Chapter 6
Combinational Logic Design

6.2 Design of Combinational Circuits

- Problem Statement
- Truth Table
- K-map
- Logic Diagram

Problem Statement

1. **The problem statement:** The door of an explosive-warehouse is operated on by four persons, namely, A, B, C, and D. Person A along with at least any one from B, C, and D can open the door. If A is absent, B, C, and D can open the door altogether. A combinational logic circuit is to be designed for controlling the door.

2. Four input lines are required for four keys operated by four persons. The four input variables are assigned letter symbols A, B, C, and D after the names of the four persons. The input variables are assigned the following logic values:

   The key is operated for opening the door: 1
   The key is operated for closing the door: 0

   One output line is required for controlling the lock of the door. The output variable is assigned the letter symbol X and the following logic values:

   The lock is opened: 1
   The lock is closed: 0

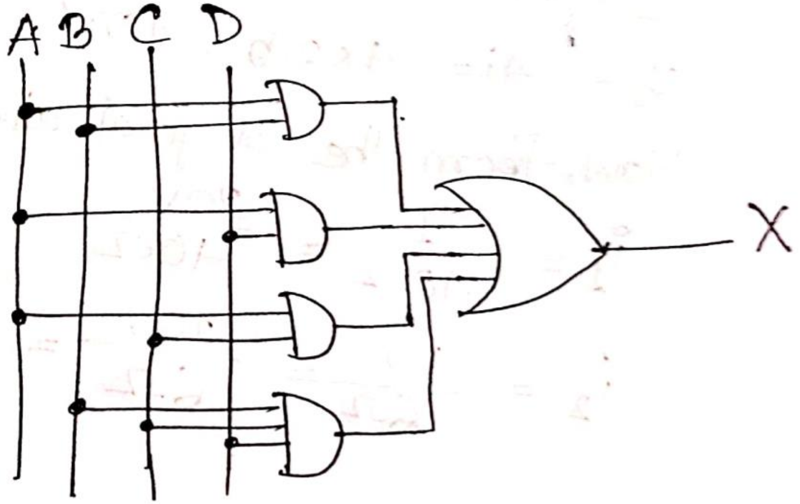
6.2 Design of Combinational Circuits

ABCD	X
0000	0
0001	0
0010	0
0011	0
0100	0
0101	0
0110	0
0111	1
1000	0
1001	1
1010	1
1011	1
1100	1
1101	1
1110	1
1111	1

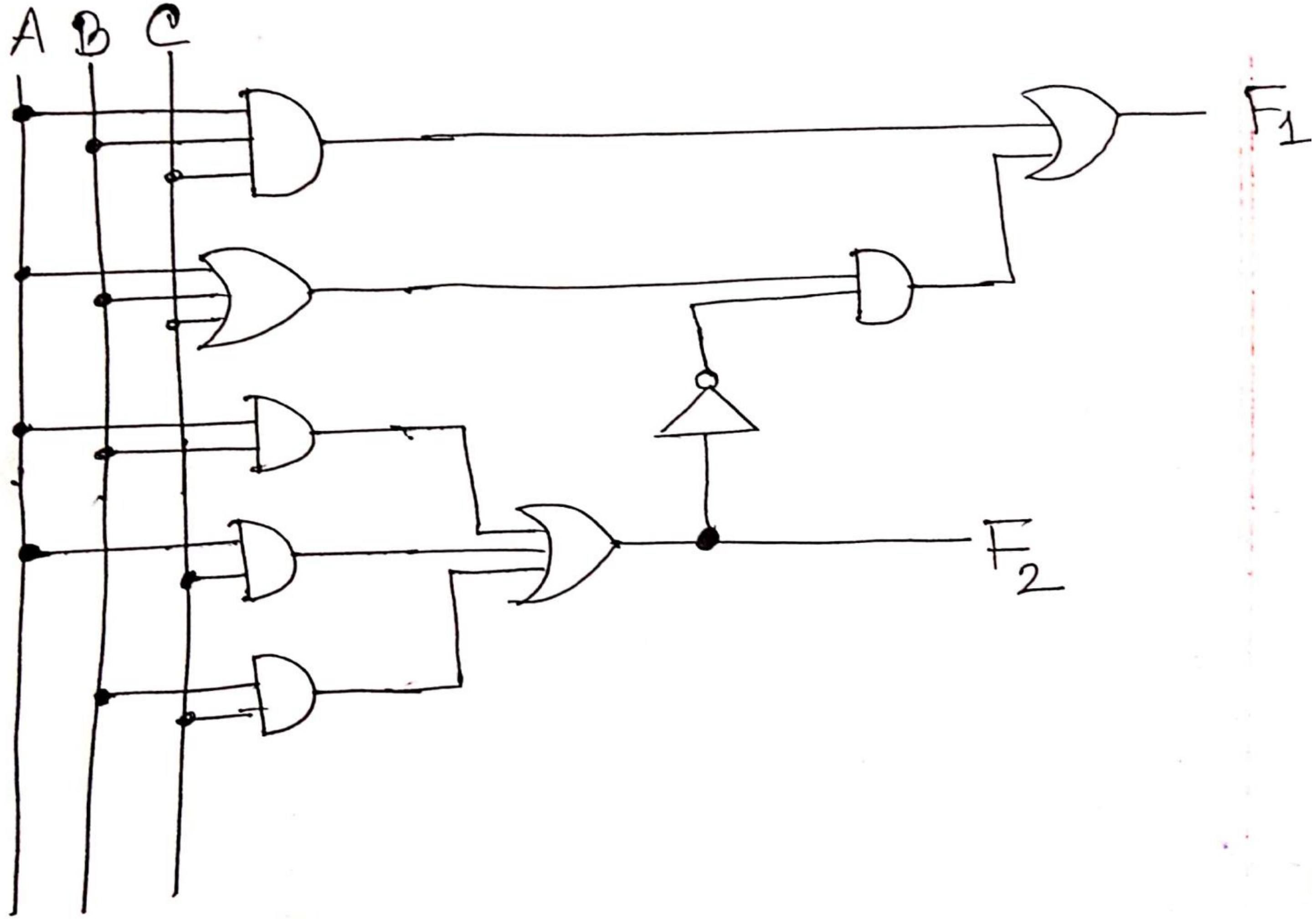
AB \ CD	00	01	11	10
00				
01			1	
11	1	1	1	1
10		1	1	1

BCD, AB, AC, AD

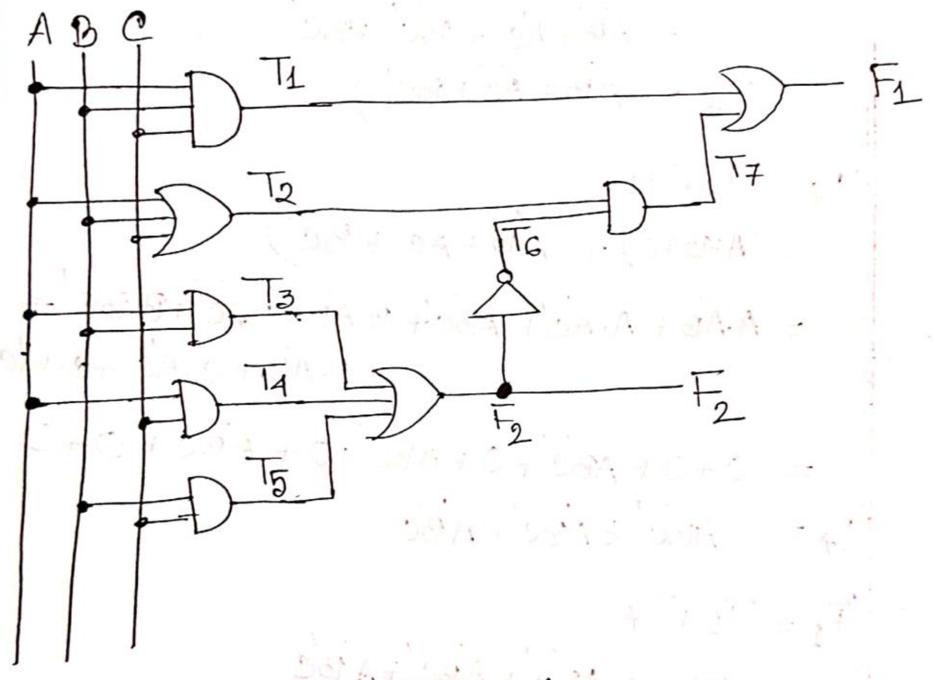
$$X = AB + AD + AC + BCD$$



6.3 Analysis of Combinational Circuits



6.3 Analysis of Combinational Circuits



Derivation of Boolean Expressions

$T_1 = ABC$

$T_2 = A+B+C$

$T_3 = AB$

$T_4 = AC$

$T_5 = BC$

$F_2 = T_3 + T_4 + T_5$

$F_2 = AB + AC + BC$

$$
\begin{aligned}
T_6 &= F_2' \\
&= (AB + AC + BC)' \\
&= (AB)' \cdot (AC)' \cdot (BC)' \\
&= (A'+B') \cdot (A'+C') \cdot (B'+C') \\
&= (A'.A' + A'C' + A'B' + B'C') \cdot (B'+C') \\
&= (A' + A'C' + A'B' + B'C') \cdot (B'+C') \\
&= \{A'(1+C'+B') + B'C'\} (B'+C') \\
&= (A'+B'C')(B'+C')
\end{aligned}
$$

$$T_6 = A'B' + A'C' + B'C'B' + B'C'C'$$

$$= A'B' + A'C' + B'C' + B'C'$$

$$T_6 = (A'B' + A'C' + B'C')$$

$$T_7 = T_2 \cdot T_6$$

$$= (A+B+C) \cdot (A'B' + A'C' + B'C')$$

$$= A.A'B' + A.A'C' + AB'C' + B.A'B' + B.A'C' + B.B'C' + C.A'B' + C.A'C' + C.B'C'$$

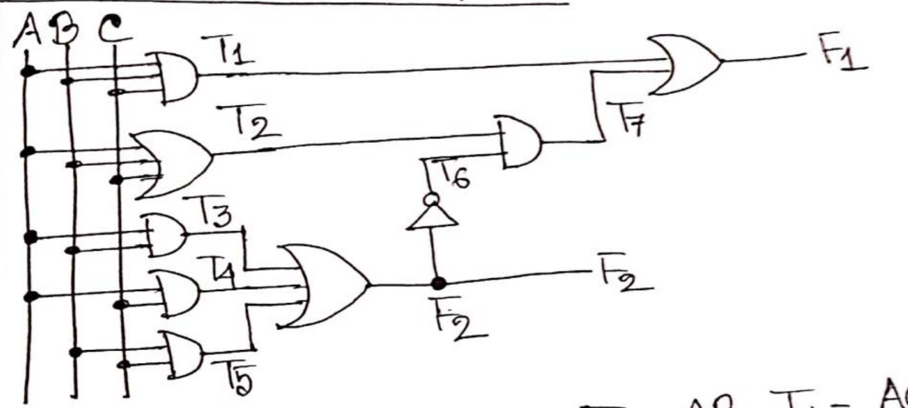
$$= 0 + 0 + AB'C' + 0 + A'BC' + 0 + A'B'C + 0 + 0$$

$$T_7 = AB'C' + A'BC' + A'B'C$$

$$F_1 = T_1 + T_7$$

$$F_1 = ABC + AB'C' + A'BC' + A'B'C$$

Derivation of Truth Table



$T_1 = ABC$, $T_2 = A+B+C$, $T_3 = AB$, $T_4 = AC$, $T_5 = BC$

$F_2 = T_3 + T_4 + T_5$

$T_6 = F_2'$

$T_7 = T_2 \cdot T_6$

$F_1 = T_1 + T_7$

ABC	T_1	T_2	T_3	T_4	T_5	F_2	T_6	T_7	F_1
000	0	0	0	0	0	0	1	0	0
001	0	1	0	0	0	0	1	1	1
010	0	1	0	0	0	0	1	1	1
011	0	1	0	0	1	1	0	0	0
100	0	1	0	0	0	0	1	1	1
101	0	1	0	1	0	1	0	0	0
110	0	1	1	0	0	1	0	0	0
111	1	1	1	1	1	1	0	0	1

Exercise

6.9 Analyze the two-output combinational circuit shown in Figure 6.25.

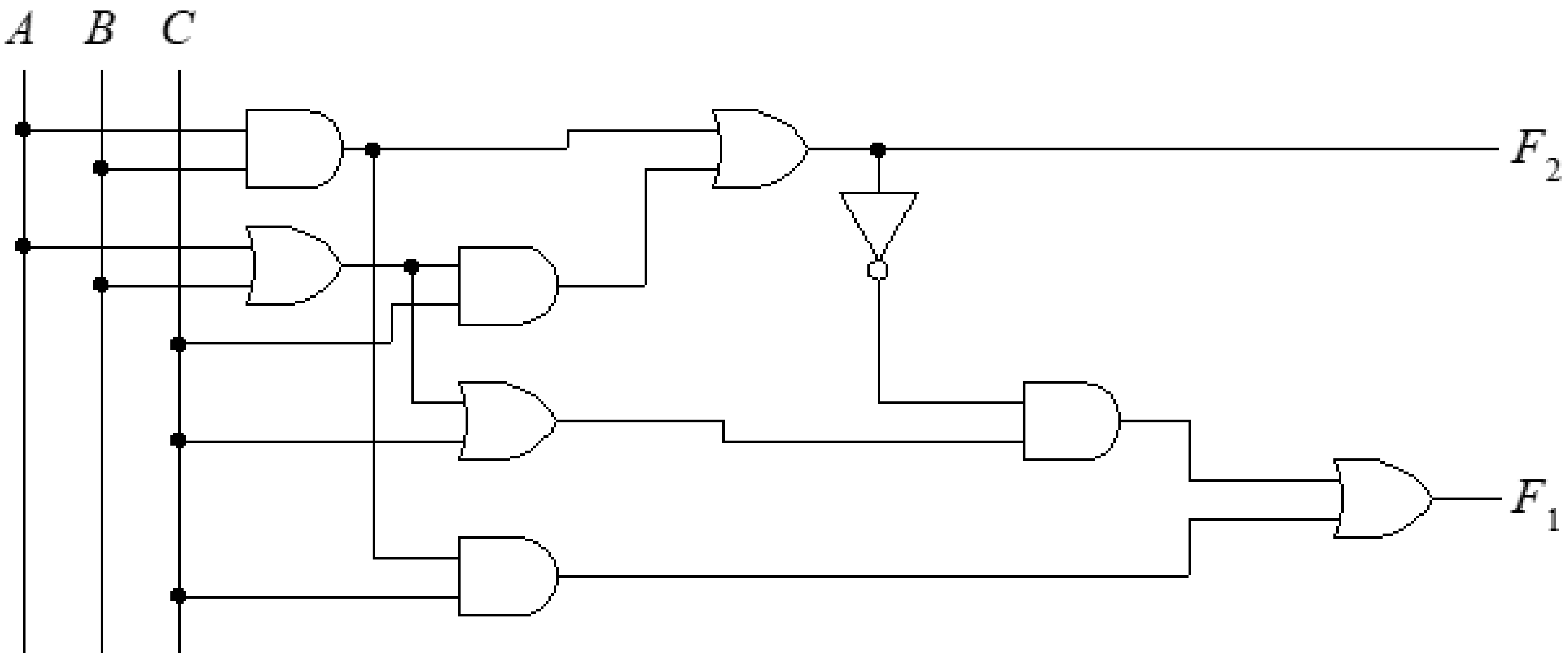


Figure 6.25 Logic diagram for analysis problem 6.9.

Exercise

6.21 Design AND-OR circuits for the following functions

(a) $F(A,B,C,D) = \sum(0,1,8,11)$

(b) $F(A,B,C,D) = \sum(1,2,4,7,9,10,13,14)$

(c) $F(A,B,C,D) = \sum(0,1,3,6,7,8,10,13)$

(d) $F(A,B,C,D) = \sum(0,1,2,3,8,11)$

(e) $F(A,B,C,D) = \sum(0,1,3,6,7,8,11)$