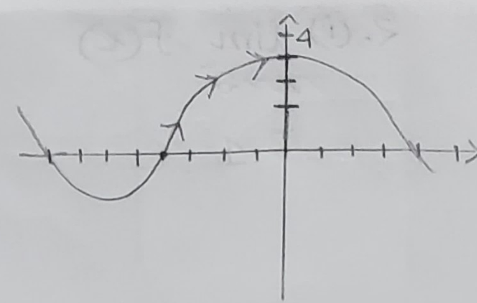
ID: 2021-3-60-016
BM Shahria Alam

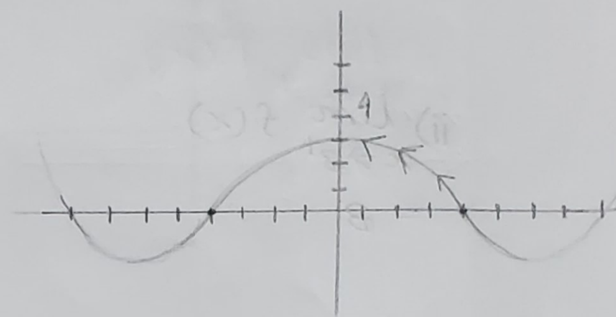
1. (i) $\lim_{x \to 0^-} f(x)$

$= 3$



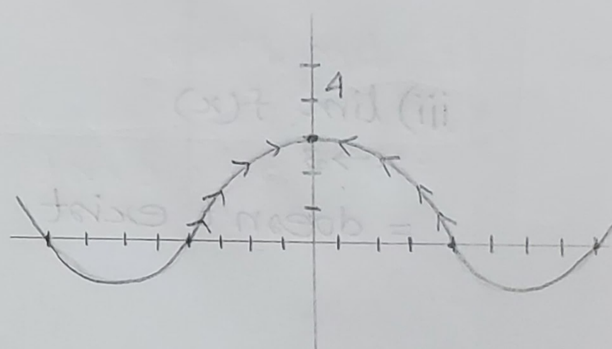
ii) $\lim_{x \to 0^+} f(x)$

$= 3$



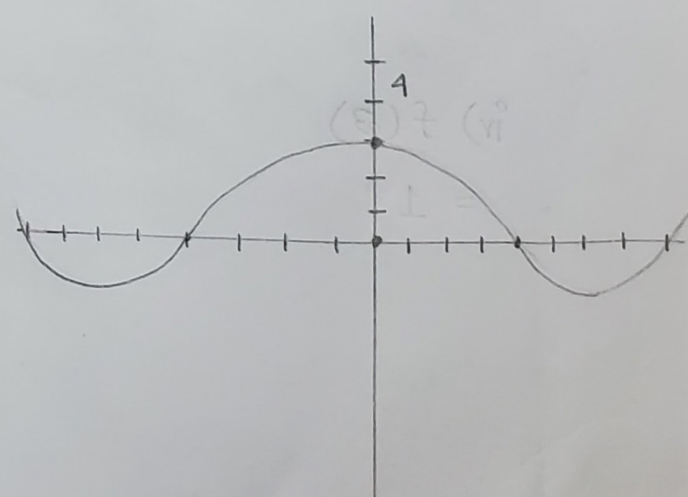
iii) $\lim_{x \to 0} f(x)$

$= 3$



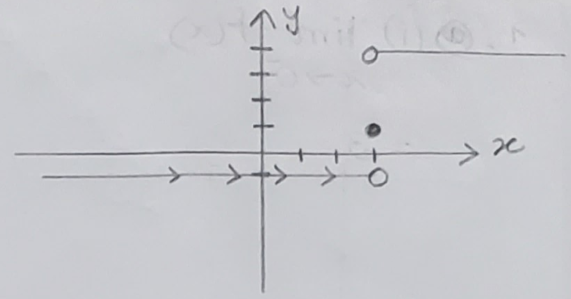
iv) $f(0)$

$= 3$



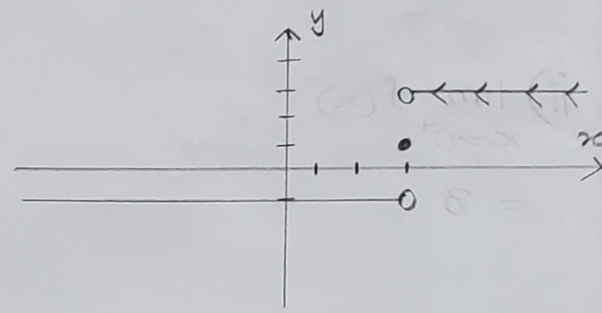
2. (i) $\lim_{x \to 3^-} f(x)$

$= -1$



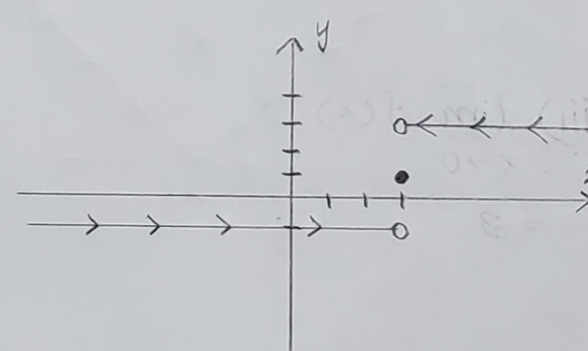
ii) $\lim_{x \to 3^+} f(x)$

$= 3$



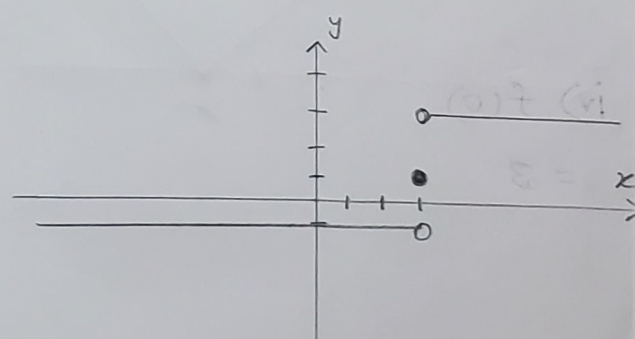
iii) $\lim_{x \to 3} f(x)$

$=$ doesn't exist



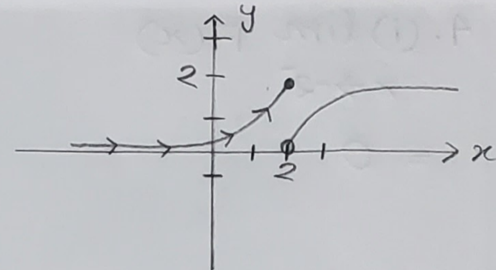
iv) $f(3)$

$= 1$



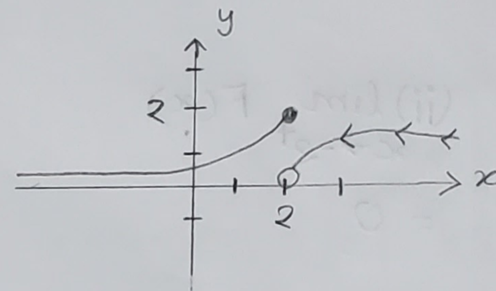
3. (i) $\lim_{x \to 2^-} f(x)$

$= 2$



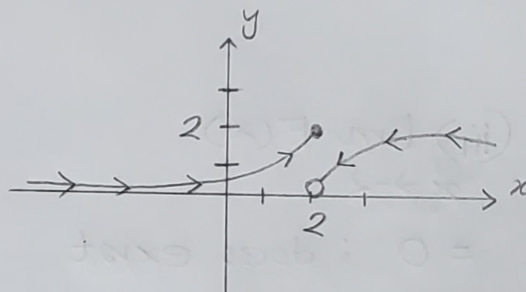
(ii) $\lim_{x \to 2^+} f(x)$

$= 0$



(iii) $\lim_{x \to 2} f(x)$

$=$ doesn't exist

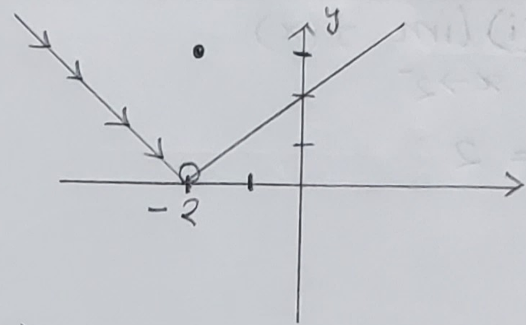


(iv) $f(2)$

$= 2$

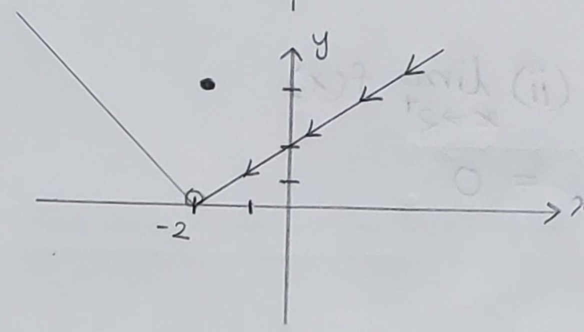
4. (i) $\lim_{x \to -2^-} F(x)$

$= 0$



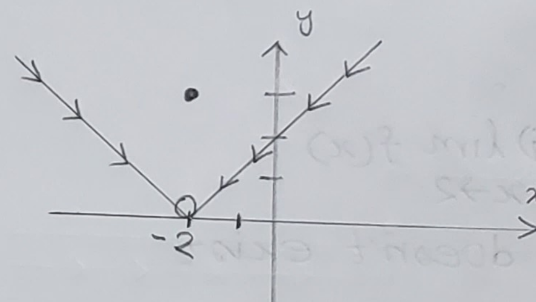
(ii) $\lim_{x \to -2^+} F(x)$

$= 0$



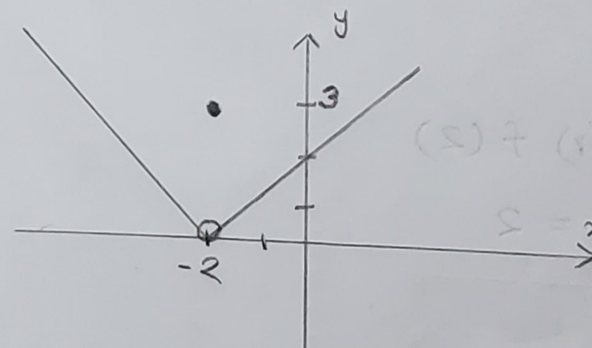
(iii) $\lim_{x \to -2} F(x)$

$= 0$; does exist



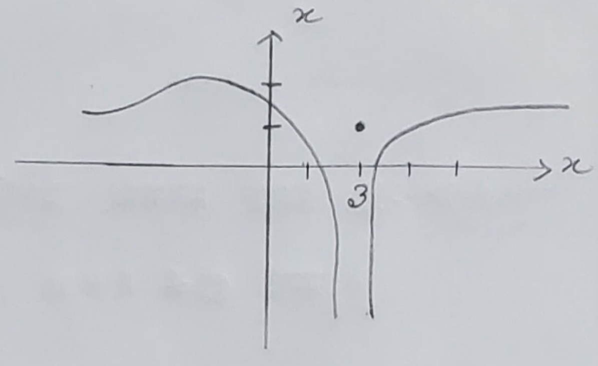
(iv) ~~lim~~ $F(-2)$

$= 3$



5. (i) $\lim_{x \to 3^-} f(x)$

$= -\infty$



(ii) $\lim_{x \to 3^+} f(x)$

$= -\infty$

(iii) $\lim_{x \to 3} f(x)$

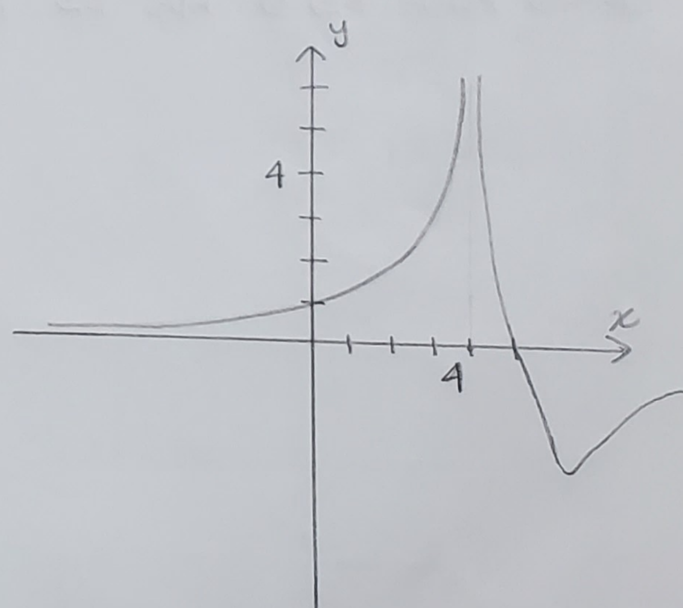
$= -\infty$

(iv) $f(3)$

$= 1$

6. (i) $\lim_{x \to 4^-} f(x)$

$= \infty$



(ii) $\lim_{x \to 4^+} f(x)$

$= \infty$

(iii) $\lim_{x \to 4} f(x)$

$= \infty$

(iv) ~~li~~ $f(4)$

= doesn't exist

7\. i) $f(x) = \sqrt{x+1} + 1$

~~∴ Df :~~

উপরোক্ত ফাংশনটিতে x এর সেই সকল মান গ্রহনযোগ্য হবে, যে সকল মান এর জন্য $x+1 \geq 0$ হয়।

অর্থাৎ, $x+1 \geq 0$

$\therefore x \geq -1$

$\therefore$ Domain $f(x) = [-1, \infty)$

আবার,

$\sqrt{x+1} \geq 0$

অর্থাৎ এই ফাংশনটির জন্য $\sqrt{x+1}$ এর মান 0 এর সমান অথবা 0 থেকে বড় হবে।

সুতরাং

Range $= [1, \infty)$

$\therefore$ ফাংশনটির জন্য,

$D_f : [-1, \infty)$

$R_f : [1, \infty)$

ii) $f(x) = \sqrt{x+1} + 1$

$\therefore f(3) = \sqrt{3+1} + 1$

$= \sqrt{4} + 1$

$= 3$

iii) $f(x) = \sqrt{x+1} + 1$

$f(t^2-1) = \sqrt{t^2-1+1} + 1$

$= t + 1$

iv) $f(x) = \sqrt{x+1} + 1$

if, $f(x) = 7$

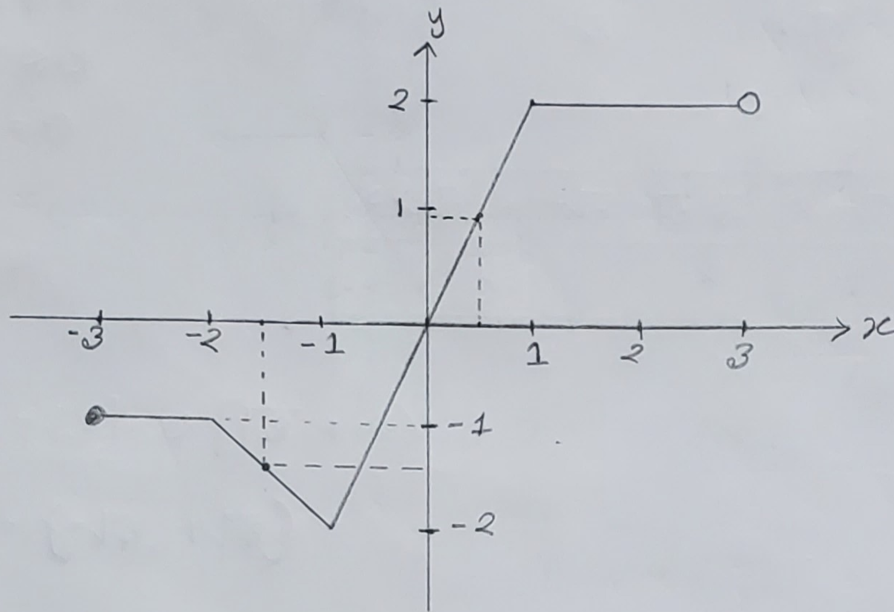
$\Rightarrow \sqrt{x+1} + 1 = 7$

$\Rightarrow \sqrt{x+1} = 6$

$\Rightarrow x+1 = 36$

$\therefore x = 35$

8.



i) domain $= [-3, 3)$

Range $= [-2, 2]$

ii) $f(-3) = -1$

iii) $f(\frac{1}{2}) = 0 \cdot 9$

iv) $f(x) = -\frac{3}{2}$

$= -1 \cdot 5$

$\therefore x = -1 \cdot 5$

$= -\frac{3}{2}$

(Ans)

9. (i) $f(x) = \frac{x^2+x}{x+1}$

এখানে,

$x+1 \neq 0$

$\therefore x \neq -1$

$\therefore D_f : \mathbb{R} - \{-1\}$

আবার,

$y = \frac{x^2+x}{x+1}$

$= \frac{x(x+1)}{x+1}$

$= x \quad ; \quad x \neq -1$

$\therefore R_f : (-\infty, -1) \cup (-1, \infty)$

(Ans)

ii)

$f(x) = \frac{1}{x-3}$

এখানে, $x-3 \neq 0$

$\therefore x \neq 3$

$\therefore D_f : \mathbb{R} - \{3\}$

আবার,

$y = \frac{1}{x-3}$

$\Rightarrow xy - 3y = 1$

$\Rightarrow xy = 1 + 3y$

$\therefore x = \frac{1+3y}{y}$

$\therefore x$ সংজ্ঞায়িত হবে যখন $y \neq 0$ হবে।

$\therefore R_f : \mathbb{R} - \{0\}$

(Ans)

iii) $f(x) = \sqrt{x^2-3}$

এখানে,

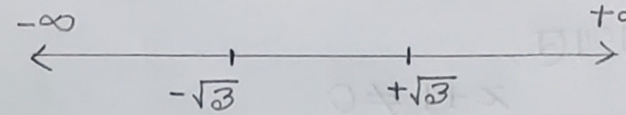
$\cancel{x^2-3 \neq 0}$

$x^2-3 \geq 0$

$\Rightarrow x^2-3=0$

$\Rightarrow x^2 = 3$

$\therefore x = \pm\sqrt{3}$



$x<-\sqrt{3}$	$x=-\sqrt{3}$	$-\sqrt{3},\sqrt{3}$	$x=\sqrt{3}$	$x>\sqrt{3}$
✓	✓	×	✓	✓

$\therefore D_f : (-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$

or, $\mathbb{R} - (-\sqrt{3}, +\sqrt{3})$

আবার,

$y = \sqrt{x^2-3}$

$\Rightarrow x^2 = y^2+3$

$\therefore y^2+3 = x^2$

$\therefore R_f : [0, +\infty)$

(Ans)

iv) $f(x) = \frac{1}{1-\sin x}$

এখানে, $1 - \sin x \neq 0$

$\Rightarrow \sin x \neq 1$

$\therefore x \neq \pi/2$

$\therefore x =$ 1; for $x = , -3\pi/2, -7\pi/2, -11\pi/2, \ldots\ldots \pi/2, 5\pi/2, 9\pi/2$

$\therefore x \neq (4n+1)\pi/2 \; ; \; n \in \mathbb{Z}$

$\therefore D_f : \mathbb{R} - \{(4n+1)\pi/2\}$

আবার,

$-1 \leq \sin x \leq 1$

$\Rightarrow 1 \geq -\sin x \geq -1$

$\Rightarrow 0 \leq 1 - \sin x \leq 2$

$\Rightarrow \frac{1}{1-\sin x} \geq \frac{1}{2}$ $\quad \left[\frac{1}{0} = \text{undefind}\right]$

$\therefore R_f : [1/2, +\infty)$

v) $f(x) = \sqrt{4-x^2}$

এখানে,

$4 - x^2 \geqslant 0$

$\Rightarrow x^2 \geqslant 4$

$\therefore x \geqslant \pm 2$

$\therefore D_f$: $[-2, +2]$

আবার,

$y = \sqrt{4-x^2}$

$\Rightarrow y^2 - 4 = -x^2$

$\Rightarrow y^2 = 4 - x^2$

$\therefore R_f$: $[0, 2]$

(Ans)

10. (i)

given, $f(x) = \sqrt{x-3}$

$g(x) = \sqrt{x^2+3}$

$\therefore fog(x) = \sqrt{\sqrt{x^2+3} - 3}$

(Ans)

$\therefore gof(x) = \sqrt{\left(\sqrt{x-3}\right)^2+3}$

$= \sqrt{x-3+3}$

$= \sqrt{x}$

(Ans)

ii) $f(x) = \dfrac{x}{1+x^2}$

$g(x) = \dfrac{1}{x}$

$$\therefore fog(x) = \frac{\frac{1}{x}}{1+\left(\frac{1}{x}\right)^2}$$

$$= \frac{\frac{1}{x}}{1+\frac{1}{x^2}}$$

$$= \frac{\frac{1}{x}}{\frac{x^2+1}{x^2}}$$

$$= \frac{1}{x} \times \frac{x^2}{x^2+1}$$

$$= \frac{x}{x^2+1}$$

$$\therefore gof(x) = \frac{1}{\frac{x}{1+x^2}}$$

$$= \frac{1+x^2}{x}$$

(Ans)

11. given,

$$f(x) = \frac{1}{1+x}$$

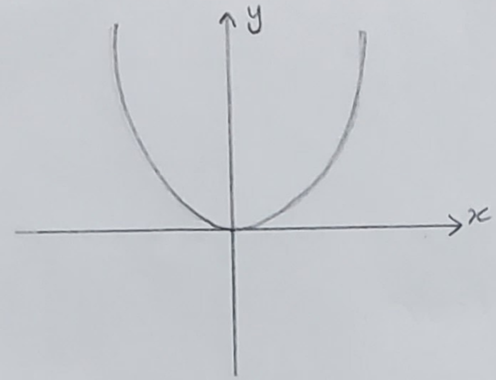
$$g(x) = x^{1/3}$$

$$h(x) = \frac{1}{x^3}$$

$$\begin{aligned}
f\circ g\circ h(x) &= f(g(h(x))) \\
&= f\left(g\left(\frac{1}{x^3}\right)\right) \\
&= f\left(\left(\frac{1}{x^3}\right)^{1/3}\right) \\
&= f\left(\frac{1}{x}\right) \\
&= \frac{1}{1+\frac{1}{x}} \\
&= \frac{1}{\frac{x+1}{x}} \\
&= \frac{x}{x+1}
\end{aligned}$$

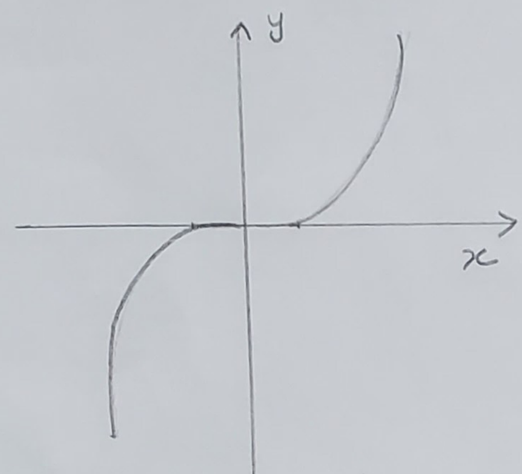
(Ans)

12.



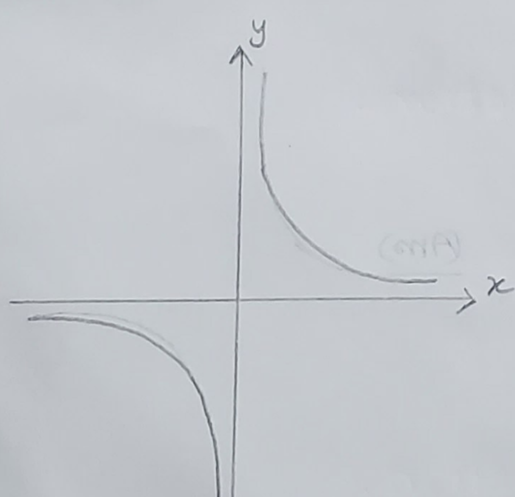
∴ According to this graph this is a mirror reflect graph of y-axix.
So, the function is even.

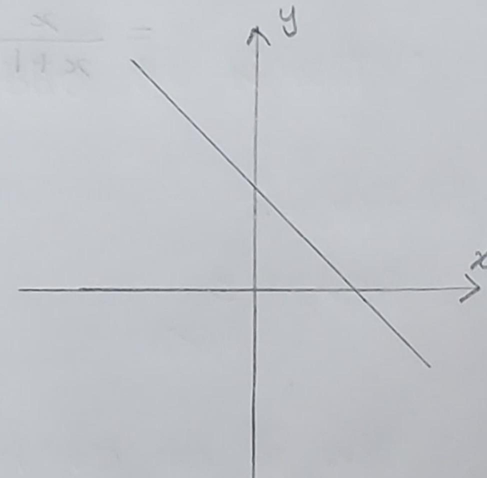
(Ans)



∴ According to this graph this is a mirror reflect graph of the origin
So, the function is odd.

(Ans)





∴ Accoding to the graphs they are not mirror reflect graph of y-axix or the origin.
So, the functions are neither.

(Ans)

13. (i) $f(x) = x + 1$

$\therefore f(-x) = (-x) + 1$
$= 1 - x$

$\therefore$ This is a neither function.

ii) $f(x) = \frac{x^5 - x}{1 + x^2}$

$\therefore f(-x) = \frac{(-x)^5 - (-x)}{1 + (-x)^2}$

$= \frac{-x^5 + x}{1 + x^2}$

$= \frac{-(x^5 - x)}{1 + x^2}$

$= -f(x)$

$\therefore$ This is an odd function.

iii) $f(x) = 2$

$\therefore f(-x) = 2$
$= f(x)$

$\therefore$ This is an even function.