13th August 2025 Lecture 18

Wednesday, August 13, 2025 8:18 PM

* Ex # 2.3

Q1) Find the discriminant of the following quadratic equations.

i) $x^2 + 6x - 27 = 0$

$a = 1$, $b = 6$, $c = -27$

$$\begin{aligned} Disc &= b^2 - 4ac \\ &= (6)^2 - 4(1)(-27) \\ &= 36 + 108 \\ &= 144 \end{aligned}$$

ii) $x^2 - x - 12 = 0$

$a = 1$, $b = -1$, $c = -12$

$$\begin{aligned} Disc &= b^2 - 4ac \\ &= (-1)^2 - 4(1)(-12) \\ &= 1 + 48 \\ &= 49 \end{aligned}$$

(iii) $8x^2 + 2x + 1 = 0$

$a = 8$, $b = 2$, $c = 1$

$$\begin{aligned} Disc &= b^2 - 4ac \\ &= (2)^2 - 4(8)(1) \\ &= 4 - 32 \\ &= -28 \end{aligned}$$

iv) $12x^2-11x-15=0$

$a=12,\ b=-11,\ c=-15$

$\text{Disc} = b^2-4ac$

$= (-11)^2-4(12)(-15)$

$= 121+720$

$= 841$

Q2) i) $x^2-2x-15=0$

$a=1,\ b=-2,\ c=-15$

$\Rightarrow \text{Disc} = b^2-4ac$

$= (-2)^2-4(1)(-15)$

$= 4+60$

$= 64$

Disc $= 64 > 0$ and perfect square

So, the given equation has two real roots and they are rational and unequal

ii) $x^2+3x-4=0$

Here $a=1,\ b=3,\ c=-4$

$\Rightarrow \text{Disc} = b^2-4ac$

$= (3)^2-4(1)(-4)$

$= 9+16$

$= 25$

Disc $= 25 > 0$ and perfect square

Disc $= 25 > 0$ and perfect square
So, the given equation has two real roots and they are rational and unequal.

(iii) $12x^2 + x - 20 = 0$
$a = 12, b = 1, c = -20$
$\Rightarrow \text{Disc} = b^2 - 4ac$
$= (1)^2 - 4(12)(-20)$
$= 1 + 960$
$= 961$
Disc $= 961 > 0$ and perfect square
So, the given equation has two real roots and they are rational and unequal.

iv) $x^2 + 2x + 8 = 0$
$a = 1, b = 2, c = 8$
$\Rightarrow \text{Disc} = b^2 - 4ac$
$= (2)^2 - 4(1)(8)$
$= 4 - 32$
$= -28$
Disc $= -28 < 0$
So, the given equation has two imaginary roots.

v) $x^2 + 3x - 9 = 0$

$a=1, b=3, c=-9$

$Disc = b^2 - 4ac$

$= (3)^2 - 4(1)(-9)$

$= 9 + 36$

$= 45$

$Disc = 45 > 0$ and not perfect square

So, the given equation has two real roots and they are irrational and unequal.

Q3) Solution:-

$9x^2 - Kx + 16 = 0$

$a=9, b=-K, c=16$

$Disc = b^2 - 4ac$

$= (-K)^2 - 4(9)(16)$

$= K^2 - 576$

Given expression is perfect square. Therefore roots are rational and equal.

$K^2 - 576 = 0$

$K^2 = 576$

$\sqrt{K^{\cancel{2}}} = \sqrt{576}$

$K = \pm 24$

Q4) Solution:-

$$x^2 + kx + 9 = 0$$

Here $a = 1, b = k, c = 9$

$$\text{Disc} = b^2 - 4ac$$
$$= (k)^2 - 4(1)(9)$$
$$= k^2 - 36$$

It is given the equation has equal roots, So

$$k^2 - 36 = 0$$
$$k^2 = 36$$
$$\sqrt{k^{\cancel{2}}} = \sqrt{36}$$
$$k = \pm 6$$

Q5) Solution:-

$$2x^2 + (mx - 1)^2 = 3$$

$\because (a-b)^2 = a^2 + b^2 - 2ab$

$$2x^2 + [(mx)^2 + (1)^2 - 2(mx)(1)] = 3$$
$$2x^2 + m^2x^2 + 1 - 2mx = 3$$
$$x^2(2 + m^2) - 2mx + 1 - 3 = 0$$
$$x^2(2 + m^2) - 2mx - 2 = 0$$

$x^2(2+m^2) - 2mx - 2 = 0$

$(m^2+2)x^2 - 2mx - 2 = 0$

Here $a = m^2+2$, $b = -2m$, $c = -2$

$$\begin{aligned} \text{Disc} &= b^2 - 4ac \\ &= (-2m)^2 - 4(m^2+2)(-2) \\ &= 4m^2 + 8m^2 + 16 \\ &= 12m^2 + 16 \end{aligned}$$

For the roots to be equal, the disc of equation must be 0.

$$12m^2 + 16 = 0$$

$$4(3m^2+4) = 0$$

$$\frac{\cancel{4}(3m^2+4)}{\cancel{4}} = \frac{0}{4}$$

$$3m^2 + 4 = 0$$

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* Ex# 2.3

Q6) (i) $x^2 - 6x + m = 0$

Here $a = 1$, $b = -6$, $c = m$

Disc $= b^2 - 4ac$

$$\text{Disc} = b^2 - 4ac$$
$$= (-6)^2 - 4(1)(m)$$
$$= 36 - 4m$$

Since roots are equal, then

$$36 - 4m = 0$$
$$36 = 4m$$
$$\frac{\cancel{36}^{\,9}}{\cancel{4}_{\,1}} = m$$
$$m = 9$$

(ii) $m^2x^2 + (2m+1)x + 1 = 0$

Here $a = m^2$, $b = 2m+1$, $c = 1$

$$\text{Disc} = b^2 - 4ac$$
$$= (2m+1)^2 - 4(m^2)(1)$$

$\because (a+b)^2 = a^2 + b^2 + 2ab$

$$= \left[(2m)^2 + (1)^2 + 2(2m)(1)\right] - 4m^2$$
$$= \cancel{4m^2} + 1 + 4m - \cancel{4m^2}$$
$$= 1 + 4m$$

Since roots are equal, then disc $= 0$

$$\Rightarrow 1 + 4m = 0$$
$$4m = -1$$
$$m = -1$$

$4m = -1$

$m = \frac{-1}{4}$

(iii) $(m+3)x^2 + (m+1)x + m+1 = 0$

Here $a = m+3$, $b = m+1$, $c = m+1$

$\text{Disc} = b^2 - 4ac$

$= (m+1)^2 - 4(m+3)(m+1)$

$= [(m)^2 + (1)^2 + 2(m)(1)] - 4[m(m+1)+3(m+1)]$

$= m^2 + 1 + 2m - 4(m^2 + m + 3m + 3)$

$= m^2 + 1 + 2m - 4(m^2 + 4m + 3)$

$= m^2 + 1 + 2m - 4m^2 - 16m - 12$

$= -3m^2 - 14m - 11$

$= -1(3m^2 + 14m + 11)$

$\text{Disc} = 3m^2 + 14m + 11$

Since roots are equal, then disc $= 0$

$3m^2 + 14m + 11 = 0$

$3m^2 + 11m + 3m + 11 = 0$

$m(3m+11) + 1(3m+11) = 0$

$(3m+11)(m+1) = 0$

$(3m+11)(m+1)=0$

$3m+11=0 \quad \text{or} \quad m+1=0$

$m=\frac{-11}{3} \quad \text{or} \quad m=-1$

Q7) Solution:-

$(a^2+b^2)x^2+2(ac+bd)x+c^2+d^2=0$

Here $a=a^2+b^2$, $b=2(ac+bd)$, $c=c^2+d^2$

Disc $= b^2-4ac$

$= [2(ac+bd)]^2-4(a^2+b^2)(c^2+d^2)$

$=4[(ac)^2+(bd)^2+2(ac)(bd)]-4[a^2(c^2+d^2)+b^2(c^2+d^2)]$

$=4a^2c^2+4b^2d^2+8abcd-4(a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$=\cancel{4a^2c^2}+\cancel{4b^2d^2}+8abcd-\cancel{4a^2c^2}-4a^2d^2-4b^2c^2-\cancel{4b^2d^2}$

$=8abcd-4a^2d^2-4b^2c^2$

$=-4a^2d^2-4b^2c^2+8abcd$

$=-(4a^2d^2+4b^2c^2-8abcd)$

$= -(4a^2d^2 + 4b^2c^2 - 8abcd)$

$= -\left[(2ad)^2 + (2bc)^2 - 2(2ad)(2bc)\right]$

$\because a^2+b^2-2ab = (a-b)^2$

$= -(2ad - 2bc)^2 < 0$

Disc < 0, So roots are imaginary

* For repeated roots, put $ad = bc$

$= -(2bc - 2bc)^2$

$= -0$

$b^2 - 4ac = 0$

So roots are repeated

Q8) Solution:-

$(ax+c)^2 = 4bx$

$\left[(ax)^2 + (c)^2 + 2(ax)(c)\right] = 4bx$

$a^2x^2 + 2acx + c^2 - 4bx = 0$

$a^2x^2 + (2ac - 4b)x + c^2 = 0$

Here $a = a^2$, $b = 2ac - 4b$, $c = c^2$

$$\text{Disc} = b^2 - 4ac$$
$$= (2ac-4b)^2 - 4(a^2)(c^2)$$
$$= \left[(2ac)^2 + (4b)^2 - 2(2ac)(4b)\right] - 4a^2c^2$$
$$= \cancel{4a^2c^2} + 16b^2 - 16abc - \cancel{4a^2c^2}$$
$$= 16b^2 - 16abc$$

Since roots are equal, then disc = 0

$$16b^2 - 16abc = 0$$
$$\frac{\cancel{16b}(b-ac)}{\cancel{16b}} = \frac{0}{16b}$$
$$b - ac = 0$$
$$b = ac$$

Hence proved