MDPI

Article

Higher Order Mining Approach for Analysis of Real-world Datasets

Shahrooz Abghari [1]*, Veselka Boeva [1] Jens Brage [2], and Håkan Grahn[1]

[1] Department of Computer Science, Blekinge Institute of Technology, Karlskrona, Sweden; {firstname.lastname}@bth.se
[2] NODA Intelligent Systems AB, Karlshamn, Sweden; jens.brage@noda.se
* Correspondence: shahrooz.abghari@bth.se

Version September 10, 2020 submitted to Journal Not Specified

Abstract: In this study, we propose a higher order mining approach that can be used for analysis of real-world datasets. The approach can be used for monitoring and identifying deviating operational behaviour of the studied phenomenon in the absence of prior knowledge about data. The proposed approach consists of several different data analysis techniques such as sequential pattern mining, clustering analysis, consensus clustering and minimum spanning tree (MST). Initially, clustering analysis is performed on the extracted patterns to model the behaviour modes of the studied phenomenon for a given time interval. The built clustering models corresponding to every two consecutive time intervals can further be assessed for mining changes in the monitored behaviour. In case some significant difference is observed, further analysis is performed by integrating the built models into a consensus clustering and applying an MST for identifying deviating behaviours. The validity and potential of the proposed approach has been demonstrated on a real-world dataset originating from a network of district heating (DH) substations. The obtained results show that our approach is capable of detecting deviating and sub-optimal behaviours of the DH substations.

Keywords: Outlier Detection; Fault Detection; Higher Order Mining; Clustering Analysis; Minimum Spanning Tree; Data Mining; District Heating Substations

1. Introduction

Fault is an abnormal state within the system that may cause a failure or a malfunction. Fault detection is the identification of an unacceptable deviation of at least one feature of the system from the expected or usual behaviour [1]. The fault detection problem has been studied in different domains and it belongs to a more general category, outlier (anomaly) detection. There are several factors such as the nature of data, availability of labeled data, constraints and requirements of the outlier detection problem that make it domain specific [2–6]. Outlier detection techniques can be classified into three groups based on availability of the labeled data, namely supervised methods, unsupervised methods, and semi-supervised methods, where the aim is to learn what is (ab)normal and only model (ab)normality [2,3,6]. In the absence of prior knowledge of data, which is often the case for more real-world datasets due to, e.g., expensiveness of the data labeling process by domain experts, the initial assumption is that normal data represents a significant portion of data.

Isermann [1,7] provided a general review for fault detection and diagnosis (FDD). The main objective of an FDD system is early detection of faults and diagnosis of their causes to reduce the maintenance cost and excessive damage to other parts of the system. Katipamula and Brambley [8,9] conducted an extensive review in two parts on FDD for building systems. The authors classified FDD methods based on the availability of a priori knowledge for formulating the diagnostics and highlighted their advantages and disadvantages.

According to this classification, FDD methods can fall into two categories: model-based methods and data-driven methods. The model-based methods require a priori knowledge of the system and can use either quantitative or qualitative models. Quantitative models are sets of mathematical relationships mainly based on physical properties, processes or models of the system. Qualitative models, on the other hand, use qualitative knowledge, e.g., including domain expert's experience as a set of rules for identifying proper and faulty operations. Data-driven methods use historical data coming from a system to build models, i.e., they are system-specific. These methods became more and more popular in the recent years. Data-driven methods are easy to develop and do not require explicit knowledge of the system, which makes them suitable for domains with uncertainty.

In this study, we propose a data-driven approach that also relies on domain experts' qualitative knowledge for setting user specific thresholds for identifying deviating behaviours. We show that the proposed approach is capable of revealing patterns representing deviating behaviours of the studied phenomenon. Such deviating behaviours can arise due to different reasons, in the case of district heating (DH) systems, they might be related to faulty equipment, sudden change of outdoor temperature, and/or social behaviour of tenants. Regardless of the reasons behind these deviating behaviours, fault detection systems should be able to provide understandable reports for domain experts. The proposed approach in this study, supplies the domain experts with patterns that are suitable for human inspection. In addition, the results are visualized from different points of view to provide supplementary information for the experts. The contributions of the proposed approach can be summarized into following three steps:

1. Build a data model of the studied phenomenon, which presents its behavioural modes for a given time interval.
2. Monitor the phenomenon's behaviour by comparing the models corresponding to two consecutive time intervals.
3. Analyse and identify deviating behaviours by integration analysis of the built models using a domain specific threshold.

This paper extends our previous work [10], where the data cleaning and preparation are improved by utilizing more suitable data preprocessing methods for time series data. We apply a *k*-Nearest neighbours based approach for missing data imputation, which takes into consideration the structure of the data. The seasonality of the time series are adjusted using the differencing method. In addition, the raw time series (sequences of numbers) are discretized into string series using the symbolic aggregate approximation method. Furthermore, we have conducted more extensive evaluations of the proposed approach for the chosen case study by considering a larger number of substations coming from different heat load categories. In addition, a more general explanation of the proposed approach is provided to demonstrate its applicability for similar problems in other domains.

2. Related work

The validity and potential of the proposed approach has been demonstrated in a use case from DH domain. Therefore in this section we mainly review literature related to outlier and fault detection approaches applied to DH and smart buildings domains.

Fontes and Pereira [11] proposed a fault detection for gas turbines using pattern recognition in multivariate time series. In another study [12], the authors proposed a general methodology for identifying faults based on time-series models and statistical process control, where faults can be identified as anomalies in the temporal residual signals obtained from the models using statistical process control charts.

In a recent review, Djenouri et al. [13] focused on the usage of machine learning for smart building applications. The authors classified the existing solutions into two main categories: 1) occupancy monitoring and 2) energy/device-centric. These categories are further divided into a number of sub-categories where the classified solutions in each group are discussed and compared.

Gadd and Werner [14] showed that hourly meter readings can be used for detecting faults at DH substations. The authors identified three fault groups: 1) low average annual temperature difference, 2) poor substation control, and 3) unsuitable heat load patterns. The results of the study showed that low average annual temperature differences are the most important issues, and that addressing them can improve the efficiency of the DH systems. However, solving unsuitable heat load patterns is probably the easiest and the most cost-effective fault category to be considered.

Xue et al. [15] applied clustering analysis and association rule mining to detect faults in substations *with* and *without* return-water pressure pumps. Clustering analysis was applied in two steps 1) to partition the substations based on monthly historical heat load variations and 2) to identify daily heat variation using hourly data. The result of the clustering analysis was used for feature discretization and preparation for association rule mining. The results of the study showed the method can discover useful knowledge to improve the energy performance of the substations. However, for temporal knowledge discovery, advanced data mining techniques are required.

Capozzoli et al. [16] proposed a statistical pattern recognition techniques in combination of artificial neural ensemble network and outlier detection methods to detect real abnormal energy consumption in a cluster of eight smart office buildings. The results of the study show the usefulness of the proposed approach in automatic fault detection and it ability in reducing the number of false alarms. Månsson et al. [17] proposed a method based on gradient boosting regression to predict an hourly mass flow of a well performing substation using only a few number of features. The built model is tested by manipulating the well performing substation data to simulate two scenarios: communication problems and a drifting meter fault. The model prediction performance is evaluated by calculating the hourly residual of the actual and the predicted values on original and faulty datasets. Additionally, cumulative sums of residuals using a rolling window that contains residuals from the last 24 hours were calculated. The results of the study showed that the proposed model can be used for continued fault detection.

Calikus et al. [18] proposed an approach for automatically discovering heat load patterns in DH systems. Heat load patterns reflect yearly heat usage in an individual building and their discovery is crucial for effective DH operations and managements. The authors applied k-shape clustering [19] on smart meter data to group buildings with similar heat load profiles. Additionally, the proposed method was shown to be capable of identifying buildings with abnormal heat profiles and unsuitable control strategies.

Sandin et al. [20] used probabilistic methods and heuristics for automated detection and ranking of faults in large-scale district energy systems. The authors studied a set of methods ranging from limit-checking and basic model to more sophisticated approaches such as regression modelling and clustering analysis on hourly energy metering.

In this study, we use a combination of data analysis techniques for modelling, monitoring, and analyzing operational behaviours of a studied phenomenon. We propose a higher order mining (HOM) approach to facilitate domain experts in identifying deviating behaviours and potential faults of the phenomenon under study. HOM is a sub-field of knowledge discovery that is applied on non-primary, derived data, or patterns to provide human-consumable results [21]. We first apply sequential pattern mining on raw data and extract patterns based on a user defined time interval. Then, the behavioural model is built by performing clustering analysis for each time interval. We further analyze and assess the similarity of the built behavioural models for every two consecutive time intervals. In case of observing any significant discrepancy (given a domain specific threshold) the built clustering models are integrated into a consensus clustering model. We additionally construct a minimum spanning tree (MST) on each consensus clustering model, considering the exemplars of the clustering solution as nodes and the distance between them as edges of the tree. Note that an MST is a spanning tree that covers all the nodes with the least traversing cost. In order to identify deviating behaviours, we cut the longest edge(s) of the MST, which turns the tree into a forest. Any small and distant trees can be interpreted as outliers, which can be further analysed by the domain expert.

3. Methods and Techniques

3.1. Sequential Pattern Mining

Sequential pattern mining is the process of finding frequently occurring patterns in a sequence dataset. The records of the sequence dataset contain sequences of events that often have chronological order. In this study, we apply the PrefixSpan algorithm [22] to extract frequent sequential patterns. PrefixSpan applies a prefix-projection method to find sequential patterns. Given a sequence dataset, minimum and maximum lengths of patterns, and a user-specified threshold, the dataset is first scanned in order to identify all frequent items with the length one in sequences. Using a divide and conquer fashion the search space is divided into a number of subsets based on the extracted prefixes. Finally, for each subset a corresponding projected dataset is created and mined recursively.

3.2. Clustering Analysis

3.2.1. Affinity Propagation

We use the affinity propagation (AP) algorithm [23] for clustering the extracted patterns. AP is based on the concept of exchanging messages between data points. The exchanged messages at each step assist AP to choose the best samples as exemplars (representatives of clusters) and which data points should choose those data points as their immediate exemplars. Unlike most clustering algorithms, such as k-means [24] which requires the number of clusters as an input, AP estimates the optimal number of clusters from the provided data or similarity matrix and the chosen exemplars are real data points. These characteristics make AP a suitable clustering algorithm for this study.

3.2.2. Consensus Clustering

Gionis et al. [25] proposed an approach for clustering that is based on the concept of aggregation. They are interested in a problem in which a number of different clustering solutions are given on some datasets of elements. The objective is to produce a single clustering of the elements that agrees as much as possible with the given clustering solutions. Consensus clustering algorithms deal with similar problems to those treated by clustering aggregation techniques. Namely, such algorithms try to reconcile clustering information about the same data phenomenon coming from different sources [26] or from different runs of the same algorithm [27]. In this study, we use the consensus clustering algorithm proposed in [26] in order to integrate the clustering solutions produced on the datasets collected for two consecutive weeks. We consider the exemplars (the representative patterns) of the produced clustering solutions. These exemplars are then divided into k groups (clusters) according to the degree of their similarity by applying the AP algorithm. Subsequently, the clusters whose exemplars belong to the same partition are merged in order to obtain the final consensus clustering.

3.3. Time Series Discretization

We apply symbolic aggregate approximation (SAX) [28] as a discretization method to transform raw time series (sequences of numbers) into symbolic strings. SAX first applies piecewise aggregate approximation (PAA) to transform a time series $Y = (y_1, y_2, \ldots, y_n)$ of length n into a PAA representation $\bar{Y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m)$ of length $m \leqslant n$, where $\bar{y}_i$ is computed as follows:

$$\bar{y}_i = \frac{m}{n} \sum_{j=\frac{n}{m}(i-1)+1}^{\frac{n}{m}i} y_i \tag{1}$$

That is, the time series Y is first divided into m equally sized windows and then for each window the mean value is computed. The final product of these values is the vector $\bar{Y}$ where it

is the data-reduced (PAA) representation of Y. The PAA representation is then descritized into a string $\hat{Y} = (\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_m)$ using alphabet A of size a, where a is any integer greater than 2.

3.4. Similarity Measures

We use two similarity measures 1) to perform pairwise comparison between extracted patterns and 2) to compute a similarity between two clustering solutions by considering all pairs of members.

3.4.1. Levenshtein Distance

The similarity between the extracted patterns are assessed with Levenshtein distance (LD) metric [29]. The LD, also known as edit distance, is a string similarity metric that measures the minimum number of editing operations (insertion, deletion and substitution) required to transform one string into the other. We use the normalized LD where score *zero* implies 100% similarity between the two patterns and *one* represents no similarity. LD is a simple algorithm capable of measuring the similarity between patterns with different lengths. Although in the presented case study the extracted patterns have similar lengths, for some scenarios it might be suitable to perform the pattern extraction with different lengths. Therefore, we choose to use LD which can provide more flexibility when patterns with different lengths are required.

3.4.2. Clustering Solution Similarities

Given two clustering solutions $C = \{C_1, C_2, \ldots, C_n\}$ and $C' = \{C'_1, C'_2, \ldots, C'_m\}$ of datasets X and X', respectively the similarity, S_w, between C and C' can be assessed as follows:

$$S_w(C, C') = \frac{\sum_{i=1}^{n}(min_{j=1}^{m} w_i.d(c_i, c'_j))}{2} + \frac{\sum_{j=1}^{m}(min_{i=1}^{n} w'_j.d(c_i, c'_j))}{2}, \tag{2}$$

where c_i and c'_j are exemplars of the clustering solutions C_i and C'_j, respectively. The weights w_i and w'_j indicate the relative importance of clusters C_i and C'_j compared to other clusters in the clustering solutions C and C', respectively. For example, a weight w_i of a cluster C_i can be calculated as the ratio of its cardinality to the cardinality of the dataset X, i.e., $w_i = |C_i|/|X|$. The S_w has values in a range of [0,1]. Scores of zero imply identical performance while scores close to one show significant dissimilarities.

3.5. Minimum Spanning Tree

Given an undirected and connected graph $G = (V, E)$, a spanning tree of the graph G is a connected sub-graph with no cycles that include all vertices. A minimum spanning tree (MST) of an edge-weighted graph (G, w), where $G = (V, E)$ is a graph and $w : E \to \mathbb{R}$ is a weight function, is a spanning tree where the sum of the weights of its edges is minimum among all the spanning trees. MSTs have been studied and applied in different fields including cluster analysis and outlier detection [30–34]. In this study we apply an MST on top of the created consensus clustering solution to further analyse the deviating substations' behaviours. We use Kruskal's algorithm [35] for building the MST. Kruskal's algorithm follows a greedy approach, i.e., at each iteration it chooses an edge which has least weight and adds it to the growing spanning tree. The algorithm first sorts the edges of (G, w) in an increasing order with respect to their weights. Then, it starts adding edges in sorted order and only those that do not form a cycle in the MST.

4. Proposed Method

We propose a higher order mining approach for modelling, monitoring, and analyzing real-world data phenomena. The proposed approach uses a combination of different data analysis techniques such as sequential pattern mining, clustering analysis, consensus clustering, and the MST algorithm. Note that the last three data mining techniques fall into the HOM paradigm, i.e., they are not applied on raw data, but on derived patterns and built models. The HOM paradigm brings new potential and perspective for knowledge discovery by generating human-understandable results and simplifying the comparative analysis among the studied phenomena. Thus, in the proposed approach the available data are initially partitioned across the time axis based on a user specified time interval. This allows for conventional data mining within each time interval and for HOM over the patterns extracted from the time intervals covering the studied period. This facilitates not only revealing similarities and interesting differences among the studied entities but also contributes to a more tractable process for the whole monitoring period.

The proposed approach consists of three main steps: 1) building a data model of the studied phenomenon that represents its behavioural modes for a given time interval, 2) monitoring the phenomenon's behaviour by comparing the models corresponding to two consecutive time intervals, and 3) analysing and identifying deviating behaviours. For the rest of this section, each step and its sub-steps will be described.

4.1. Building Behavioural Model

This step consists of two sub-steps. Initially, the data collected for a given time interval is analysed and frequent sequential patterns are extracted using the PrefixSpan algorithm discussed in Section 3.1. In order to build a model representing the phenomenon behavioural modes for the considered time interval, the extracted patterns are further analysed and partitioned into a number of groups. This is performed by applying the Affinity Propagation algorithm (see Section 3.2.1). The similarity between the patterns are calculated using LD. The built clustering model, at each time interval, represents the operational behaviour of the studied phenomenon for a specific time period. Note that the exemplars of each cluster serves as the representative operational behaviour for that cluster, i.e., every clustering solution can express the behaviour of the studied phenomenon for a specific time interval with the a set of exemplars.

4.2. Monitoring of Behaviour

The goal of this step is to make use of built behavioural models in the previous step to monitor the phenomenon behaviour. As we mentioned earlier each time interval is represented by a behavioural model. These models together can be used for monitoring purposes. Considering the fact that most real-world datasets contain unlabeled data, the similarity between the detected behaviours can be analyzed and assessed with the neighbouring time intervals, i.e., every two consecutive intervals. This is done through pairwise comparison of the exemplars of the clustering solutions using equation 2. The assessed similarities can be used for measuring, e.g., a discrepancy between observed performances in the two intervals. When the discrepancy is significant (above a domain specific threshold) further analysis (see Section 4.3) is preformed by integrating the produced clusters into a *consensus clustering* solution, i.e., AP is applied only on the exemplars of the two clustering solutions. In addition, it is important to mention that a significant discrepancy can be interpreted as deviating behaviour, e.g., newly observed patterns in the time interval $t+1$ are absolutely different from the ones in the time interval t.

4.3. Data Analysis

The created consensus clustering solution is meant to group the observed behavioural patterns (the clusters' exemplars) in the two consecutive time intervals based on their similarities. The observed

number of clusters can be considered as an indication of how much the extracted patterns in the two compared time intervals are similar. In order to identify deviating behaviours, first an MST is built, by applying Kruskal's algorithm, on top of each consensus clustering solution, where the exemplars are tree nodes and the distances between them represent the tree edges. The longest edge(s) of the built MST can lead us to groups of patterns with distinct behaviour. Therefore, in the next step the longest edge(s) of the built MST is removed. This turns the tree into a forest where the smallest and distant trees created by the cut can be interpreted as outliers. Nevertheless, the identified outliers need be further analyzed by the domain experts. In addition, the assessed similarities in the previous step, all together, can be used for building up a performance signature profile of the studied phenomenon for the given time period. In addition, such performance profiles can be applied for comparing different phenomena belonging to the same category.

5. Real-world Case Study

District Heating (DH) is an energy service based on circulating heated fluid from available heat sources such as natural geothermal, combustible renewable, and excess heat from industrial processes to customers [36]. A DH system provides heat and domestic hot water (DHW) for a number of *consumer units* (buildings) in a limited geographical area. The heat is produced at a *production unit* and circulated through a *distribution network* to reach the consumers. This part of the system is referred to as *primary* side. The consumer unit consists of a heat exchanger, a circulation network, and radiators for space heating in the rooms. This part of the system is called *secondary* side. The provided heat and DHW produced at the primary side transfer through a substation into the consumer unit, i.e., the secondary side. The substation makes the water temperature and pressure at the primary side suitable for the secondary side. A DH substation involves several components, each a potential source of faults. For example, a fault can consist of a stuck valve, a fouled heat exchanger, less than optimal temperature transmitters, a poorly calibrated control system, and many more [17,36]. Gadd and Werner [14] classify the possible faults of substations and secondary systems into three categories as follows: 1) faults resulting in comfort problems such as insufficient heating, or physical issues such as water leakage, 2) faults with a known cause but unsolved due to cost, and 3) faults that require advanced fault detection techniques for their discovery, which also includes faults caused by humans, such as unsuitable settings in building operating systems.

Substations are designed to meet heat demands despite possible faults degrading their performance. For example, poor heat transfer can to some extent be compensated by increasing the flow through the substation, meeting the heat demand at a higher cost for the energy company operating the network. In addition, ownership of the substation (often part of the building) and the subsequent high cost of customer interaction makes such situations difficult to address in a traditional manner. Consequently, early detection and classification of faults and other deviations from preferred behaviour can be used to reduce the overall cost, for example, by lowering maintenance costs, by reducing the need for the energy company to compensate poor performance with an increased flow, and by streamlining customer interactions. Finally, higher performing substations makes it possible to lower the system's overall temperature, which in turn makes it possible to use a greater amount of heat from renewable and other low value energy sources such as excess heat from subway stations.

6. Experimental Design

6.1. Dataset

The data used in this study is provided by an energy company located in Southern Sweden. The dataset consists of hourly average measurements from 82 buildings equipped with the company's smart system. The collected data was obtained during February 2014 until December 2018. This means 43,800 instances per building (24 instances per day). However, since most of the buildings have a high percentage of missing days and hourly missing values in the time span of 2014 to 2015, we limit our

analysis to 47 buildings (that conform the largest DH network in the studied dataset) for the period covering the recent three years (2016, 2017, and 2018). The building are divided into four categories: 1) company (C), 2) residential (R), 3) mixture of both residential and company (R-C), and 4) school (S). Table 1 summarizes the number of buildings in each category.

Table 1

Building type	count
Company (C)	9
Residential (R)	14
Residential and company (R-C)	20
School (S)	4
Total	47

Since we are monitoring the operational behaviour of the substations based on outdoor temperature, 5 out of 10 features that have a strong negative correlation with the outdoor temperature are selected. These features are 1) Secondary temperature difference ($\Delta T_{2^{nd}}$), 2) Primary temperature difference ($\Delta T_{1^{st}}$), 3) Primary power ($Q_{1^{st}}$), 4) Primary mass flow rate ($G_{1^{st}}$), and 5) Substation effectiveness (E_s^T). The substation effectiveness is calculated by considering features from both primary and secondary sides as follows:

$$E_s^T = \frac{\Delta T_{1^{st}}}{T_{s,1^{st}} - T_{r,2^{nd}}}, \tag{3}$$

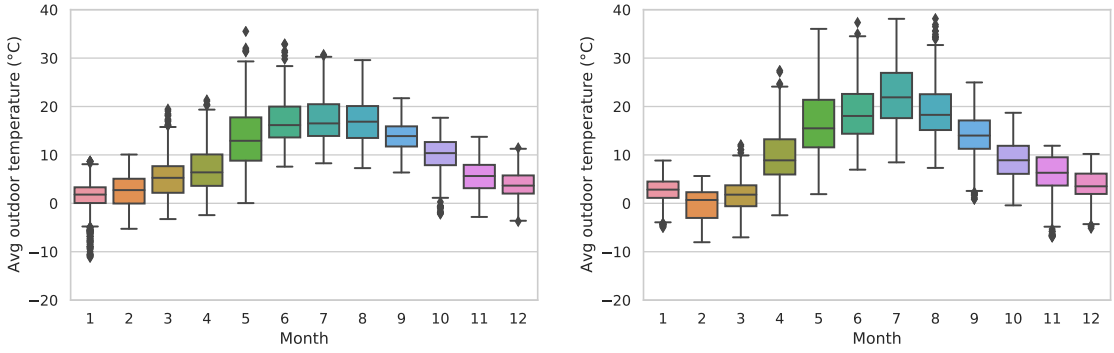
where $\Delta T_{1^{st}}$ is the difference between primary supply and return temperatures, $T_{s,1^{st}}$ is the primary supply temperature, and $T_{r,2^{nd}}$ is the return temperature at the secondary side. The substation effectiveness has a value within a range 0.0 and 1.0. The efficiency of a well-performed substation should be close to 1.0 in a normal setting. However, due to the affect of the domestic hot water generation on the primary return temperature, the E_s^T can be higher than 1.0. Table 2 shows all features included in the dataset. The features, 4-6, 9, and 10 in bold font are selected.

Table 2. Features included in the dataset

No.	Feature	Notation	Unit/Format
1	T_o	Outdoor temperature	°C
2	$T_{s,1^{st}}$	Primary supply temperature	°C
3	$T_{r,1^{st}}$	Primary return temperature	°C
4	$\mathbf{\Delta T_{1^{st}}}$	**Primary temperature difference**	**°C**
5	$\mathbf{Q_{1^{st}}}$	**Primary power**	**kW**
6	$\mathbf{G_{1^{st}}}$	**Primary mass flow rate**	**l/h**
7	$T_{s,2^{nd}}$	Secondary supply temperature	°C
8	$T_{r,2^{nd}}$	Secondary return temperature	°C
9	$\mathbf{\Delta T_{2^{nd}}}$	**Secondary temperature difference**	**°C**
10	$\mathbf{E_s^T}$	**Substation effectiveness**	**%**

Note. Features in bold font are selected due to their strong correlation with outdoor temperature.

In this study we only assess the substations' behaviour while space heating is needed. Figure 1 shows the yearly seasonality of outdoor temperature recorded for one building in two consecutive years. As one can see the average outdoor temperature in 2017 during January - April and November - December is below 10 °C. The heating season in 2018 mainly includes January - April and October - December.



(a) 2017 **(b)** 2018

Figure 1. Yearly seasonality of outdoor temperature for one specific building.

6.2. Data Preprocessing

Missing values can occur due to different reasons such as connection problems of measuring instruments, e.g., energy meters. Since we are interested in identifying deviating operational behaviour on a weekly basis, daily time series with more than 25% missing values (or six missing hours within a 24-hour period) are discarded. There are different imputation methods such as mean substitution, hot-deck imputation [37], regression analysis, and multiple imputation [38]. We apply a k-Nearest neighbours (kNN) based approach [39] to impute the missing values. That is, for each feature and year, the missing hours are imputed using the values of the five nearest neighbours (days) and the same hours (we let the number of neighbours, k, to be five which is a default value for the used library). The method identifies the nearest neighbours with the help of Euclidean distance and the missing values for each hour are weighted by distance to each neighbour.

Faults in measurement tools can appear as extreme values or sudden jumps in the measured data. We use a Hampel filter [40] which is a median absolute deviation (MAD) based estimation to detect and smooth out such extreme values. The filter computes the median, MAD, and the standard deviation (SD) over the data in a local window. We apply the filter with default parameters, i.e, the size of the window is considered to be seven which yields 3-neighbours on each side of a sample and the threshold for extreme value detection is set to be three. Therefore, in each window a sample with the distance three times the SD from its local median is considered as an extreme value and is replaced by the local median.

Time series data may contain seasonality, a repeating pattern within a fixed time period. The process of identifying and removing a time series seasonal effects is called seasonal adjustment. Seasonal effects can mask other interesting characteristics of the data. Seasonal adjustment helps better reveal any interesting components and allows better data analysis. In our case study, the data contains yearly seasonality. We apply differencing to adjust the seasonality. That is, each observation is subtracted by the value from previous year. We consider three years of data (2016, 2017, and 2018) where this gives us two years of data after seasonality adjustment (the first year of data is skipped to adjust the seasonality). Additionally, since 2016 is a leap year, February 29 is excluded while differencing.

As it was mentioned earlier, the proposed approach partitions the available data across the time axis on a weekly basis in order to extract patterns within each week. Therefore, it is necessary to convert the continuous features to categorized or nominal features, i.e., *data discretization* must be conducted. We apply SAX to transform the time series into symbolic representation. In order to have a meaningful comparison between time series with different offsets and amplitudes, the time series need to be normalized, i.e., to have mean of zero and standard deviation of one. Therefore, each yearly time series (feature vector) are normalized with z-score normalization. This step is performed automatically while applying SAX. We consider five categories (alphabet size) for the SAX transformation process

as follows: *low*, *low_medium*, *medium*, *medium_high*, and *high*. Nevertheless, the alphabet size can be adjusted based on the available data. In the previous study [10], we considered four categories, i.e., the same as the number of season periods. However, due to the risk of losing information the fifth category *medium* is added.

6.3. Data Segmentation and Pattern Extraction

The size of the time window (partition) for pattern extraction is important for further analysis. The proper partition length leads us to monitor operational behaviour of the substations rather than the residents' behaviour. Therefore, after performing some preliminary tests and having discussions with domain experts, the time window is set to be a week. The PrefixSpan algorithm is used for identifying frequent sequential patterns with a desired length. Any patterns that satisfy the user-specified threshold are considered as frequent. For the studied problem, the user-specified threshold is set to be one, i.e., any patterns that appear at least once will be considered. In addition, due to the importance of the selected features, the desired length of pattern is set to be equal to number of features, which is five.

6.4. Affinity Propagation Parameters Tuning

AP has a number of parameters. In this study we adjust two of these parameters, namely *affinity* and *damping*. The *affinity* parameter is set to be *pre-computed* since the algorithm is fed with a similarity matrix. The *damping* factor can be regarded as a slowly converging learning rate to avoid numerical oscillations. It is within a range 0.5 to 1.0. We always apply AP with damping factor equal to 0.5, in case the convergence does not occur the damping factor will be increased by 0.05 units and AP will be rerun with a new damping factor until convergence occurs.

6.5. Implementation and Availability

The proposed approach is implemented in Python version 3.6. The Python implementations of the PrefixSpan algorithm and the edit distance are fetched from [22] and [29], respectively. The AP algorithm and the *k*-means-based discretization are adopted from the scikit-learn module [41]. For constructing and manipulating an MST the NetworkX package is used [42]. The package uses Kruskal's algorithm for constructing the MST. The implemented code and the experimental results are available at GitHub[1].

7. Results and Discussion

We have studied substations' operational behaviour of 47 buildings during a period of two years (2017 and 2018). For each building, we first model the substation's weekly operational behaviour. This is performed by grouping the extracted frequent patterns into clusters of similar patterns. In order to monitor the substation's performance, we analyze and assess the similarity between substation's behaviours for every two consecutive weeks. When the bi-weekly comparison shows more than 25% (a user-specified threshold) difference and if the average temperature be less than or equal to 10 °C, further analysis is conducted by integrating the produced clustering solutions into a consensus clustering. The obtained consensus clustering solution is used for building an MST, where the exemplars are tree nodes and the distances between them represent the tree edges. In order to identify unusual behaviours, the longest edge(s) of the MST is removed. The smallest sub-trees created by the cut are interpreted as faults or deviations.

[1] https://github.com/shahrooz-abghari/HOM-Real-World-Datasets

7.1. Substations bi-weekly performance signature

As we mentioned earlier in Section 4.3, the assessed similarities of a substation's operational behaviour can be used for building the substation performance signature profile for the entire studied period. Additionally, such profiles can be used for comparing the substations belonging to the same heat load category. We studied four types of buildings: C, R, R-C, and S (see Table 1 for more information). Figure 2a shows the signature profiles of 14 residential buildings in 2018. The area between the two vertical dashed lines represents the non-heating season in all figures. Figure 2b depicts the substations' performance signature profiles of nine company buildings for the same year. Figure 2c contains the largest number of buildings belonging to the R-C category, i.e., 20 in total. Figure 2d represents the signature profiles of four schools. All the studied buildings are located in the same city. As one can see, Figures 2a-2d contain signatures that are quite similar in the period of week 1 until week 18 (January 1 - May 6, 2018) and week 45 to week 46 (November 5-18, 2018).

Although, the expectation was to observe similar performance signatures from the buildings that are in the same category, there are substations showing quite different behaviours. The main reasons can be related to the difference between average outdoor temperature within two weeks in different areas of the city. Our further analysis shows that buildings of the same type and close together tend to have similar performance signature profiles during heating seasons. In addition, social behaviour of people, special holidays, and/or faulty substations and equipment have high impact on substations' performance. It is also the case that buildings of same category behave differently mostly due to installation issues, unsuitable configurations, or different brand of equipment. Nevertheless, this requires further analysis by domain experts.

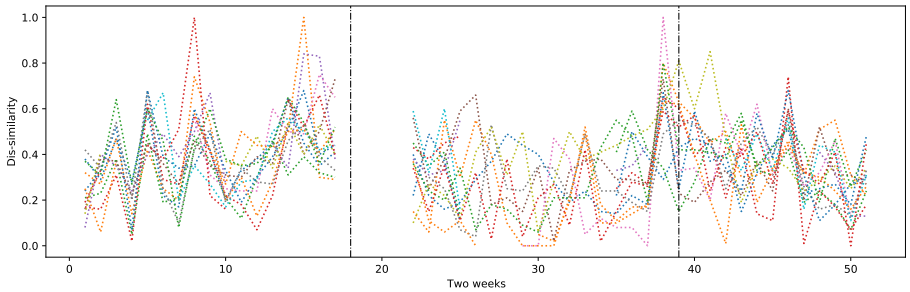
7.2. Modelling Substation Operational Behaviour

Weekly operational behaviour of a substation can be modeled by clustering the extracted patterns based on their similarities into groups. Using the AP algorithm, each cluster can be recognized by its exemplar, a representative pattern of the whole group. Each cluster models the substation's operational behaviour for some hours up to a couple of days, based on its frequency. The number of clusters in each clustering solution can be interpreted as different operational modes of the substation for the studied week. High number of clusters may due to the same reasons as ones discussed in the foregoing section such as the difference between outdoor temperature during days and nights. The extracted patterns contain five features. Each feature can belong to one of five available categories: *low*, *low_medium*, *medium*, *medium_high*, and *high*.

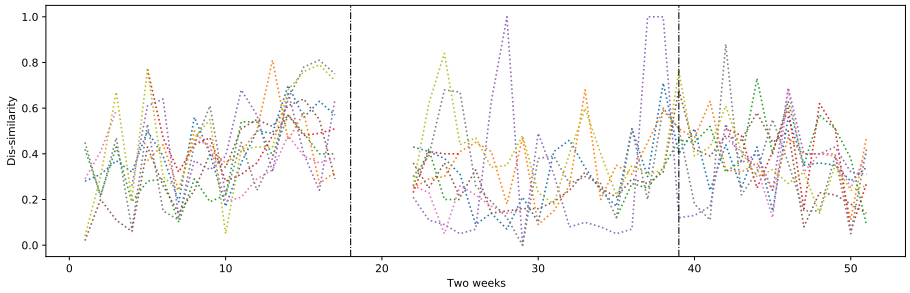
Figure 3 shows the operational behaviour of B_3 substation during weeks 2 and 3 in 2017, where *low* is represented by one and *high* by five, respectively. Note that each category is the result of differencing to adjust yearly seasonality for each feature. Figure 3a represents 4 operational behaviour modes of B_3 substation during the second week of 2017. Cluster 2 covers 57 hours, the most number of hours among the others, while cluster 1 covers only 26 hours, which is the least number of hours. In week 3 (see Figure 3b) 6 operational behaviour modes are detected. Cluster 3 in this week, similar to cluster 2 in week 2, covers 57 hours. Cluster 4 with the least number of hours covers only 11 hours.

We further analyze the operational behaviour models of weeks 2 and 3 by calculating the similarity between the exemplars of the corresponding clustering solutions. The calculated dissimilarity is above 25% and the average weekly outdoor temperature below 10 °C. Therefore, the proposed method integrates the clustering solutions into consensus clustering. Figure 3c represents the substation's operational behaviour model for the studied two weeks. The model contains 3 clusters. In order to detect deviating behaviour as explained previously in Section 4.3, first an MST is built on top of the consensus clustering solution. Next, the longest edge(s) of the tree is removed and sub-tree(s) with the smallest size and far from majority of data can be marked as deviating behaviour. In Figure 3c, cluster 1 (framed in red) is detected as an outlier.

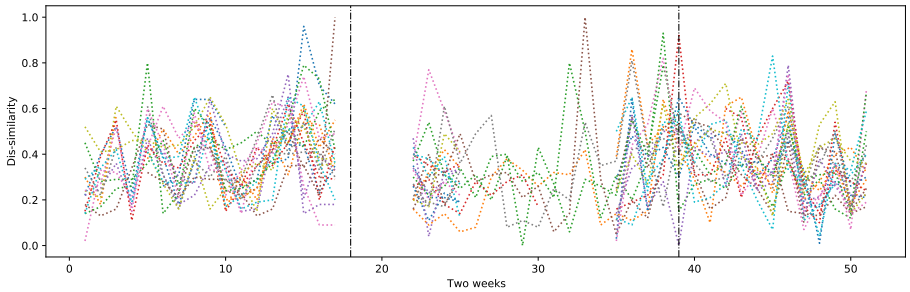
Tables 3 and 4 show the distribution of each weekly cluster together with the number of days and hours that they cover across the consensus clustering solution respectively. As one can see, in Table 3,



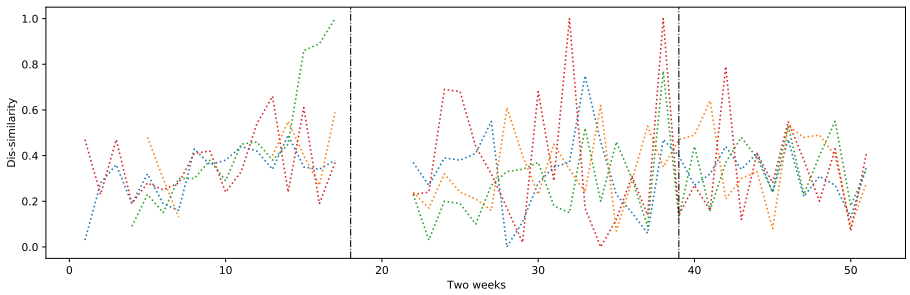
(a) Subsections profiles type R: residential in 2018.



(b) Subsections profiles type C: company in 2018.



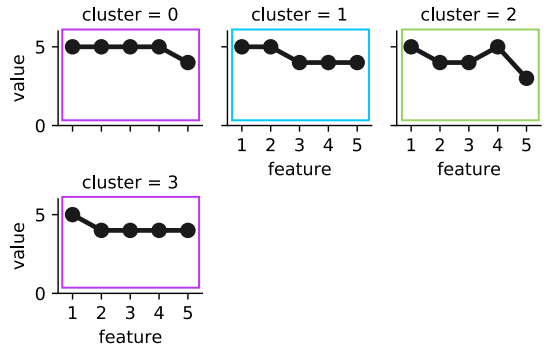
(c) Subsections profiles type R-C: residential and company in 2018.



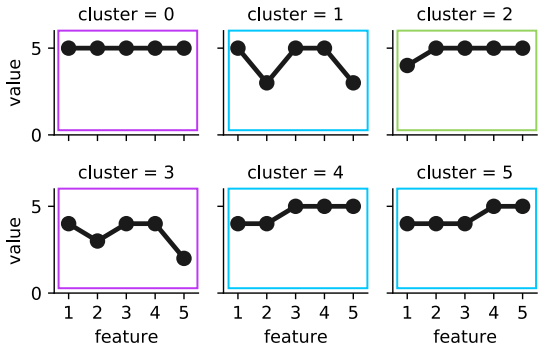
(d) Subsections profiles type S: school in 2018.

Figure 2. Substations profiles based on buildings types. Due to missing values some of the bi-weekly comparisons are missing. The area between two vertical dashed lines in each plot represents the non-heating season.

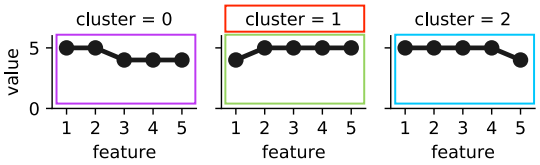
the detected operational behaviours in week 2 are divided into four groups. While week 3 contains six categories of different operational behaviours. In week 2, the majority of the behaviours appear across the whole week except cluster 1, which covers only 6 days. In week 3, on the other hand, clusters 1 and 2 each contain operational behaviours observed within 4 days. Clusters 4 and 5 each cover 5 days



(a) The substation's behaviour on week 2.



(b) The substation's behaviour on week 3.



(c) The consensus clustering integrating the clustering solutions for weeks 2 and 3.

Figure 3. The B_3's substation operational behaviour in weeks 2 and 3 in 2017. Each cluster is shown by its exemplar. The colored frames represent the consensus clustering solution, where purple = cluster 0, green = cluster 1, and blue = cluster 2. The exemplars of clusters 0 and 2 are chosen from week 2 and cluster 1 is chosen from week 3. After building an MST on top of the consensus clustering solution, cluster 1 is identified as the deviating behaviour of the substation due to its small size and distance from majority of the data.

and the last two remaining clusters cover 6 days each. Considering the consensus clustering solution, cluster 1 contains the least number of days, 11, while clusters 0 and 2 each include 12 and 14 days. Note that, weekly clustering solutions can have daily overlap, however, they cover different hours.

Table 3. Number of identified daily deviating behaviours in weeks 2 and 3 for B_3 in 2017

	Consensus cluster (CC)		
Weekly cluster	CC0	CC1	CC2
W2,C0			7
W2,C1	6		
W2,C2		7	
W2,C3			7
W3,C0			6
W3,C1	4		
W3,C2		4	
W3,C3			6
W3,C4	5		
W3,C5	5		
Total days	12	11	14

Table 4 shows the number of hours covered by each weekly cluster and the total hours for each bi-weekly cluster (consensus cluster). As one can see, consensus cluster 2 contains the most number of hours, 168. Cluster 0 and 1 cover 88 and 80 hours, respectively, within weeks 2 and 3. As it was mentioned earlier, by cutting the longest edge(s) of the built MST on top of consensus clustering solution the smallest and distant cluster can be considered as deviating behaviour, i.e., consensus cluster 1. This cluster appears in 11 days (Table 3, consensus cluster CC1) and in total 80 times (Table 4,

consensus cluster CC1) out of 336 (*24 hours* $\times$ *14 days*). The data collected for these particular days can be further analyzed by domain experts to get better insight and understanding of the identified deviating behaviour.

Table 4. Identified hourly deviating behaviours in weeks 2 and 3 for B_3 in 2017.

	Consensus cluster (CC)		
Weekly cluster	CC0	CC1	CC2
W2,C0			37
W2,C1	26		
W2,C2		57	
W2,C3			48
W3,C0			26
W3,C1	30		
W3,C2		23	
W3,C3			57
W3,C4	11		
W3,C5	21		
Total hours	88	80	168

In general, an increase or decrease in the number of observed clusters in one week in comparison to its neighbouring week, can be interpreted as an indication of deviating behaviours. This can occur for different reasons such as sudden drop in outdoor temperature. Therefore, in order not to take into account every single change as deviating behaviour, we consider using a performance measure called overflow. The measure expresses a substation's performance in terms volume flow per unit of energy flow. In the district heating domain overflow of a well-performed substation is expected to be 20 $\frac{l}{kWh}$. Therefore, by computing a substation's weekly overflow, any bi-weekly detected deviations in conjunction with weekly overflow above 20 can be flagged as real changes in the substation.

Table 5 represents substations with bi-weekly deviating behaviours on an hourly basis and overflow more than 20 in 2017 and 2018. In total substations that belong to the *Residential* category have the most number of deviating behaviours in both years, i.e., four substations with 95 and three substations with a total 86 detected deviating behaviours in 2017 and 2018 respectively. In the *Residential-Company* category there is only one substation which in both years contains considerable number of deviating behaviours, i.e., 88 and 64. The *Company* category contains four substations with 47 and five substations with 38 identified deviating behaviours in 2017 and 2018 respectively. For the *Schools* category, all four substations contain in total 11 detected deviating behaviours in 2017, while in 2018 only one of these substations contain one deviating behaviour. In Table 5 those substations that appear in both years are shown in bold.

Figure 4 summarizes the statistics of Table 5 on a daily basis for the four categories in 2017 and 2018. The plot can help the domain expert in identifying categories with the most number of daily deviating behaviours for a specific time period. For example, substations belong to *Company* and *Residential* categories had the most number of deviating behaviours during weeks 42 to 45 in 2017.

7.3. Patterns representative of deviating behaviours

Extracted patterns can provide meaningful information for the domain experts, i.e., each pattern represents the status of the five selected features at a specific time period. Note that each category shows the status of a feature at time t in comparison to its value in time $365 - t$. As it was mentioned in Section 6.2, the yearly seasonality of the data is adjusted by the differencing method.

Table 6 shows the top 10 weekly patterns detected as deviating behaviours in 2017 and 2018. These patterns are exemplars (representative) of the bi-weekly consensus clustering solutions. As one can see in all patterns except pattern number 7, features 3 and 4 (shown underlined in table) primary heat and primary mass flow rate, respectively, hold similar values.

Table 5. Substations with bi-weekly deviating behaviours on an hourly basis and overflow more than 20 in 2017 and 2018.

Year	Substation	C	R	R-C	S
2017	B_L	2			
	B_3			**88**	
	E_D_32_A		**14**		
	G_3		6		
	M_S		**14**		
	O_S				6
	P_S				1
	R_4	**4**			
	S_1		**61**		
	S_10	**40**			
	S_2A_C	1			
	S			1	
	V_S				**3**
	Total	47	95	88	11
Year	Substation	C	R	R-C	S
2018	**B_3**			**64**	
	C_T	2			
	E_D_32_A		**2**		
	F_12_18	21			
	M_S		**46**		
	R_4	**1**			
	S_		**38**		
	S_10	**9**			
	S_2D-F	5			
	V_S				**1**
	Total	38	86	64	1

Note. Substations in bold font are those that had deviating behaviours in both 2017 and 2018. C: company, R: residential, R-C: residential and company, S: school.

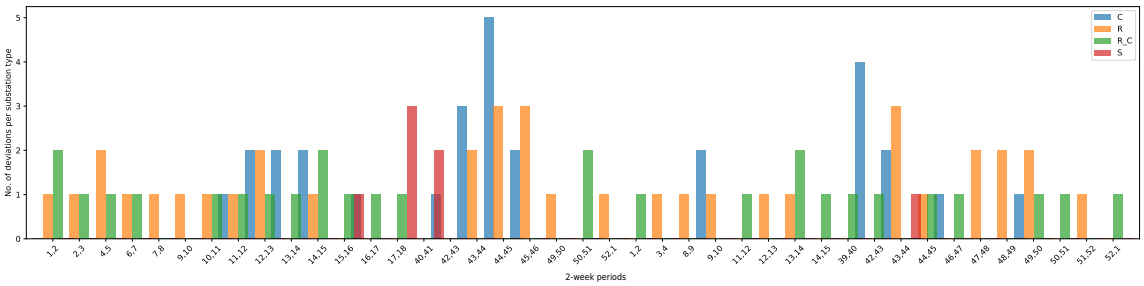


Figure 4. Number of substations with bi-weekly deviating behaviours on a daily basis and their category during heating season in 2017 and 2018.

Table 7 shows top weekly patterns based on four categories of the substations. The majority of patterns have only occurred for specific types of substations, except pattern "medium, medium, medium, medium, medium_high", which is observed for types R-C and S in 2017 (row 6 and 8) and R in 2018 (row 2). Patterns belonging to *Residential-Company* and *Residential* categories are in total the most frequent in count. In addition, some of these patterns re-occurred in both 2017 and 2018 for the same category of substations, as shown in bold in table.

Table 6. Top 10 weekly patterns detected as outliers in 2017 and 2018 in total

No.	Pattern	Count
1	high,high,high,high,high	80
2	high,high,high,high,medium_high	63
3	medium_high,medium,medium_high,medium_high,medium	39
4	medium,medium,medium,medium,medium_high	39
5	low,high,low_medium,low_medium,high	21
6	medium,medium_high,medium,medium,high	15
7	medium_high,low_medium,medium,medium_high,low_medium	13
8	medium_high,medium_high,medium,medium,medium	13
9	low_medium,low_medium,medium,medium,medium	11
10	medium,medium,low,low,medium	7

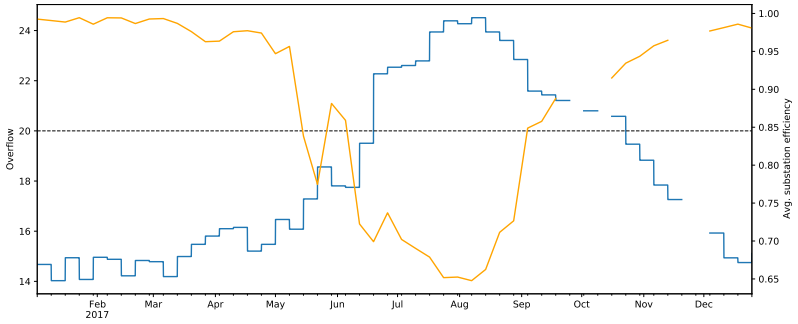
Table 7. Top weekly patterns detected as outliers with respect to substation types in 2017 and 2018

Year	No.	Type	Pattern	Count
2017	1	R-C	**high,high,high,high,medium_high**	**57**
	2	R	**high,high,high,high,high**	**55**
	3	C	medium_high,low_medium,medium,medium_high,low_medium	13
	4	R	low_medium,low_medium,medium,medium,medium	11
	5	C	medium_high,medium_high,medium,medium,medium	10
	6	R-C	**medium,medium,medium,medium,medium_high**	**8**
	7	C	medium,medium,low,low,medium	7
	8	S	**medium,medium,medium,medium,medium_high**	**6**
	9	R-C	medium,high,low_medium,low_medium,high	5
	10	R-C	medium,low_medium,low_medium,low_medium,low_medium	5
2018	1	R-C	medium_high,medium,medium_high,medium_high,medium	39
	2	R	**medium,medium,medium,medium,medium_high**	**25**
	3	R	**high,high,high,high,high**	**24**
	4	C	low,high,low_medium,low_medium,high	21
	5	R	medium,medium_high,medium,medium,high	15
	6	R-C	**high,high,high,high,medium_high**	**6**
	7	R	high,medium,high,high,medium	5
	8	C	low,low,medium_high,high,low_medium	5
	9	C	high,low_medium,medium,medium_high,low_medium	4
	10	R-C	medium_high,high,high,medium,high	4
	11	R-C	medium_high,medium_high,medium_high,medium_high,high	4

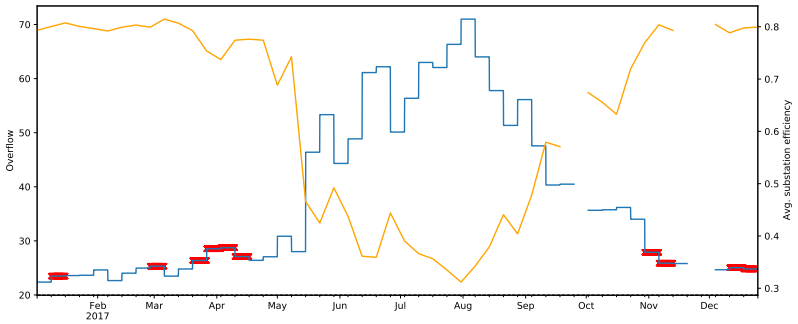
Note. Patterns in bold font occurred in both years.

7.4. Substation Performance

Substation efficiency, E_s^T, can be used as an indicator to assess a substation's operational behaviour throughout the entire year. Figure 5 depicts the detected deviations for two substations belonging to the *Residential* category using their average efficiency and overflow for year 2017. Figure 5a, represents a well-performed substation. Notice that the substation's efficiency on average is around 98%. In addition, the weekly substation's overflows for the whole heating season (January-May and November-December) are below 20. Figure 5b, on the other hand, shows substation with sub-optimal performance during 2017. As one can see the substation's efficiency on average is around 80% during heating season. Moreover, the weekly overflows for the whole year are above 20. In addition, the proposed approach identified deviating behaviours in 10 weeks which are marked with red.



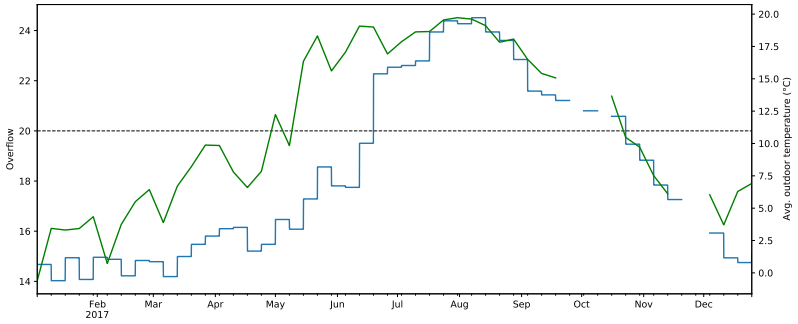
(a) Well performed substation: overflow (blue line) below 20 during heating season vs. substation effectiveness (orange line).



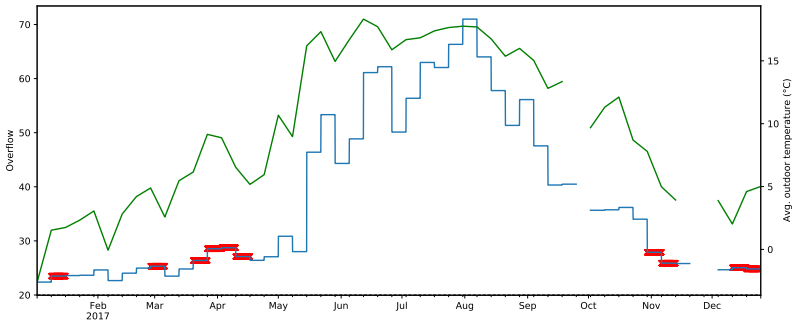
(b) Sub-optimal performed substation: overflow (blue line) above 20 during heating season vs. substation effectiveness (orange line). The red marks represent identified deviating behaviours in 10 weeks.

Figure 5. Example of well and sub-optimal performed substations in 2017. Overflow vs. Substation's efficiency. Due to missing values some of the weeks are missing

Figure 6 represents the performance of the same substations based on weekly overflow and outdoor temperature. Something that is noticeable in Figure 6b is that whenever the outdoor temperature has a sudden change the purposed approach observes that as deviating behaviour.



(a) Well performed substation: overflow (blue line) below 20 during heating season vs. outdoor temperature (green line).



(b) Sub-optimal performed substation: overflow (blue line) above 20 during heating season vs. outdoor temperature (green line). The red marks represent identified deviating behaviours in 10 weeks.

Figure 6. Example of well and sub-optimal performed substations in 2017. Overflow vs. outdoor temperature. Due to missing values some of the weeks are missing

Notice that in this study we only consider the smallest sub-tree(s), after cutting the longest edge(s) of an MST, as outlier(s). Nevertheless, one can consider sorting the sub-trees based on their size from smallest to the largest for further analysis. Alternatively, by defining a domain specific threshold any edges with a distance greater than the threshold can be removed and further analysis can be performed on smaller sub-trees.

8. Conclusion and Future Work

We have proposed a higher order data mining approach for analyzing real-world datasets. The proposed approach combines different data analysis techniques for 1) building a behavioural model of the studied phenomenon, 2) using the built model for monitoring the phenomenon's behaviour, and 3) finally identifying and further analyzing deviating behavioural patterns. At each step, adequate information is supplied which can facilitate the domain experts in decision making and inspection.

The approach has been demonstrated and evaluated on a case study from the district heating (DH) domain. For this purpose, we have used data collected for a three-year period of 47 operational substations belonging to four different heat load categories. The results have shown that the method is able to identify and analyze deviating and sub-optimal behaviours of the DH substations. In addition, the proposed approach provides different techniques for monitoring and data analysis, which can facilitate domain experts to better understand and interpret the DH substations' operational behaviour and performance.

For future work we aim to pursue further analysis and evaluation of the proposed approach on similar scenarios in different application domains, e.g., fleets of wind turbines. In addition, we

want to extend the proposed approach with means for root-cause analysis and diagnosis of detected deviations.

Author Contributions: conceptualization, S.A. and V.B.; methodology, S.A. and V.B.; software, S.A.; validation, S.A and J.B.; formal analysis, S.A.; investigation, S.A., J.B.; data curation, S.A.; writing–original draft preparation, S.A.; writing–review and editing, S.A., V.B., J.B., and H.G.; visualization, S.A.; funding acquisition, H.G.

Funding: This work is part of the research project "*Scalable resource-efficient systems for big data analytics*" funded by the Knowledge Foundation (grant: 20140032) in Sweden.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

AP	Affinity Propagation
C	Company
DH	District Heating
DHW	Domestic Hot Water
FDD	Fault Detection and Diagnosis
HOM	Higher Order Mining
LD	Levenshtein Distance
MAD	Median Absolute Deviation
MST	Minimum Spanning Tree
PAA	Piecewise Aggregate Approximation
R	Residential
R-C	Residential and Company
S	School
SAX	Symbolic Aggregate Approximation
SD	Standard Deviation

References

1. Isermann, R. *Fault-diagnosis systems: an introduction from fault detection to fault tolerance*; Springer Science & Business Media, 2006.
2. Hodge, V.; Austin, J. A survey of outlier detection methodologies. *Artificial Intelligence Review* **2004**, *22*, 85–126.
3. Chandola, V.; Banerjee, A.; Kumar, V. Anomaly detection: A survey. *ACM Computing Surveys* **2009**, *41*, 15.
4. Zhang, Y.; Meratnia, N.; Havinga, P. Outlier detection techniques for wireless sensor networks: A survey. *IEEE Communications Surveys & Tutorials* **2010**, *12*, 159–170.
5. Gupta, M.; Gao, J.; Aggarwal, C.C.; Han, J. Outlier detection for temporal data: A survey. *IEEE Transactions on Knowledge and Data Engineering* **2014**, *26*, 2250–2267.
6. Aggarwal, C.C. Outlier analysis. Data mining. Springer, 2015, pp. 237–263.
7. Isermann, R. Supervision, fault-detection and fault-diagnosis methods—an introduction. *Control engineering practice* **1997**, *5*, 639–652.
8. Katipamula, S.; Brambley, M.R. Methods for fault detection, diagnostics, and prognostics for building systems-A review, part I. *Hvac&R Research* **2005**, *11*, 3–25.
9. Katipamula, S.; Brambley, M.R. Methods for fault detection, diagnostics, and prognostics for building systems-A review, part II. *Hvac&R Research* **2005**, *11*, 169–187.
10. Abghari, S.; Boeva, V.; Brage, J.; Johansson, C.; Grahn, H.; Lavesson, N. Higher order mining for monitoring district heating substations. 2019 IEEE International Conference on Data Science and Advanced Analytics (DSAA). IEEE, 2019, pp. 382–391.
11. Fontes, C.H.; Pereira, O. Pattern recognition in multivariate time series–A case study applied to fault detection in a gas turbine. *Engineering Applications of Artificial Intelligence* **2016**, *49*, 10–18.

12. Sánchez-Fernández, A.; Baldán, F.; Sainz-Palmero, G.; Benítez, J.; Fuente, M. Fault detection based on time series modeling and multivariate statistical process control. *Chemometrics and Intelligent Laboratory Systems* **2018**, *182*, 57–69.
13. Djenouri, D.; Laidi, R.; Djenouri, Y.; Balasingham, I. Machine Learning for Smart Building Applications: Review and Taxonomy. *ACM Computing Surveys* **2019**, *52*, 24.
14. Gadd, H.; Werner, S. Fault detection in district heating substations. *Applied Energy* **2015**, *157*, 51–59.
15. Xue, P.; Zhou, Z.; Fang, X.; Chen, X.; Liu, L.; Liu, Y.; Liu, J. Fault detection and operation optimization in district heating substations based on data mining techniques. *Applied Energy* **2017**, *205*, 926–940.
16. Capozzoli, A.; Lauro, F.; Khan, I. Fault detection analysis using data mining techniques for a cluster of smart office buildings. *Expert Systems with Applications* **2015**, *42*, 4324–4338.
17. Månsson, S.; Kallioniemi, P.O.J.; Sernhed, K.; Thern, M. A machine learning approach to fault detection in district heating substations. *Energy Procedia* **2018**, *149*, 226–235.
18. Calikus, E.; Nowaczyk, S.; Sant'Anna, A.; Gadd, H.; Werner, S. A Data-Driven Approach for Discovery of Heat Load Patterns in District Heating. *arXiv preprint arXiv:1901.04863* **2019**.
19. Paparrizos, J.; Gravano, L. k-shape: Efficient and accurate clustering of time series. Proceedings of the 2015 ACM SIGMOD International Conference on Management of Data. ACM, 2015, pp. 1855–1870.
20. Sandin, F.; Gustafsson, J.; Delsing, J. *Fault detection with hourly district energy data: Probabilistic methods and heuristics for automated detection and ranking of anomalies*; Svensk Fjärrvärme, 2013.
21. Roddick, J.F.; Spiliopoulou, M.; Lister, D.; Ceglar, A. Higher order mining. *ACM SIGKDD Explorations Newsletter* **2008**, *10*, 5–17.
22. Pei, J.; Han, J.; Mortazavi-Asl, B.; Pinto, H.; Chen, Q.; Dayal, U.; Hsu, M. Prefixspan: Mining sequential patterns efficiently by prefix-projected pattern growth. Proc. of the 17th Int'l Conf. on Data Engineering, 2001, pp. 215–224.
23. Frey, B.J.; Dueck, D. Clustering by passing messages between data points. *Science* **2007**, *315*, 972–976.
24. MacQueen, J.; others. Some methods for classification and analysis of multivariate observations. Proc. of the Fifth Berkeley Symp. on Mathematical Statistics and Probability. Oakland, CA, USA., 1967, Vol. 1, pp. 281–297.
25. Gionis, A.; Mannila, H.; Tsaparas, P. Clustering Aggregation. *ACM Transaction of Knowledge Discovery Data* **2007**, *1*.
26. Boeva, V.; Tsiporkova, E.; Kostadinova, E., Analysis of Multiple DNA Microarray Datasets. In *Springer Handbook of Bio-/Neuroinformatics*; Springer Berlin Heidelberg, 2014; pp. 223–234.
27. Goder, A.; Filkov, V. Consensus Clustering Algorithms: Comparison and Refinement. ALENEX, 2008, pp. 109–234.
28. Lin, J.; Keogh, E.; Wei, L.; Lonardi, S. Experiencing SAX: a novel symbolic representation of time series. *Data Mining and knowledge discovery* **2007**, *15*, 107–144.
29. Levenshtein, V.I. Binary codes capable of correcting deletions, insertions, and reversals. Soviet physics doklady, 1966, Vol. 10, pp. 707–710.
30. Aggarwal, C.C.; Yu, P.S. Outlier detection for high dimensional data. ACM Sigmod Record. ACM, 2001, Vol. 30, pp. 37–46.
31. Jiang, M.F.; Tseng, S.S.; Su, C.M. Two-phase clustering process for outliers detection. *Pattern Recognition Letters* **2001**, *22*, 691–700.
32. Müller, A.C.; Nowozin, S.; Lampert, C.H. Information theoretic clustering using minimum spanning trees. Joint DAGM (German Association for Pattern Recognition) and OAGM Symp. Springer, 2012, pp. 205–215.
33. Wang, X.; Wang, X.L.; Wilkes, D.M. A minimum spanning tree-inspired clustering-based outlier detection technique. Ind. Conf. on Data Mining. Springer, 2012, pp. 209–223.
34. Wang, G.W.; Zhang, C.X.; Zhuang, J. Clustering with Prim's sequential representation of minimum spanning tree. *Applied Mathematics and Computation* **2014**, *247*, 521–534.
35. Kruskal, J.B. On the shortest spanning subtree of a graph and the traveling salesman problem. *Proc. of the American Mathematical Society* **1956**, *7*, 48–50.
36. Frederiksen, S.; Werner, S. *District heating and cooling*; Vol. 579, Studentlitteratur Lund, 2013; chapter 10, p. 465.
37. Ford, B.L. An overview of hot-deck procedures. *Incomplete data in sample surveys* **1983**, *2*, 185–207.

38. Rubin, D.B. Multiple imputations in sample surveys-a phenomenological Bayesian approach to nonresponse. Proceedings of the survey research methods section of the American Statistical Association. American Statistical Association, 1978, Vol. 1, pp. 20–34.
39. Troyanskaya, O.; Cantor, M.; Sherlock, G.; Brown, P.; Hastie, T.; Tibshirani, R.; Botstein, D.; Altman, R.B. Missing value estimation methods for DNA microarrays. *Bioinformatics* **2001**, *17*, 520–525.
40. Hampel, F.R. A general qualitative definition of robustness. *The Annals of Mathematical Statistics* **1971**, pp. 1887–1896.
41. Pedregosa, F.; Varoquaux, G.; Gramfort, A.; Michel, V.; Thirion, B.; Grisel, O.; Blondel, M.; Prettenhofer, P.; Weiss, R.; Dubourg, V.; Vanderplas, J.; Passos, A.; Cournapeau, D.; Brucher, M.; Perrot, M.; Duchesnay, E. Scikit-learn: Machine Learning in Python. *J. of Machine Learning Research* **2011**, *12*, 2825–2830.
42. Hagberg, A.; Swart, P.; S Chult, D. Exploring network structure, dynamics, and function using NetworkX. Technical report, Los Alamos National Lab.(LANL), Los Alamos, NM (United States), 2008.