Name:- Abdullah al Tamim.

ID:- 2020-1-60-127

Final exam script

1) Given,

$V_i = \{0.09, 0.23, 0.49, 0.88, 0.31\}$

$\mu = 10, \quad \sigma = 2$

$P[X < x_1] = V_1 \Rightarrow P\left[Z < \frac{x_1 - 10}{2}\right] = 0.09$

$\Rightarrow P\left[Z < \frac{x_1 - 10}{2}\right] = P[Z < -1.34]$

$\Rightarrow \frac{x_1 - 10}{2} = -1.34 \quad \therefore x_1 = 7.32$

$\therefore x_i = Z_{u_i} \times \sigma + \mu$

$\therefore x_2 = Z_{0.23} \times 2 + 10 \qquad x_3 = Z_{0.49} \times 2 + 10$

$\Rightarrow x_2 = 8.52 \qquad \Rightarrow x_3 = 9.95$

$x_4 = 12.35 \qquad x_5 = 9.0083$

$\therefore$ Random numbers that follow Normal distribution

are, $x_i = \{7.32, 8.52, 9.95, 12.35, 9.0083\}$

b) $\int_2^{10} (x-5)^2+3 \, dx$

$= (10-2) \times \frac{1}{n} \sum_{i=1}^{n} f\left(2+(10-2)\times U_i\right)$

$= \frac{8}{n} \sum_{i=1}^{n} f\left(2+8U_i\right)$

$= \frac{8}{n} \sum_{i=1}^{n} \left((2+8U_i)-5\right)^2+3 = \frac{8}{n} \sum_{i=1}^{n} 64U_i^2-48U_i+12$

~~When,~~

$\cancel{U_1 = 0.09, \ I_1 = \frac{8}{5}\left((2+8\times0.09)-5\right)^2+3 = 11.32}$

$\cancel{U_2 = 0.23}$

When,

$U_1 = 0.09, \ I_1 = \frac{8}{5}\left(64\times0.09^2-48\times0.09+12\right) = 13.12$

$U_2 = 0.23, \ I_2 = 6.95$

$U_3 = 0.49, \ I_3 = 6.15$

$U_4 = 0.88, \ I_4 = 30.91$

$U_5 = 0.31, \ I_5 = 5.23$

$\therefore I = \sum I_i = 62.37$

2] a]

$$TPM = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.3 & 0.2 & 0.5 \\ 1 & 0.1 & 0.6 & 0.3 \\ 2 & 0.4 & 0.4 & 0.2 \end{array}$$

b] Here,

$$\pi_0 + \pi_1 + \pi_2 = 1 \quad \text{--- (1)}$$

$$\pi_0 = 0.3\pi_0 + 0.1\pi_1 + 0.4\pi_2 \quad \text{--- (2)}$$

$$\pi_1 = 0.2\pi_0 + 0.6\pi_1 + 0.4\pi_2 \quad \text{--- (3)}$$

Equatin eqn (1) and putting in (2) and (3)

$$\pi_0 = 0.3\pi_0 + 0.1(1-\pi_0-\pi_2) + 0.4\pi_2 \quad \text{--- (4)}$$

$$1-\pi_0-\pi_2 = 0.2\pi_0 + 0.6(1-\pi_0-\pi_2) + 0.4\pi_2 \quad \text{--- (5)}$$

Now, equatin (5) and (6):

$$\pi_0 = 0.24 \quad \pi_2 = 0.32$$

Now, putting in (1):

$$\pi_1 = 0.4436$$

3] a] $\bar{p} = \frac{578}{20 \times 90} = 0.321$

$se(\bar{p}) = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.049$

$\therefore UCL = \bar{p} + 3 \times se(\bar{p}) = 0.47$

$LCL = \bar{p} - 3 \times se(\bar{p}) = 0.17$

$\therefore$ Center line $= \frac{0.47 + 0.17}{2} = 0.32$

b) Given,

$n = 90$

$\therefore$ proportion of 39 failures, $P_{39} = \frac{39}{90} = 0.43$

" " 13 ", $P_{13} = \frac{13}{90} = 0.14$

~~So these point~~

Here, $0.43 < UCL$ and $0.14 \ngtr LCL$

So the point 13 indicate an out of control condition.

4) a) Here, mean $= 4$ $\therefore \lambda = \frac{1}{4} = 0.25$

$$P[X<1] = \int_0^1 0.25 \cdot e^{-0.25x} \, dx = 0.2212$$

On 22.12% of policies will they have to pay a claim.

b) $P[X<a] = 0.2 \Rightarrow \int_0^a 0.25 e^{-0.25x} \, dx = 0.2$

$\therefore a = 0.89257$ year

If they are ready to pay 20% customers, their warrenty period should be 0.8925 year