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1. Here, $f(x) = \dfrac{x^2}{x^3+8}$

$$\therefore f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx}\left(\frac{x^2}{x^3+8}\right)$$

$$= \frac{(x^3+8)\frac{d}{dx}x^2 - x^2\frac{d}{dx}(x^3+8)}{(x^3+8)^2}$$

$$= \frac{(x^3+8)\times 2x - x^2(3x^2)}{(x^3+8)^2}$$

$$= \frac{2x(x^3+8) - 3x^4}{(x^3+8)^2}$$

$$= \frac{2x^4 + 16x - 3x^4}{(x^3+8)^2}$$

$$= \frac{16x - x^4}{(x^3+8)^2}$$

at $x = 0,\ 2\sqrt[3]{2}$; $f'(x) = 0$

So, these points are critical points which are the stationary points.

2. Here, $f(x) = \sqrt[3]{x^2-25}$

So $f'(x) = \frac{d}{dx} f(x)$

$= \frac{d}{dx} \sqrt[3]{x^2-25}$

$= \frac{1}{3(x^2-25)^{2/3}} \frac{d}{dx}(x^2-25)$

$= \frac{1}{3(x^2-25)^{2/3}} \times 2x$

$= \frac{2x}{3(x^2-25)^{2/3}}$

at $x=0$; $f'(x)=0$

So, $x=0$ is the critical point which is the stationar point.

(Ans)

3 Here, $f(x) = x^4 - 12x^3$

So, $f'(x) = \frac{d}{dx} x^4 - 12x^3$

$= 4x^3 - 36x^2$

at, $x = 0, 9$; $f'(x) = 0$

again. $f''(x) = \frac{d}{dx} 4x^3 - 36x^2$

$= 12x^2 - 72x$

Here,

at, $x = 0$ $f''(x) = 0$ So, this is inconclusive

at, $x = 9$ $f''(x) = 324$ So, this is relative minima.

(Ans)

4 Here $f(x) = 2x^3 + 3x^2 - 12x$; interval ~~[0, π]~~ $[-3, 2]$

So, $f'(x) = \frac{d}{dx} 2x^3 + 3x^2 - 12x$

$= \frac{d}{dx} 2x^3 + \frac{d}{dx} 3x^2 - \frac{d}{dx} 12x$

$= 6x^2 + 6x - 12$

at $x = 1, -2$; $f'(x) = 0$

again,

$$f''(x) = \frac{d}{dx} f'(x)$$

$$= \frac{d}{dx} 6x^2 + 6x - 12$$

$$= 12x + 6$$

$\therefore f''(1) = 18$

$\therefore f''(-2) = -18$

$\therefore$ Relative maxima at $x = -2$

$\therefore$ Relative minima at $x = 1$

$f(1) = -7$

$f(-2) = 20$

$\therefore$ Absolute maximum at $x = -2$ which is 20

$\therefore$ Absolute minimum at $x = 1$ which is -7

(Ans)

5

Here, $f(x) = \sin x - \cos x$; interval $[0, \pi]$

So, $f'(x) = \frac{d}{dx}(\sin x - \cos x)$

$= \cos x + \sin x$

$x = 135° + 180° n$; $f'(x)$

we can take only one value $x = 135°$ because we can't consider other values cause they are out of the interval $[0, \pi]$

Hence, $x = \frac{3\pi}{4}$

Again, $f''(x) = \frac{d}{dx}(\cos x + \sin x)$

$= -\sin x + \cos x$

$\therefore f''\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

In this interval there is ~~no~~ relative minima.

$\therefore$ Relative maxima at $x = \frac{3\pi}{4}$

$\therefore$ Relative minima at $x = (0, \pi)$

$\therefore f\left(\frac{3\pi}{4}\right) = \sqrt{2}$; $f(0) = -1$; $f(\pi) = -1$

$\therefore$ Absolute maximum at $x = \frac{3\pi}{4}$ which is $\sqrt{2}$

Absolute minimum at $x = \pi$ which is -1

(Ans)

6 Using the accompaning graph the interval [-3,3]

There are 3 values of c

Those are c = -2, 0, 2

So, the conclusion is there will at least one point where we'll get a tengent line

We find 3 point where find horizontal tengent line.

7

Here, $f(x) = \frac{x}{2} - \sqrt{x}$; $[0,4]$

Rolle's theorem satisfied on interval $[0,4]$.
So, c in the interval $(0,4)$ such that $f'(c)=0$

$$f'(x) = \frac{d}{dx} f(x)$$
$$= \frac{d}{dx} \frac{x}{2} - \sqrt{x}$$
$$= \frac{d}{dx}\left(\frac{x}{2}\right) - \frac{d}{dx}\sqrt{x}$$
$$= \frac{1}{2} - \frac{1}{2\sqrt{x}}$$

if, $f'(x) = 0$

$$\Rightarrow \frac{1}{2} - \frac{1}{2\sqrt{x}} = 0$$

$$\Rightarrow 2\sqrt{x} = 2$$

$$\therefore x = 1$$

$$\therefore c = 1$$

So, $c = 1$ is a point in the interval $(0,4)$ at which $f'(c) = 0$

(Ans)

8

Here, $f(x) = x^3 + x - 4$; $[-1, 2]$

The slope of the secant line joining $[-1,2]$

$$= \frac{f(2) - f(-1)}{2+1}$$

$$= \frac{6+6}{3}$$

$$= 4$$

Now, we'll find the values of c;

$$f'(x) = \frac{d}{dx} f(x)$$

$$= \frac{d}{dx} x^3 + x - 4$$

$$= 3x^2 + 1$$

So, $3x^2 + 1 = 4$

$\Rightarrow 3x^2 = 3$

$\Rightarrow x^2 = 1$

$\therefore x = \pm 1$

So, the value of $c = \pm 1$ in the given interval

(Ans)

9

given,

$$z = f(x,y) = (2x-y)^2$$

a) $$\frac{\delta^3 z}{\delta y\,\delta x\,\delta y} = \frac{\delta^2}{\delta y\,\delta x}\left[\frac{\delta}{\delta y}(2x-y)^2\right]$$

$$= \frac{\delta^2}{\delta y\,\delta x}\left[\frac{\delta}{\delta y}\,4x^2-4xy+y^2\right]$$

$$= \frac{\delta^2}{\delta y\,\delta x}(2y-4x)$$

$$= \frac{\delta}{\delta y}\left[\frac{\delta}{\delta x}\,2y-4x\right]$$

$$= \frac{\delta}{\delta y}(-4)$$

$$= 0$$

b) $$\frac{\delta^3 z}{\delta x^2\,\delta y} = \frac{\delta^2}{\delta x^2}\left[\frac{\delta}{\delta y}\,4x^2-4xy+y^2\right]$$

$$= \frac{\delta^2}{\delta x^2}\cancel{(8x-4y)}\,(2y-4x)$$

$$= \frac{\delta}{\delta x}\left[\frac{\delta}{\delta x}\,2y-4x\right]$$

$$= \frac{\delta}{\delta x}(-4)$$

$$= 0$$

10 given,

$$z = f(x, y) = e^y \cos x$$

a) $\dfrac{\delta^2 z}{\delta x\, \delta y} \Big|_{(\frac{\pi}{4}, 0)}$

$= \dfrac{\delta}{\delta x} \left[\dfrac{\delta}{\delta y} e^y \cos x \right] \Big|_{(\frac{\pi}{4}, 0)}$

$= \dfrac{\delta}{\delta x} (\cos x \; e^y) \Big|_{(\frac{\pi}{4}, 0)}$

$= -e^y \sin x \Big|_{(\frac{\pi}{4}, 0)}$

As, $x = \frac{\pi}{4}$, $y = 0$

$$\frac{\delta^2 z}{\delta x\, \delta y} = -e^0 \sin \frac{\pi}{4}$$

$$= -\frac{\sqrt{2}}{2}$$

(Ans)

b) $\dfrac{\delta^3 z}{\delta x^2 \delta y} \Big|_{(\frac{\pi}{4}, 0)}$

$= \dfrac{\delta}{\delta x} \left(\dfrac{\delta^2 z}{\delta x\, \delta y} \right) \Big|_{(\frac{\pi}{4}, 0)}$

$= \dfrac{\delta}{\delta x} (-e^y \sin x) \Big|_{(\frac{\pi}{4}, 0)}$

$= -e^y \cos x \Big|_{(\frac{\pi}{4}, 0)}$

As $x = \frac{\pi}{4}$; $y = 0$

So, $\dfrac{\delta^3 z}{\delta x^2 \delta y} = -e^0 \cos \frac{\pi}{4}$

$$= -\frac{\sqrt{2}}{2}$$

(Ans)