

Chapter 2

Number Systems and Codes

Binary Number System

2.4 Binary Number System

3	2	1	0
2^3	2^2	2^1	2^0
8	4	2	1

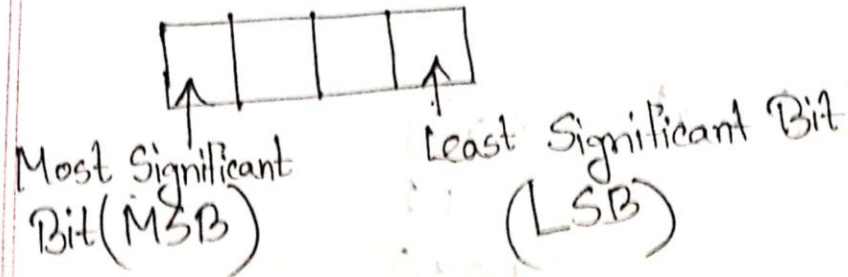
Positional Weight

Binary to Decimal

$$\begin{array}{cccc} (1101)_2 \\ 8 & 4 & 2 & 1 \end{array}$$

$$8 + 4 + 1 = (13)_{10}$$

$$\begin{array}{ccc} (100)_2 & = & (4)_{10} \\ 4 & 2 & 1 \end{array}$$



Decimal to Binary

$$(9)_{10} = \begin{array}{cccc} (8 & 4 & 2 & 1) \\ (1 & 0 & 0 & 1) \end{array}_2$$

$$(7)_{10} = (111)_2 = (0111)_2$$

$n=4 \Rightarrow 2^4 = 16$ Combinations

Binary 8 4 2 1	Decimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

MSB LSB

Binary Addition:

$4+8$
 $4 \rightarrow 0100$
 $+ 8 \rightarrow 1000$

 $12 \rightarrow 1100$

$5+12$
 $5 \rightarrow 0101$
 $+12 \rightarrow 1100$

 $17 \rightarrow 10001$

1's Complement

$$(10110)_2 \xrightarrow{1's \text{ complement}} (01001)_2$$

$$(1010)_2 \xrightarrow{1's \text{ complement}} (0101)_2$$

2's Complement

$$(0110)_2 \xrightarrow{1's \text{ complement}} \begin{array}{r} 1001 \\ + 1 \\ \hline (1010)_2 \end{array}$$

$$\text{Another Way: } (0110)_2 \xrightarrow{2's \text{ complement}} (1010)_2$$

$$(0011)_2 \xrightarrow{1's \text{ complement}} \begin{array}{r} 1100 \\ + 1 \\ \hline (1101)_2 \end{array}$$

$$\text{Another Way: } (0011)_2 \xrightarrow{2's \text{ complement}} (1101)_2$$

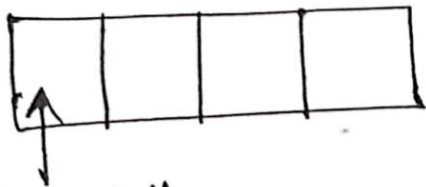
2's complement Signed Number System

Range: -2^{n-1} to $+(2^{n-1}-1)$

$$n=4 \Rightarrow -2^{4-1} \text{ to } +(2^{4-1}-1)$$

$$= -2^3 \text{ to } +(2^3-1)$$

$$= -8 \text{ to } +7$$



Sign Bit

Sign Bit = 0 \Rightarrow Positive Number

Sign Bit = 1 \Rightarrow Negative Number

Binary - 8 4 2 1	Decimal
0000	+0
0001	+1
0010	+2
0011	+3
0100	+4
0101	+5
0110	+6
0111	+7
1000	-8
1001	-7
1010	-6
1011	-5
1100	-4
1101	-3
1110	-2
1111	-1

Algebraic Addition

$x + y$	x	y
	+	+
	+	-
	-	+
	-	-

$\boxed{++}$

$x = 3, y = 2$

$3 \rightarrow 00^111$

$2 \rightarrow 0010$

$5 \rightarrow 0101$

$x = 4, y = 5$

$4 \rightarrow 01^100$

$5 \rightarrow 0101$

$9 \rightarrow 1001 (\text{Overflow})$

$$\boxed{+-} \quad x=3, y=-2$$

2 →

$$3 \rightarrow 0011$$

$$-2 \rightarrow 1110$$

$$(2)_{10} = (0010)_2 \xrightarrow{2's} (1110)_2 = (-2)_{10}$$

$$3 \rightarrow \overset{1}{0}\overset{1}{0}11$$

$$-2 \rightarrow 1110$$

$$1 \rightarrow 10001$$

↓
Discard

$$x = 3, y = -4$$

$$(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$$

$$3 \rightarrow 0011$$

$$-4 \rightarrow 1100$$

$$-1 \rightarrow 1111$$

$$\boxed{-+} \quad x = -3, y = 4$$

$$(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2 = (-3)_{10}$$

$$-3 \rightarrow \overset{1}{1}101$$

$$4 \rightarrow 0100$$

$$1 \rightarrow \boxed{1}10001$$

↓
Discard

-- $x = -3, y = -4$

$$(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2 = (-3)_{10}$$

$$(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$$

$$-3 \rightarrow \overset{1}{1}101$$

$$-4 \rightarrow 1100$$

$$-7 \rightarrow \overset{1}{\downarrow} 11001$$

Discard

$$x = -4, y = -5$$

$$(4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$$

$$(5)_{10} = (0101)_2 \xrightarrow{2's} (1011)_2 = (-5)_{10}$$

$$\begin{array}{rcl} -4 & \rightarrow & 1100 \\ -5 & \rightarrow & 1011 \end{array}$$

$$-9 \rightarrow \boxed{1}0111 \text{ (Underflow)}$$

↓
Discard

Subtraction:

$x - y = x + (-y)$ where $-y = 2$'s complement of y

++ $x = 4, y = 2$

$$x - y = 4 - 2 = 4 + (-2)$$

$$4 \rightarrow \overset{1}{0}100$$

$$-2 \rightarrow 1110$$

$$2 \rightarrow \boxed{1}0010$$



Discard

$$(2)_{10} = (0010)_2 \xrightarrow{2's} (1110)_2$$
$$= (-2)_{10}$$

$$x = 2, y = 4$$

$$x - y = 2 - 4 = 2 + (-4) \quad \left| \quad (4)_{10} = (0100)_2 \xrightarrow{2's} (1100)_2 = (-4)_{10}$$

$$2 \rightarrow 0010$$

$$-4 \rightarrow 1100$$

$$-2 \rightarrow 1110$$

$$\boxed{+-} \quad x = 3, y = -2$$

$$x - y = 3 - (-2) = 3 + 2$$

$$3 \rightarrow 00\overset{1}{0}11$$

$$2 \rightarrow 0010$$

$$5 \rightarrow 0101$$

$$x = 4, y = -5$$

$$x - y = 4 - (-5) = 4 + 5$$

$$4 \rightarrow 0\overset{1}{0}100$$

$$5 \rightarrow 0101$$

$$9 \rightarrow 1001 (\text{overflow})$$

$\boxed{-4}$

$$x = -3, y = 2$$

$$x - y = (-3) - (2) = (-3) + (-2)$$

$$-3 \rightarrow \overset{1}{1}101$$

$$-2 \rightarrow 1110$$

$$-5 \rightarrow \overset{1}{1}1011$$

↓
Discard

$$(3)_{10} = (0011)_2 \xrightarrow{2's} (1101)_2 = (-3)_{10}$$

$$(2)_{10} = (0010)_2 \xrightarrow{2's} (1110)_2 = (-2)_{10}$$

$$x = -5, y = 4$$

$$x - y = (-5) - 4 = (-5) + (+4)$$

$$-5 \rightarrow 1011$$

$$-4 \rightarrow 1100$$

$$-9 \rightarrow 10111 \text{ (Underflow)}$$

↓
Discard

$$\begin{aligned} (5)_{10} &= (0101)_2 \xrightarrow{2's} (1011)_2 \\ &= (-5)_{10} \\ (4)_{10} &= (0100)_2 \xrightarrow{2's} (1100)_2 \\ &= (-4)_{10} \end{aligned}$$

$$\boxed{- -} \quad x = -3, y = -2$$

$$x - y = (-3) - (-2) = (-3) + (2)$$

$$-3 \rightarrow 1101$$

$$2 \rightarrow 0010$$

$$-1 \rightarrow 1111$$

$$x = -3, y = -4$$

$$x - y = (-3) - (-4) = (-3) + (4)$$

$$-3 \rightarrow \overset{1}{1}101$$

$$4 \rightarrow 0100$$

$$1 \rightarrow \boxed{1}0001$$

Discard

$$(3)_{10} = (0011)_2 \xrightarrow{2^3} (1101)_2 = (-3)_{10}$$

Examples

Addition in 2's Complement Signed Number System

Example 1: $7 \rightarrow \underline{0111}$ Minuend
 $-5 \rightarrow \underline{(+)} 1011$ 2's complement of subtrahend

 $+2 \rightarrow \underline{(1)0010}$
 ↓
 Discard end carry

The answer is 0010, which is equivalent to $(+2)_{10}$.

Example 2: $5 \rightarrow \underline{0101}$ Minuend
 $-7 \rightarrow \underline{(+)} 1001$ 2's complement of subtrahend

 $-2 \rightarrow \underline{1110}$

Examples

Subtraction in 2's Complement Signed Number System

Example 1: $48 - 23 = 48 + (-23) = +25$

$$\begin{array}{rcl} 48 & \rightarrow & \underline{00110000} \text{ 8-bit 2's complement representation of } +48 \\ + (-23) & \rightarrow (+) & \underline{11101001} \text{ 8-bit 2's complement representation of } -23 \\ \hline +25 & & (1)\underline{00011001} \text{ 8-bit 2's complement representation of } +25 \\ & \searrow & \text{Discard end carry} \end{array}$$

Example 2: $23 - 48 = 23 + (-48) = -25$

$$\begin{array}{rcl} 23 & \rightarrow & \underline{00010111} \text{ 8-bit 2's complement representation of } +23 \\ + (-48) & \rightarrow (+) & \underline{11010000} \text{ 8-bit 2's complement representation of } -48 \\ \hline -25 & & \underline{11100111} \text{ 8-bit 2's complement representation of } -25 \end{array}$$

Examples

Example 3: $48 - (-23) = 48 + 23 = +71$

$$\begin{array}{rcl} 48 & \rightarrow & \underline{00110000} \text{ 8-bit 2's complement representation of } +48 \\ +23 & \rightarrow (+) & \underline{00010111} \text{ 8-bit 2's complement representation of } +23 \text{ or } -(-23) \\ \hline +71 & & \underline{01000111} \text{ 8-bit 2's complement representation of } +71 \end{array}$$

Example 4: $-48 - 23 = (-48) + (-23) = -71$

$$\begin{array}{rcl} -48 & \rightarrow & \underline{11010000} \text{ 8-bit 2's complement representation of } -48 \\ -23 & \rightarrow (+) & \underline{11101001} \text{ 8-bit 2's complement representation of } -23 \\ \hline -71 & \underline{(1)} & 10111001 \text{ 8-bit 2's complement representation of } -71 \end{array}$$

↓
Discard end carry