ID: 2021-3-60-016
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1. Here, $f(x) = \sqrt[3]{12+\sqrt{x}}$

$$f'(x) = \frac{d}{dx} f(x)$$
$$= \frac{d}{dx} \sqrt[3]{12+\sqrt{x}}$$
$$= \frac{1}{3} \times (12+\sqrt{x})^{-2/3} \times \frac{d}{dx}(12+\sqrt{x})$$
$$= \frac{1}{3(12+\sqrt{x})^{2/3}} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{1}{6\sqrt{x}\,(12+\sqrt{x})^{2/3}}$$

(Ans)

2. Here,
$f(x) = \sin(3x^2)$

$$f''(x) = \frac{d}{dx}\left\{\frac{d}{dx} f(x)\right\}$$
$$= \frac{d}{dx}\left\{\frac{d}{dx} \sin(3x^2)\right\}$$
$$= \frac{d}{dx}\left\{\cos(3x^2)\cdot\frac{d}{dx}(3x^2)\right\}$$
$$= \frac{d}{dx}\left\{\cos(3x^2)\cdot 6x\right\}$$
$$= 6\frac{d}{dx}\left\{\cos(3x^2)\, x\right\}$$
$$= 6\left\{x\cdot\frac{d}{dx}\cos(3x^2) + \cos(3x^2)\frac{d}{dx}x\right\}$$
$$= 6\left\{x\cdot(-\sin(3x^2)\cdot 6x) + \cos(3x^2)\right\}$$
$$= 6\left\{-\sin(3x^2)\cdot 6x^2 + \cos(3x^2)\right\}$$
$$= 6\left\{-6x^2\sin(3x^2) + \cos(3x^2)\right\}$$

(Ans)

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3. $f(x) = \tan\left(\frac{1}{x}\right)$

$$\therefore f''(x) = \frac{d}{dx}\left\{\frac{d}{dx} f(x)\right\}$$

$$= \frac{d}{dx}\left\{\frac{d}{dx} \tan\left(\frac{1}{x}\right)\right\}$$

$$= \frac{d}{dx}\left\{ \sec^2\left(\frac{1}{x}\right) \frac{d}{dx}\left(1/x\right)\right\}$$

$$= \frac{d}{dx}\left\{\sec^2\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right\}$$

$$= \frac{d}{dx}\left\{\frac{-\sec^2(1/x)}{x^2}\right\}$$

$$= -\frac{x^2 \frac{d}{dx}(\sec^2(1/x)) - \sec^2(1/x)\, d/dx\, x^2}{(x^2)^2}$$

$$= -\frac{x^2\left(2\sec(1/x) \times \sec(1/x) \cdot \tan(1/x) \times \left(-\frac{1}{x^2}\right)\right) - 2x\sec^2(1/x)}{x^4}$$

$$= -\frac{x^2\left(-\frac{2\sec^2(1/x)\tan\left(\frac{1}{x}\right)}{x^2}\right) - 2x\sec^2(1/x)}{x^4}$$

$$= -\frac{-\left(2\sec^2\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right) + 2x\sec^2\left(\frac{1}{x}\right)\right)}{x^4}$$

$$= \frac{2\sec^2\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right) + 2x\sec^2\left(\frac{1}{x}\right)}{x^4}$$

(Ans)

4. $f(x) = \ln(\ln(\ln x))$

$$f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx} \ln(\ln(\ln x))$$

$$= \frac{1}{\ln(\ln x)} \frac{d}{dx} (\ln(\ln x))$$

$$= \frac{1}{\ln(\ln x)} \times \frac{1}{\ln(x)} \times \frac{d}{dx} \ln x$$

$$= \frac{1}{\ln(\ln x)} \times \frac{1}{x \cdot \ln(x)}$$

$$= \frac{1}{x \cdot \ln(x) \{\ln(\ln x)\}}$$

(Ans)

5. $\frac{d}{dx}\left[\ln\left((\cos^2 x)\sqrt{1+x^4}\right)\right]$

$$= \frac{1}{(\cos^2 x)(\sqrt{1+x^4})} \times \frac{d}{dx}\left((\cos^2 x) \times \sqrt{1+x^4}\right)$$

$$= \frac{1}{(\cos^2 x)(\sqrt{1+x^4})} \times \left\{\cos^2 x \frac{d}{dx}(\sqrt{1+x^4}) + (\sqrt{1+x^4}) \frac{d}{dx}(\cos^2 x)\right\}$$

$$= \frac{1}{(\cos^2 x)(\sqrt{1+x^4})} \times \left\{\cos^2 x \times \frac{2x^3}{\sqrt{1+x^4}} + (\sqrt{1+x^4}) \times 2\cos x\,(-\sin x)\right\}$$

$$= \frac{1}{(\cos^2 x)(\sqrt{1+x^4})} \times \left\{\frac{2x^3 \cos^2 x}{\sqrt{1+x^4}} - 2\sin(2x)(\sqrt{1+x^4})\right\}$$

$$= \frac{1}{(\cos^2 x)(\sqrt{1+x^4})} \times \left\{-2\sin(2x)(\sqrt{1+x^4}) + \frac{2x^3 \cos^2 x}{\sqrt{1+x^4}}\right\}$$

$$= -\frac{2\sin(2x)}{\cos^2 x} + \frac{2x^3 \cos^2 x}{\cos^2 x(1+x^4)}$$

(Ans)

6 Here,

$$f(x) = y = \sqrt[5]{\frac{x-1}{x+1}}$$

taking ln both side,

$$\therefore \ln y = \ln\left(\sqrt[5]{\frac{x-1}{x+1}}\right)$$

$$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} \ln\left(\sqrt[5]{\frac{x-1}{x+1}}\right)$$

$$\Rightarrow \frac{dy}{dx} \times \frac{1}{y} = \frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \frac{d}{dx}\left(\sqrt[5]{\frac{x-1}{x+1}}\right)$$

$$\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = \frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{1}{5} \times \frac{1}{\left(\frac{x-1}{x+1}\right)^{4/5}} \frac{d}{dx}\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{1}{5\left(\frac{x-1}{x+1}\right)^{4/5}} \times \frac{(x+1)-(x-1)}{(x+1)^2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{1}{5\left(\frac{x-1}{x+1}\right)^{4/5}} \times \frac{2}{(x+1)^2}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{2}{5(x-1)^{4/5} \times (x+1)^{6/5}}$$

$$\Rightarrow \frac{dy}{dx} = y \times \left\{\frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{2}{5(x-1)^{4/5}(x+1)^{6/5}}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt[5]{\frac{x-1}{x+1}} \left\{\frac{1}{\sqrt[5]{\frac{x-1}{x+1}}} \times \frac{2}{5(x-1)^{4/5}(x+1)^{6/5}}\right\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{5(x-1)^{4/5}(x+1)^{6/5}}$$

$$\therefore f'(x) = \frac{2}{5(x-1)^{4/5}(x+1)^{6/5}}$$

(Ans)

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7.

Here, $y = \ln(-x)$; $x = -e$

$\Rightarrow y = \ln(e)$

$\therefore y = 1$

Again,

$y' = \frac{d}{dx} \ln(-x)$

$y' = \frac{1}{x}$

$\therefore$ slope of tangent line at $\cancel{(e,0)}$ $(-e, 1)$; $m = \frac{1}{x}$

$= \frac{1}{-e}$

$\therefore$ Tangent line;

$y - 1 = -\frac{1}{e}(x - e)$

$\Rightarrow y - 1 = -\frac{x}{e} + 1$

$\Rightarrow y = -\frac{x}{e} + 2$

$\therefore y + \frac{x}{e} = 2$

(Am)

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8 Here,

$$y^3 + yx^2 + x^2 - 3y^2 = 0$$

Differantiation with respect to x;

$$\frac{d}{dx}(y^3 + yx^2 + x^2 - 3y^2) = 0$$

$$\Rightarrow 3y^2 \frac{dy}{dx} + \left(y \cdot \frac{d}{dx} x^2 + x^2 \cdot \frac{d}{dx} y\right) + 2x - 6y \frac{dy}{dx} = 0$$

$$\Rightarrow 3y^2 \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} + 2x - 6y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(3y^2 + x^2 - 6y) = -2xy - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x(y+1)}{3y^2 + x^2 - 6y}$$

At $(0,3)$;

$$m = \frac{dy}{dx}$$

$$= \frac{-2 \times 0 (3+1)}{3 \times 9 + 0 - 18}$$

$$= \frac{0}{27 - 18}$$

$$= 0$$

$\therefore$ Equation of the tangent line ;

$$y - 3 = 0(x - 0)$$

$$\therefore y = 3$$

(Ans)

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2 Here, $x^{2/3} + y^{2/3} = 4$

Differantiation with respect to x;

$$\frac{d}{dx}\left(x^{2/3} + y^{2/3}\right) = 4 \times \frac{d}{dx}$$

$$\Rightarrow \frac{d}{dx} x^{2/3} + \frac{d}{dx} y^{2/3} = 0$$

$$\Rightarrow \frac{2}{3x^{1/3}} + \frac{2}{3} \times y^{-\frac{1}{3}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3x^{1/3}} + \frac{2}{3y^{1/3}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{3x^{1/3}} \times \frac{3y^{1/3}}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

~~At (-1, 3√3~~

At $(-1, 3\sqrt{3})$;

$$m = \frac{dy}{dx}$$

$$= -\frac{\sqrt[3]{3\sqrt{3}}}{\sqrt[3]{-1}}$$

Equation of the tangent line:

$$y - 3\sqrt{3} = \left(-\frac{\sqrt[3]{3\sqrt{3}}}{\sqrt[3]{-1}}\right)(x+1)$$

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10 (i) $f(x) = x^4 - 5x^3 + 9x^2$

Here,

$$f'(x) = 4x^3 - 15x^2 + 18x$$
$$= x(4x^2 - 15x + 18)$$

again,

$$f''(x) = 12x^2 - 30x + 18$$

ques {a & b}

for $x = 0, \frac{15}{8} + \frac{3\sqrt{7}}{8}i \,;\, \frac{15}{8} - \frac{3\sqrt{7}}{8}i \;;\; f'(x) = 0$

Intervals: $(-\infty, 0)\ (0, \frac{15}{8} - \frac{3\sqrt{7}}{8}i), (\frac{15}{8} - \frac{3\sqrt{7}}{8}i, \frac{15}{8} + \frac{3\sqrt{7}}{8}i),$
$(\frac{15}{8} + \frac{3\sqrt{7}}{8}i, \infty)$

At $(-\infty, 0)$; $f'(x) < 0$

$\therefore f$ is decreasing at interval: $(-\infty, 0)$

At $(0, \frac{15}{8} - \frac{3\sqrt{7}}{8}i)$; $f'(x) > 0$

$\therefore f$ is increasing at interval: $(0, \frac{15}{8} - \frac{3\sqrt{7}}{8}i)$

At $(\frac{15}{8} - \frac{3\sqrt{7}}{8}i, \frac{15}{8} + \frac{3\sqrt{7}}{8}i)$; $f'(x) < 0$

$\therefore f$ is decreasing at interval: $(\frac{15}{8} - \frac{3\sqrt{7}}{8}i, \frac{15}{8} + \frac{3\sqrt{7}}{8}i)$

At $(\frac{15}{8} + \frac{3\sqrt{7}}{8}i, \infty)$; $f'(x) > 0$

$\therefore f$ is increasing at interval: $(\frac{15}{8} + \frac{3\sqrt{7}}{8}i, \infty)$

Finally, $f'(x)$ is increasing at: $(0, \frac{15}{8} - \frac{3\sqrt{7}}{8}i) \cup (\frac{15}{8} + \frac{3\sqrt{7}}{8}i, \infty)$
and decreasing at: $(-\infty, 0) \cup (\frac{15}{8} - \frac{3\sqrt{7}}{8}i, \frac{15}{8} + \frac{3\sqrt{7}}{8}i)$

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{ques c & d}

$$f''(x) = 12x^2 - 30x + 18$$

At $x = \frac{3}{2}, 1$; $f''(x) = 0$

So, interval : ~~$(-\infty, \frac{3}{2})$~~ $(-\infty, 1)$, $(1, \frac{3}{2})$, $(\frac{3}{2}, \infty)$

At $(-\infty, 1)$; $f''(x) > 0$

$\therefore$ f is concave up at the interval : $(-\infty, 1)$

At $(1, \frac{3}{2})$; $f''(x) < 0$

$\therefore$ f is concave down at the interval: $(1, \frac{3}{2})$

At $(\frac{3}{2}, \infty)$; $f''(x) > 0$

$\therefore$ f is concave up at the interval : $(\frac{3}{2}, \infty)$

So, f is concave up at : $(-\infty, 1) \cup (\frac{3}{2}, \infty)$

and concave down at : $(1, \frac{3}{2})$

(Ans)

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(ii) $f(x) = \frac{x}{x^2+2}$

Here, $f'(x) = \frac{-x^2+2}{(x^2+2)^2}$

$f''(x) = -\frac{2x(-x^2+6)}{(x^2+2)^3}$

{ques a & b}

at, $x = \sqrt{2}, -\sqrt{2}$; $f'(x) = 0$

intervals : $(-\infty, -\sqrt{2})$ $(-\sqrt{2}, \sqrt{2})$ $(\sqrt{2}, \infty)$

at $(-\infty, -\sqrt{2})$; $f'(x) < 0$

$\therefore$ f is decreasing at interval : $(-\infty, -\sqrt{2})$

at $(-\sqrt{2}, \sqrt{2})$; $f'(x) > 0$

$\therefore$ f is increasing at interval : $(-\sqrt{2}, \sqrt{2})$

at $(\sqrt{2}, \infty)$; $f'(x) < 0$

$\therefore$ f is decreasing at interval : $(\sqrt{2}, \infty)$

$\therefore$ Finally, f is increasing at : $(-\sqrt{2}, \sqrt{2})$

f is decreasing at : $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

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{ques c & d}

$$f''(x) = -\frac{2x(-x^2+6)}{(x^2+2)^3}$$

Here,

at $x = 0, -\sqrt{6}, \sqrt{6}$; $f''(x) = 0$

intervals : $(-\infty, -\sqrt{6})$, $(-\sqrt{6}, 0)$; $(0, \sqrt{6})$, $(\sqrt{6}, \infty)$

At $(-\infty, -\sqrt{6})$; $f''(x) < 0$

$\therefore$ f is concave ~~up~~ down at interval : $(-\infty, -\sqrt{6})$

At $(-\sqrt{6}, 0)$; $f''(x) > 0$

$\therefore$ f is concave up at interval : $(-\sqrt{6}, 0)$

At $(0, \sqrt{6})$; $f''(x) < 0$

$\therefore$ f is concave down at interval : $(0, \sqrt{6})$

At $(\sqrt{6}, \infty)$; $f''(x) > 0$

$\therefore$ f is concave up at interval : $(\sqrt{6}, \infty)$

Finally f is concave up at : $(-\sqrt{6}, 0) \cup (\sqrt{6}, \infty)$

f is concave down : $(-\infty, -\sqrt{6}) \cup (0, \sqrt{6})$

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(iii) $f(x) = x^{4/3} - x^{1/3}$

Here,
$$f'(x) = \frac{4x-1}{3x^{2/3}}$$

$$f''(x) = \frac{12x^{2/3}\sqrt[3]{x} - 2(4x-1)}{9\sqrt[3]{x}\ x^{4/3}}$$

{ques a & b}

at $x = \frac{1}{4}$; $f'(x) = 0$

intervals : $(-\infty, \frac{1}{4})$, $(\frac{1}{4}, \infty)$

at $(-\infty, \frac{1}{4})$; $f'(x) > 0$

So, f is increasing at interval : $(-\infty, \frac{1}{4})$

at $(\frac{1}{4}, \infty)$; $f'(x) > 0$

So, f is increasing at interval : $(\frac{1}{4}, \infty)$

Finally, f is increasing at interval : $(-\infty, \frac{1}{4}) \cup (\frac{1}{4}, \infty)$

(Ans)

{ques c & d}

$$f''(x) = \frac{12x^{2/3}\sqrt[3]{x} - 2(4x-1)}{9\sqrt[3]{x}\ x^{4/3}}$$

Here, at $x = -\frac{1}{2}$; $f''(x) = 0$

intervals : $(-\infty, -\frac{1}{2})$, $(-\frac{1}{2}, \infty)$

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at $(-\infty, -\frac{1}{2})$; $f''(x) > 0$

$\therefore$ So, f is concave up at interval : $(-\infty, -\frac{1}{2})$

at $(-\frac{1}{2}, \infty)$; $f''(x) > 0$

$\therefore$ So, f is concave up at interval : $(-\frac{1}{2}, \infty)$

finally, f concave up at intervals : $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$