Homework #2 Part I:
Prove that as a function of θ...

$$\propto \exp\left[-\frac{1}{2}\left(\left(\sum_{i=1}^{n}\phi(x_i-\theta)^2\right)+\tau(\theta-\theta_0)^2\right)\right]$$

$$\propto \exp\left(-\frac{1}{2}(\tau+n\phi)\left(\theta-\frac{1}{\tau+n\phi}\left(\tau\theta_0+\phi\sum_{i=1}^{n}x_i\right)\right)^2\right)$$

$\propto \exp\left[-\frac{1}{2}\left(\left(\sum_{i=1}^{n}\varphi(x_i-\theta)^2\right)+\tau(\theta-\theta_0)^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n}\varphi(x_i^2-2x_i\theta+\theta^2)+\tau(\theta^2-2\theta\theta_0+\theta_0^2)\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n}(\varphi x_i^2-2x_i\varphi\theta+\varphi\theta^2)+\tau\theta^2-2\tau\theta\theta_0+\tau\theta_0^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\varphi n\theta^2-2\theta\sum_{i=1}^{n}\varphi x_i+\sum_{i=1}^{n}\varphi x_i^2+\tau\theta^2-2\tau\theta\theta_0+\tau\theta_0^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\theta^2(\tau+n\varphi)-2\theta\left(\varphi\sum_{i=1}^{n}x_i+\tau\theta_0\right)+\sum_{i=1}^{n}\varphi x_i^2+\tau\theta_0^2\right)\right]$

$\propto \exp\left[-\frac{1}{2}\left(\theta^2(\tau+n\varphi)-2\theta\left(\varphi\sum_{i=1}^{n}x_i+\tau\theta_0\right)\right)\right]$ Keep terms only dependent on θ.

$\propto \exp\left[-\frac{1}{2}(\tau+n\varphi)\left(\theta^2-\frac{2\theta\left(\varphi\sum_{i=1}^{n}x_i+\tau\theta_0\right)}{\tau+n\varphi}\right)\right]$ Factor out $\tau+n\theta$

$\propto \exp\left[-\frac{1}{2}(\tau+n\varphi)\left((\theta)^2-2\cdot\theta\cdot\frac{\varphi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+n\varphi}+\left(\frac{\varphi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+n\varphi}\right)^2-\left(\frac{\varphi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+n\varphi}\right)^2\right)\right]$ ← Complete the square

$\propto \exp\left[-\frac{1}{2}(\tau+n\varphi)\left(\theta-\frac{\varphi\sum_{i=1}^{n}x_i+\tau\theta_0}{\tau+n\varphi}\right)^2\right.$

$\propto \exp\left\{-\frac{1}{2}(\tau+n\varphi)\left(\theta-\frac{1}{\tau+n\varphi}\left(\tau\theta_0+\varphi\sum_{i=1}^{n}x_i\right)\right)^2\right\}$ Rewrite

Q.E.D.