6th August 2025 Lecture 14

Ex# 2.2

Q2) ix) $4\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-11=0$

Solution:-

$4\left(x^2+\frac{1}{x^2}\right)-\left(x-\frac{1}{x}\right)-11=0$

Let $x-\frac{1}{x}=y$

$$\left(x-\frac{1}{x}\right)^2=(y)^2$$

$$x^2+\frac{1}{x^2}-2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)=y^2$$

and $x^2+\frac{1}{x^2}=y^2+2$

$$4(y^2+2)-y-11=0$$

$$4y^2+8-y-11=0$$

$$4y^2-y-3=0$$

$$4y^2-4y+3y-3=0$$

$$4y(y-1)+3(y-1)=0$$

$$(4y+3)(y-1)=0$$

$4y+3=0$ or $y-1=0$

$y=\frac{-3}{4}$ or $y=1$

Putting back $y=x-\frac{1}{x}$

Putting back $y = x - \frac{1}{x}$

$x - \frac{1}{x} = -\frac{3}{4}$ or $x - \frac{1}{x} = 1$

$x^2 - 1 = -\frac{3}{4}x$ or $x^2 - 1 = x$

$4(x^2 - 1) = -3x$ or $x^2 - x - 1 = 0$

$4x^2 - 4 = -3x$ or $x^2 - x - 1 = 0$

$4x^2 + 3x - 4 = 0$ or $x^2 - x - 1 = 0$

$a = 4, b = 3, c = -4$ or $a = 1, b = -1, c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-4)}}{2(4)}$ or $x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{9 + 64}}{8}$ or $x = \frac{1 \pm \sqrt{1 + 4}}{2}$

$x = \frac{-3 \pm \sqrt{73}}{8}$ or $x = \frac{1 \pm \sqrt{5}}{2}$

Solution set $= \left\{ \frac{1 \pm \sqrt{5}}{2}, \frac{-3 \pm \sqrt{73}}{8} \right\}$

x) $2^{2x} - 34 \times 2^{x} + 64 = 0$

Solution:-

$2^{2x} - 34 \times 2^{x} + 64 = 0$

let $2^{x} = y$

and $(2^{x})^{2} = (y)^{2}$

$2^{2x} = y^{2}$

$y^{2} - 34y + 64 = 0$

$y^{2} - 32y - 2y + 64 = 0$

$y(y - 32) - 2(y - 32) = 0$

$(y - 32)(y - 2) = 0$

$y - 32 = 0$ or $y - 2 = 0$

$y = 32$ or $y = 2$

Putting back $y = 2^{x}$

$2^{x} = 32$ or $2^{x} = 2$

$2^{x} = 2^{5}$ or $2^{x} = 2^{1}$

$x = 5$ or $x = 1$

Solution set $= \{1, 5\}$

xi) $3^{2x} - 12 \times 3^{x} + 27 = 0$

Solution:-

$3^{2x} - 12 \times 3^{x} + 27 = 0$

Let $3^{x} = y$

$(3^{x})^2 = (y)^2$

and $3^{2x} = y^2$

$y^2 - 12y + 27 = 0$

$y^2 - 9y - 3y + 27 = 0$

$y(y-9) - 3(y-9) = 0$

$(y-9)(y-3) = 0$

$y - 9 = 0$ or $y - 3 = 0$

$y = 9$ or $y = 3$

Putting back $y = 3^{x}$

$3^{x} = 9$ or $3^{x} = 3$

$3^{x} = 3^{2}$ or $3^{x} = 3^{1}$

$x = 2$ or $x = 1$

Solution set = $\{1, 2\}$

xii) $5^{2x} - 150 \times 5^{x} + 3125 = 0$

Solution:-

Let $5^{x} = y$

Solution:

let $5^x = y$

$5^{2x} = y^2$

$y^2 - 150y + 3125 = 0$

$y^2 - 125y - 25y + 3125 = 0$

$y(y-125) - 25(y-125) = 0$

$(y-25)(y-125) = 0$

$y - 25 = 0$ or $y - 125 = 0$

$y = 25$ or $y = 125$

Putting back $y = 5^x$

$5^x = 25$ or $5^x = 125$

$5^x = 5^2$ or $5^x = 5^3$

$x = 2$ or $x = 3$

Solution set = $\{2, 3\}$

xiii) $(x+2)(x-3)(x+10)(x+5) = -396$

Solution:-

$(x+2)(x+5)(x-3)(x+10) = -396$

$[x(x+5) + 2(x+5)][x(x+10) - 3(x+10)] = -396$

$(x^2 + 5x + 2x + 10)(x^2 + 10x - 3x - 30) = -396$

$(\ldots)(x^2 + 7x - 30) = -396$

$(x^2+5x+2x+10)(x^2+10x-3x-3\ldots$

$(x^2+7x+10)(x^2+7x-30) = -396$

let $x^2+7x = y$

$(y+10)(y-30) = -396$

$y(y-30)+10(y-30) = -396$

$y^2-30y+10y-300 = -396$

$y^2-20y-300+396=0$

$y^2-20y+96=0$

$y^2-12y-8y+96=0$

$y(y-12)-8(y-12)=0$

$(y-12)(y-8)=0$

$y-12=0$ or $y-8=0$

$y=12$ or $y=8$

Putting back $y = x^2+7x$

$x^2+7x=12$ or $x^2+7x=8$

$x^2+7x-12=0$ or $x^2+7x-8=0$

$a=1, b=7, c=-12$ or $x^2+8x-x-8=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ or $x(x+8)-1(x+8)=0$

or $(x+8)(x-1)=0$

$x^2+7x-12=0$

$x^2+4x+3x-12=0$

$x(x+4)+3(x-4)$

$2a$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-12)}}{2(1)} \quad \text{or} \quad (x+8)(x-1)=0$$

$$x+8=0 \quad \text{or} \quad x-1=0$$

$$x = \frac{-7 \pm \sqrt{49+48}}{2} \quad \text{or}$$

$$x = \frac{-7 \pm \sqrt{97}}{2} \quad \text{or} \quad x=-8 \quad \text{or} \quad x=1$$

Solutionset $= \left\{ 1, -8, \frac{-7 \pm \sqrt{97}}{2} \right\}$

7th August 2025 Lecture 15

* Ex# 2.2

Q2) xiv) $x(x-1)(x+2)(x+3)=40$

Solution:-

$$x(x+2)(x-1)(x+3)=40$$

$$(x^2+2x)[x(x+3)-1(x+3)]=40$$

$$(x^2+2x)(x^2+3x-x-3)=40$$

$$(x^2+2x)(x^2+2x-3)=40$$

let $x^2+2x=y$

$$y(y-3)=40$$

$y(y-3)=40$

$y^2-3y-40=0$

$y^2-8y+5y-40=0$

$y(y-8)+5(y-8)=0$

$(y-8)(y+5)=0$

$y-8=0$ or $y+5=0$

$y=8$ or $y=-5$

Putting back $y=x^2+2x$

$x^2+2x=8$ or $x^2+2x=-5$

$x^2+2x-8=0$ or $x^2+2x+5=0$

Here $a=1, b=2, c=5$

$x^2+4x-2x-8=0$ or $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x(x+4)-2(x+4)=0$

$(x+4)(x-2)=0$ or $x=\frac{-2\pm\sqrt{(2)^2-4(1)(5)}}{2(1)}$

$x+4=0$ or $x-2=0$ or $x=\frac{-2\pm\sqrt{4-20}}{2}$

$x=-4$ or $x=2$

or $x=\frac{-2\pm\sqrt{-16}}{2}$

$x=\frac{-2\pm\sqrt{-1\times16}}{2}$

$$2$$

$$x = \frac{-2 \pm \sqrt{-1} \times \sqrt{16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$$x = \frac{\cancel{2}(-1 \pm 2i)}{\cancel{2}}$$

$$x = -1 \pm 2i$$

Solution set $= \{2, -4, -1 \pm 2i\}$

xv) $(x-2)(x-6)(x+4)(x+8) + 256 = 0$

Solution:-

$(x-2)(x+4)(x-6)(x+8) + 256 = 0$

$[x(x+4) - 2(x+4)][x(x+8) - 6(x+8)] + 256 = 0$

$(x^2 + 4x - 2x - 8)(x^2 + 8x - 6x - 48) + 256 = 0$

$(x^2 + 2x - 8)(x^2 + 2x - 48) + 256 = 0$

let $y = x^2 + 2x$

$(y-8)(y-48) + 256 = 0$

$y(y-48) - 8(y-48) + 256 = 0$

$y^2 - 48y - 8y + 384 + 256 = 0$

$y^2 - 48y - 8y + 384 + 256 = 0$

$y^2 - 56y + 640 = 0$

$y^2 - 16y - 40y + 640 = 0$

$y(y-16) - 40(y-16) = 0$

$(y-16)(y-40) = 0$

$y - 16 = 0$ or $y - 40 = 0$

$y = 16$ or $y = 40$

Putting back $y = x^2 + 2x$

$x^2 + 2x = 16$ or $x^2 + 2x = 40$

$x^2 + 2x - 16 = 0$ or $x^2 + 2x - 40 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ or $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 1$, $b = 2$, $c = -16$ or Here $a = 1$, $b = 2$, $c = -40$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-16)}}{2(1)}$ or $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-40)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{4 + 64}}{2}$ or $x = \dfrac{-2 \pm \sqrt{4 + 160}}{2}$

$x = \dfrac{-2 \pm \sqrt{68}}{2}$ or $x = \dfrac{-2 \pm \sqrt{164}}{2}$

$$x = \frac{-2 \pm \sqrt{68}}{2} \quad \text{or} \quad x = \frac{-2 \pm \sqrt{164}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \times 17}}{2} \quad \text{or} \quad x = \frac{-2 \pm \sqrt{4 \times 41}}{2}$$

$$x = \frac{-2 \pm \sqrt{4} \times \sqrt{17}}{2} \quad \text{or} \quad x = \frac{-2 \pm \sqrt{4} \times \sqrt{41}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{17}}{2} \quad \text{or} \quad x = \frac{-2 \pm 2\sqrt{41}}{2}$$

$$x = \frac{\cancel{2}(-1 \pm \sqrt{17})}{\cancel{2}} \quad \text{or} \quad x = \frac{\cancel{2}(-1 \pm \sqrt{41})}{\cancel{2}}$$

$$x = -1 \pm \sqrt{17} \quad \text{or} \quad x = -1 \pm \sqrt{41}$$

Solution Set $= \{-1 \pm \sqrt{17},\ -1 \pm \sqrt{41}\}$

8th August 2025 — Lecture 16

* Ex: 2.2

Q3) Solve the following equations by factoring without using substitution.

Without using substitution.

i) $2x^4 - 3x^2 + 1 = 0$

Solution:-

$2x^4 - 3x^2 + 1 = 0$

$2x^4 - 2x^2 - x^2 + 1 = 0$

$2x^2(x^2 - 1) - 1(x^2 - 1) = 0$

$(2x^2 - 1)(x^2 - 1) = 0$

$2x^2 - 1 = 0$ or $x^2 - 1 = 0$

$2x^2 = 1$ or $x^2 = 1$

$x^2 = \frac{1}{2}$ or $x^2 = 1$

$\sqrt{x^{\cancel{2}}} = \sqrt{\frac{1}{2}}$ or $\sqrt{x^{\cancel{2}}} = \sqrt{1}$

$x = \pm \frac{1}{\sqrt{2}}$ or $x = \pm 1$

Solution set $= \left\{ \pm 1, \pm \frac{1}{\sqrt{2}} \right\}$

(ii) $8x^6 - 7x^3 - 1 = 0$ (Find only real roots)

Solution:-

$8x^6 - 7x^3 - 1 = 0$

$8x^6 - 8x^3 + x^3 - 1 = 0$

$8x^3(x^3 - 1) + 1(x^3 - 1) = 0$

$(8x^3 + 1)(x^3 - 1) = 0$

$[(2x)^3 + (1)^3][(x)^3 - (1)^3] = 0$

$(2x+1)(x^3-1)=0$

$$\left[(2x)^3+(1)^3\right]\left[(x)^3-(1)^3\right]=0$$

$\because\ a^3+b^3=(a+b)(a^2-ab+b^2)$

$\because\ a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\left[(2x+1)\left((2x)^2-(2x)(1)+(1)^2\right)\right]\left[(x-1)\left(x^2+(x)(1)+(1)^2\right)\right]=0$$

$$\left[(2x+1)(4x^2-2x+1)\right]\left[(x-1)(x^2+x+1)\right]=0$$

$2x+1=0$ or $x-1=0$

$2x=-1$ or $x=1$

★ $x=\frac{-1}{2}$ or $x=1$

$4x^2-2x+1=0$ or $x^2+x+1=0$

Here $a=4, b=-2, c=1$ or Here $a=1, b=1, c=1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ or $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(4)(1)}}{2(4)}$ or $x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$x=\frac{2\pm\sqrt{4-16}}{8}$ or $x=\frac{-1\pm\sqrt{1-4}}{2}$

$x=\frac{2\pm\sqrt{-12}}{8}$ or $x=\frac{-1\pm\sqrt{-3}}{2}$

$x = \frac{2 \pm \sqrt{-12}}{8}$ or $x = \frac{-1 \pm \sqrt{-3}}{2}$

$x = \frac{2 \pm \sqrt{-1 \times 12}}{8}$ or $x = \frac{-1 \pm \sqrt{-1 \times 3}}{2}$

$x = \frac{2 \pm \sqrt{12}\, i}{8}$ or $x = \frac{-1 \pm \sqrt{3}\, i}{2}$

(Neglected) (Neglected)

Solutionset $= \{1, -\frac{1}{2}\}$

(iii) $x^2 + \frac{1}{x^2} = 2$

Solution:-

$x^2 + \frac{1}{x^2} - 2 = 0$

$(x)^2 + \left(\frac{1}{x}\right)^2 - 2(x)\left(\frac{1}{x}\right) = 0$

$\because a^2 + b^2 - 2ab = (a-b)^2$

$\left(x - \frac{1}{x}\right)^2 = 0$

$\sqrt{\left(x - \frac{1}{x}\right)^{\cancel{2}}} = \sqrt{0}$

$x - \frac{1}{} = 0$

x

$$x \times x - \frac{1}{x} \times x = 0 \times x$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\sqrt{x^{\cancel{2}}} = \sqrt{1}$$

$$x = \pm 1$$

iv) $4 \times 2^{2x} - 4 \times 2^x + 1 = 0$

Solution:-

$$4 \times 2^{2x} - 4 \times 2^x + 1 = 0$$

$$2^2 \times 2^{2x} - 2^2 \times 2^x + 1 = 0$$

$$(2 \times 2^x)^2 - 2(2 \times 2^x)(1) + (1)^2$$

$$\because (a^2 - 2ab + b^2) = (a-b)^2$$

$$(2 \times 2^x - 1)^2 = 0$$

$$\sqrt{(2 \times 2^x - 1)^{\cancel{2}}} = \sqrt{0}$$

$$2 \times 2^x - 1 = 0$$

$$2 \times 2^x = 1$$

$$2^x = \frac{1}{2^1}$$

$$2^x = 2^{-1}$$

$x = -1$

Solution set $= \{-1\}$

9th August 2025 Lecture 17

* Ex# 2.2

Q4) Write two examples of equations that are not quadratic but can be written in quadratic form.

Solution:-

i) $x^4 - 7x^2 + 4 = 0$

This equation is not quadratic because of x^4 term. However, by substituting $y = x^2$ we can rewrite it as a quadratic equation in terms of y i.e

$y^2 - 7y + 4 = 0$

ii) $(x^2+1)^2 - 4(x^2+1) - 5 = 0$

This equation is not quadratic because of binomial in place of single variable. However, by substituting $y = (x^2+1)$ we can rewrite it as a quadratic equation in terms of y i.e

$y^2 - 4y - 5 = 0$

Q5) How would you solve the equation $(y-4)^2 - 7(y-4) = -6$? Write explanation and then solve the equation.

Solution:-

Steps:-

1) Write the equation:
$(y-4)^2 - 7(y-4) = -6$

2) Substitution:
Let $x = y-4$

3) Rewrite the equation:
Replace every instance of $(y-4)$ with x.

4) Solve quadratic equation:
Solve the resulting quadratic equation for x.

5) Back-Substitute:
Substitute back the value of $y-4 = x$

6) Solve for y:
Solve for y to find the solutions to original equation.

Solution:-
$(y-4)^2 - 7(y-4) = -6$

$(y-4)^2 - 7(y-4) = -6$

let $y-4 = x$

$x^2 - 7x = -6$

$x^2 - 7x + 6 = 0$

$x^2 - 6x - x + 6 = 0$

$x(x-6) - 1(x-6) = 0$

$(x-6)(x-1) = 0$

$x-6 = 0$ or $x-1 = 0$

$x = 6$ or $x = 1$

Putting back $y-4 = x$

$y-4 = 6$ or $y-4 = 1$

$y = 6+4$ or $y = 1+4$

$y = 10$ or $y = 5$

Solution set $= \{5, 10\}$

Q6) Solve $y^3 = 125$

Solution:-

$y^3 = 125$

$y^3 - 125 = 0$

$(y)^3 - (5)^3 = 0$

By using formula

$\ldots(\ldots^2 + ab + b^2)$

By using formula

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$(y-5)\left((y)^2+(y)(5)+(5)^2\right) = 0$$

$$(y-5)(y^2+5y+25)=0$$

$y-5=0$ or $y^2+5y+25=0$

Here $a=1$, $b=5$, $c=25$

$y=5$ or

$$y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$y = \frac{-5 \pm \sqrt{(5)^2-4(1)(25)}}{2(1)}$$

$$y = \frac{-5 \pm \sqrt{25-100}}{2}$$

$$y = \frac{-5 \pm \sqrt{-75}}{2}$$

$$y = \frac{-5 \pm \sqrt{-1 \times 75}}{2}$$

$$y = \frac{-5 \pm i\sqrt{75}}{2}$$

$$y = \frac{-5 \pm i\sqrt{3 \times 25}}{}$$

2

$$y = \frac{-5 \pm 5i\sqrt{3}}{2}$$

$$y = \frac{-5 \pm 5\sqrt{3}\,i}{2}$$

Solution set $= \left\{ 5, \ \frac{-5 \pm 5\sqrt{3}}{2} i \right\}$

★ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discrimincant $= b^2 - 4ac$

Nature $= ?$

disc < 0 (negative) $\rightarrow$ imaginary

disc $= 0$ $\rightarrow$ Rational, equal, repeated

disc > 0 (positive, perfect square) $\sqrt{25}, \sqrt{4}$

$\downarrow$

rational, unequal

disc > 0 (positive, not perfect square)

$\downarrow$

irrational, unequal $\sqrt{11}, \sqrt{2}$

★ Example:- Pg 43

[illegible]

* Example:- Pg 45

Find the value of m, when $x^2-3x+m=0$ has equal roots.

Solution:-

$$x^2-3x+m=0$$

Here $a=1$, $b=-3$, $c=m$

$$\text{Disc} = b^2-4ac = 0$$

$$(-3)^2-4(1)(m)=0$$

$$9-4m=0$$

$$9=4m$$

$$m=\frac{9}{4}$$