

★ Ex# 3.3

$$\text{Q1) i) } (a, -b) = (7, 1)$$

Solution

$$a = 7 \quad , \quad -b = 1$$

$$a = 7 \quad , \quad b = -1$$

$$\text{(ii) } (2a, 2b+3) = (-10, -b)$$

$$2a = -10 \quad , \quad 2b+3 = -b$$

$$\frac{2a}{2} = \frac{-10}{2} \quad , \quad 2b+b+3 = 0$$

$$a = -5 \quad , \quad 3b = -3$$

$$a = -5 \quad , \quad b = \frac{-3}{3}$$

$$a = -5 \quad , \quad b = -1$$

$$\text{(iii) } (2a-4, 6) = (8, -b+1)$$

Solution:-

$$2a-4 = 8 \quad , \quad 6 = -b+1$$

$$2a = 8+4 \quad , \quad -b = 6-1$$

$$2a = 12 \quad , \quad -b = 5$$

$$2a = 12 \quad ; \quad -b = 5$$

$$a = \frac{12}{2} = 6 \quad ; \quad b = -5$$

$$a = 6 \quad , \quad b = -5$$

iv) $(x+2y, y-3) = (2, 5)$

Solution:-

$$x+2y = 2 \quad \text{---(i)}$$

$$y-3 = 5 \quad \text{---(ii)}$$

from eq (ii)

$$y-3 = 5$$

$$y-8+3 = 5+3$$

$$y = 8$$

Putting value of y in eq (i)

$$x+2y = 2$$

$$x+2(8) = 2$$

$$x+16 = 2$$

$$x = 2 - 16$$

$$x = -14$$

$$x = -14, y = 8$$

v) $(2x-y, y-3x) = (4, 2)$

Solution:-

$$2x - y = 4 \quad \text{--- (i)}$$

$$y - 3x = 2 \quad \text{--- (ii)}$$

Adding Eq (i) and (ii)

$$\begin{array}{r} 2x - y = 4 \\ -3x + y = 2 \\ \hline -x = 6 \end{array}$$

$$\Rightarrow x = -6$$

Putting value of x in Eq (ii)

$$y - 3x = 2$$

$$y - 3(-6) = 2$$

$$y + 18 = 2$$

$$y = 2 - 18$$

$$y = -16$$

$$x = -6, y = -16$$

$$\text{vi) } (4x + 6y, x - 12y) = (6, -3)$$

Solution:-

$$4x + 6y = 6 \quad \text{--- (i)}$$

$$x - 12y = -3 \quad \text{--- (ii)}$$

Multiplying 2 with Eq (i)

Multiplying 2 with eq (i)

$$2(4x + 6y = 6)$$

$$8x + 12y = 12 \quad \text{--- (iii)}$$

Adding eq (ii) and (iii)

$$\begin{array}{r} x - 12y = -3 \\ 8x + 12y = 12 \\ \hline 9x \qquad \qquad = 9 \end{array}$$

$$\Rightarrow 9x = 9$$

$$x = \frac{9}{9}$$

$$x = 1$$

Putting value of x in eq (i)

$$4x + 6y = 6$$

$$4(1) + 6y = 6$$

$$4 + 6y = 6$$

$$6y = 6 - 4$$

$$6y = 2$$

$$y = \frac{2}{6}$$

$$y = \frac{1}{3}$$

$$y = \frac{1}{3}$$

$$\Rightarrow x = 1, \quad y = \frac{1}{3}$$

vii) $(5x+y, -x+y) = (6, 1)$

Solution:-

$$5x+y = 6 \quad \text{--- (i)}$$

$$-x+y = 1 \quad \text{--- (ii)}$$

Subtract eq (ii) from (i)

$$\begin{array}{r}
 5x + y = 6 \\
 -x + y = 1 \\
 \hline
 6x = 5
 \end{array}$$

$$\Rightarrow 6x = 5$$

$$x = \frac{5}{6}$$

Putting value of x in eq (ii)

$$-x+y = 1$$

$$-\frac{5}{6} + y = 1$$

$$y = 1 + \frac{5}{6}$$

$$y = \frac{6+5}{6}$$

$$J^- = \frac{6}{6}$$

$$y = \frac{11}{6}$$

$$\Rightarrow x = \frac{5}{6}, \quad y = \frac{11}{6}$$

Q2) Let $A = \{1, 4, 8\}$, $B = \{1, 0\}$. Find

(i) $A \times B$

Solution:-

$$A \times B = \{1, 4, 8\} \times \{1, 0\}$$

$$= \{(1, 1), (1, 0), (4, 1), (4, 0), (8, 1), (8, 0)\}$$

$$\text{Number of elements in } A \times B = n(A \times B)$$

$$= 3 \times 2$$

$$= 6$$

(ii) $B \times A$

Solution:-

$$B \times A = \{1, 0\} \times \{1, 4, 8\}$$

$$= \{(1, 1), (1, 4), (1, 8), (0, 1), (0, 4), (0, 8)\}$$

$$\text{Number of elements in } B \times A = n(B \times A)$$

$$= 2 \times 3$$

$$= 6$$

(iii) $A \times A$

Solution:-

$$\begin{aligned}A \times A &= \{1, 4, 8\} \times \{1, 4, 8\} \\&= \{(1, 1), (1, 4), (1, 8), (4, 1), (4, 4), \\&\quad (4, 8), (8, 1), (8, 4), (8, 8)\}\end{aligned}$$

$$\begin{aligned}\text{Number of elements in } A \times A &= n(A \times A) \\&= 3 \times 3 \\&= 9\end{aligned}$$

iv) $B \times B$

Solution:-

$$\begin{aligned}B \times B &= \{1, 0\} \times \{1, 0\} \\&= \{(1, 1), (1, 0), (0, 1), (0, 0)\}\end{aligned}$$

$$\begin{aligned}\text{Number of elements in } B \times B &= n(B \times B) \\&= 2 \times 2 \\&= 4\end{aligned}$$



* Ex # 3.3

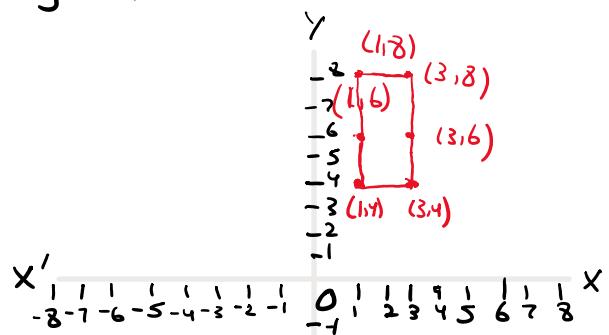
(Q3) Let $E = \{1, 3\}$ and $F = \{4, 6, 8\}$. Express $E \times F$, $F \times E$, $E \times E$, $F \times F$ graphically.

1) $E \times F$

Solution:-

$$E \times F = \{1, 3\} \times \{4, 6, 8\}$$

$$E \times F = \{(1, 4), (1, 6), (1, 8), (3, 4), (3, 6), (3, 8)\}$$

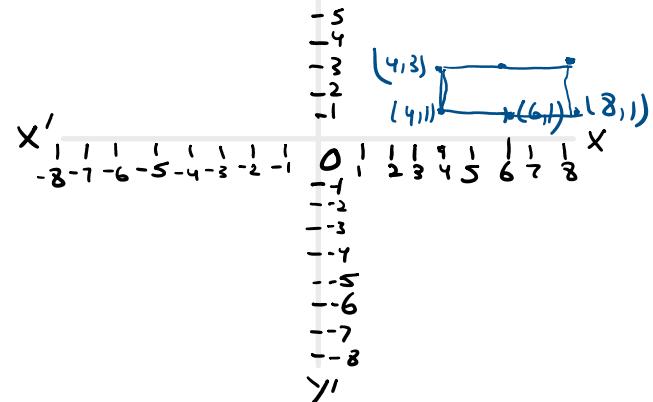


$F \times E$

Solution:-

$$F \times E = \{4, 6, 8\} \times \{1, 3\}$$

$$= \{(4, 1), (4, 3), (6, 1), (6, 3), (8, 1), (8, 3)\}$$

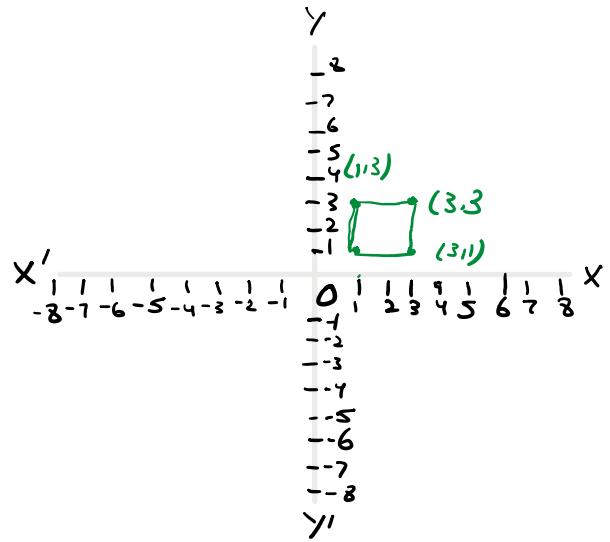


$E \times E$

Solution:-

$$E \times E = \{1, 3\} \times \{1, 3\}$$

$$= \{(1, 1), (1, 3), (3, 1), (3, 3)\}$$



$F \times F$

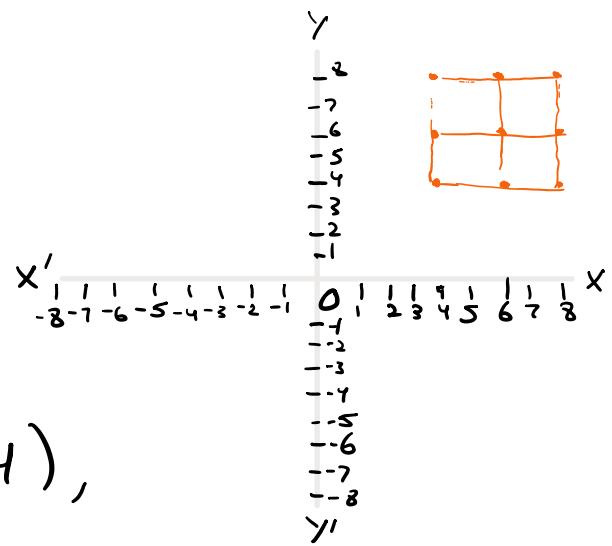
Solution:-

$$F \times F = \{4, 6, 8\} \times \{4, 6, 8\}$$

$$= \{(4, 4), (4, 6), (4, 8),$$

$$(6, 4), (6, 6), (6, 8), (8, 4),$$

$$(8, 6), (8, 8)\}$$



Q4) $L \times M = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$, find $L, M, M \times L$

Solution:-

$$L = \{0, 1\}, M = \{2, 3, 4\}$$

$$M \times L = \{2, 3, 4\} \times \{0, 1\}$$

$$= \{(2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)\}$$

$$(Q5) A = \{1, 3, 5\}, B = \{2, 4\}, C = \{6, 7\}$$

$$(i) A \times (B \cup C)$$

Solution:-

$$\begin{aligned} B \cup C &= \{2, 4\} \cup \{6, 7\} \\ &= \{2, 4, 6, 7\} \end{aligned}$$

$$\begin{aligned} A \times (B \cup C) &= \{1, 3, 5\} \times \{2, 4, 6, 7\} \\ &= \{(1, 2), (1, 4), (1, 6), (1, 7), \\ &\quad (3, 2), (3, 4), (3, 6), (3, 7), \\ &\quad (5, 2), (5, 4), (5, 6), (5, 7)\} - (i) \end{aligned}$$

$$(ii) \text{ Find } (A \times B) \cup (A \times C)$$

$$\begin{aligned} \Rightarrow (A \times B) &= \{1, 3, 5\} \times \{2, 4\} \\ &= \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)\} \end{aligned}$$

$$\begin{aligned} \Rightarrow (A \times C) &= \{1, 3, 5\} \times \{6, 7\} \\ &= \{(1, 6), (1, 7), (3, 6), (3, 7), (5, 6), \\ &\quad (5, 7)\} \end{aligned}$$

$$\begin{aligned} (A \times B) \cup (A \times C) &= \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), \\ &\quad (5, 4)\} \cup \{(1, 6), (1, 7), (3, 6), (3, 7), (5, 6), \\ &\quad (5, 7)\} \end{aligned}$$

$$\begin{aligned}
 & (5,7) \\
 &= \{(1,2), (1,4), (1,6), (1,7), (3,2), (3,4), \\
 &\quad (3,6), (3,7), (5,2), (5,4), (5,6), (5,7)\} \\
 &\qquad\qquad\qquad\text{`}(ii)
 \end{aligned}$$

(iii) Verify $A \times (B \cup C) = (A \times B) \cup (A \times C)$
from (i) and (ii)

$$\text{L.H.S} = \text{R.H.S}$$

$$A \times (B \cup C) = (A \times B) \cup (A \times C) \quad (\text{Proved})$$

$$\text{Q6) } D = \{a, e, i\}, E = \{a, c\}, F = \{b, c\}$$

$$(i) \text{ Find } D \times (E \cap F)$$

Solution:-

$$\begin{aligned}
 E \cap F &= \{a, c\} \cap \{b, c\} \\
 &= \{c\}
 \end{aligned}$$

$$\begin{aligned}
 D \times (E \cap F) &= \{a, e, i\} \times \{c\} \\
 &= \{(a, c), (e, c), (i, c)\} - (i)
 \end{aligned}$$

$$(ii) \text{ Find } (D \times E) \cap (D \times F)$$

Solution:-

$$(D \times E) = \{a, e, i\} \times \{a, c\}$$

SOLUTION:-

$$\begin{aligned} D \times E &= \{a, e, i\} \times \{a, c\} \\ &= \{(a, a), (a, c), (e, a), (e, c), (i, a), (i, c)\} \end{aligned}$$

$$\begin{aligned} D \times F &= \{a, e, i\} \times \{b, c\} \\ &= \{(a, b), (a, c), (e, b), (e, c), (i, b), (i, c)\} \end{aligned}$$

$$\begin{aligned} (D \times E) \cap (D \times F) &= \{(a, a), (a, c), (e, a), (e, c), (i, a), \\ (i, c)\} \cap \{(a, b), (a, c), (e, b), (e, c), (i, b), \\ (i, c)\} \end{aligned}$$

$$(D \times E) \cap (D \times F) = \{(a, c), (e, c), (i, c)\} - (ii)$$

(iii) Verify $D \times (E \cap F) = (D \times E) \cap (D \times F)$

from (i) and (ii)

$$D \times (E \cap F) = (D \times E) \cap (D \times F)$$

$$L.H.S = R.H.S$$

It is verified



9th August 2025

* Ex: 3.3

Lecture 15

Q7) Given $A = \{x | x \in \mathbb{N}, x < 3\}$, $B = \{y | y \in \mathbb{W}, y < 2\}$, $C = \{0, 2, 4\}$

$$(i) \text{ Verify } A \times (B - C) = (A \times B) - (A \times C)$$

$$(ii) \text{ Verify } (A - B) \times C = (A \times C) - (B \times C)$$

Solution:-

$$A = \{1, 2\}, B = \{0, 1\}, C = \{0, 2, 4\}$$

$$(i) A \times (B - C) = (A \times B) - (A \times C)$$

Taking L.H.S

$$\Rightarrow A \times (B - C)$$

$$\Rightarrow B - C = \{0, 1\} - \{0, 2, 4\} \\ = \{1\}$$

$$\Rightarrow A \times (B - C) = \{1, 2\} \times \{1\} \\ = \{(1, 1), (2, 1)\} - (i)$$

Taking R.H.S

$$(A \times B) - (A \times C)$$

$$\Rightarrow A \times B = \{1, 2\} \times \{0, 1\} \\ = \{(1, 0), (1, 1), (2, 0), (2, 1)\}$$

$$\Rightarrow A \times C = \{1, 2\} \times \{0, 2, 4\} \\ = \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), (2, 4)\}$$

$\therefore \{(1, 1), (2, 1)\} - \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), (2, 4)\} =$

$$\begin{aligned}
 &= \{(1,0), (1,2), (1,4), (2,0), (2,2), (2,4)\} - \\
 (A \times B) - (A \times C) &= \{(1,0), (1,1), (2,0), (2,1)\} - \\
 \{(1,0), (1,2), (1,4), (2,0), (2,2), (2,4)\} \\
 &= \{(1,1), (2,1)\} \quad -(ii)
 \end{aligned}$$

from eq (i) and (ii)

$$LHS = RHS$$

$$A \times (B-C) = (A \times B) - (A \times C)$$

$$(ii) (A-B) \times C = (A \times C) - (B \times C)$$

Solution:-

Taking LHS.

$$\Rightarrow (A-B) \times C$$

$$\begin{aligned}
 \Rightarrow (A-B) &= \{1,2\} - \{0,1\} \\
 &= \{2\}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow (A-B) \times C &= \{2\} \times \{0,2,4\} \\
 &= \{(2,0), (2,2), (2,4)\} \quad -(i)
 \end{aligned}$$

Taking R.H.S

$$\Rightarrow (A \times C) - (B \times C)$$

$$\begin{aligned}
 \Rightarrow A \times C &= \{1,2\} \times \{0,2,4\} \\
 &= \{(1,0), (1,2), (1,4), (2,0), (2,2), (2,4)\}
 \end{aligned}$$

$$\Rightarrow B \times C = \{0,1\} \times \{0,2,4\}$$

$$\begin{aligned}
 \Rightarrow B \times C &= \{0, 1\} \times \{0, 2, 4\} \\
 &= \{(0, 0), (0, 2), (0, 4), (1, 0), (1, 2), (1, 4)\} \\
 (A \times C) - (B \times C) &= \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), \\
 &\quad (2, 4)\} - \{(0, 0), (0, 2), (0, 4), (1, 0), (1, 2), (1, 4)\} \\
 &= \{(2, 0), (2, 2), (2, 4)\} \quad \text{--- (ii)}
 \end{aligned}$$

from (i) and (ii)

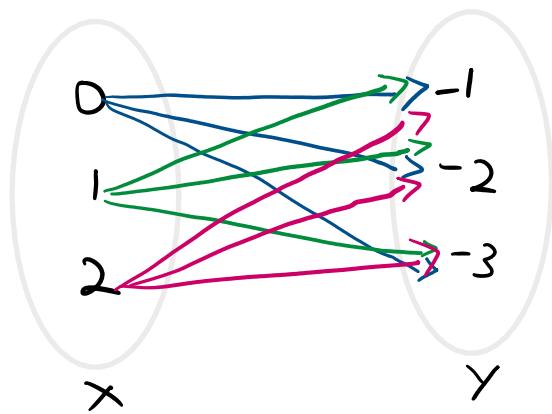
$$\text{LHS} = \text{RHS}$$

$$(A - B) \times C = (A \times C) - (B \times C)$$

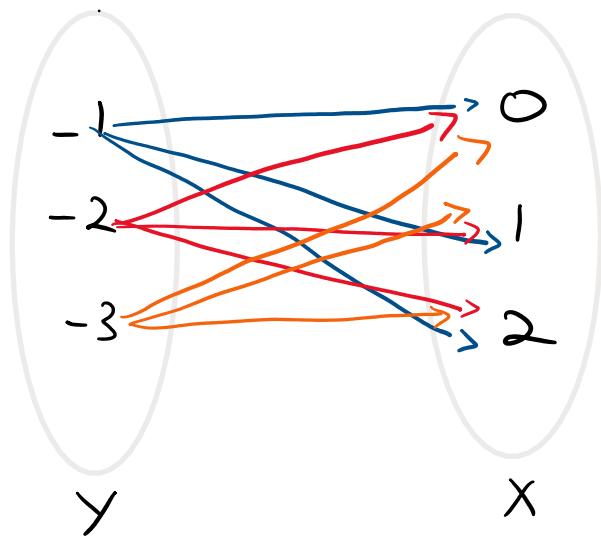
$$\begin{aligned}
 \text{(Q8)} \quad X &= \{x \mid x \in \mathbb{W}, x \leq 2\}, Y = \{-1, -2, -3\} \\
 X \times Y \text{ and } Y \times X \text{ (arrow diagram)}
 \end{aligned}$$

Solution:-

$$\begin{aligned}
 X &= \{0, 1, 2\}, \quad Y = \{-1, -2, -3\} \\
 X \times Y &= \{0, 1, 2\} \times \{-1, -2, -3\} \\
 &= \{(0, -1), (0, -2), (0, -3), (1, -1), (1, -2), \\
 &\quad (1, -3), (2, -1), (2, -2), (2, -3)\} \\
 X \times Y
 \end{aligned}$$



$$\begin{aligned}
 Y \times X &= \{-1, -2, -3\} \times \{0, 1, 2\} \\
 &= \{(-1, 0), (-1, 1), (-1, 2), (-2, 0), (-2, 1), \\
 &\quad (-2, 2), (-3, 0), (-3, 1), (-3, 2)\}
 \end{aligned}$$



* Example 18:-

Pg 63

$$\text{If } P = \{2, 3\}, Q = \left\{\frac{1}{2}, \frac{1}{3}\right\}$$

Solution:-

Number of binary relations in $P \times Q = 2^{n(P \times Q)}$
 $\sim (2 \times 2)$

$$\begin{aligned}
 \text{Number of binary relations in } R \times Q &= 2^{(2 \times 2)} \\
 &= 2^4 \\
 &= 16
 \end{aligned}$$

$$\begin{aligned}
 R \times Q &= \{2, 3\} \times \left\{\frac{1}{2}, \frac{1}{3}\right\} \\
 &= \{(2, \frac{1}{2}), (2, \frac{1}{3}), (3, \frac{1}{2}), (3, \frac{1}{3})\}
 \end{aligned}$$

$$R_1 = \{\}$$

$$R_2 = \{(2, \frac{1}{2})\}$$

$$R_3 = \{(2, \frac{1}{3})\}$$

$$R_4 = \{(3, \frac{1}{2})\}$$

$$R_5 = \{(3, \frac{1}{3})\}$$

$$R_6 = \{(2, \frac{1}{2}), (2, \frac{1}{3})\}$$

$$R_7 = \{(2, \frac{1}{2}), (3, \frac{1}{2})\}$$

$$R_8 = \{(2, \frac{1}{2}), (3, \frac{1}{3})\}$$

$$R_9 = \{(2, \frac{1}{3}), (3, \frac{1}{2})\}$$

$$R_{10} = \{(2, \frac{1}{3}), (3, \frac{1}{3})\}$$

$$R_{11} = \{(3, \frac{1}{2}), (3, \frac{1}{3})\}$$

$$R_{12} = \{(2, \frac{1}{2}), (2, \frac{1}{3}), (3, \frac{1}{2})\}$$

- - - - - / / / / / / / /

$$R_{12} = \{ (2, \frac{1}{2}), (2, \frac{1}{3}), (3, \frac{1}{2}) \}$$

$$R_{13} = \{ (2, \frac{1}{2}), (2, \frac{1}{3}), (3, \frac{1}{3}) \}$$

$$R_{14} = \{ (2, \frac{1}{2}), (3, \frac{1}{2}), (3, \frac{1}{3}) \}$$

$$R_{15} = \{ (2, \frac{1}{3}), (3, \frac{1}{2}), (3, \frac{1}{3}) \}$$

$$R_{16} = \{ (2, \frac{1}{2}), (2, \frac{1}{3}), (3, \frac{1}{2}), (3, \frac{1}{3}) \}$$