarrow & \underline{00110000}\ \text{8-bit 2's complement representation of} + 48 \\
+\,(- 23) \longrightarrow (+) & 11101001\ \ \text{8-bit 2's complement representation of} - 23 \\
\text{---} & \text{------------} \\
+\,\underline{25}\qquad & (1)00011001\ \ \text{8-bit 2's complement representation of} + 25
\end{array}
$$

$\llcorner\!\!\rightarrow$ Discard end carry

Example 2: $23 - 48 = 23 + (-48) = - 25$

$$
\begin{array}{rl}
23 \longrightarrow & \underline{00010111}\ \ \text{8-bit 2's complement representation of} + 23 \\
+\,(- 48) \longrightarrow (+) & \underline{11010000}\ \ \text{8-bit 2's complement representation of} - 48 \\
\text{---} & \text{------------} \\
-\,25\qquad & \underline{11100111}\ \ \text{8-bit 2's complement representation of} - 25
\end{array}
$$

Examples

Example 3: $48 - (-23) = 48 + 23 = +71$

$$
\begin{array}{rl}
48 \rightarrow & \underline{00110000} \text{ 8-bit 2's complement representation of} + 48 \\
+23 \rightarrow (+) & \underline{00010111} \text{ 8-bit 2's complement representation of} + 23 \text{ or } - (-23) \\
\text{---} & \text{------------} \\
+71 & \underline{01000111} \text{ 8-bit 2's complement representation of} + 71
\end{array}
$$

Example 4: $-48 - 23 = (-48) + (-23) = -71$

$$
\begin{array}{rl}
-48 \rightarrow & \underline{11010000} \text{ 8-bit 2's complement representation of} - 48 \\
-23 \rightarrow (+) & \underline{11101001} \text{ 8-bit 2's complement representation of} - 23 \\
\text{---} & \text{------------} \\
-71 \underline{\qquad} & (1)10111001 \text{ 8-bit 2's complement representation of} - 71
\end{array}
$$

$\longrightarrow$ Discard end carry