24/10/2021 Sunday | Mat 101

Natural number (N)

Negative এবং fraction বাদে সংখ্যা গুলো (0,1 . . . . . ∞)

ধনাত্মক এবং ঋনাত্মক সংখ্যা Integer. (পূর্ণ সংখ্যা)

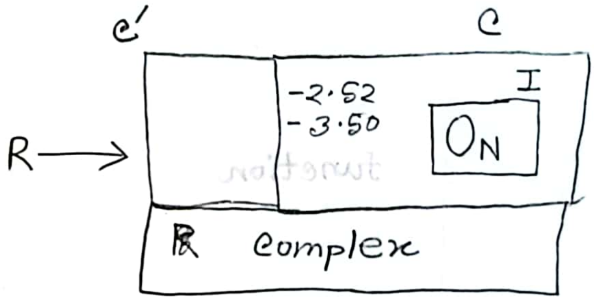
মূলদ সংখ্যা (Rational number) Q ($\frac{p}{q}$)

Improper fraction (Q')

-2.72 real number

$N \subset I$ । Natural number এবং Integer উভয়ই মূলদ সংখ্যার Sub set ।

$\begin{cases} x \in R \text{ হলে } \pm \\ x \in N \text{ হলে } + \end{cases}$



$5/0 \neq \infty$

Rational	Irrational
* If any number can be expressed as the ratio of two integer is called Rational number. 5, -3.2	π, √2, √3, √5
দশমিকের পরে recurrent value থাকবে । পুনপুনিক হবে ।	দশমিকের পরে recurrent value থাকবে না । পুনপুনিক হবে না ।

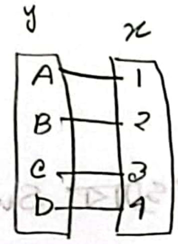
$\square^{\circ} = 1$

$\square \times 0 = 0$

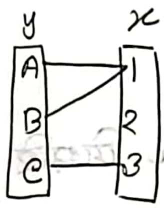
$\frac{\square}{0}$ = undefined

function

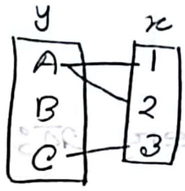
one-one

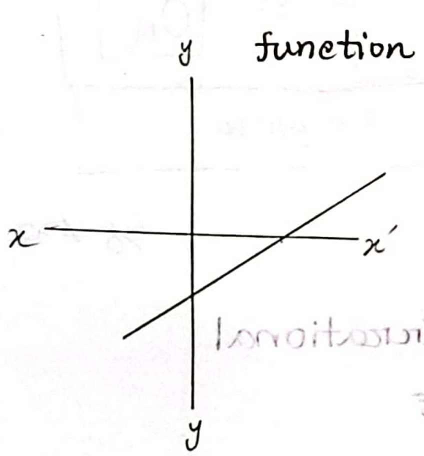


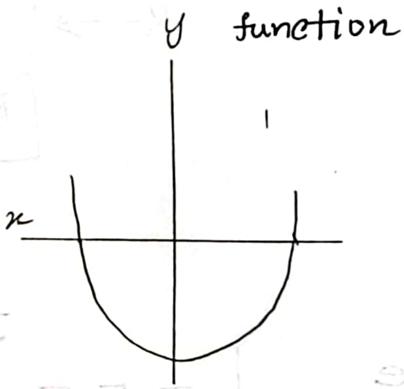
Not a function

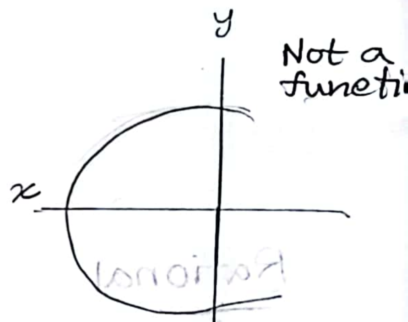


On to









*Virctical line Test করবো function mark করার জন্য গ্রাফ থে

*Horizontal line test করবো ~~f~~ one-one এবং one-two functio mark করার জন্য।

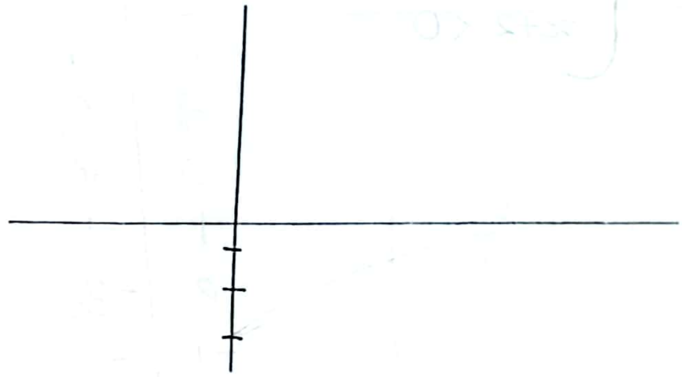
#Linier function অর্থাৎ x এর সর্বোচ্চ মান 1 সেগুলো

one-one function। $f(x) = x+2$

#এর মান >1 হলে onto function

$f(x) = x^2 - 3$

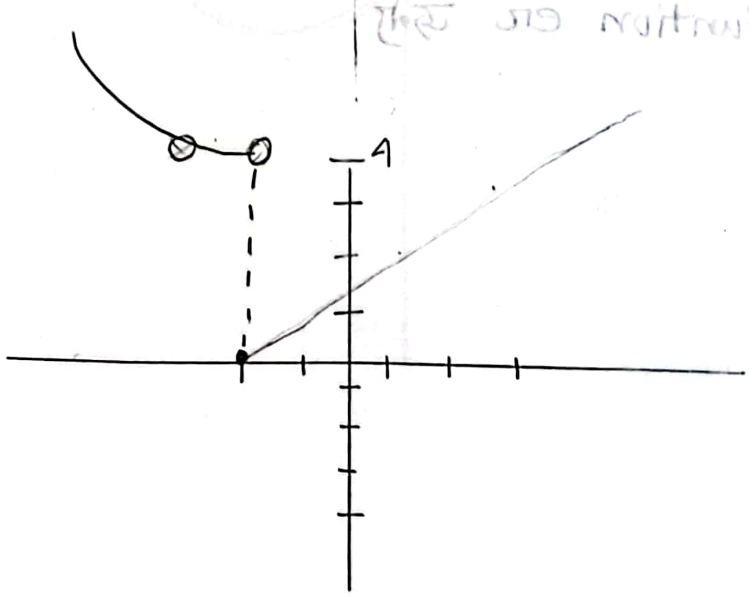
x	y
0	-3
1	-2
4	-1
9	0
16	1
25	2



Piece - wise function:

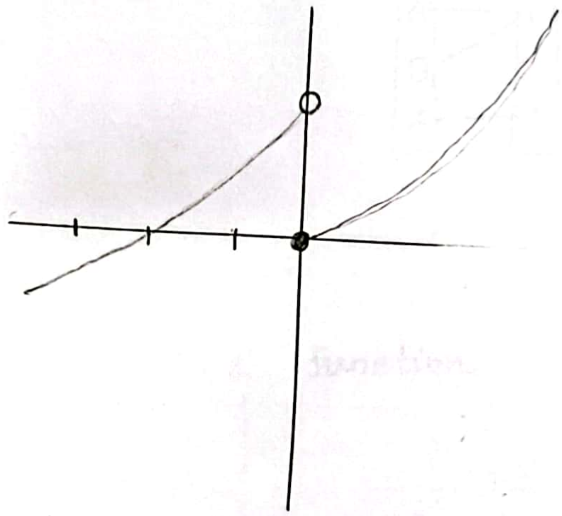
$$f(x) = \begin{cases} x+2 \ ; \ x \geq -2 \\ x^2 < -2 \end{cases}$$

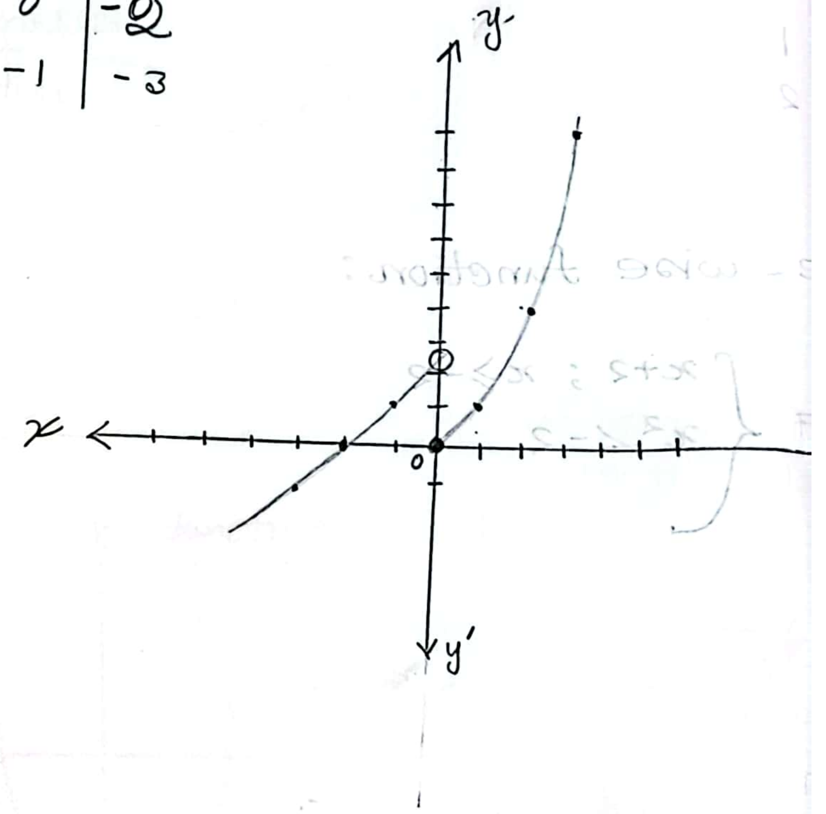
x	y
0	2
1	3
2	4
-1	1
-2	0
-3	9
-4	16



$$f(x) = \begin{cases} x^2 & \geq 0 \\ x+2 & < 0 \end{cases}$$

y	x
2	0
1	1
4	2
9	3
1	-1
0	-2
-1	-3

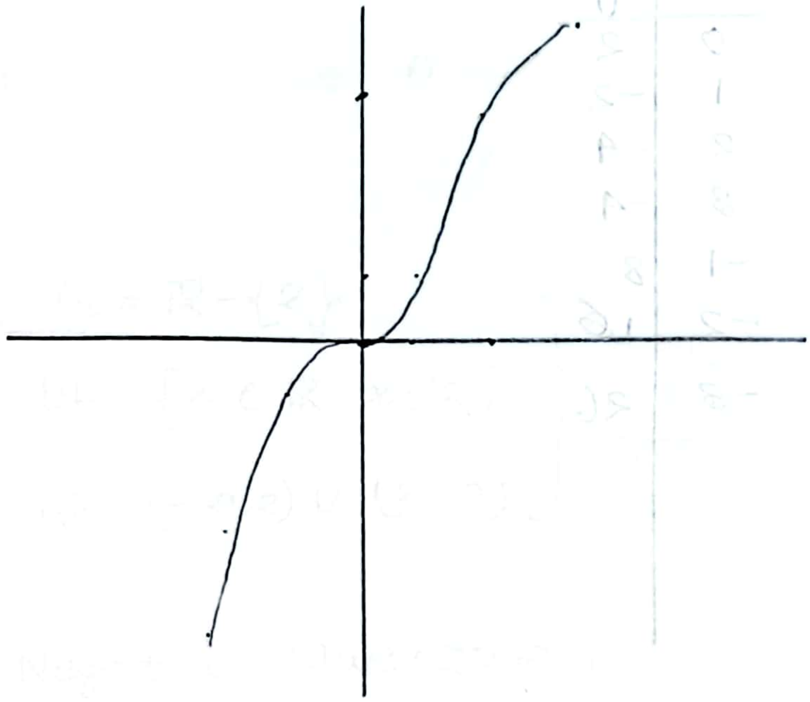




জাংশন Point এর আগে পাছে বেশি Point নিতে হবে pie
funtion er জন্য

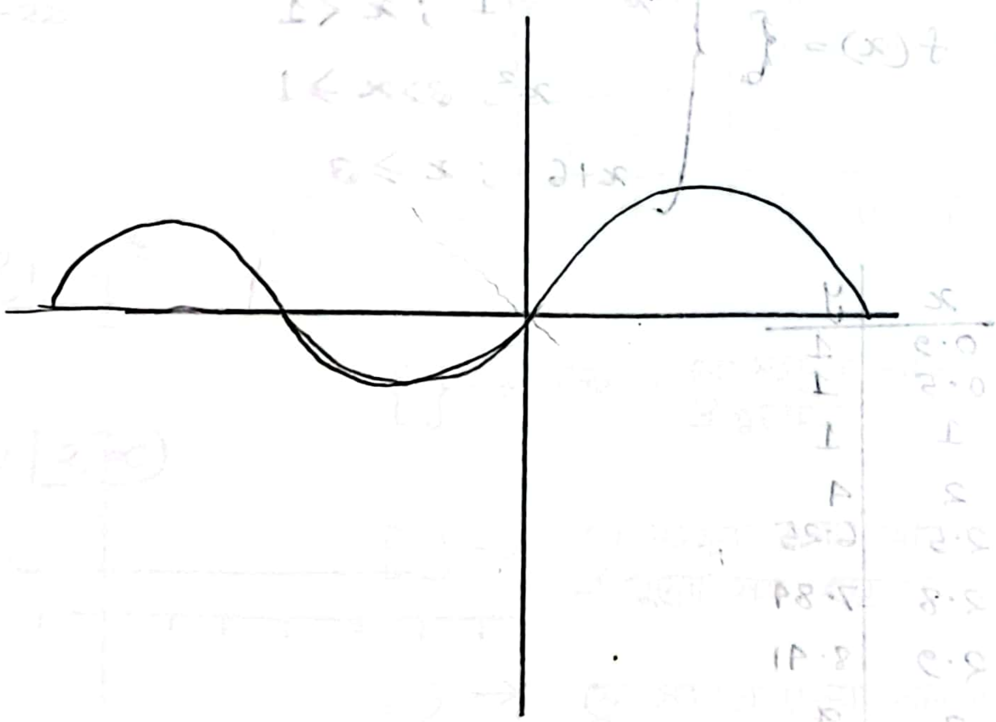
$f(x) = x^3$

x	y
0	0
1	1
2	8
3	9
-1	-1
-2	-8
-3	-27



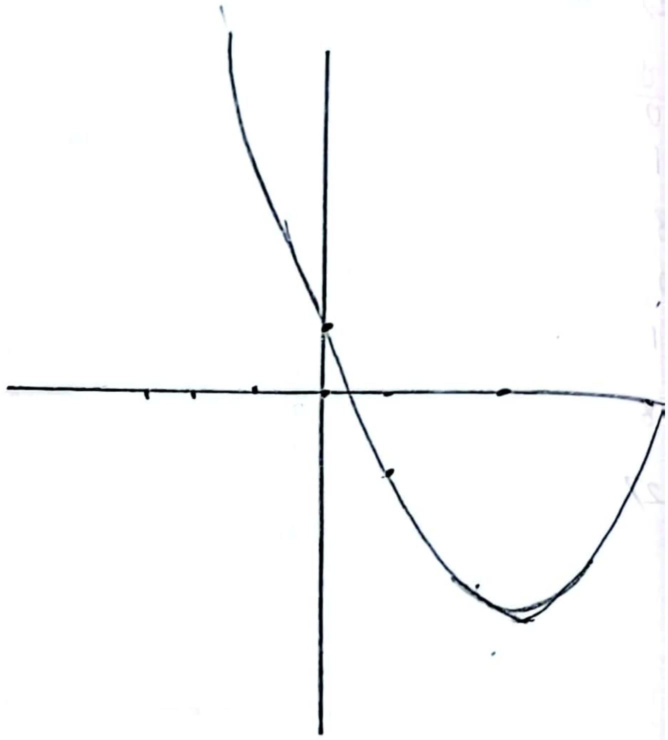
$f(x) = \sin(x)$

x	y
0	0
30	$1/2$
45	$\frac{1}{\sqrt{2}}$
60	$\frac{\sqrt{3}}{2}$



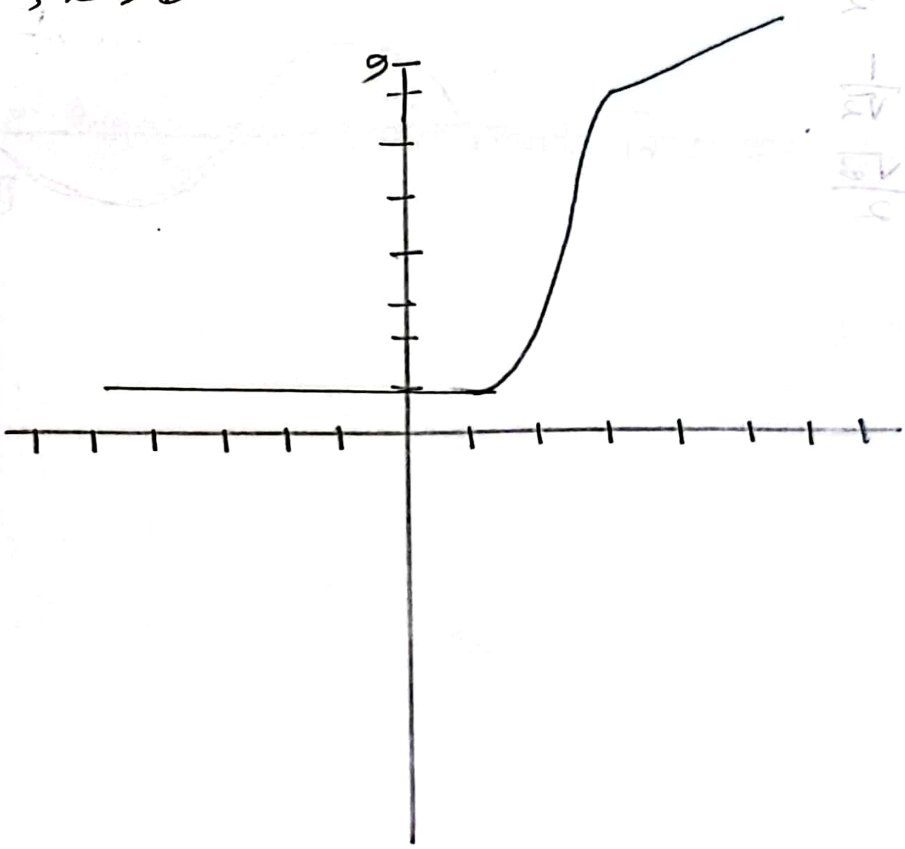
$f(x) = x^2 - 5x + 2$

x	y
0	2
1	-2
2	-4
3	-4
-1	8
-2	16
-3	26



$$f(x) = \begin{cases} 1 & ; x < 1 \\ x^2, & 3 > x \geq 1 \\ x+6 & ; x \geq 3 \end{cases}$$

x	y
0.9	1
0.5	1
1	1
2	4
2.5	6.25
2.8	7.84
2.9	8.41
3	9
4	10
5	11



* Domain

Restrictions –

① $\dfrac{}{\boxed{0}}$ হর / dinominator can't be 0

e.x: $f(x) = \dfrac{x+1}{x-2}$

we know,

$x-2 \neq 0$

$\therefore x \neq 2$

So, $D_f = \mathbb{R} - \{2\}$

$D_f = \{x \in \mathbb{R}, x \neq 2\}$

$D_f = (-\infty, 2) \cup (2, \infty)$

৩ ভাবেই Ans লিখা যায়

(ii) $\sqrt{(-2)}$ $\sqrt{\ }$ এর ভিতরে Negetive value হবে না।

e.x – $f(x) = \sqrt{x^2-2x}$

$x^2 - 2x$

$\Rightarrow x(x-2)$

	x	$x-2$	$x(x-2)$	
$-\infty, 0$	$-$	$-$	$+$	(Acceptable)
$0, 2$	$+$	$-$	$-$	(X)
$2, \infty$	$+$	$+$	$+$	(Acceptable)

So, $D_f = \mathbb{R} - (0, 2)$

আবার

$D_f : (-\infty, 0] \cup [2, \infty)$

$\{\ \} \rightarrow$ সেট ; যে সংখ্যা থাকবে ঐ গুলোই

$[\ ] \rightarrow$ যে সংখ্যা গুলো থাকবে ঐ গুলো সহ হবে মান

$(\ ) \rightarrow$ যে সংখ্যা গুলো থাকবে ঐ গুলো ব্যতিত হবে। এবং তার মধ্যবর্তী সংখ্যা

$(0, 2) \rightarrow 0 < x < 2$

$[0, 2] \rightarrow 0 \leq x \leq 2$

1. $f(x) = x^2 + 5x$ $\therefore D_f = \mathbb{R}$

2. $f(x) = \sqrt{x} - 3$ $\therefore D_f = [0, \infty)$

3. $f(x) = \tan(x)$ $\therefore D_f = \mathbb{R} - \{\pm (2n-1)\frac{\pi}{2}\}$; এখানে n হলো natural num

4. $f(x) = \frac{1}{(x-1)(x-3)}$ $\therefore D_f : \mathbb{R} - \{1, 3\}$

5. $f(x) = \sqrt{x^2 - 5x + 6}$ $\therefore D_f : \mathbb{R} - (2, 3)$

$x^2 - 5x + 6$

$x^2 - 3x + 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$

$(x-3)(x-2) = 0$

3. $\tan(x) = \frac{\sin x}{\cos x}$

$\therefore \cos(x) \neq 0$

$\therefore D_f = \mathbb{R} - \{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots \ldots$
$-\frac{\pi}{2}, -\frac{3\pi}{2} \ldots\}$

$= \mathbb{R} - \{\pm (2n-1)\frac{\pi}{2}\}$ এখানে n হলো natural number

$f(x) = \sec x$

$= \frac{1}{\cos x}$

$\cos x \neq 0$

$\therefore D_f = \mathbb{R} - \{\pm (2n-1)\frac{\pi}{2}\}$; n = natural number

$\therefore R_f = \mathbb{R} - (-1, 1)$ / $R_f = (-\infty, -1] \cup [1, \infty)$

* $f(x) = \cot x = \frac{\cos x}{\sin x}$

এখানে,

$\sin x \neq 0$

$\sin x = 0$; for $x = 0, \pi, 2\pi, 3\pi \ldots$

$-\pi, -2\pi, -3\pi$

$\therefore D_f = \mathbb{R} - \{n\pi\}$ here 'n' is all integer

আবার,

$D_f = \mathbb{R} - \{\pm (n-1)\pi\}$ here 'n' is all natural.

$R_f : \mathbb{R}$

(Ans)

* $f(x) = \frac{x^2-4}{x-2}$

$= \frac{(x-2)(x+2)}{(x-2)}$

$= x+2$; $x \neq 2$

a) $f(x) = 2+\sqrt{x-1}$

এখানে,

$x-1 \nless 0$ or,

$x \nless 1$

$D_f : \mathbb{R} - (-\infty, 1)$

আবার,

$x-1 \geqslant 0$

$x \geqslant 1$

$\therefore D_f : [1, \infty)$

b) $f(x) = (x+1)/(x-1)$

$f(x)$ এখানে,

$x-1 \neq 0$

$\therefore x \neq 1$

$\therefore D_f : \mathbb{R} - \{1\}$

Range :

i) $f(x) = x$

$\therefore D_f : \mathbb{R}$

$\therefore R_f : \mathbb{R}$

ii) $f(x) = x^2$

$D_f : \mathbb{R}$

$R_f : [0, \infty)$

iii) $f(x) = x^3$

$D_f = \mathbb{R}$

$R_f = \mathbb{R}$

iv) $f(x) = \sin x$

$R_f : [-1, 1]$

$D_f :$ ~~$\mathbb{R} - \{\pm (2n-1)\pi\}$; n = natural number~~ $\mathbb{R}$

v) $f(x) = \cos x$

$R_f : [-1, 1]$

$D_f : \mathbb{R}$

vi) $f(x) = \tan x$

$D_f : \mathbb{R} - \{\pm (2n-1)\frac{\pi}{2}\}$

$R_f : \mathbb{R}$

vii) $f(x) = \sqrt{x}$

$D_f : [0, \infty)$

$R_f : [0, \infty)$

* একটি $\sqrt{\ }$ এর ভিতরের কোন ফাংশন থাকলে তার $R_f : [0, \infty]$

$f(x) = \sqrt{x-1} + 4$

$D_f : [1, \infty)$

$R_f : [4, \infty)$

06/11/2021 | Math
Saturday | 101

7. a) $f(x) = 3x^2 - 2$

x	y
0	−2

$x^2 - 6x + 8 = -10$

$\cancel{\frac{x^2}{-10} + \frac{x}{5/3}}$

Q.23.(d) y এর maximum value নেই।
$y = x^2 - 6x + 8$ এই সমীকরণে y এর min value -1 যখন x এর মান 3।

এখানে, $x^2 - 6x + 8 \geq 0$
$\Rightarrow (x-4)(x-2) \geq 0$

	$x-4$	$x-2$	$(x-4)(x-2)$
$-\infty, 2$	−	−	+
2,4	−	+	−
$4, \infty$	+	+	+

~~$D_f : (-\infty, 2] \cup [4, \infty)$~~ অর্থাৎ $x = 4, 2$ বিন্দুতে y এর মান 0 হবে। অর্থাৎ $x = 2, 4$ এর মাঝেই y এর min value রয়েছে।

* দ্বি-পদী সমীকরণে y এর maximum value infiniti (∞)।

Sunday / Math

* $\frac{\square}{\square}$ লব < হর ; এই ধরনের faction গুলোর মান $(-1,1)$

লব > হর ; এই ধরনের faction গুলোর মান $(-\infty,-1]\cup[1,\infty)$

* $f(x)=x^2-2x+1 = (x-1)^2$ * quadratic equation এর $D_f=\mathbb{R}$ হবে

$D_f : \mathbb{R}$

$R_f : [0,\infty)$

* $f(x)=x^2-2x-2 \Rightarrow x^2-2x+1-3$

$\therefore D_f : \mathbb{R} \quad \Rightarrow (x-1)^2-3$

$\therefore R_f : [-3,\infty)$

* $f(x)=\sqrt{x-1}$

$D_f : [1,\infty)$ অথবা, $D_f : \mathbb{R}-[0,-\infty)$

$R_f : [0,\infty)$

* $f(x)=\sqrt{x^2-2x}+2$

$=\sqrt{x(x-2)}+2$

$\therefore$ ~~D_f :~~

$\therefore D_f : (-\infty,0]\cup[2,\infty)$

$\therefore R_f : [2,\infty)$

Odd and even function

$f(-x) = f(x)$ even function

e.g

$f(-x) = -f(x)$ odd function

e.g $f(x) = x^3 + x$

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -(x^3 + x)$$
$$= -f(x) \quad \text{odd function}$$

$f(x) = x^2 + x + 2$

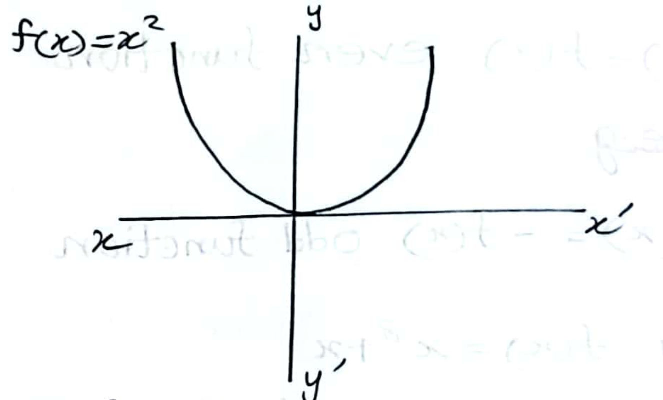
$$f(-x) = (-x)^2 - x + 2$$
$$= x^2 - x + 2 \quad \text{neither function}$$

* কোনো ফাংশনের সাথে যদি constant থাকে তাহলে তা সাধারনত neither function. even function ও হতে পারে কিন্তু Odd function হবে না।

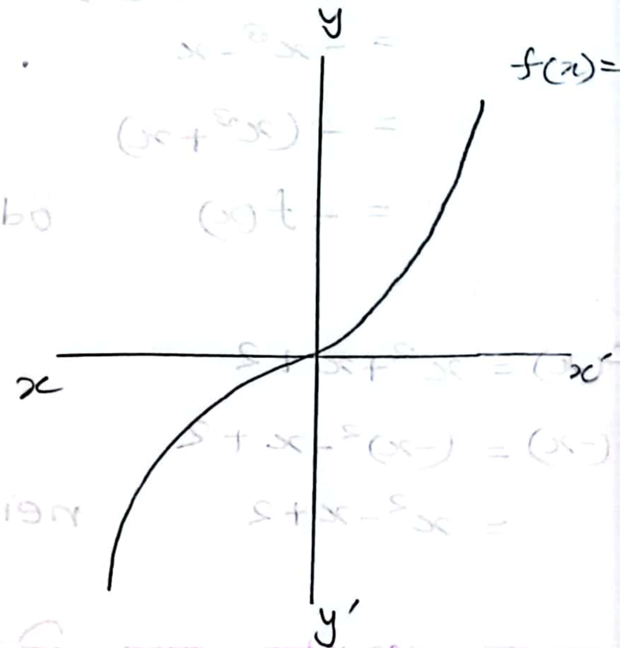
ব্যাতিক্রম:

$$f(x) = x^2 + 2$$
$$f(-x) = (-x)^2 + 2$$
$$= x^2 + 2$$
$$= f(x) \quad \text{even function.}$$

* y - axix এর mirror হলে তা even function.



(0,0) বা মূল বিন্দু
* ~~x-axix~~ এর mirror হলে odd function.
Origin



Q. 57/

~~$f(x) = x - 3x^2 - 3$~~

f(x) is a neither function

g(x) is a odd function

h(x) is a even function

Q. 58.

$f(x) = x^2+2$

$g(x) = \sqrt{x-1}$

i) $\cancel{gof}\ fog$

$\cancel{fog}(x) = \sqrt{x^2+2-1}$

$f(g(x)) = \sqrt{x^2+1}$

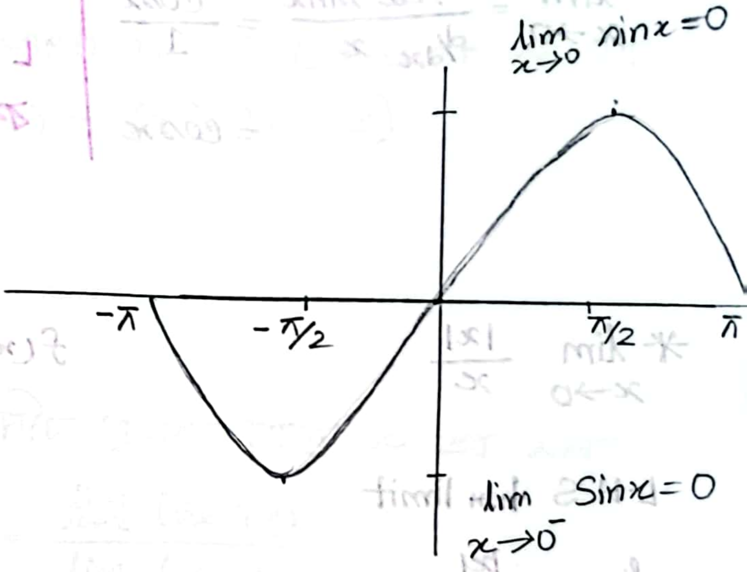
ii) $\cancel{fog}\ gof$

$\cancel{gof}(x) = x-1+2$

$g(f(x)) = x+1$

Limit:

$\lim_{x\to 0} \sin x = 0$



যদি L·H Limit ও R·H Limit same হয় তাহলে ঐটা লিমিট এবং দুইটার লিমিটের মান যা হবে তাই limit

09/11/21 Tuesday / Math 101

* $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

$= \frac{(\sqrt{x}+1)(x-1)}{x-1}$

$= \sqrt{x}+1 \ ; \ x \neq 1$

$= 2$ (Ans)

$f(1)$ undefind

* লপিটালস Rule : কোনো ফাংশন $\frac{0}{0}$ আকারে চলে আসলে উপরে নিচে পৃথক ভাবে ~~ভিন্ন~~ $\frac{d}{dx}$ করতে হবে।

* $\lim_{x\to 0} \frac{\sin x}{x}$

Using L Hopital's rule

$\lim_{x\to 0} = \frac{d/dx \sin x}{d/dx \ x} = \frac{\cos x}{1}$

$= \cos x$

Only undefind situation এ L Hospital's rules Apply করা যাবে।

* $\lim_{x\to 0} \frac{|x|}{x}$

L.H.S ~~lmi~~ limit

$\lim_{x\to \bar{0}} \frac{|x|}{x}$.

$\lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$f(x) = \frac{|x|}{x}$

$= \begin{cases} \frac{x}{x}, & x>0 \\ \frac{-x}{x}, & x<0 \end{cases}$

$= \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$

$\therefore$ L.H.S lim

$\lim_{x\to 0^-} \frac{|x|}{x}$

$= -1$

$\therefore$ R.H.S lim

$\lim_{x\to 0^+} \frac{|x|}{x}$

$= 1$

$\therefore$ L.H.S $\neq$ R.H.S

(7) Df: $[1,\infty)$ Rf: $[-1,\infty)$

(8) Df: $[-3,3]$ Rf: $[-2,2]$

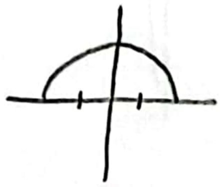
Mid-2

Mid - 2 Start

16 - 11 - 21 / Math 101
Tuesday

$$f(x) = \frac{x^2+2x+1}{x+1} = \frac{(x+1)^2}{(x+1)}$$

$\sqrt{a^2-x^2}$; এই ধরনের equation কে half circle বলে। এটি ব্যাতিক্রম (0,0) radius.



* Polynomial function:

কোনো $\sqrt{\ }$ থাকবে না simple function । ~~যেমন~~ direct x এর মান বসিয়ে দেয়া যাবে। $f(x) = (x^2 - 4x + 3)$

* Rational function

এ ধরনের ফাংশনে উপরে এবং নিচে পৃথক ভাবে x এর মান বসাতে হবে।

$$\lim_{x\to 2} \frac{5x^3+4}{x-3} = \frac{\lim_{x\to 2}(5x^3+4)}{\lim_{x\to 2}(x-3)}$$

* $\frac{1}{0}$ কোনো ফাংশনে x এর মান বসালে এমন আসার পরিক্রমা হলে,

$$\lim_{x\to 0^+} f(x) = +\infty$$

$$\lim_{x\to 0^-} f(x) = -\infty$$

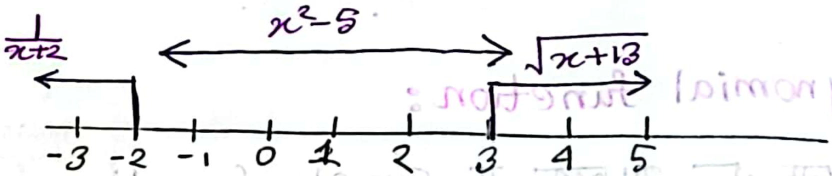
$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}-1} = \lim_{x\to 1} \sqrt{x}+1 = 2$ (Ans)

$f(x) = \begin{cases} \frac{1}{x+2}, & x < -2 \\ x^2-5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$

(i) $\lim_{x\to -2} f(x)$ এখানে,

(ii) $\lim_{x\to 0} f(x)$

(iii) $\lim_{x\to 3} f(x)$



(i) এখানে,

$\lim_{x\to -2^-} f(x) = -\infty$

$\lim_{x\to -2^+} f(x) = (-2)^2 - 5$
$= -1$

$\therefore \lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$

does n't exist

(ii) $\lim_{x\to 0} f(x) = 0^2 - 5$
$= -5$

(iii) $\lim_{x\to 3} f(x) = 4$; does exist

এখানে,

$\lim_{x\to 3^-} f(x) = 3^2 - 5$
$= 4$

↓

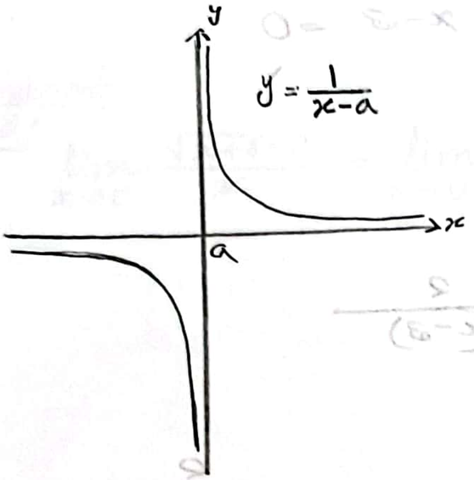
বাম দিক থেকে অগ্রসর এর সময় ২য় ভোমেনে পেয়েছে।

আবার,

$\lim_{x\to 3^+} f(x) = \sqrt{3+13}$
$= \sqrt{16}$
$= 4$

↓

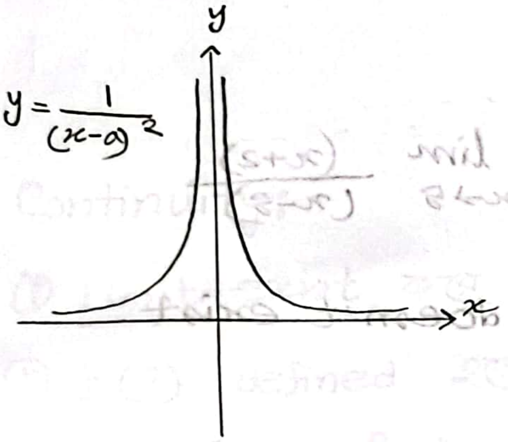
ডান দিক থেকে অগ্রসর এর সময় ৪র্থ ডোমেন পেয়েছে।



এই গ্রাফের ক্ষেত্রে, ডান দিক থেকে যত a এর দিকে যাব তত $+\infty$ এবং বাম দিক থেকে যত a এর দিকে যাব তত $-\infty$

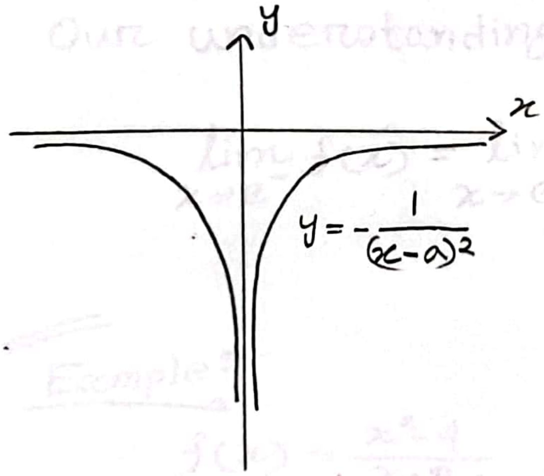
$$\lim_{x\to a^+}\frac{1}{x-a}=+\infty$$

$$\lim_{x\to a^-}\frac{1}{x-a}=-\infty$$



এখানে,

$$\lim_{x\to a}\frac{1}{(x-a)^2}=+\infty$$



এখানে,

$$\lim_{x\to a}-\frac{1}{(x-a)^2}=-\infty$$

a) $\lim_{x\to 3} \frac{x^2-6x+9}{x-3} = \lim_{x\to 3} \frac{(x-3)^2}{x-3} = \lim_{x\to 3} x-3 = 0$

কিন্তু $f(3) = \text{undefined}$

b) $\lim_{x\to -4} \frac{2x+8}{x^2+x-12} = \lim_{x\to -4} \frac{2(x+4)}{(x-3)(x+4)} = \lim_{x\to -4} \frac{2}{(x-3)}$

$= \frac{\lim_{x\to -4} 2}{\lim_{x\to -4} (x-3)} = -\frac{2}{7}$

c) $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-10x+25} = \lim_{x\to 5} \frac{(x+2)(x-5)}{(x-5)^2} = \lim_{x\to 5} \frac{(x+2)}{(x-5)}$

এখানে,

$\lim_{x\to 5^+} \frac{(x+2)}{(x-5)} = +\infty$

$= \text{doesn't exist}$

$\lim_{x\to 5^-} \frac{(x+2)}{(x-5)} = -\infty$

\# $f(x) = \begin{cases} \frac{1}{x+2}, & x < -2 \\ x^2-5, & -2 < x \le 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$

i) $\lim_{x\to -2} f(x)$

এখানে,
$f(-2) = \text{undefined}$

ii) $\lim_{x\to 0} f(x)$

iii) $\lim_{x\to 3} f(x)$

২৯/১১/২১/ Math
Sunday / 101

Assignment
3/ $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0}\frac{\frac{1}{2\sqrt{x+4}}}{1} = \lim_{x\to 0}\frac{1}{2\sqrt{x+4}} = \frac{1}{4}$

ল্যাপিটাল Rule ব্যবহার করে।

23/11/21/ Math
Tuesday / 101

Continuity:

(I) Limit exist করে কিনা। অর্থাৎ L.H.L = R.H.L হতে হবে।

(II) f (c) defined হতে হবে।

(III) $\lim_{x\to c} f(x) = f(c)$

Our understanding:

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

Exmple:

$$f(x) = \frac{x^2-4}{x-2}$$

এখানে f(x) discontinuous when $x=2$

$$L.H.L = \frac{x^2-4}{x+2} = x+2$$

$\therefore x=2$; $2+2 = 4$

$$g(x) = \begin{cases} \frac{x^2-4}{x-2} & , x \neq 2 \\ 3 & , x = 2 \end{cases}$$

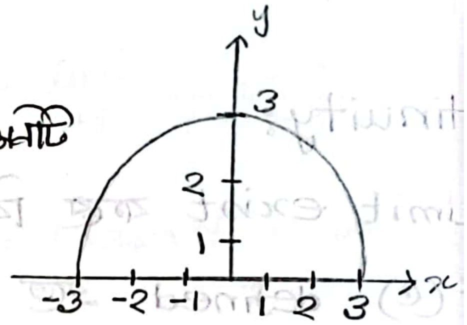
যেহেতু, $\frac{x^2-4}{x-2}$ এর মান $\neq 3$

তাই $x=2$ তে $g(x)$ discontinuous

Exmple: 2

$f(x)=\sqrt{9-x^2}$ এই ধরনের ফাংশনের ক্ষেত্রে প্রথমে ডোমেইন নির্ণয় করতে হবে এবং প্রান্তিক value গুলো check করতে হবে। এই সমীকরনের domain $[-3, 3]$।

Exmple: 3 এই ধরনের যাতে প্রান্তিক বিন্দু হলো $-3, 3$। এখানে $-3, 3$ বিন্দুতে ফাংশনটি continuous. সুতরাং $f(x)=\sqrt{9-x^2}$ ফাংশনটি continuous.



Exmple: 3

$$y = \frac{x^2-9}{x^2-5x+6}$$; এখানে $D_f : \mathbb{R} - \{2, 3\}$

Exmple: 21. $f(x) = \begin{cases} 2x+3 & ; x \le 4 \\ 7+\frac{16}{x} & ; x > 4 \end{cases}$

$$\lim_{x \to 4^-} f(x) = 2\times4+3 = 11$$

$$\lim_{x \to 4^+} f(x) = 7 + \frac{16}{4} = 11$$

$$\therefore f(4) = 2\times4+3 = 11$$

$$\therefore \lim_{x\to4^-} f(x) = \lim_{x\to4^+} f(x) = f(4)$$

$\therefore$ There is no point at which $f(x)$ is discontinuous.

30. (a) $f(x) = \begin{cases} 9-x^2 ; & x \geq -3 \\ \dfrac{k}{x^2} ; & x < -3 \end{cases}$

$$\lim_{x \to -3^-} f(x) = \frac{k}{(-3)^2} = \frac{k}{9}$$

$$\lim_{x \to -3^+} f(x) = 9 - (-3)^2 = 11$$

$$f(-3) = 0$$

to be ~~contubus~~ continuous at everywhere,

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x) = f(-3)$$

so, the function is continuous.

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28.11.21 Saturday / Math 101

differentiation mean the ~~value~~ rate of change. calculation.

instant velocity formula $\lim_{\Delta t \to 0} \frac{V_t - V_0}{\Delta t}$

Exam tangent line formula $\boxed{m_{tan} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$

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Secant line → Avarage rate of change.

Tangent line → Instant rate of change.

* The perpose of differentiation is to find ou rate of change with respect to that variab

Example: 1

$f(x) = x^2 \;;\; x_0 = 2$

$$m_{tan} = \lim_{x \to 2} \frac{f(x) - f(x_0)}{x - x_0}$$
$$= \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$
$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{(x-2)}$$
$$= \lim_{x \to 2} (x+2)$$
$$= 2+2$$
$$= 4$$

$m_{tan} = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$

$= \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

$= \lim_{x \to 2} \frac{(x+2)(x-2)}{x-2}$

$= \frac{x^2-4}{x-2}$

$= x+2 = 4$

$y = (2, 2)$

$y = mx + c$

$2 = 4 + c$

$\therefore c = -2$

$y = 2x - 2$

$y = 2(x-1)$

$y = 4x - 4$ } $y = x^2$ এর জন্য

$f(t) = 1 + 5t - 2t^2$

$\frac{10 - 26}{3 - 5}$

$=$

b) $2x+1$

$f(x) = (x)^2 + 1$

$s = 3t + c$

$y = 3x + c$

$s = 3t \cdot 7$

(4)

30.11.21 / Math 101
Tueday /

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1(a) $m_{tan} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{50-10}{15-5}$

$= 4\ m/s$

(Ans)

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1. $y = 4x^7$

$\Rightarrow \frac{dy}{dx} = 28x^6$

(Ans)

3. $y = 3x^8 + 2x + 1$

$\frac{dy}{dx} = 24x^7 + 2$

(Ans)

* $y = \frac{1}{5}(x^2+1)$

$\frac{dy}{dx} = \frac{2}{5}x$

$f''(x) = \frac{d}{dx}\left(\frac{d}{dx} f(x)\right)$

$= \frac{d^2}{dx^2} f(x)$

* $f(x) = \sqrt{x} + \frac{1}{x}$

$f'(x) = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{x}\right)$

$= \cancel{\frac{d}{dx}}\left(\cancel{x}\frac{1}{2\sqrt{x}} - \frac{1}{x^2}\right)$

$= \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

(Ans)

* $f(x) = -3x^{-8} + 2\sqrt{x}$

$\therefore f'(x) = 24x^{-9} + \frac{1}{\sqrt{x}}$

(Ans)

* $f(x) = (1-x)(1+x)(1+x^2)(1+x^4)$

$f'(x) = \cancel{1 \times (1) \times 3} \times$

$= \frac{d}{dx}(1^2 - x^2)(1+x^2)(1+x^4)$

$= \frac{d}{dx}(1-x^4)(1+x^4)$

$f'(x) = \frac{d}{dx}(1-x^8)$

$= -8x^7$

$f'(1) = -8$

(Ans)

*

27.

a) $s = 0.3 \times t^3$

$= 0.3 \times 40^3$

$= 19200$

(Ans)

(b) $\frac{19200}{40} = 480$ | Here, from a we find total distance ~~from~~ at 40s, As we know average velocity = $\frac{\text{total dis}}{\text{total time}}$

(c) ~~t = 14.9~~ $s = 0.3t^3 \Rightarrow 1000 = 0.3t^3 \therefore t = 14.9$

average velocity = total distance / total time

$= 1000 / 14.9$

$= 67.1$

(d) 1440 (Ans) / $s = 0.3t^3$

$\frac{d}{dx} s = 0.3 \times 3 \times t^2$

$= 0.3 \times 3 \times (40)^2$

$= 1440$ (Ans)

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61. $F = \frac{GmM}{r^2}$

$\frac{d}{dx} F = -2\, mMG / r^{\cancel{4}-3}$

$\frac{d}{dx} F = \frac{-2GMm}{r^3}$

(Ans)

05/12/21 / Math 101
Sunday

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Q:

Exmple:

$y = (4x^2-1)(7x^3+\cancel{x})$

$\frac{dy}{dx} = (4x^2-1)\frac{d}{dx}(7x^3+\cancel{x}) + (7x^3+\cancel{x})\frac{d}{dx}(4x^2-1)$

$= (4x^2-1) \times (21x^2+1) + (7x^3+\cancel{x})(8x)$

$= 84x^4 - 21x^2 \cancel{+56} + 4x^2 - 1 + 56x^4 + 8x^2$

$= 140x^4 - 9x^2 - 1$

(Ans)

Q: $y = \dfrac{x^3+2x^2-1}{x+5}$

$$\frac{dy}{dx} = \frac{(x+5)\frac{d}{dx}(x^3+2x^2-1) - (x^3+2x^2-1)\frac{d}{dx}(x+5)}{(x+5)^2}$$

$$= \frac{(x+5)(3x^2+4x) - (x^3+2x^2-1)\times 1}{(x+5)^2}$$

$$= \frac{3x^3+15x^2+4x^2+20x-x^3-2x^2+1}{(x+5)^2}$$

$$= \frac{2x^3+17x^2+20x+11}{(x+5)^2}$$

(Ans)

Q: $f(x) = \dfrac{x^2-1}{x^4+1}$

make rough estimates of the locations of the horizontal tangent lines

x-axix এর Parallal slop $m=0$ হবে

$$\frac{dy}{dx} = \frac{dy}{dx}\left(\frac{x^2-1}{x^4+1}\right)$$

$$= \frac{2x(x^4+1)-(x^2-1)4x^3}{(x^4+1)^2}$$

if

$$\frac{2x(x^4+1)-(x^2-1)4x^3}{(x^4+1)^2} = 0$$

$$\Rightarrow 2x(x^4+1) - (x^2-1)4x^3 = 0$$

$$\Rightarrow 2x^5+2x - 4x^5+4x^3 = 0$$

$$\Rightarrow 2x - 2x^5 + 4x^3 = 0$$

$$\Rightarrow 2x(-x^4+2x^2+1) = 0$$

$\Rightarrow$ হয়,

$2x = 0$

$\therefore x = 0$

অথবা;

$-x^4 + 2x^2 + 1 = 0$

$\cancel{x^2(2-x^2) = -1}$

$\Rightarrow x^4 - 2x^2 - 1 = 0$

$\Rightarrow (x^2)^2 - 2(x^2) - 1 = 0$

$$\therefore x^2 = \frac{-(-2) \pm \sqrt{(4+4)}}{2}$$

$\Rightarrow x^2 = 1 \pm \sqrt{2}$

$\Rightarrow x^2 = 2.414 \ , \ -0.414$

As, x^2 can't take any negative value.

So, $x^2 = 2.414$

Hence, $x = 0 \ , \ \pm\sqrt{2.414}$

$= 0, -1.55, 1.55$

(Ans)

Q: Find the point of x at which the tangent line of the function $f(x) = \frac{x^2+1}{x+1}$ is parallel to $y=$

$$f(x) = \frac{x^2+1}{x+1}$$

$$\therefore \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{(x+1)\frac{d}{dx}(x^2+1) - (x^2+1) \times \frac{d}{dx}(x+1)}{(x+1)^2} = 1$$

$$\Rightarrow \frac{(x+1)2x - (x^2+1)}{(x+1)^2} = 1$$

$$\Rightarrow 2x^2 + 2x - x^2 - 1 = x^2 + 2x + 1$$

$\Rightarrow$

As, There is no point of x at which tangent line of the function $f(x) = \frac{x^2+1}{x+1}$ is parallel to $y = x$

Page: 120
te rules
ase

Q: $y = x \sin x$

$$\frac{dy}{dx} = \cancel{\cos x}\; x \cdot \cos x + \sin x$$

(Ans)

Q: $y = \frac{\sin x}{1 + \cos x}$

$$\frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x + \cos^2 x + \sin^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x + 1}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{1}{1+\cos x}$$

(Ans)

Page; 196

Q:

31: $\sin\theta = \frac{x}{10}$

$\Rightarrow \sin\theta = \frac{x}{10}$

$\Rightarrow \cos\theta \times 10 = x$

$\Rightarrow \cos 60 \times 10 = x$

$\therefore x = 5$ ft/rad

$\Rightarrow x = 5 \times \pi/180$ ft/deg

$= 0.087$ ft/deg

(Ans)

* 1 rad $= 180/\pi$

07/12/21 / Math

Tuesday / 101

Chain rule:

* $\frac{d}{dx}(\tan(x^2+1)) = \sec^2(x^2+1) \times \frac{d}{dx}(x^2+1)$

$= 2x \sec^2 x$

* $\frac{d}{dx}\sqrt{x^3 + \operatorname{cosec} x} = \frac{1}{2\sqrt{x^3+\operatorname{cosec} x}} \times \frac{d}{dx}(x^3 + \operatorname{cosec} x)$

$= \frac{1}{2\sqrt{x^3+\operatorname{cosec} x}} \times (3x^2 - \operatorname{cosec} x \times \cot x)$

CamScanner

$$* \frac{d}{dx} \sin\sqrt{1+\cos x}$$

$$= \cos\sqrt{1+\cos x} \times \frac{1}{2\sqrt{1+\cos x}} \times (-\sin x)$$

$$= -\frac{\sin x \times \cos\sqrt{1+\cos x}}{2\sqrt{1+\cos x}}$$

(Ans)

70. a) $F = \dfrac{\mu W}{\cos\theta + \mu\sin\theta}$

$$F = \frac{45}{\cos\theta + 0.3\sin\theta}$$

$$\frac{dF}{d\theta} = \frac{-45(-\sin\theta + 0.3\cos\theta)}{(\cos\theta + 0.3\sin\theta)^2}$$

if, $\theta = 30°$

$$\frac{dF}{d\theta} = \frac{-45(-\sin 30° + 0.3\cos 30)}{(\cos 30 + 0.3\sin 30)^2}$$

$$\frac{dF}{d\theta} = 10.47 \text{ lb/rad}$$

$$= 0.183 \text{ lb/degree}$$

$$\frac{dF}{dt} = \frac{dF}{d\theta} \times \frac{d\theta}{dt}$$

$$= (0.18)(-0.8)$$

$$= -0.09 \text{ lb/S}$$

log

long Method

$$* \frac{d}{dx}\left[\ln\left(\frac{x^2\sin x}{\sqrt{1+x}}\right)\right]$$

$$= \frac{d}{dx}\left[\ln(x^2\sin x) - \ln(\sqrt{1+x})\right]$$

$$= \frac{d}{dx}\left[\ln(x^2) + \ln(\sin x) - \ln(\sqrt{1+x})\right]$$

$$= \frac{d}{dx}\left[2\ln(x) + \ln(\sin x) - \frac{1}{2}\ln(1+x)\right]$$

$$= \frac{2}{x} + \frac{1}{\sin x} \times \cos x - \frac{1}{2} \times \frac{1}{1+x}$$

$$= \frac{2}{x} + \cot x - \frac{1}{2(1+x)}$$

(Ans)

* $\frac{d}{dx}\ \frac{x^2\sqrt[3]{(7x-14)}}{(1+x^2)^4} = \frac{dy}{dx}\ y$

$\Rightarrow$ Take ln on both side,

$$\ln(y) = 2\ln(x) + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)$$

$$\Rightarrow \frac{d}{dx}\ln(y) = \frac{d}{dx}\left[2\ln(x) + \frac{1}{3}\ln(7x-14) - 4\ln(1+x^2)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{1}{3} \times \frac{1}{7x-14} \times 7 - \frac{4}{1+x^2} \times 2x$$

$$\Rightarrow \frac{dy}{dx} = y \times \left[\frac{2}{x} + \frac{7}{21x-42} - \frac{8x}{1+x^2}\right]$$

$$\therefore \frac{dy}{dx} = \frac{x^2\sqrt[3]{(7x-14)}}{(1+x^2)^4} \times \left[\frac{2}{x} + \frac{7}{21x-42} - \frac{8x}{1+x^2}\right]$$

(Ans)

Find $\frac{dy}{dx}$, when $y = x\sqrt[3]{(1+x^3)}$

long Methord

$$\frac{dy}{dx} = x \times \frac{d}{dx}\left(\sqrt[3]{(1+x^3)}\right) + \left(\sqrt[3]{(1+x^3)}\right)\frac{d}{dx}$$

$$\frac{dy}{dx} = x \times \frac{1}{3}(1+x^3)^{-2/3} \times (3x^2) + \sqrt[3]{(1+x^2)} \times$$

$$= x^3(1+x^3)^{-2/3} + \cancel{3x^2} \sqrt[3]{(1+x^3)}$$

$$= \frac{x^3}{(1+x^3)^{2/3}} + \sqrt[3]{(1+x^2)}$$

(Ans)

$\boxed{\text{Short Methord}}$ Taking ln both sides.

$$\ln y = \ln x + \frac{1}{3}\ln(1+x^3)$$

$$\Rightarrow \frac{dy}{dx}\ln(y) = \frac{1}{x} + \frac{1}{3} \times \frac{1}{(1+x^3)}(3x^2)$$

$$\Rightarrow \frac{d}{dx}\ln(y) = \frac{1}{x} + \frac{x^2}{(1+x^3)}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{x^2}{(1+x^3)}$$

$$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{1}{x} + \frac{x^2}{(1+x^3)}\right)$$

$$\Rightarrow \frac{dy}{dx} = x\sqrt[3]{(1+x^3)} \times \left(\frac{1}{x} + \frac{x^2}{(1+x^3)}\right)$$

(Ans)

১২/১২/২১ / Math 10
Sunday /

Explicit function:

exm:

$y = x^2$ $\qquad$ $y = f(x)$

$y = \sin x$

$y = \frac{2}{x} = f(x)$

একটি Variable এর মাধ্যমে অ
Variable কে প্রকাশ করা যায়
ফাংশনকে Explicit functi
বলে। যেমন: $y = f(x) = x$

Implicit function:

exm:

$x^2 + y^2 = 25$

$xy + x^2 = 5$

Implicit function কে Explic
function এ Convart করা সম্
Convart না করে উভয় পক্ষে
d/dx করে দিতে হবে

Differentiation of implicit function

Q: Find $\frac{dy}{dx}$ from the implicit function; $x^2 + y^2 = 25$

$\Rightarrow \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 25$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = -\frac{x}{y}$

(Ans)

Q: Find $\frac{dy}{dx}$ of $xy + x^2 = 5$

$\Rightarrow \frac{d}{dx}(xy + x^2) = \frac{d}{dx} 5$

$\Rightarrow \frac{d}{dx}(xy) + 2x = 0$

$\Rightarrow x\frac{d}{dx}(y) + y\frac{d}{dx}(x) + 2x = 0$

$$\Rightarrow x\frac{dy}{dx} + y + 2x = 0$$

$$\therefore \frac{dy}{dx} = \frac{-(2x+y)}{x}$$

(Ans)

Find $\frac{d^2y}{dx^2}$ from the implicit function $x\cos y = y$

$$\Rightarrow \frac{d}{dx}(x\cos y) = \frac{dy}{dx}$$

$$\Rightarrow \cos y - x\sin(y)\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \frac{d}{dx}\left(\cos y - x\sin(y)\frac{dy}{dx}\right) = \frac{dy}{dx}$$

$$\Rightarrow -\sin y\frac{dy}{dx} - \left(\sin y\frac{dy}{dx} + x\left(\cos y\cdot\frac{dy^2}{dx} + \sin y\frac{d^2y}{dx^2}\right)\right) = \frac{d^2y}{dx^2}$$

$$=$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos y}{1+x\sin y}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\cos y}{1+x\sin y}\right)$$

$$\frac{d^2y}{dx^2} = \frac{(1+x\sin y)\frac{d}{dx}(\cos y) - \cos y\frac{d}{dx}(1+x\sin y)}{(1+x\sin y)^2}$$

$$= \frac{(1+x\sin y)(-\sin y)\frac{\cos y}{1+x\sin y} - \cos y\left(x\frac{d}{dx}\sin y + \sin y\right)}{(1+x\sin y)^2}$$

$$= \frac{(1+x\sin y)(-\sin y)\times\left(\frac{\cos y}{1+x\sin y}\right) - \cos y\left(x\cos y\, \frac{dy}{dx} + \sin y\right)}{(1+x\sin y)^2}$$

$$= \frac{-\sin y\cos y - \cos y\left(x\cos y\,\frac{\cos y}{1+x\sin y} + \sin y\right)}{(1+x\sin y)^2}$$

$$= \frac{-\sin y\cos y - \frac{x\cos^3 y}{1+x\sin y} - \sin y\cos y}{(1+x\sin y)^2}$$

$$= \frac{-2\sin y\cos y - \frac{x\cos^3 y}{1+x\sin y}}{(1+x\sin y)^2}$$

$$= \frac{-2\sin y\cos y\,(1+x\sin y) - x\cos^3 y}{(1+x\sin y)^3}$$

$$= -\frac{2\sin y\cos y + 2x\sin^2 y\cos y + x\cos^3 y}{(1+x\sin y)^3}$$

$$= -\frac{2x\sin^2 y\cos y + x\cos^3 y + 2\cos y\sin y}{(1+x\sin y)^3}$$

(Ans)

Use implicit differentiation to find the slope of the tangent line to the curve at the specified point

$$2(x^2+y^2)^2 = 25\,(x^2-y^2) \qquad \text{at } (3,1)$$

Solve:

$$\frac{d}{dx}\left(2\,(x^2+y^2)^2\right) = \frac{d}{dx}\,25\,(x^2-y^2)$$

$$\Rightarrow 2\,\frac{d}{dx}(x^2+y^2)^2 = 25\,\frac{d}{dx}(x^2-y^2)$$

$$\Rightarrow 2\left(2(x^2+y^2)\frac{d}{dx}(x^2+y^2)\right) = 25\left(2x - 2y\frac{dy}{dx}\right)$$

$$\Rightarrow 4\,(x^2+y^2)\left(2x + 2y\frac{dy}{dx}\right) = 25\left(2x - 2y\frac{dy}{dx}\right)$$

$$\Rightarrow 8x^3 + 8xy^2 + 8x^2y\frac{dy}{dx} + 8y^3\frac{dy}{dx} = 50x - 50y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(8x^2y+8y^3+50y\right) = 50x - 8x^3 - 8xy^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{50x-8x^3-8xy^2}{8x^2y+8y^3+50y}$$

at $(3,1)$

$$m = \frac{dy}{dx} = \frac{150-216-24}{72+8+50} = -\frac{90}{130} = -\frac{9}{13}$$

$$\therefore\ y-1 = -\frac{9}{13}(x-3)$$

$$\Rightarrow 13y - 13 = -9x+27$$

$$\Rightarrow 9x+13y = 40$$

(Ans)

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Increasing, decreasing function:

Q: $f(x) = x^2 - 4x + 3$

Differentiating with respect to x

$$f'(x) = 2x - 4$$

for $x = 2$; $f'(x) = 0$

Intervals: $(-\infty, 2)$, $(2, \infty)$

At, $(-\infty, 2)$ $f'(x) < 0$

f is decreasing at interval $(-\infty, 2)$

Again. At $(2, \infty)$ $f'(x) > 0$

f is ~~decreasing~~ increasing at interval $(2, \infty)$

Q: Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing

$D_f : R$

De:

Differentiation with respect to x

$$f'(x) = 3x^2$$

For, $x = 0$ $f'(x) = 0$

Intervals: $(-\infty, 0)$, $(0, \infty)$

At $(-\infty, 0)$ $f'(x) > 0$

f is increasing at interval $(-\infty, 0)$

At $(0, \infty)$ $f'(x) > 0$

f is increasing at interval $(0, \infty)$

The function is ~~e~~ increasing everywhere except $x = 0$

Q: Find the intervals on which $f(x) = 3x^4 + 4x^3 - 12x^2$ is increasing and the intervals on which is decreasing.

Solution:

$$f'(x) = 12x^3 + 12x^2 - 24x$$
$$= 12x(x^2 + x - 2)$$

At, $x = -2, 0, 1$ $f'(x) = 0$

So intervals are: $(-\infty, -2)$; $(-2, 0)$, $(0, 1)$, $(1, \infty)$

At $(-\infty,-2)$; $f'(x) < 0$

Hence, f is decreasing at $(-\infty,-2)$

At $(-2,0)$; $f'(x) > 0$

Hence, f is increasing $(-2,0)$

At $\cancel{(-1,0)}$ $(0,1)$; $f'(x) < 0$

Hence, f is decreasing at $(0,1)$

At $(1,\infty)$; $f'(x) > 0$

Hence, f is increasing at $(1,\infty)$

Finally, f is increasing at $(-2,0) \cup (1,\infty)$

and decresing at $(-\infty,-2) \cup (0,1)$

(Am)

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Note

a) If $f'(x) > 0$ then increasing

b) If $f'(x) < 0$ then decreasing

c) If $f'(x) = 0$ then constant

14/12/21 / Math 101
Tuesday

Concave up & down:

Q: $f(x) = x^2 - 4x + 3$

solution:

$$f'(x) = 2x - 4$$
$$f''(x) = 2$$

As the $f''(x)$ is showing positive value of all the valu of x, So, f is concave up in its entire domain

(Ans)

Q: $f(x) = x^3$

Solution:

$$f'(x) = 3x^2$$
$$f''(x) = 6x$$

At $x=0$, $f''(x) = 0$

So, there are two different intervals: $(-\infty, 0), (0, \infty$

At $(-\infty, 0)$; $f''(x) < 0$

So, f is concave down at the interval $(-\infty, 0)$

At $(0, \infty)$; $f''(x) > 0$

So, f is concave up at the interval $(0, \infty)$

(Ans)

Q. $y = f(x) = \cancel{x^2 - 4x + 3}\ 3x^4 + 4x^3 - 12x^2 + 2$

Solution:

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 2$$

$$f'(x) = 12x^3 + 12x^2 - 24x$$

$$f''(x) = 36x^2 + 24x - 24$$

$$= 12(3x^2 + 2x - 2)$$

$$= 12(x + 1.22)(x - 0.55)$$

At $x = 0.55, -1.22$; $f''(x) = 0$

Inflection point: $0.55, -1.22$

So, the intervals are: $(-\infty, -1.22)$, $(-1.22, 0.55)$, $(0.55, \infty)$

Intervals	$(x + 1.22)$	$(x - 0.55)$	$12(x+1.22)(x-0.55)$	Decision
$(-\infty, -1.22)$	$-$	$-$	$+$	Concave up
$(-1.22, 0.55)$	$+$	$-$	$-$	Concave down
$(0.55, \infty)$	$+$	$+$	$+$	Concave up

So, f is concave up at $(-\infty, -1.22) \cup (0.55, \infty)$

and concave down at $(-1.22, 0.55)$

(Ans)

extra:

$$y = xe^{-\frac{x^2}{2}}$$

$$\ln y = \ln(x) + \ln(e^{-\frac{x^2}{2}})$$

$$= \ln(x) - \frac{x^2}{2}$$

$$\frac{dy}{dx} = y\left(\frac{1}{x} - \frac{2x}{2}\right)$$

$$= y\left(\frac{1}{x} - x\right)$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}\left(\frac{1}{x} - x\right) + y\left(-\frac{1}{x^2} - 1\right)$$

$$= y\left(\frac{1}{x}-x\right)\left(\frac{1}{x}-x\right) - \frac{1+x^2}{x^2} \times y$$

$$\frac{d^2y}{dx^2} = xe^{-\frac{x^2}{2}}\left[\left(\frac{1}{x}-x\right)\left(\frac{1}{x}-x\right) - \frac{1+x^2}{x^2}\right]$$

$$\frac{d^2y}{dx^2} = xe^{-\frac{x^2}{2}}\left[\frac{1+x^4-2x^2}{x^2} - \frac{1+x^2}{x^2}\right]$$

$$\frac{d^2y}{dx^2} = xe^{-\frac{x^2}{2}}(x-\sqrt{3})(x+\sqrt{3})$$

At $x = -\sqrt{3}, 0, \sqrt{3}$; $\frac{d^2y}{dx^2} = 0$

So, the inflection points are : $-3, 0, \sqrt{3}$

Thats means, the intervals : $(-\infty, -\sqrt{3})$, $(-\sqrt{3}, 0)$, $(0, \sqrt{3})$
$(\sqrt{3}, \infty)$

At $(-\infty, -\sqrt{3})$; $\frac{d^2y}{dx^2} \cancel{>0} < 0$

So, $\frac{d^2y}{dx^2}$ is concave ~~up~~ down at the interval : $(-\infty, -\sqrt{3})$

At $(-\sqrt{3}, 0)$; $\frac{d^3y}{dx^2} > 0$

So, $\frac{d^2y}{dx^2}$ is concave up at the interval : $(-\sqrt{3}, 0$

At $(0, \sqrt{3})$; $\frac{d^2y}{dx^2} < 0$

So, $\frac{d^2y}{dx^2}$ is concave down at the interval : $(0, \sqrt{3})$

At $(\sqrt{3}, \infty)$; $\frac{d^2y}{dx^2} > 0$

So, $\frac{d^2y}{dx^2}$ is concave up at the interval : $(\sqrt{3}, \infty$

Note

a) If $f''(x) > 0$ then concave up

b) If $f''(x) < 0$ then concave down

Final Start

19/12/21 / Math
Sunday / 101

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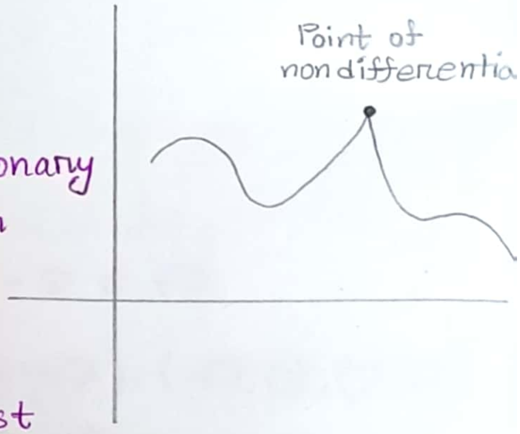
Relative Maxima and minima:

* যে সকল Point sharp change হয়ে ঐ Point গুলো Differention করা যায় না। অর্থাৎ যে সকল Point এ tangent line exist করে এ ধরনের Point কে nondifferentiable Point বলে।

non differentiable Point কে non stationary point ও বলে।



Sharp changing point একটি non stationary point কারন sharp point কে differen-tiation করা যায় না।

Note: [কোন Point Stationary point হলে 1st $dy/dx = 0$ হবে।] [কোন Point inflection point হলে $f''(x) = 0$ হবে]

* Inflection point
  Critical point
  Stationary point

Inflection point: Double differention এর মান যেখানে শূন্য হয় সেই Point কে Inflection Point বলে।
Concave up / down point গুলোকে Inflection point ~~বলে~~ নয়। Inflection point হলো concave and concave down এর মাঝে changing point.

Critical Point: গ্রাফের যে কোনো বিন্দুতে যদি মানের কিছুমাত্র পরিবর্তন ঘটে তবে ঐ গ্রাফের ঐ Point কে Critical Point বলে।

Stationary Point: যে সকল Point কে differentiation করা যায় তাদেরকে Stationary Point বলে।

Sharp changing point একটি non stationary Point।

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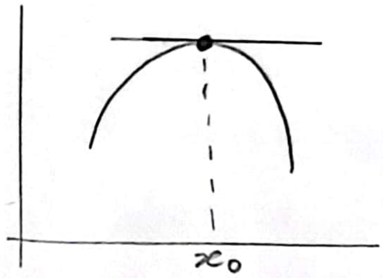
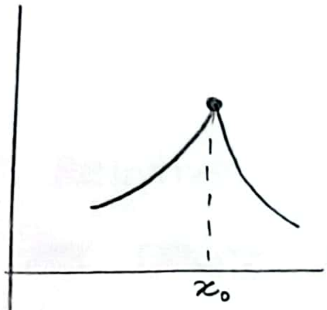


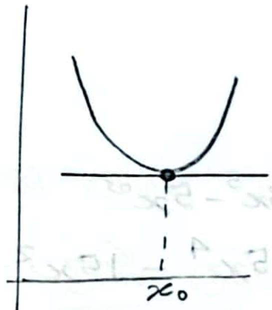
Critical Point
Stationary point
Relative maximum

Concave down এর মাঝে
এখানে, $f'(x) = 0$
এবং $f''(x) < 0$ হবে।



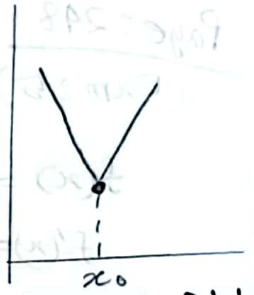
Critical Point
not a Stationary point
Relative maximum

এখানে, $f'(x)$ এবং $f''(x)$ doesn't exist হবে।
$f'(x) = f''(x) = $ undefind



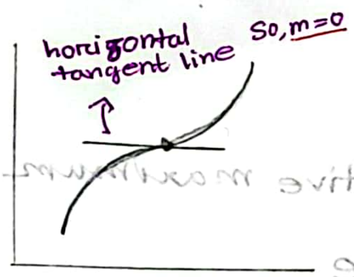
Critical Point
Stationary point
Relative minimum

Concave up এর মাঝে
এখানে, $f'(x) = 0$
$f''(x) > 0$ হবে



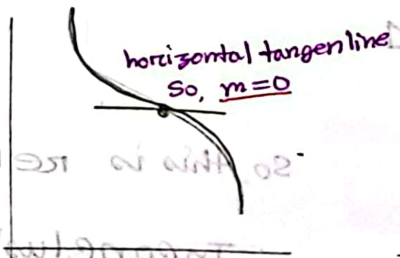
Critical Point
Not a stationary Point
Relative minimu

এখানে, $f'(x)$ ও $f''(x)$ উভয়ই does not exist হবে।



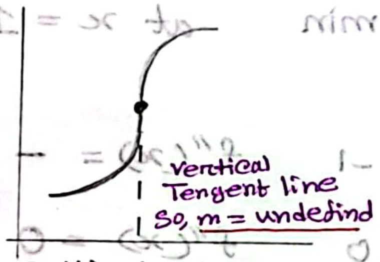
- Critical Point
- Stationary point
- Inflection Point
- not a relative extremum

এখানে, $f'(x)$ ও $f''(x)$ উভয়ই 0 হবে।



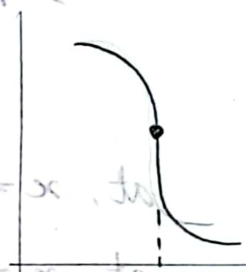
- Critical Point
- Stationary Point
- Inflection Point
- Not a relative extremum

এখানে, $f'(x)$ ও $f''(x)$ উভয়ই 0 হবে।



- Critical Point
- Not a stationary Point
- Inflection Point
- Not a relative Extremum

যেহেতু, m = undefind
So, $f'(x) = $ undefind
$f''(x) = 0$



- Critical point
- Not a stationary point
- Inflection point
- Not a relative extremum

যেহেতু, m = undefind
So, $f'(x) = $ undefind
$f''(x) = 0$

Polynomial function এ non stationary point থাকে না।

At which point of x, $f'(x)$ and $f''(x)$ is undefined. [here we'll get sharp change] [we will get relative max / relative min]

At which point of x, $f'(x)$ undefind and $f''(x)$ defind. [Here we'll get inflection point]

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(Exm: 5)

$f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$f''(x) = 60x^3 - 30x$

Here

$f'(x) = 15x^4 - 15x^2$

$= 15x^2(x^2 - 1)$

$= 15x^2(x+1)(x-1)$

at, $x = 0, -1, 1$; $f'(x) = 0$

Here, maximum at $x = -1$
min at $x = 1$

at, $x = -1$ $f''(x) = -$ So this is relative maximum

at $x = 0$ $f''(x) = 0$ Inconclusive

at $x = 1$ $f''(x) = +$ relative minimum

Point to be Noted

a) If $f'(x_0) = 0$ and $f''(x_0) > 0$; f has relative minimum at x_0

b) If $f'(x_0) = 0$ and $f''(x_0) < 0$; f has a relative maximum at x_0

c) If $f'(x_0) = 0$ and $f''(x_0) = 0$; the test is inconclusive ;

Another Point about absolute maxima and minima

a) If f has a relative minimum at x_0, then $f(x_0)$ is the absolute minimum of f on the interval.

b) If f has a relative maximum at x_0, then $f(x_0)$ is the absolute maximum of f the interval.

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* যে Point গুলোতে Relative minima আছে সেই Point গুলোর $f(x)$ এর মান যেখানে সবচেয়ে ~~বেশি~~ কম সেই Point কে Absolute minima হবে।

* যে Point গুলোতে Relative maxima আছে সেই Point গুলোর $f(x)$ এর মান যেখানে সবচেয়ে বেশি সেই Point কে Absolute maxima বলে।

Relative maxima / minima and Absolute maxima / minima নির্ণয়ের জন্য প্রথমে stationary এবং non stationary point নির্ণয় করতে হবে।

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$f(x) = 2x^3 - 15x^2 + 36x$

$f'(x) = 6x^2 - 30x + 36$
$= 6(x^2 - 5x + 6)$
$= 6(x-2)(x-3)$

At $x = 2, 3$; $f'(x) = 0$

$f''(x) = 12x - 30$

At $x = 2$, $f''(x) = -6$. So, at $x = 2$, f has a relative maxima

At $x = 3$, $f''(x) = 6$, So, at $x = 3$, f has a relative minima

So, f has an absolute maximum at $x = 2$ and absolute minimum at $x = 3$

2

$$f(x) = 3x^4 + 4x^3$$

$$f'(x) = 12x^3 + 12x^2$$
$$= 12x^2(x+1)$$

at, $x = 0, -1$; $f'(x) = 0$

$$f''(x) = 36x^2 + 24x$$
$$= 12x(3x+2)$$

At $x = 0$, $f''(x) = 0$; So, at $x = 0$, in conclusive

At $x = -1$, $f''(x) = 12$; So, at $x = -1$, f has a relative minim

So, there is no absolute maximum point of this function but has an absolute minimum point at $x = -1$

Q:

$f(x) = (3x^4 - 4x^3)(x^2+4x+4)$

$f'(x) = \frac{d}{dx}(3x^4-4x^3)(x^2+4x+4) + \frac{d}{dx}(x^2+4x+4)(3x^4-4x^3)$

$= (12x^3 - 12x^2)(x^2+4x+4) + (2x+4)(3x^4-4x^3)$

$= 18x^5 + 40x^4 - 16x^3 - 48x^2$

[0 method]

$= 2x^2(9x^3+20x^2-8x-24)$

$= 2x^2(x+2)(9x^2+2x-12)$

at, $x = 0, -2, -1.27, 1.04$

~~$f''(x) = 90x^4 + 80x^3$~~

~~$f''(x) =$~~

$f''(x) = 90x^4 + 160x^3 - 48x^3 - 96x$

$= x(90x^3 + 160x^2 - 40x - 96)$

$\therefore f''(0) = 0$

$\therefore f''(-2) = 160$

$\therefore f''(-1.27) = -49.11$

$\therefore f''(1.04) = 133.51$

$f(-2) = 0$

$f(1.04) = -9.19$

$f(-1.27) = 8.53$

$\therefore$ Relative maxima at $x = -1.27$

$\therefore$ Inconclusive at $x = 0$

$\therefore$ Relative minima at $x = -2, 1.04$

$\therefore$ Absolute maximum at $x = -1.27$ which is 8.53

$\therefore$ Absolute minimum at $x = 1.04$ which is -9.19

(Ans)

CamScanner

Q: $f(x) = 3x^{5/3} - 15x^{2/3}$

$$f'(x) = 5x^{2/3} - 10x^{-1/3}$$

$$= 5x^{2/3} - \frac{10}{x^{1/3}}$$

$$= 10x^{-\frac{1}{3}}\left(\frac{x}{2} - 1\right)$$

$$= \frac{10}{x^{1/3}}\left(\frac{x}{2} - 1\right)$$

Stationary point $(x = 2)$

non stationary point $(x = 0)$

$$f''(x) = \frac{10}{3}x^{-\frac{1}{3}} + \frac{10}{3}x^{-\frac{4}{3}}$$

$$= \frac{10}{3}x^{-\frac{4}{3}}(x+1)$$

$$= \frac{10}{3x^{4/3}}(x+1)$$

$f''(0)$ undefined

$f''(2) = 3.96$ that means, at $x = 2$ f has a relative mini

যদি vartical tangent line হয় তাহলে ঐ point এর আগে দুইটি increasing অথবা দুটি decreasing value হবে।

sharp change হলে ঐ Poin এর আগে একটি value incre আরেকটি decreasing value হবে।

$$\therefore f(-1) = 3(-1)^{5/3} - 15(-1)^{2/3}$$
$$= -3 - 15$$
$$= -18$$

$$\therefore f(1) = 3(1)^{5/3} - 15(1)^{2/3}$$
$$= -12$$

$\therefore f'(-1) = 15$ [$x = -1$, f is increasing]

$\therefore f'(1) = -5$ [$x = 1$, f is decreasing]

$\therefore$ Thats mean, at $x = 0$; f has a relative maxima.

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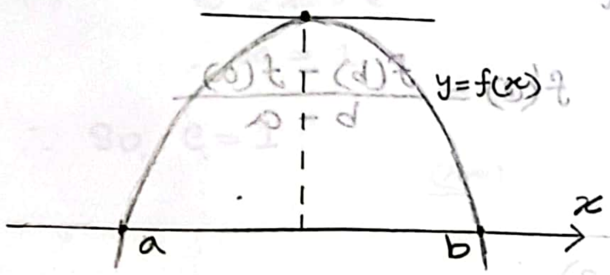
⊞ Rolle's theorem:

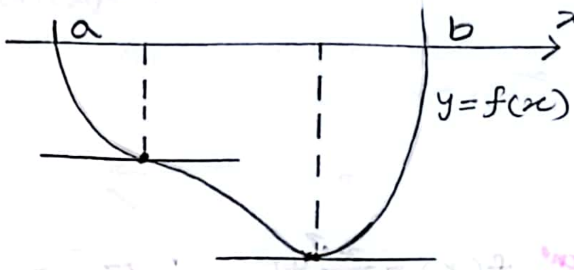
(i) Continuous at [a, b]

(ii) Differentiable at (a, b)

(iii) a and b will cut x-axis, Means $f(a) = f(b) = 0$

Then we will get at least one point c, for which $f'(c) = 0$





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Exm: 1 Find the two x-intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.

Solve: Here, $f(x) = x^2 - 5x + 4$

$$= (x-1)(x-4)$$

So, the x-intercepts at 1, 4. Rolle's theorem satisfied on the interval [1, 4]. So, c in the interval (1, 4) such that $f'(c) = 0$.

$$f'(x) = 2x - 5$$

if $f'(x) = 0$; $2x - 5 = 0$

$$\therefore x = \frac{5}{2}$$

$$\therefore c = \frac{5}{2}$$

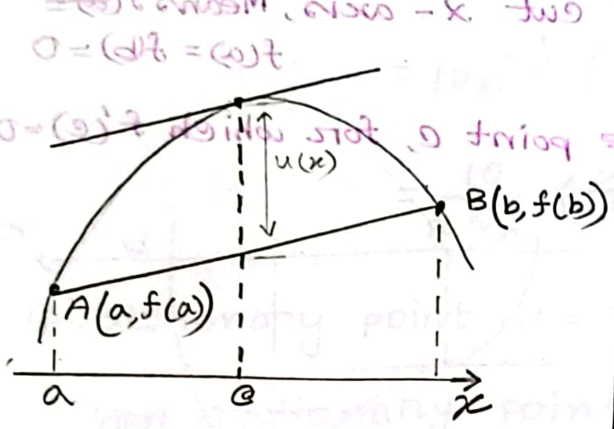
So, $c = 5/2$ is a point in the interval (1, 4) at which $f'(c) = 0$

The mean-value theorem:

① Continuous at $[a,b]$

⑪ Differentiable at (a,b)

Then we will get at least one point c, at which we will get tangent line parallel to the secant line



If closed interval $[a,b]$ and differenti on the open interval (a,b). Then there is at least one point c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Exm: $f(x) = \frac{x^3}{4}+1$; $[0,2]$; $(0,2)$

The slope of the secant line joining $[0,2] = \frac{f(2)-f(0)}{2-0} = \frac{3-1}{2} = 1$

Now, we will find the values of c,

$$f'(x) = \frac{3}{4}x^2 = 1$$

$$\Rightarrow x^2 = 4/3$$

$$\therefore \quad x = \pm\frac{2}{\sqrt{3}}$$

we can't consider $x = -\frac{2}{\sqrt{3}}$; cause, this is out of the given interval $[0,2]$

Hence, $c = \frac{2}{\sqrt{3}}$

(Ans)

Ques-2 $f(x) = x^2 - x$; i) Find a point b such that the slope of the seca line through $(0,0)$ and $(b, f(b))$ is 1

ii) Find all values of c that satisfy the conclusion

Solution: 1

we know, $\frac{f(b)-0}{b-0} = 1$

$$\Rightarrow b^2 - b = b$$

$$\Rightarrow b^2 - 2b = 0$$

$\Rightarrow b(b-2)=0$

Either,

$b=0$ or, $b=2$

Then $f(2) = 2^3-2$

$= 2$

(Ans)

(ii)

$f'(x) = 2x-1 = 1$

$\Rightarrow 2x = 2$

$\therefore x = 1$

$\therefore$ So, $c = 1$

(Ans)

Partial derivative:

Rules:

$$f(x) = \frac{\delta}{\delta x} f(x,y)$$

$$f(y) = \frac{\delta}{\delta y} f(x,y)$$

$$f(xx) = \frac{\delta^2}{\delta x^2} f(x,y)$$

$$f(yy) = \frac{\delta^2}{\delta y^2} f(x,y)$$

$$f(xy) = \frac{\delta}{\delta y}\left[\frac{\delta}{\delta x} f(x,y)\right]$$

Q. $f(x,y) = 2xy^3 + 4x^3y$

① $f_x = \frac{\delta}{\delta x} f(x,y) = 2y^3 + 12x^2y$

② $f_y = \frac{\delta}{\delta y} f(x,y) = 6xy^2 + 4x^3$

③ $f_{xx} = \frac{\delta^2}{\delta x^2} f(x,y) = \frac{\delta}{\delta x}(2y^3+12x^2y) = 24xy$

④ $f_{yy} = \frac{\delta^2}{\delta y^2} f(x,y) = \frac{\delta}{\delta y}(6xy^2+4x^3) = 12xy$

⑤ $f_{xy} = \frac{\delta}{\delta y}\left[\frac{\delta}{\delta x} f(x,y)\right] = \frac{\delta}{\delta y}\left[2y^3+12x^2y\right]$

$= 6y^2+12x^2$

28/12/21 Tuesday / Math 101

Indefinite Integral

* Monte Carlo Integration এর basic use করে integral করা হয়।

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

i) $\int x^2 dx = x^3/3 + c$

ii) $\int \sqrt{x}\, dx = \frac{x^{3/2}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$

iii) $\int \frac{1}{x} dx = \ln(x) + c$

iv) $\int \sin x\, dx = -\cos x + c$

v) $\int \cos x\, dx = \sin x + c$

vi) $\int \sec^2 x\, dx = \tan x + c$

vii) $\int \sec x \tan x\, dx = \sec x + c$

viii) $\int e^x dx = e^x + c$

ix) $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$

x) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + c$

xi) $\int a^x dx = \frac{a^x}{\ln(a)}$

xii) $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$

*xiii) $\int \sin^{-1}x\, dx = \frac{1}{\sqrt{1-x^2}} + c$

xiv) $\int \tan^{-1}x\, dx = \frac{1}{1+x^2} + c$

Q. $\int (x^2 - x)\, dx$

$= \int x^2 dx - \int x\, dx$

$= \frac{x^3}{3} - \frac{x^2}{2} + c$

$= \frac{2x^3 - 3x^2}{6} + c$

Q. $\int 10 \cos x\, dx$

$= 10 \sin x + c$

Q. $\int \frac{\cos x}{\sin^2 x} dx$

$= \int \frac{\cos x}{\sin x} \times \frac{1}{\sin x} dx$

$= \int \cot x \operatorname{cosec} x\, dx$

$= -\operatorname{cosec} x + c$

iv) $\int \frac{x^2 - 2x^4}{x^3} dx$

$= \int \frac{1}{x} dx - \int 2x\, dx$

$= \ln(x) - x^2 + c$

v) $\int \frac{x^2}{x^2+1} dx$

$= \int \frac{x^2+1-1}{x^2+1} dx$

$= \int 1\, dx - \int \frac{1}{x^2+1} dx$

$= x - \tan^{-1}x + c$

Chain rule of differentiation:

$$\frac{d}{dx} \sin(5x) = \cos(5x) \times 5$$

Integration by substitution:

① $\int e^{mx} dx$

Lets take, $mx = z$

$x = \frac{z}{m}$

Differentian with respect to z

$\frac{dx}{dz} = \frac{d}{dz}\left(\frac{z}{m}\right) = \frac{1}{m}$

$dx = \frac{dz}{m}$

$= \int e^{z} dx$

$= \int e^{z} \frac{dz}{m}$

$= \frac{1}{m} \int e^{z} dz$

$= \frac{e^{z}}{m}$

$= \frac{e^{mx}}{m}$

2. $\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$

Lets take $f(x) = z$

Differentiation with respect to z

Now,

$\int \frac{1}{z} dz = \ln(z).$

$= \ln(f(x))$

$\Rightarrow \int \frac{f'(x)dx}{f(x)} = \ln(f(x))$

Q. Evaluate $\int (x^2+1)^{50}\, 2x\, dx$

Lets take,

$$x^2+1 = z$$

Differentiation with respect to x

$$2x = \frac{dz}{dx}$$

$$dz = 2x\,dx$$

Now,

$$\int z^{50}\, dz = \frac{z^{51}}{51}$$

$$= \frac{(x^2+1)^{51}}{51} + C$$

Q. $\int \frac{dx}{\left(\frac{x}{3}-8\right)^5}$

Lets take,

$$\frac{x}{3} - 8 = z$$

Differentiation with respect to x

$$\frac{1}{3} = \frac{dz}{dx}$$

$$dz = 3\,dz$$

Now,

$$\int \frac{3\,dz}{z^5} = \frac{3z^{-4}}{-4}$$

$$= -\frac{3}{4\left(\frac{x}{3}-8\right)^4} + C$$

Q. $\int \frac{dx}{1+4x^2}$

$$= \int \frac{dx}{1+(2x)^2}$$

Lets take

$$2x = z$$

differantiation with respect to x

$$2 = \frac{dz}{dx}$$

$$dz = 2\,dx$$

$$dx = \frac{dz}{2}$$

Now,

$$\int \frac{dz}{2(1+z^2)} = \frac{1}{2}\tan^{-1} z$$

$$= \frac{1}{2}\tan^{-1}(2x) + C$$

Q $\int \left(\frac{1}{x} + \sec^2(\pi x)\right) dx$

Let's take $\pi x = z$

differentiation with respect to x

$\pi = \frac{dz}{dx}$

$dx = \frac{dz}{\pi}$

$= \ln(x) + \int \sec^2 z \frac{dz}{\pi}$

$= \ln(x) + \frac{1}{\pi} \tan(z)$

$= \ln(x) + \frac{1}{\pi}(\tan(\pi x)) + c$

Q $\int \sin^2 x \cos x \, dx$

let's take $\sin x = z$

Differentiating with respect to x

$\cos x = \frac{dz}{dx}$

$\cos x \, dx = dz$

Now, $\int z^2 dz = \frac{z^3}{3}$

$= \frac{1}{3}\sin^3 x + c$

Q $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Lets take $\sqrt{x} = z$

Differentiating with respect to x

$\frac{1}{2\sqrt{x}} = \frac{dz}{dx}$

$\frac{dx}{\sqrt{x}} = 2dz$

Now, $\int e^z 2dz = 2e^z$

$= 2e^{\sqrt{x}} + c$

Q $\int x^3 \sqrt[3]{3 - 4x^4} \, dx$

Lets take $3 - 4x^4 = z$

Differentiating with respect to x

$-16x^3 = \frac{dz}{dx}$

$x^3 dx = \frac{dz}{-16}$

Now, $\int \sqrt[3]{z} \frac{dz}{-16} = -\frac{3}{4 \times 16} z^{4/3}$

$= -\frac{3}{64}(3 - 4x^4)^{4/3} + c$

Q $\int \sin^3 x\, dx$

$= \int (1-\cos^2 x)\sin x\, dx$

Let $z = \cancel{1-\cos^2}\cos x$

$\frac{dz}{dx} = -\sin x$

$dz = dx \; -\sin x$

$-\sin x = \frac{dz}{dx}$

$\sin x\, dx = -dz$

Now,

$= -\int (1-z^2) \cdot dz$

$\cancel{= -\frac{z^3}{3} \times dz}$

$= -\int dz + \int z^2 dz$

$= -z + \frac{z^3}{3}$

$= -\cos x + \frac{\cos^3 x}{3} + C$

Q. $\int \frac{dx}{4+9x^2} = \int \frac{dx}{4\left(1+\frac{9x^2}{4}\right)}$

$= \frac{1}{4}\int \frac{dx}{1+\left(\frac{3x}{2}\right)^2}$

Lets take $\frac{3x}{2} = z$

differenting with respect to x

$\frac{3}{2} = \frac{dz}{dx}$

$dz = \frac{2}{3} dz$

Now,

$\frac{2}{4\times 3} \times \int \frac{dz}{1+z^2} = \frac{1}{6}\tan^{-1} z$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) + C$

Q $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{dx}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}$

lets take $\frac{x}{2} = z$

Differenting with respec x

$dx = 2dz$

$= \frac{1}{4}\int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$

Now, $\frac{2}{4}\int \frac{dz}{\sqrt{1-z^2}} = \frac{1}{2}\sin^{-1} z$

$= \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$

u. তাকে ধরবো যাকে $\frac{d}{dx}$ করলে ছোট হয়ে যায়। 04/01/22 / Tuesday / Math 101

Q.

$$\int x \cos x \, dx$$ ~~Let's take.~~

$$= x \int \cos x \, dx - \int \left[\frac{d}{dx}(x) \int \cos x \, dx\right] dx$$

$$= x \sin x - \int 1 \times \sin x \, dx$$

$$= x \sin x + \cos x + c$$

Q.

$$\int x^2 \cos x \, dx$$

$$= x^2 \int \cos x \, dx - \int \left[\frac{d}{dx}(x^2) \int \cos x \, dx\right] dx$$

$$= x^2 \sin x - \int [2x \times \sin x] \, dx$$

$$= x^2 \sin x - 2x \int \sin x \, dx + 2 \int \left[\frac{d}{dx}(x) \times \int \sin x \, dx\right] dx$$

$$= x^2 \sin x - 2x(-\cos x) + 2\int 1 (-\cos x) \, dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Q.

$$\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x|$$

$$\int \sec x \, dx = \ln(\sec x + \tan x)$$

* $$\frac{d}{dx}(\ln(\sec x + \tan x)) = \frac{1}{\sec x + \tan x} \times \frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{\sec x \cdot \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x}$$

$$= \sec x$$

* $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$

$$\int \sec x \, dx = \int \sec x \, \frac{\sec x + \tan x}{\sec x + \tan x} dx$$

$$= \int \frac{\sec x \cdot \tan x + \sec^2 x}{\sec x + \tan x} dx$$

Lets take, $\sec x + \tan x = z$

differentiating with respect to x

$$\sec x \cdot \tan x + \sec^2 x = \frac{dz}{dx}$$

$$dx(\sec x \cdot \tan x + \sec^2 x) = dz$$

Now,

$$\int \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} dx = \int \frac{dz}{z}$$

$$= \ln|z|$$

$$= \ln|\sec x + \tan x| + C$$

Q

$$\int x e^x dx$$

$$= x \int e^x dx - \int \left[\frac{d}{dx}(x) \times \int e^x dx\right] dx$$

$$= x \cdot e^x - \int 1 e^x dx$$

$$= xe^x - e^x$$

$$= e^x (x-1) + C$$

Q $\int \ln(x) dx$

$$= \ln(x) \int 1 \, dx - \int \left[\frac{d}{dx}(\ln(x)) \int 1 \cdot dx\right] dx$$

$$= x \ln(x) - \int \left[\frac{1}{x} \times x\right] dx$$

$$= x \ln(x) - x$$

$$= x(\ln(x) - 1) + C$$

Q. $\int x^2 e^{-x} dx$

$= x^2 \int e^{-x} dx - \int [\frac{d}{dx}(x^2) \int e^{-x} dx] dx$

$= -x^2 e^{-x} - \int [2x \; (-e^{-x})] dx$

$= -x^2 e^{-x} + 2\int x e^{-x} dx$

$= -x^2 e^{-x} + x2(-e^{-x}) - 2\int [(-e^{-x})] dx$

$= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x}$

$= -e^{-x}(x^2+2x+2) + c$

$\int e^x dx = e^x$

$\int e^{mx} dx = \frac{e^{mx}}{m}$

$\int e^{-x} dx = \frac{e^{-1x}}{-1} = -e^{-x}$

$\int e^{-mx} dx = \frac{e^{-mx}}{-m}$

Q. $\int_0^5 x^2 dx = [\frac{x^3}{3}]_0^5 = \frac{5^3}{3} - \frac{0^3}{3}$

$= \frac{125}{3}$

Q. $\int_1^3 x e^x dx = [e^x x - \int e^x dx]_1^3$

$= [e^x x - e^x]_1^3$

$= (3e^3 - e^3) - (1e^1 - e^1)$

$= 2e^3$

Q. $\int_0^1 \tan^{-1} x \, dx$ ~~$= [\frac{1}{1+x^2}]_0^1$~~

$= \tan^{-1}x \int 1 \, dx - \int [\frac{d}{dx}(\tan^{-1}x) \int 1 \, dx] dx$

$= x\tan^{-1}x - \int [\frac{1}{1+x^2} \times x] dx$

$= [x\tan^{-1}x]_0^1 - \int [\frac{1}{1+x^2} x] dx$

CamScanner

Now,

$$[x\tan^{-1}x]_0^1 - \int\left[\frac{1}{1+x^2}\, x\right]dx$$

Lets take
$1+x^2 = z$

Differentiating with respect to x

$$2x = \frac{dz}{dx}$$

$$x\,dx = \frac{dz}{2}$$

$$= -\int \frac{dz}{2z}$$

$$= -\frac{\ln|z|}{2}$$

$$= [x\tan^{-1}x]_0^1 - \left[\frac{\ln|1+x^2|}{2}\right]_0^1$$

$$= \frac{\pi}{4} - \left(\frac{\ln(2)}{2} - \frac{\ln(1)}{2}\right)$$

$$= \frac{\pi}{4} - \frac{\ln(2)}{2}$$

Q $\int \operatorname{cosec}^3 x\, dx$

$$\therefore I = \int \operatorname{cosec}^3 x\, dx$$

$$\Rightarrow I = \int \operatorname{cosec} x \cdot \operatorname{cosec}^2 x\, dx$$

$$\Rightarrow I = \int \operatorname{cosec} x \int \operatorname{cosec}^2 x\, dx - \int\left\{\frac{d}{dx}\operatorname{cosec} x \int \operatorname{cosec}^2 x\, dx\right\} dx$$

$$\Rightarrow I = \operatorname{cosec} x\,(-\cot x) - \int\{\operatorname{cosec} x \cdot \cot x \cdot \cot x\, dx\}$$

$$\Rightarrow I = -\operatorname{cosec} x \cdot \cot x - \int \operatorname{cosec} x\,(\operatorname{cosec}^2 x - 1)\,dx$$

$$\Rightarrow I = -\operatorname{cosec} x \cdot \cot x - \int \operatorname{cosec}^3 x\,dx + \int \operatorname{cosec} x \cdot dx$$

$$\Rightarrow I = -\operatorname{cosec} x \cdot \cot x - I + \log\{\operatorname{cosec} x - \cot x\} + C$$

$$\Rightarrow 2I = -\operatorname{cosec} x \cdot \cot x + \log\{\operatorname{cosec} x - \cot x\} + C$$

$$\Rightarrow I = \frac{-\operatorname{cosec} x \cdot \cot x}{2} + \frac{1}{2}\log|\operatorname{cosec} x - \cot x| + \frac{C}{2}$$

Area Between $y = f(x)$ and $y = g(x)$

09-01-22 / Math 101
Sunday

Q: $x = y^2$
$x = y + 2$

$y^2 - y - 2 = 0$

$\therefore y = 2, -1$

$\therefore x = 4, 1$

$$\text{Area } \int_{-1}^{2} (y + 2 - y^2)\, dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 4.5$$

Another system:

$y = \pm\sqrt{x}$
$y = x - 2$

$x = 4, 1$

$$\therefore A_1 = \int_0^1 \sqrt{x} - (-\sqrt{x})\, dx$$

$$= 2\int \sqrt{x}\, dx$$

$$= \frac{2 \times 2}{3}\left[x^{3/2}\right]_0^1$$

$$= \frac{4}{3}$$

$$\therefore A_2 = \int_1^4 \sqrt{x} - (x - 2)\, dx$$

$$= \int_1^4 (\sqrt{x} - x + 2)\, dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x\right]_1^4$$

$$= \frac{19}{6}$$

$$\therefore \text{Area} = A_1 + A_2$$

$$= \frac{4}{3} + \frac{19}{6}$$

$$= 4.5$$

Example-1

$y = x + 6$
$y = x^2$
$x = 0$
$x = 2$

$$\text{Area} = \int_0^2 [(x + 6) - x^2]\, dx$$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_0^2$$

$$= \frac{34}{3} - 0$$

$$= \frac{34}{3}$$

(Ans)

6.1 Page: 443

(i) $\int_{-1}^{2}(x^2-1-x)dx$

$= \left[\frac{x^3}{3} + x - \frac{x^2}{2}\right]_{-1}^{2}$

$= \frac{9}{3}$

(iii) Area $= \int_{1}^{2}\left(y - \frac{1}{y^2}\right)dy$

$= \left[\frac{y^2}{2} + \frac{1}{y}\right]_{1}^{2}$

$= \left(\frac{4}{2} + \frac{1}{2}\right) - \frac{3}{2}$

$= 1$

Again,

Area $= \int_{1/4}^{2}$

Another

$y^2 = \frac{1}{x}$

$y = \pm\frac{1}{\sqrt{x}}$

$y = x$

$\therefore A_1 = \int_{1/4}^{1}\left(2 - \frac{1}{\sqrt{x}}\right)dx$

$= \left[2x - 2\sqrt{x}\right]_{1/4}^{1}$

$= (2\cdot1 - 2\cdot1) - \left(\frac{2}{4} - \frac{2}{2}\right)$

$= \frac{1}{2}$

$A_2 = \int_{1}^{2}(2x - x)dx$

$= \left[2x - \frac{x^2}{2}\right]_{1}^{2}$

$= \frac{1}{2}$

$\therefore$ Area $= A_1 + A_2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

(ii) $\int_{0}^{4}\sqrt{x} - \left(-\frac{1}{4}x\right)dx$

$= \int_{0}^{4}\sqrt{x} + \frac{1}{4}x\,dx$

$= \left[\frac{2}{3}x^{3/2} + \frac{1}{8}x^2\right]_{0}^{4}$

$= \frac{22}{3}$

(iv) $\int_{0}^{2}(2 - y^2 - (-y))dy$

$= \int_{0}^{2}(2 - y^2 + y)\,dy$

$= \left[2y - \frac{y^3}{3} + \frac{y^2}{2}\right]_{0}^{2}$

$= \frac{10}{3}$

Another

$y = -x$

$y^2 = 2 - x$

$\Rightarrow y = \pm\sqrt{(2-x)}$

$A_1 = \int_{0}^{2}\sqrt{2-x}\,dx$

Lets take $2 - x = z$

Differentiation with respect to x

$-1 = \frac{dz}{dx}$

$dx = -dz$

$= \int_{0}^{2} -\sqrt{z}\,dz$

$= -\left[\frac{2}{3}(2-x)^{3/2}\right]_{0}^{2}$

$= \frac{4\sqrt{2}}{3}$

$A_2 = \int_{-2}^{0}\left(\sqrt{2-x} - (-x)\right)dx$

$= \int_{-2}^{0}\left(\sqrt{2-x} + x\right)dx$

$= \left[-\frac{2}{3}(2-x)^{3/2} + \frac{x^2}{2}\right]_{-2}^{0}$

$= \frac{10}{3} - \frac{4\sqrt{2}}{3}$

$\therefore$ Area $= A_1 + A_2$

$= \frac{4\sqrt{2}}{3} + \frac{10}{3} - \frac{4\sqrt{2}}{3}$

$= \frac{10}{3}$

11-01-22 Monday / Math 101

Q. $y = x^2$

$y = 2x + 4$

solve:

$x^2 = 2x + 4$

$x^2 - 2x - 4 = 0$

$\therefore x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 (-4)}}{2}$

$= -1.23 \; ; \; 3.23$

$\text{Area} = \int_{-1.23}^{3.23} (2x + 4 - x^2) dx$

$= \left[x^2 + 4x - \frac{x^3}{3} \right]_{-1.23}^{3.23}$

$= \left(3.23^2 + 4 \times 3.23 - \frac{3.23^3}{3}\right) - \left((-1.23)^2 + 4 \times (-1.23) - \frac{(-1.23)^3}{3}\right)$

$= 14.91$ (Ans)

Another Approach

$A_1 = \int_0^{10.4} (\sqrt{y} - \frac{y}{2} + 2) dy$

$= 16.12$

$A_2 = \int_0^{1.5} (-\sqrt{y} - \frac{y}{2} + 2) dy$

$= \cancel{-}1.98$

$\therefore \text{Area} = A_1 - A_2$

$= 16.12 - 1.98$

$= 14.15$

(Ans)

Another Approach

$x = -1.23, \; 3.23$

$y = 1.5, \; 10.4$

$y = x^2$

$\therefore x = \pm\sqrt{y}$

$y = 2x + 4$

$\therefore x = \frac{y}{2} - 2$

$\therefore A_1 = \int_0^{1.5} \sqrt{y} - (-\sqrt{y}) dy$

$= \int_0^{1.5} 2 \times \frac{2}{3} \left[y^{3/2}\right]_0^{1.5}$

$= 2.45$

$\therefore A_2 = \int_{1.5}^{10.4} (\sqrt{y} - \frac{y}{2} + 2) dy$

$= \left[\frac{2}{3} y^{3/2} - \frac{y^2}{4} + 2y\right]_{1.5}^{10.4}$

$= \left(\frac{2}{3} \times (10.4^{3/2} - \frac{10.4^2}{4} + 2 \times 10.4\right) - \left(\frac{2}{3}(1.5^{3/2} - \frac{1.5^2}{4} + 2 \times 1.5\right)$

$= 16.12 - 3.66$

$= 12.46$

$\therefore \text{The Area} = A_1 + A_2$

$= 2.45 + 12.26$

$= 14.91$ (Ans)

Q

$$\text{Area} = \int_0^1 (\sqrt{x} - x^2)\,dx$$
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$
$$= \frac{1}{3}$$

Exer-6

$$x = \frac{y^2}{4}$$
$$x = \frac{y}{2} + 2$$
$$\text{Area} = \int_{-2}^{4} \left(\frac{y^2}{4} - y/2 + 2\right) dy$$
$$= \left[\frac{y^3}{12} - y^2/4 - 2y\right]_{-2}^{4}$$
$$= -9$$

Another approach

$$y = \pm 2\sqrt{x}$$
$$y = 2x - 4$$
$$A_1 = \int_0^1 (2\sqrt{x} - (-2\sqrt{x}))\,dx$$
$$= \int_0^1 (4\sqrt{x})\,dx$$
$$= \left[4/3\, x^{3/2}\right]_0^1$$
$$= \frac{8}{3}$$
$$A_2 = \int_1^4 2\sqrt{x} - (2x-4)\,dx$$
$$= \int_1^4 (2\sqrt{x} - 2x + 4)\,dx$$
$$= \left[4/3\, x^{3/2} - x^2 + 4x\right]_1^4$$
$$= (4/3 \times 8 - 16 + 16) - (4/3 - 1 + 4)$$
$$= \frac{19}{3}$$
$$\therefore \text{Area} = A_1 + A_2 = 8/3 + 19/3 = 9$$

Gamma function:

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$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$$

$\star$ $\Gamma(n) = (n-1)!$

$\therefore \Gamma(10) = (10-1)!$
$= 9!$

Exm

$$\int_0^\infty x^4 e^{-x}\, dx$$

$$= \int_0^\infty x^{5-1} e^{-x}\, dx$$

$$= \Gamma(5)$$

$$= (5-1)!$$

$$= 24$$

Exm

$$\int_0^\infty x^2 e^{-\sqrt{x}}\, dx$$

Lets take $\sqrt{x} = z$

Differentiating with respect to x

$$\frac{1}{2\sqrt{x}} = \frac{dz}{dx}$$

$$dx = 2\sqrt{x}\, dz$$

$$= 2z\, dz$$

Now,

$$\int_0^\infty x^2 e^{-\sqrt{x}}\, dx$$

$$= \int_0^\infty z^4 e^{-z}\, 2z\, dz$$

$$= 2 \int_0^\infty z^5 e^{-z}\, dz$$

$$= 2 * \Gamma(6)$$

$$= 2 \times 5!$$

$$= 240$$

Exm.

$$\int_0^\infty x^5 e^{-2x}\, dx$$

Letstake

$z = -2x$

differentiating with res x

$$\frac{dz}{dx} = -2$$

$$dx = \frac{dz}{-2}$$

Now

$$\int_0^\infty \frac{z^5 e^z}{64}\, dz$$

$$= \frac{1}{64}\int_0^\infty (z^5 e^z)\, dz$$

$$= \frac{5!}{2^6} \quad \frac{1}{2^6} \quad \Gamma(6)$$

$$= \frac{5!}{2^6}$$

$$= \frac{15}{8}$$