

Answer all the following questions.

1. Find an equation for the tangent line to the graph at the specified value of x :

$$y = \sec^3\left(\frac{\pi}{2} - x\right), x = -\frac{\pi}{2} \quad [2]$$

2. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^3y^2 - 5x^2y + x = 1$ [2]

3. Consider the following function:

$$f(x) = x^4 - 5x^3 + 9x^2$$

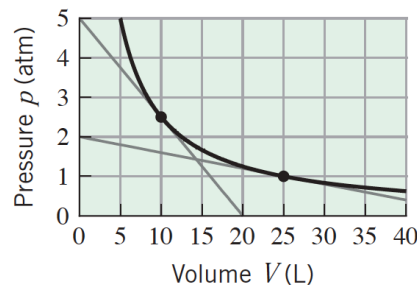
- i) Determine the intervals on which $f(x)$ is increasing and decreasing. [2]

- ii) Determine the intervals on which $f(x)$ is concave up and concave down. [2]

4. Find the derivative of the following functions using logarithmic differentiation.

$$y = \frac{x^3 \sqrt[3]{3x-4}}{(1+x^3)^3} \quad [2]$$

5. The accompanying figure shows the graph of the pressure p in atmospheres (atm) versus the volume V in liters (L) of 1 mole of an ideal gas at a constant temperature of 300 K (kelvins). Use the line segments shown in the figure to estimate the rate of change of pressure with respect to volume at the points where $V = 10$ L. [2]



6. A searchlight is trained on the side of a tall building. As the light rotates, the spot it illuminates moves up and down the side of the building. That is, the distance D between ground level and the illuminated spot on the side of the building is a function of the angle θ formed by the light beam and the horizontal (see the accompanying figure). If the searchlight is located 50 m from the building, find the rate at which D is changing with respect to θ when $\theta = 60^\circ$. Express your answer in units of meters/degree. [3]

