

① Write a short note on (a) Post Correspondence problem

(b) Classes of P and NP

(c) Quantum Computation with Example

② Prove that Every language that is accepted in multi-tape Turing machine is also accepted by Standard Turing machine.

① (a) defn: The PCP can be Stated as follows:

Given two Sequence of n Strings on Some Alphabet Σ Says

$A = w_1, w_2, w_3, \dots, w_n$ and $B = v_1, v_2, v_3, \dots, v_n$

We say that there exists a post Correspondence Solution for pair (A, B) if there is a non empty Sequence of integers i_1, i_2, \dots, i_k such that

$w_{i_1} w_{i_2} \dots w_{i_k} = v_{i_1} v_{i_2} \dots v_{i_k}$

The post Correspondence problem is to device an algorithm, that will inform us, for any (A, B) whether or not there exists a PC-Solution.

(b) classes of P and NP

def: A TM M is said to be of Complexity $T(n)$ if the following holds: Given an i/p w of length n , M halts after making at most $T(n)$ moves

def: A language L is in class P if there exists Some polynomial $T(n)$ such that $L \in T(n)$ for Some deterministic TM M of time complexity $T(n)$

def: A language L is in class NP if there is a non deterministic TM M and a polynomial time Complexity $T(n)$ such that $L = T(M)$ and M Executes at most $T(n)$ moves for every i/p w of length n .

(c) def. A quantum computer is a system built from quantum computers circuits, containing wires and elementary quantum gates to carry out manipulation of quantum information.

The digital computers data is represented using binary states 0 & 1 whereas the data in quantum computers is represented mathematically as shown below:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Example: a two-qubit system has four computational basis states, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ & quantum states are represented as:

$$|\psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

where

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

2) Proof: Every language L is accepted by a k -tape TM M , we simulate M with a single tape TM with $2k$ tracks. The second, fourth, ... $(2k)^{\text{th}}$ tracks hold the contents of the k -tapes. The first, third, ... $(2k-1)^{\text{th}}$ tracks hold a head marker to indicate the position of the respective tape head.

In the figure below the symbols A_2 and B_5 are the current symbols to be scanned and so the head marker X is above the two symbols.

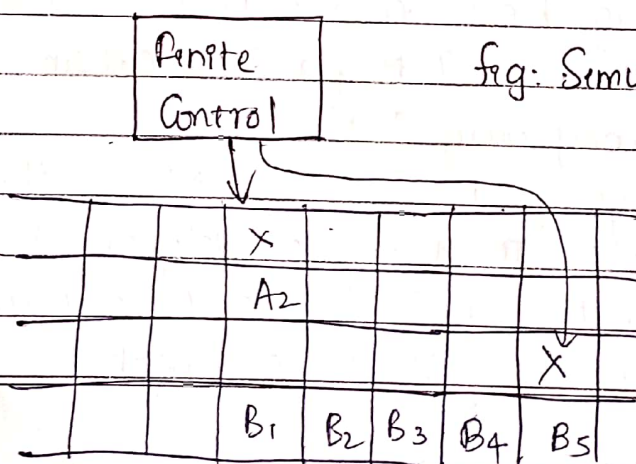


Fig: Simulation of multi-tape TM

Initially the contents of tapes 1 and 2 of M are stored on the second and fourth tracks of M_1 . The head markers of the first and third tracks are at the cells containing the first symbol.

To simulate a move of M , the 2k track TM M_1 has to visit the two head markers and store the scanned symbols in its control - keeping it in the finite control of M_1 . The finite control of M_1 has also the information about the states of M and its moves. After visiting both head markers, M_1 knows the tape symbols being scanned by the two heads of M .

Now M_1 revisits each of the head markers.

- (i) it changes its tape symbol in the corresponding track of M_1 based on the information regarding the move of M corresponding to the state (of M) and its tape symbol in the corresponding tape M .
- (ii) it moves the head markers to the left or right.
- (iii) M_1 changes the state of M in its control. This is the

Simulation of a single move of M at the end of this M_1 is ready to implement its next move based on the revised position of its head markers and the changed state available in its control.

Running time of M : the running time of M on input w , is the number of steps and M takes before halting, if M does not halt on an input strings w , then its running time of M on w is infinite.

Time complexity of M : Time complexity of M is the fnc $T(n)$, n being the input size, where $T(n)$ is defined as the maximum of the running time of M over all inputs w of size n .