Solution to the Classwork Problem

Problem: If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14zy\hat{\jmath} + 20xz^2\hat{k}$, then evaluate $\int_C \vec{A} \cdot d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the following path C:

C: the straight lines from $(0,0,0)$ to $(1,0,0)$, then to $(1,1,0)$ and then to $(1,1,1)$.

Solution:

Along the straight line from $(0,0,0)$ to $(1,0,0)$ we get,

$< (0,0,0) + (1,0,0)t >= < (t, 0,0) >$ where $0 \leq t \leq 1$

So that, $x = t, y = 0$ and $z = 0$

that is $dx = dt, dy = 0$ and $dz = 0$

So we get

$$\vec{A} = 3t^2\,\hat{\imath}$$

and $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k} = dt\hat{\imath}$

Now, $\int_C \vec{A} \cdot d\vec{r} = \int_0^1 3t^2\,dt = 1$

Along the straight line from $(1,0,0)$ to $(1,1,0)$ we get,

$< (1,0,0) + (0,1,0)t >= < (1, t, 0) >$ where $0 \leq t \leq 1$

So that, $x = 1, y = t$ and $z = 0$

that is $dx = 0$, $dy = dt$ and $dz = 0$

So we get

$$\vec{A} = (3 + 6t)\,\hat{\imath}$$

and $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k} = dt\hat{\jmath}$

Now, $\int_C \vec{A} \cdot d\vec{r} = 0$

Along the straight line from $(1,1,0)$ to $(1,1,1)$ we get,

$< (1,1,0) + (0,0,1)t >=< (1,1,t) >$ where $0 \leq t \leq 1$

So that, $x = 1, y = 1$ and $z = t$

that is $dx = 0$, $dy = 0$ and $dz = dt$

So we get

$$\vec{A} = 9\,\hat{\imath} - 14t\hat{\jmath} + 20t^2\hat{k}$$

and $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k} = dt\hat{k}$

Now, $\int_C \vec{A} \cdot d\vec{r} = \int_0^1 20t^2 dt = \frac{20}{3}$

Adding, $\int_C \vec{A} \cdot d\vec{r} = 1 + 0 + \frac{20}{3} = \frac{23}{3}$