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Course : MAT 102

Section : 01

Home-Assignment No : 01

1. a)

$$y'' - y' - 6y = e^x + 3x^2 \quad ------ (i)$$

Lets $y = e^{mx}$ is the solution of $y'' - y' - 6y = 0 \quad ---- (ii)$

So, equation (ii) becomes,

$$m^2 e^{mx} - m e^{mx} - 6e^{mx} = 0$$

$$\Rightarrow e^{mx}(m^2 - m - 6) = 0$$

Here, $e^{mx} \neq 0$

So, A.E is $m^2 - m - 6 = 0$

$$\Rightarrow m^2 - 3m + 2m - 6 = 0$$

$$\Rightarrow m(m-3) + 2(m-3) = 0$$

$$\Rightarrow (m-3)(m+2) = 0$$

$$\therefore m = -3, 2$$

$\therefore$ Complementary function is,

$$y_c = c_1 e^{-3x} + c_2 e^{2x}$$

Let,
Particular solution is,

$$y_p = Ae^x + Bx^2 + Cx + E$$

$$y'_p = Ae^x + 2Bx + C$$

$$y''_p = Ae^x + 2B$$

$\therefore$ Equation (i) becomes,

$$Ae^x + 2B - Ae^x - 2Bx - C - 6Ae^x - 6Bx^2 - 6Cx - 6E = e^x + 3x^2$$

Equiting the coefficient of e^x, x^2, x and const from the both sides,

$$-6A = 1 \qquad -6B = 3 \qquad -2B - 6C = 0 \qquad 2B - C - 6E = 0$$

$$\therefore A = -\frac{1}{6} \qquad \therefore B = -\frac{1}{2} \qquad \therefore C = \frac{1}{6} \qquad \therefore E = \frac{7}{36}$$

$$\therefore y_p = -\frac{1}{6}e^x - \frac{1}{2}x^2 + \frac{1}{6}x + \frac{7}{36}$$

$\therefore$ General solution is,

$$y = c_1 e^{-3x} + c_2 e^{2x} - \frac{1}{6} e^{x} - \frac{1}{2} x^2 + \frac{1}{6} x + \frac{7}{36}$$

1. b) $y''' + 9y' = -2\sin 3x + \cos 3x + 3x$ ..... (i)

Lets, $y = e^{mx}$ is the solution of $y''' + 9y' = 0$ --- (ii)

equation (ii) becomes,

$$m^3 e^{mx} + 9me^{mx} = 0$$

$$\Rightarrow e^{mx}(m^2 + 9m) = 0$$

Here, $e^{mx} \neq 0$

A·E is $m^3 + 9m = 0$

$$\Rightarrow m(m^2+9) = 0$$

$\therefore m = 0$ Again, $m^2 + 9 = 0$

$\therefore m = \pm 3i$

$m = 0, 3i, -3i$; $\alpha = 0$, $\beta = 3$

Complementary function is,

$$y_c = c_1 + e^0 [c_2 \cos 3x + c_3 \sin 3x]$$

$$= c_1 + c_2 \cos 3x + c_3 \sin 3x$$

Lets,

Particular solution is, $y_p = x(A\cos 3x + B\sin 3x) + x(cx + E)$

$$y_p' = (A\cos 3x + B\sin 3x) + x(3B\cos 3x - 3A\sin 3x) + 2cx + E$$

$$y''_p = (-3A\sin 3x + 3B\cos 3x) + (3B\cos 3x - 3A\sin 3x) + x(-9A\cos 3x - 9B\sin 3x) + 2c$$

$$y''' = (-9A\cos 3x - 9B\sin 3x) + (-9B\sin 3x - 9A\cos 3x) + x(27A\sin 3x - 27B\cos 3x) + (-9A\cos 3x - 9B\sin 3x)$$

Equation (i) becomes,

$-9B\sin 3x - 9A\cos 3x - 9A\cos 3x - 9B\sin 3x - 9A\cos 3x - 9B\sin 3x + 27x A\sin 3x - 27x B\cos 3x + 9A\cos 3x + 9B\sin 3x + 27x B\cos 3x - 27x A\sin 3x + 18cx + 9E = -2\sin 3x + \cos 3x + 3x$

$$\Rightarrow -18A\cos 3x - 18B\sin 3x + 18Cx + 9E = -2\sin 3x + \cos 3x + 3x$$

Equiting the coefficient of $\cos 3x$, $\sin 3x$ and x, constant From both side;

$-18A = 1$ $\quad$ $-18B = -2$ $\quad$ $18C = 3$ $\quad$ $9E = 0$

$\Rightarrow A = -\frac{1}{18}$ $\quad$ $\therefore B = \frac{1}{9}$ $\quad$ $\therefore C = \frac{1}{6}$ $\quad$ $\therefore E = 0$

$$\therefore y_p = x\left(-\frac{1}{18}\cos 3x + \frac{1}{9}\sin 3x\right) + x\left(\frac{x}{6}\right)$$

$$= x\left(-\frac{1}{18}\cos 3x + \frac{1}{9}\sin 3x\right) + \frac{x^2}{6}$$

The general solution is,

$$y = C_1 + C_2\cos 3x + C_3\sin 3x - \frac{x}{18}\cos 3x + \frac{x}{9}\sin 3x + \frac{x^2}{6}$$

e) $y'' - y' - 2y = (x-4)e^x + x$ ...... (i)

Lets, $y = e^{mx}$ is the solution of $y'' - y' - 2y = 0$ ..... (ii)

equation (ii) becomes,

$$m^2 e^{mx} - m e^{mx} - 2e^{mx} = 0$$

$$\Rightarrow e^{mx}(m^2 - m - 2) = 0$$

Here, $e^{mx} \neq 0$

A.E is $m^2 - m - 2 = 0$

$\Rightarrow m^2 - 2m + m - 2 = 0$

$\Rightarrow m(m-2) + 1(m-2) = 0$

$\Rightarrow (m-2)(m+1) = 0$

$\therefore m = 2, -1$

$\therefore$ Complementory function is, $y_c = c_1 e^{2x} + c_2 e^{-x}$

$\therefore$ Particular solution, $y_p = (Ax - B)e^x + Cx + E$

$$y'_p = (Ax - B)e^x + Ae^x + C$$

$$y''_p = (Ax - B)e^x + 2Ae^x$$

equation (i) becomes,

$$Axe^x - Be^x + 2Ae^x - Axe^x + Be^x - Ae^x - C - 2Axe^x + 2Be^x - 2Cx - 2E = (x-4)e^x + x$$

$$\Rightarrow -2Ae^x x + Ae^x + 2Be^x - 2Cx - C - 2E = xe^x - 4e^x + x$$

Equiting the coefficient of xe^x, e^x, x, constant from both side;

$-2A = 1$ $\quad$ $A + 2B = -4$ $\quad$ $-2C = 1$ $\quad$ $-C - 2E = 0$

$\therefore A = -\frac{1}{2}$ $\quad$ $\therefore B = -\frac{7}{4}$ $\quad$ $\therefore C = -\frac{1}{2}$ $\quad$ $\therefore E = \frac{1}{4}$

$\therefore y_p = \left(-\frac{1}{2}x - \frac{7}{4}\right)e^x - \frac{1}{2}x + \frac{1}{4}$

$\therefore$ The general solution is; $y = c_1 e^{2x} + c_2 e^{-x} - \frac{1}{2}xe^x - \frac{7}{4}e^x - \frac{x}{2} + \frac{1}{4}$

d) $y^4 - y'' = 2e^{-x} + 5 \quad \ldots\ldots (i)$

Lets, $y = e^{mx}$ is the solution $y^4 - y'' = 0 \quad \ldots (ii)$

equation (ii) becomes,

$$m^4 e^{mx} - m^2 e^{mx} = 0$$

$$\Rightarrow e^{mx}(m^4 - m^2) = 0$$

Here, $e^{mx} \neq 0$

A.E is $m^4 - m^2 = 0$

$$m^2(m^2 - 1) = 0$$

Here, $m^2 - 1 = 0$

$$\therefore m = \pm 1$$

$$\therefore m = 0, 0, 1, -1$$

$\therefore$ Complementary function is $y_c = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-x}$

Lets,

Particular solution is, $y_p = Axe^{-x} + Bx^2$

$$y'_p = Ae^{-x} - Axe^{-x} + 2Bx$$

$$y''_p = -Ae^{-x} - Ae^{-x} + Axe^{-x} + 2B$$

$$y'''_p = Ae^{-x} + Ae^{-x} + Ae^{-x} - Axe^{-x}$$

$$y^4_p = -Ae^{-x} - Ae^{-x} - Ae^{-x} - Ae^{-x} + Axe^{-x}$$

Equation (i) becomes,

$$-4Ae^{-x} + Axe^{-x} + 2Ae^{-x} - Axe^{-x} - 2B = 2e^{-x} + 5$$

$$\Rightarrow -4Ae^{-x} + 2Ae^{-x} - 2B = 2e^{-x} + 5$$

Equiting the coefficient of e^{-x}, constant from both side.

$-4A + 2A = 2$ $\qquad$ $-2B = 5$

$\Rightarrow -2A = 2$ $\qquad$ $\therefore B = -\frac{5}{2}$

$\therefore A = -1$

$$y_p = -xe^{-x} - \frac{5}{2}x^2$$

$\therefore$ General solution is, $y = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-x} - xe^{-x} - \frac{5}{2}x^2$