Solutions to the Homework Problems

1. Find the volume of the parallelepiped whose edges are represented by $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, $\vec{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$.

Solution:

Given,

$$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

$$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k},$$

$$\vec{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

We know,

Volume = $|\vec{a} \cdot (\vec{b} \times \vec{c})|$

= $7\ unit^3$

2. The position vectors of A, B, C and D are $2\hat{\imath} + 4\hat{k}$, $5\hat{\imath} + 3\sqrt{3}\hat{\jmath} + 4\hat{k}$, $-2\sqrt{3}\hat{\jmath} + \hat{k}$ and $2\hat{\imath} + \hat{k}$ respectively. Show that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel and $CD = \frac{2}{3}AB$.

Solution:

Given,

$$\overrightarrow{OA} = 2\hat{\imath} + 4\hat{k}$$

$$\overrightarrow{OB} = 5\hat{\imath} + 3\sqrt{3}\hat{\jmath} + 4\hat{k}$$

$$\overrightarrow{OC} = -2\sqrt{3}\hat{\jmath} + \hat{k}$$

$$\overrightarrow{OD} = 2\hat{\imath} + \hat{k}.$$

Now,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= 3\hat{\imath} + 3\sqrt{3}\,\hat{\jmath}$$

$$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$$

$$= 2\hat{\imath} + 2\sqrt{3}\hat{\jmath}$$

$$\therefore\ \overrightarrow{AB} \times \overrightarrow{CD} = \vec{0}$$

$\therefore \overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

Then,

$$\left|\overrightarrow{AB}\right| = AB = 6$$

$$\left|\overrightarrow{CD}\right| = CD = 4$$

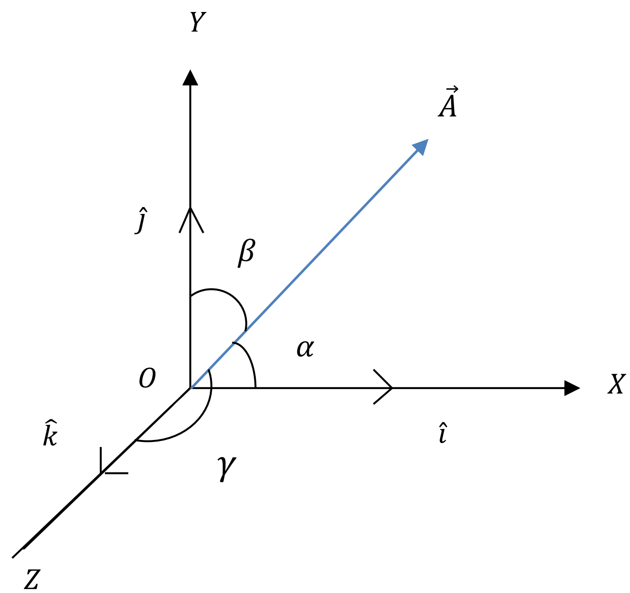
Now,

$$CD = {}^{2}/_{3} \times A\mathrm{B}$$

$$\Rightarrow \quad 4 = {}^{2}/_{3} \times 6$$

$$\Rightarrow \quad 4 = 4$$

3. Find the angles α, β, γ, which the vector $\vec{A} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ makes with the coordinates axes and also show that $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$.



<u>**Solution:**</u>

Now,

$$\cos\alpha = \frac{\vec{A}.\hat{\imath}}{|\vec{A}||\hat{\imath}|}$$

$$= {}^{3}/_{7}$$

$$\therefore \alpha = \cos^{-1} {}^{3}/_{7}$$

$$\cos\beta = \frac{\vec{A}.\hat{\jmath}}{|\vec{A}||\hat{\jmath}|}$$

$$= -{}^{6}/_{7}$$

$$\therefore \beta = \cos^{-1}(-{}^{6}/_{7})$$

$$\cos\gamma = \frac{\vec{A}.\hat{k}}{|\vec{A}||\hat{k}|}$$

$= {}^{2}/_{7}$

$\therefore \gamma = \cos^{-1} {}^{2}/_{7}$

Now,

$cos^2\alpha + cos^2\beta + cos^2\gamma$

$= ({}^{3}/_{7})^2 + ({}^{-6}/_{7})^2 + ({}^{2}/_{7})^2$

$= 1$

4. If the position vectors of the three points A, B and C are (2, 4, -1), (1, 2, -3) and (3, 1, 2) respectively. Find a vector perpendicular to the plane ABC.

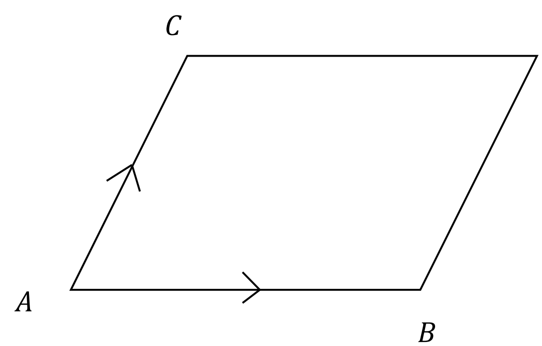
Solution:

Given,

$$\overrightarrow{OA} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}$$

$$\overrightarrow{OB} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$$\overrightarrow{OC} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$$



Now,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= -\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$$

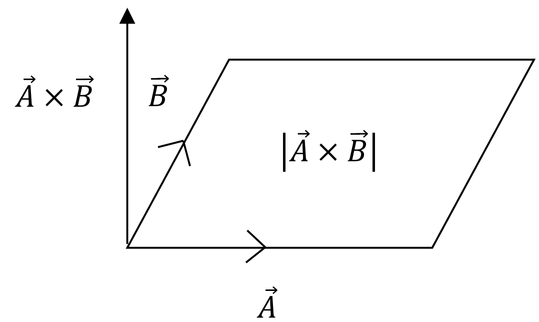
$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$$

$$= \hat{\imath} - 3\hat{\jmath} + 3\hat{k}$$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = -12\hat{\imath} + \hat{\jmath} + 5\hat{k}$$

5. Determine a unit vector perpendicular to the plane of $\vec{A} = 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$ and $\vec{B} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

<u>Solution:</u>



$$\vec{A} \times \vec{B} = 15\hat{\imath} - 10\hat{\jmath} + 30\hat{k}$$

$$\left|\vec{A} \times \vec{B}\right| = 35$$

$$\text{Required unit vector} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

$$= \frac{3}{7}\hat{\imath} - \frac{2}{7}\hat{\jmath} + \frac{6}{7}\hat{k}$$