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Instructions:

1) ***Provide complete solutions in the space provided.***

2) ***Complete questions 8 and 9 using SCILAB. Copy/paste the output or use screenshot and attach it to this assignment.***

3) ***Late assignments will not be accepted***. **Due date is: October 1st, 2021.**

1. **Use Jordan-Gauss elimination to solve the following system of linear equations: [5]**

$$\begin{aligned} 2x - y \quad -3z &= 6 \\ x - 2y \quad + z &= -9 \\ -x + y \quad -2z &= 1 \end{aligned}$$

1)

$$\begin{aligned} 2x - y - 3z &= 6 \\ x - 2y + z &= -9 \\ -x + y - 2z &= 1 \end{aligned} \Rightarrow \left[\begin{array}{ccc|c} 2 & -1 & -3 & 6 \\ 1 & -2 & 1 & -9 \\ -1 & 1 & -2 & 1 \end{array}\right]$$

$$\begin{aligned} R_1' &= R_1 - R_2 \\ R_2' &= R_2 + R_3 \\ R_3' &= R_3 + R_1' \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & -4 & 15 \\ 0 & -1 & -1 & -8 \\ 0 & 2 & -6 & 16 \end{bmatrix}$$

$$\begin{aligned} R_2'' &= R_2' \times 2 \\ R_3'' &= R_3' - 2R_2' \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & -4 & 15 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & -8 & 0 \end{bmatrix}$$

$\times -1/8$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -4 & 15 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

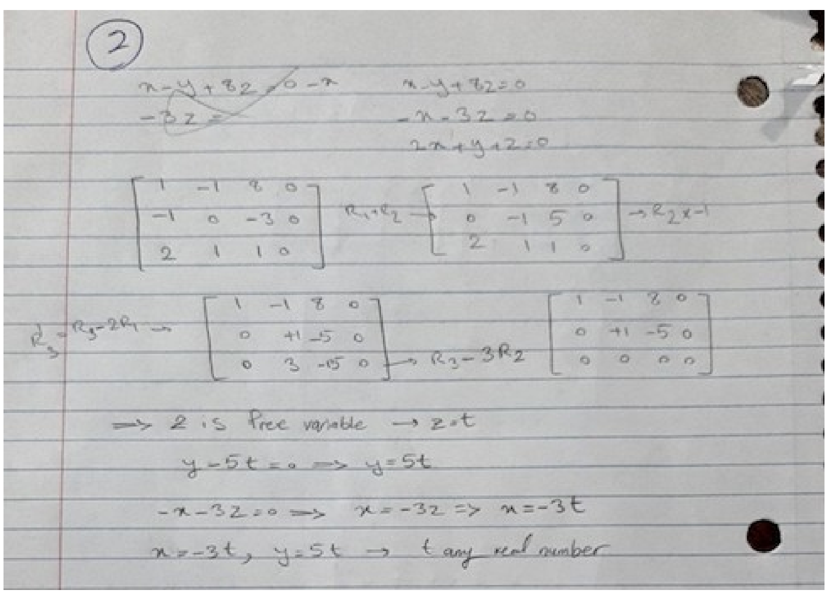
$$\begin{aligned} R_1' &= R_1 + 4R_3 \\ R_2' &= R_2 - R_3 \end{aligned} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 15 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_1' = R_1 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow x = 7,\ y = 8,\ z = 0$$

2. **For the following system of homogenous linear equations:**

$$\begin{aligned} x-y \quad &+8z=0 \\ -x \quad &-3z=0 \\ 2x+y \quad &+z=0 \end{aligned}$$

Use elementary row operations to solve the above system. Identify the free variable(s) if any. [5]



2

$\cancel{x-y+8z=0 \ -x}$
$\cancel{-3z=}$

$x-y+8z=0$
$-x-3z=0$
$2x+y+z=0$

$$\begin{bmatrix} 1 & -1 & 8 & 0 \\ -1 & 0 & -3 & 0 \\ 2 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} 1 & -1 & 8 & 0 \\ 0 & -1 & 5 & 0 \\ 2 & 1 & 1 & 0 \end{bmatrix} \rightarrow R_2 \times -1$$

$$R_3 = R_3 - 2R_1 \rightarrow \begin{bmatrix} 1 & -1 & 8 & 0 \\ 0 & +1 & -5 & 0 \\ 0 & 3 & -15 & 0 \end{bmatrix} \xrightarrow{R_3-3R_2} \begin{bmatrix} 1 & -1 & 8 & 0 \\ 0 & +1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ z is free variable $\rightarrow z=t$

$y-5t=0 \Rightarrow y=5t$

$-x-3z=0 \Rightarrow x=-3z \Rightarrow x=-3t$

$x=-3t,\ y=5t \rightarrow$ t any real number

3. **Given the following matrices:**

$$A=\begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix},\ B=\begin{bmatrix} -1 & 2 \\ -4 & 7 \end{bmatrix}, C=\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix}, D=\begin{bmatrix} 7 & 3 & -2 \\ 0 & 1 & 3 \\ 4 & 1 & 2 \end{bmatrix},$$

a. Prove that **[5]**

3 a

$(AB)^{-1} = B^{-1}A^{-1}$

$$AB = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} \underbrace{[5\times(-1)]}_{-5} + \underbrace{((-3)\times(-4))}_{12} & \underbrace{[5\times 2}_{10} + \underbrace{(-3)\times 7]}_{-21} \\ \underbrace{[2\times(-1)]}_{-2} + \underbrace{[(-1)\times(-4)]}_{4} & \underbrace{[2\times 2}_{4} + \underbrace{(-1)\times 7]}_{-7} \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -11 \\ 2 & -3 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{\cancel{-21-(-22)}} \begin{bmatrix} -3 & +11 \\ -2 & 7 \end{bmatrix} \Rightarrow (AB)^{-1} = \begin{bmatrix} -3 & 11 \\ -2 & 7 \end{bmatrix}$$

$22 - 21$

$$B^{-1} = \frac{1}{\cancel{-7+8}} \begin{bmatrix} 7 & -2 \\ +4 & -1 \end{bmatrix} \qquad A^{-1} = \frac{1}{\cancel{-5+6}} \begin{bmatrix} -1 & +3 \\ -2 & 5 \end{bmatrix} \quad //$$

$$B^{-1}A^{-1} = \begin{bmatrix} 7 & -2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -7+4 & 21-10 \\ -4+2 & 12-5 \end{bmatrix} = \begin{bmatrix} -3 & 11 \\ -2 & 7 \end{bmatrix}$$

b. Prove that **[5]**

3b

$$(CD)^T = D^T C^T$$

$$CD = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 5 \\ -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 7 & 3 & -2 \\ 0 & 1 & 3 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1\times7+3\times0+2\times4 & 1\times3+3\times1+2\times1 & 1\times(-2)+3\times3+2\times2 \\ 4\times7+0\times0+5\times4 & 4\times3+0\times1+5\times1 & 4\times(-2)+0\times3+5\times2 \\ -1\times7+3\times0+2\times4 & -1\times3+3\times1+2\times1 & -1\times(-2)+3\times3+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 8 & 11 \\ 48 & 17 & 2 \\ 1 & 2 & 15 \end{bmatrix} \Rightarrow (CD)^T = \begin{bmatrix} 15 & 48 & 1 \\ 8 & 17 & 2 \\ 11 & 2 & 15 \end{bmatrix}$$

$$D^T = \begin{bmatrix} 7 & 0 & 4 \\ 3 & 1 & 1 \\ -2 & 3 & 2 \end{bmatrix} \qquad C^T = \begin{bmatrix} 1 & 4 & -1 \\ 3 & 0 & 3 \\ 2 & 5 & 2 \end{bmatrix}$$

$$D^T C^T = \begin{bmatrix} 7\times1+0\times3+4\times2 & 7\times4+0\times0+4\times5 & 7\times(-1)+0\times3+4\times2 \\ 3\times1+1\times3+1\times2 & 3\times4+1\times0+1\times5 & 3\times(-1)+1\times3+1\times2 \\ -2\times1+3\times3+2\times2 & -2\times4+3\times0+2\times5 & -2\times(-1)+3\times3+2\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 48 & 1 \\ 8 & 17 & 2 \\ 11 & 2 & 15 \end{bmatrix} \Rightarrow (CD)^T = D^T C^T$$

4. For the following system of non-homogenous linear equations:

$$\begin{aligned} x+y \quad &+3z=-5 \\ -x \quad &+z=2 \\ 3x-2y \quad &-z=1 \end{aligned}$$

a. Use elementary row operations to find A^{-1} **[4]**

4

$x+y+3z=-5$

$-x+z=2$

$3x-2y-z=1$

$$Ax=b \rightarrow \begin{bmatrix} 1 & 1 & 3 \\ -1 & 0 & 1 \\ 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 3 & -2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R'_2 = R_2 + R_1 = 0 \quad 1 \quad 4 \quad 1 \quad 1 \quad 0$

$R'_3 = R_3 - 3R_1 = 0 \quad -5 \quad -10 \quad -3 \quad 0 \quad 1$

$$\rightarrow \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 1 & 0 \\ 0 & -5 & -10 & -3 & 0 & 1 \end{bmatrix}$$

$R''_3 = R'_3 + 5R'_2 \rightarrow 0 \quad 0 \quad 10 \quad 2 \quad 5 \quad 1$

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 1 & 0 \\ 0 & 0 & \frac{10}{10} & \frac{2}{10} & \frac{5}{10} & \frac{1}{10} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{5} & \frac{1}{2} & \frac{1}{10} \end{bmatrix}$$

4: $\frac{4}{5} \quad 2 \quad \frac{4}{10} \quad \frac{2}{5}$

3: $\frac{3}{5} \quad \frac{3}{2} \quad \frac{3}{10}$

$R'_2 = R_2 - 4R_3 \rightarrow 0 \quad 1 \quad 0 \quad \frac{1}{5} \quad -1 \quad -\frac{2}{5}$

$R'_1 = R_1 - 3R_3 \rightarrow 1 \quad 1 \quad 0 \quad \frac{2}{5} \quad -\frac{3}{2} \quad -\frac{3}{10}$

$$\begin{bmatrix} 1 & 1 & 0 & \frac{2}{5} & -\frac{3}{2} & -\frac{3}{10} \\ 0 & 1 & 0 & \frac{1}{5} & -1 & -\frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{1}{2} & \frac{1}{10} \end{bmatrix} \xrightarrow{R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 & \frac{1}{5} & -\frac{1}{2} & +\frac{1}{10} \\ 0 & 1 & 0 & \frac{1}{5} & -1 & -\frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{1}{2} & \frac{1}{10} \end{bmatrix}$$

A^{-1}

b. Use $X = A^{-1}B$ to solve the above system of equation. **[2]**

-- 4

$$X = A^{-1}B = \begin{bmatrix} 1/5 & -1/2 & 1/10 \\ 1/5 & -1 & -2/5 \\ 1/5 & 1/2 & 1/10 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -19/10 \\ -17/5 \\ 1/10 \end{bmatrix} \Rightarrow \quad x = \frac{-19}{10},\ y = \frac{-17}{10},\ z = \frac{1}{10}$$

$$1/5 \times (-5) + (-1/2) \times 2 + 1/10 \times 1 = -1 - 1 + 1/10 = \frac{-19}{10}$$

$$1/5 \times (-5) + (-1) \times 2 + (-2/5) = -1 - 2 - 2/5 = -3 - 2/5 = -17/5$$

$$1/5 \times (-5) + (1/2) \times 2 + 1/10 = -1 + 1 + 1/10 = 1/10$$

5. Given the following non-invertible matrices, solve for m and n, where:

$$M = \begin{bmatrix} 5m+2n-6 & 3 \\ 0 & 8 \end{bmatrix}, N = \begin{bmatrix} -2 & 0 \\ 15 & 4m-3n+7 \end{bmatrix}$$

[4]

(5) $M = \begin{bmatrix} 5m+2n-6 & 3 \\ 0 & 8 \end{bmatrix} \quad N = \begin{bmatrix} -2 & 0 \\ 15 & 4m-3n+7 \end{bmatrix}$

M & N are non-invertible $\Rightarrow ad - bc = 0$

$$\Rightarrow \begin{aligned} 5m + 2n - 6 &= 0 \\ 4m - 3n + 7 &= 0 \end{aligned} \Rightarrow \begin{array}{c} 3\times \\ 2\times \end{array} \begin{bmatrix} 5 & 2 & 6 \\ 4 & -3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 15 & 6 & 18 \\ 8 & -6 & -14 \end{bmatrix} \xrightarrow{R_2' = R_1 + R_2} \begin{bmatrix} 15 & 6 & 18 \\ 23 & 0 & 4 \end{bmatrix}$$

$\Rightarrow 23m = 4 \Rightarrow m = 4/23$

$\frac{20}{23} + 2n = 6 \rightarrow n = 59/23$

6. Given the following information find A:

a) $(A^2)^{-1} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 49 \end{bmatrix}$ **[2]**

b) $(3A^{-1})^T = \begin{bmatrix} 0 & 1 \\ 7 & -3 \end{bmatrix}$ **[3]**

6

a) $(A^2)^{-1} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 49 \end{bmatrix}$

$((A^2)^{-1})^{-1} = \begin{bmatrix} 1/9 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/49 \end{bmatrix} = A^2$

$A = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & 1/7 \end{bmatrix}$

b) $(3A^{-1})^T = \begin{bmatrix} 0 & 1 \\ 7 & -3 \end{bmatrix}$

$3A^{-1} = ((3A^{-1})^T)^T = \begin{bmatrix} 0 & 7 \\ 1 & -3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 7/3 \\ 1/3 & -1 \end{bmatrix}$

$A = (A^{-1})^{-1}$

$\Rightarrow A = \frac{1}{0 - 7/9} \begin{bmatrix} -1 & -7/3 \\ -1/3 & 0 \end{bmatrix} = \frac{-9}{7} \begin{bmatrix} -1 & -7/3 \\ -1/3 & 0 \end{bmatrix}$

$A = \begin{bmatrix} +9/7 & +3 \\ 3/7 & 0 \end{bmatrix}$

7. Use the *Adjoint Theorem* to find the inverse of the matrix A.

[4]

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

7

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 0 & 2 \\ 1 & 1 & -1 \end{bmatrix} \qquad A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

$$\det(A) = 2 \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} + 0 \cancel{\begin{vmatrix} & \end{vmatrix}}$$

$$= 2\underbrace{(0-2)}_{-4} + \underbrace{(1-2)}_{-1} = -5$$

$C_{11} = -2$ $\quad C_{12} = +1$ $\quad C_{13} = -1$

$C_{21} = -1$ $\quad C_{22} = -2$ $\quad C_{23} = -3$

$C_{31} = -2$ $\quad C_{32} = -4$ $\quad C_{33} = -1$

$$\rightarrow \begin{bmatrix} -2 & 1 & -1 \\ -1 & -2 & -3 \\ -2 & -4 & -1 \end{bmatrix}$$

$$\operatorname{Adj}(A) = \begin{bmatrix} -2 & -1 & -2 \\ 1 & -2 & -4 \\ -1 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2/5 & 1/5 & 2/5 \\ -1/5 & 2/5 & 4/5 \\ 1/5 & 3/5 & 1/5 \end{bmatrix}$$

8. Use *SCILAB* to solve the following system of equations: **[3]**

$$\begin{aligned} -2x - 3y + z - 7w &= 6 \\ x - 5y - 2z + w &= -7 \\ 3x + 2y - 2z - 5w &= -10 \\ x - y + 6z + 10w &= 45 \end{aligned}$$

```
--> A= [-2 -3 1 -7 6; 1 -5 -2 1 -7; 3 2 -2 -5 -10; 1 -1 6 10 45]
 A  =

  -2.  -3.   1.  -7.    6.
   1.  -5.  -2.   1.   -7.
   3.   2.  -2.  -5.   -10.
   1.  -1.   6.   10.   45.


--> rref(A)
 ans  =

   1.   0.   0.   0.   2.
   0.   1.   0.   0.  -1.
   0.   0.   1.   0.   7.
   0.   0.   0.   1.   0.
```

x = 2, y = -1, z = 7, w = 1

9. Use *SCILAB* to solve the following systems simultaneously: [3]

$$2x - y + z = b_1$$

$$-x + 3y + z = b_2$$

$$x - 2z = b_3$$

a) $b_1 = 0\, b_2 = -5\, b_3 = 13$

b) $b_1 = -5\, b_2 = 5\, b_3 = -2$

```
--> B = [2 -1 1 0 -5; -1 3 1 -5 5; 1 0 -2 13 -2]
 B  =

   2.  -1.   1.   0.   -5.
  -1.   3.   1.  -5.    5.
   1.   0.  -2.   13.  -2.


--> rref(B)
 ans  =

   1.   0.   0.   3.  -2.
   0.   1.   0.   1.   1.
   0.   0.   1.  -5.   0.
```

x = 3 , y = 1 , z = -5 and x = -2, y= 1, z = 0