Assignment 3

(1) define context free grammer and write CFG for the following languages (i) $L=\{a^i b^j c^k ; i+j=k, i\geq 0, j\geq 0\}$
(ii) $L=\{a^n b^m c^k ; n+m=k\}$

→ Context free grammer is grammer that Consisting of 4 tuples. i.e $G=(V,T,P,S)$
where V is Set of Variables / non terminals
T is Set of terminals
P is Set of production
Each production is of the form $\alpha \to \beta$ be ε, but β is String from $(V \cup T)^*$ Hence it can include ε also.

(i) - when $i, j \geq 0$.

$S \to aSc \mid X$
$X \to bXc \mid \varepsilon$

$S \to X$
$\to \varepsilon$

$S \to aSc$ $\quad$ $S \to X$
$\to aXc$ $\quad$ $\to bXc$
$\to ac$ $\quad$ $\to bc$
when $i=1$ $j=0$ $\quad$ when $i=0$ $j=1$

$S \to aSc$
$\to aXc$
$\to abXcc$
$\to abcc$

(ii) Must add a 'c' for each 'a' and 'b'
Production
$S \to aSc$
$S \to S_1$
$S \to \epsilon$
$S_1 \to bS_1c$
$S_1 \to \epsilon$

② Consider the grammar G, with productions:

$S \to AbB$

$A \to aA \mid \epsilon$

$B \to aB \mid bB \mid \epsilon$

Give the LMD, RMD and parse tree for String aaabab

⇒ LMD

$S \xRightarrow{LM} AbB$

$\Rightarrow aAbB \; (A \to aA)$

$\Rightarrow aaAbB \; (A \to aA)$

$\Rightarrow aaaAbB \; (A \to aA)$

$\Rightarrow aaabB \; (A \to \varepsilon)$

$\Rightarrow aaabaB \; (B \to aB)$

$\Rightarrow aaababB \; (B \to bB)$

$\Rightarrow aaabab \; (B \to \varepsilon)$

RMD

$S \xRightarrow{RM} AbB$

$\Rightarrow AbaB \; (B \to aB)$

$\Rightarrow AbabB \; (B \to bB)$

$\Rightarrow Abab \; (B \to \varepsilon)$

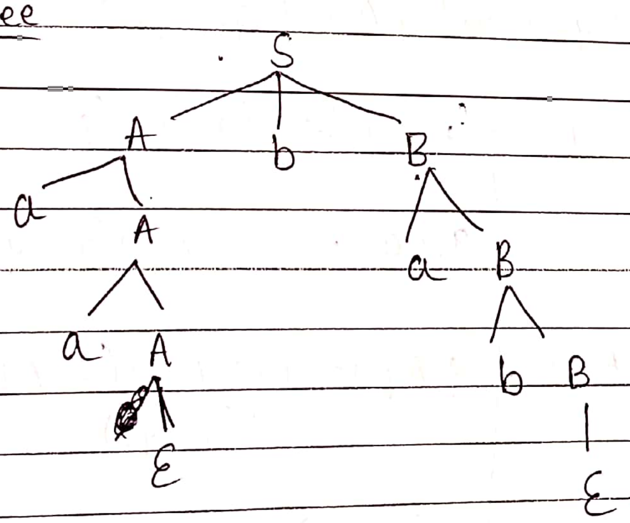
$\Rightarrow aAbab \; (A \to aA)$

$\Rightarrow aaAbab \; (A \to aA)$

$\Rightarrow aaaAbab \; (A \to aA)$

$\Rightarrow aaabab \; (A \to \varepsilon)$

Parse tree



③ Obtain a CFG for PDA $M = (\{q_0, q_1\}, \{a, b\}, \{A, Z\}, \delta, q_0, Z, \{q_1\})$ with the transitions

$\delta(q_0, a, Z) = (q_0, AZ)$

$\delta(q_0, b, A) = (q_0, AA)$

$\delta(q_0, a, A) = (q_1, \epsilon)$

→ Now the transition $\delta(q_0, a, A), (q_1, \epsilon)$ can be converted

$\delta(q_i, a, Z) = (q_j, \epsilon) \rightarrow RP\ (q_i Z q_j) \rightarrow a$

$\delta(q_0, a, A), (q_1, \epsilon)$ R.P

$$\boxed{(q_0 A q_1) \rightarrow a}$$

Now the transition

$\delta(q_0, a, Z) = (q_0, AZ)$

$\delta(q_0, b, A) = (q_0, AA)$

general form:

$\delta(q_i, a, Z) = (q_j, AB)$

RP: $(q_i Z q_k) \rightarrow a (q_j A q_l)(q_l B q_k)$

$\delta(q_0, a, Z), (q_0, AZ)$	$(q_0 Z q_0) \rightarrow a (q_0 A q_0)(q_0 Z q_0) \mid a (q_0 A q_1)(q_1 Z q_0)$
	$(q_0 Z q_1) \rightarrow a (q_0 A q_0)(q_0 Z q_1) \mid a (q_0 A q_1)(q_1 Z q_1)$
$\delta(q_0, b, A), (q_0 AA)$	$(q_0 A q_0) \rightarrow b (q_0 A q_0)(q_0 A q_0) \mid b (q_0 A q_1)(q_1 A q_0)$
The Start Symbol is $q_0 Z q_1$	$(q_0 A q_1) \rightarrow b (q_0 A q_0)(q_0 A q_1) \mid b (q_0 A q_1)(q_1 A q_1)$

④ Explain the following terms (i) Pushdown automata (PDA)
(ii) Languages of a PDA

(i) Push down automata is a finite automata with memory Element is called PDA which consists of 6 tuples they are

$M(Q, \Sigma, \delta, q, f, \Gamma)$

- Q → States
- Σ → Alphabets
- δ → transition
- q → ~~Final~~ Start Symbol
- f → final State
- Γ → Stack Symbol

(ii) Languages of a PDA.

There are two ways to accept the inputs

→ The PDA accepts an input if after reading the input & the machine empty stack.

→ The Set of inputs accepted by the PDA is the language accepted by empty Stack.

→ Some States of the PDA are final States. The PDA accepts an input if the machine Enters a final State.

→ The Set of inputs accepted by the PDA is the language accepted by final State

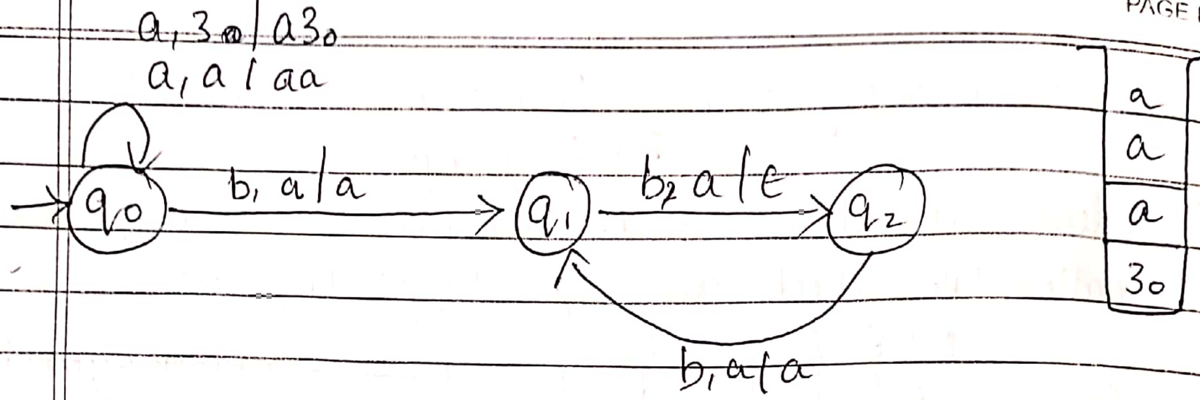
5 a) define CFG. write a CFG to Specify
(i) all String over {a, b} that are even and odd palindrome
(ii) $L = \{a^n b^{2n}$ over $\Sigma = \{a, b\}$ $n \geq 1\}$

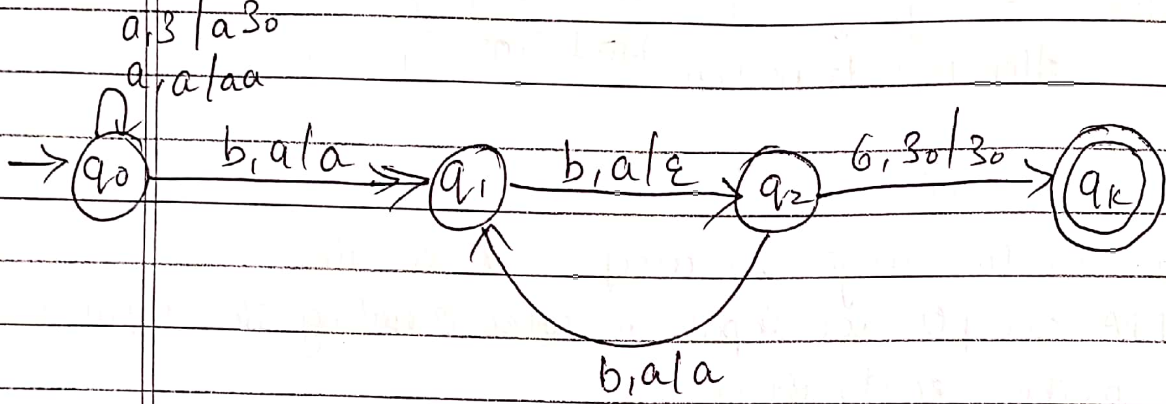
→ (i) $S \to aSa \mid bSb \mid \epsilon$

Even length palindrome	odd length palindrome.
$S \to aSa$	$S \to bSb$
$\to abSba$	$\to baSab$
$\to abba$	$\to ba$

(ii)

~~a~~	~~a~~	~~a~~	~~b~~	~~b~~	b	b	b	b	ϵ





⑥ 6 "design a PDA for language that accepts the String with $n_a(w) < n_b(w)$ where $W \in (a+b)^*$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

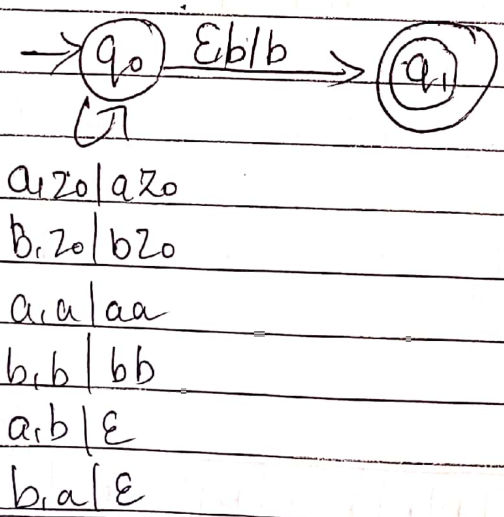
$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, \varepsilon)$

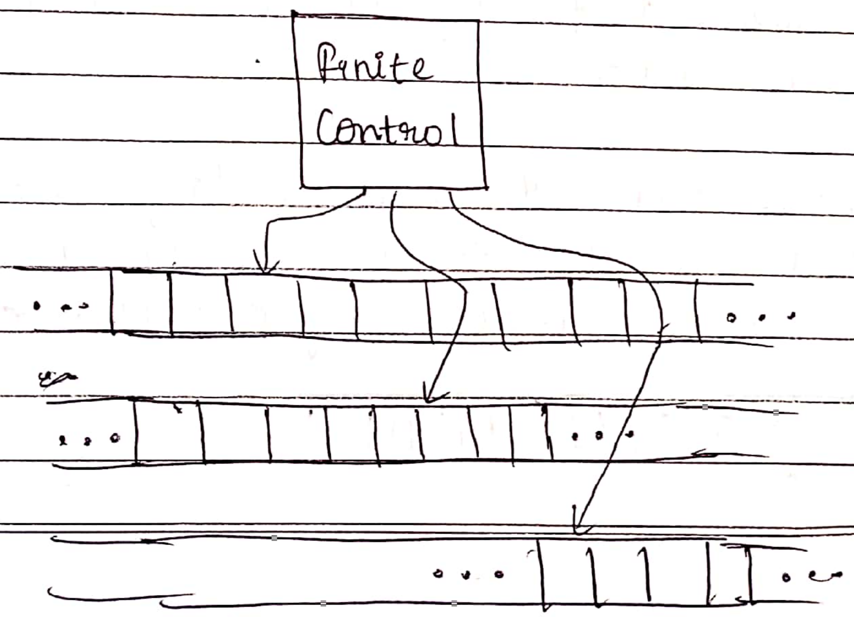
$\delta(q_0, b, a) = (q_0, \varepsilon)$

$\delta(q_0, \varepsilon, b) = (q_1, b)$



⑦ Explain the following with Example
① decidability, decidable languages, undecidable languages.
① we can Say that turing machine (TM) halts when M reaches a State q and Current Symbol to be Scanned.
* There are TM'S that never halt on Some i/p in any of these ways.
* There exists a destinction b/w the language accepted by TM that halts on all i/p String and TM that never halts on Some i/p String.
② Decidable language is a problem with two answers (yes/No) is decidable if the Corresponding language is recursive in this case, the language 'L' is also called decidable language.
③ undecidable language is a problem or language is undecidable if it is not decidable then it is considered as undecidable language

⑧ Explain the following ① multiple TM ⓘⓘ post Correspondence problem
→ i) multiple Turing machine.
→ A multiple Turing machine is nothing but a Standard Turing machine with more number of tapes.



The Components of multiple TM are.

*} Finite Control

*} multiple R/W heads.

→ Each tape is divided into Cell which can hold any Symbol from the given alphabet to Start with the TM Should be in Start State q_0.

→ for eg: if no. of tapes in TM is 3

$$\delta(q, a, b, c) = (p, x, y, z, L, R, S)$$

→ The TM in State q will be moved to State P only when the first R/w head points to a, the Second R/w head points to b and third R/w head points to C.

(ii) Post Correspondence problem

def:- The PCP Can be Stated as follows:

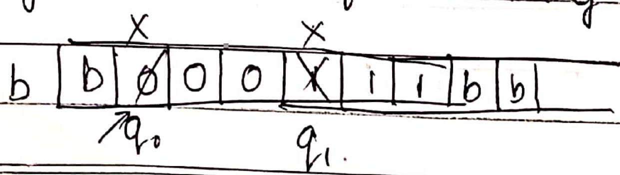
given two Sequence of n Strings on Some alphabets Σ Says $A = w_1, w_2, \ldots, w_n$ and $B = V_1, V_2, \ldots, V_n$

we Say that there exists a post Correspondence Solution for Pair (A, B) if there is a non empty Sequence of integer i, j, k.. Such that

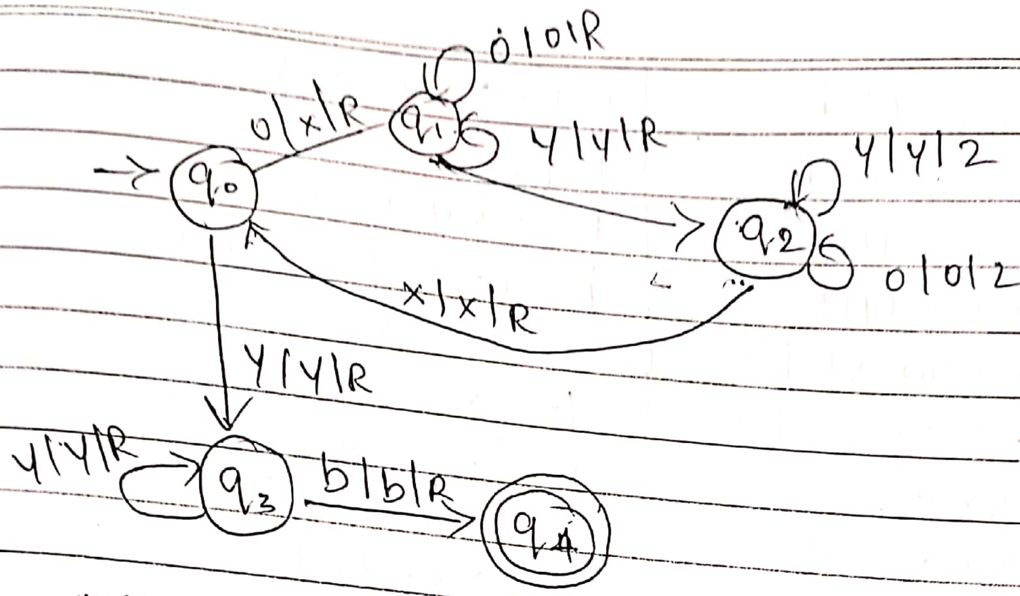
$$W_i W_j \ldots W_k = V_i V_j \ldots V_k$$

The post Correspondence problem is to devia an algorithm that will inform W, for any (A, B) whether or not there exists a PC-Solution.

(9) design a Turing machine to accept the language $L = \{0^n 1^n \mid n \geq 0\}$ draw the transition diagram Show the moves made by this machine for the String 000111 & 0011



Scanned with CamScanner



instantaneous description for the String 000111

$q_0 \underline{0} 00111b$
$\vdash (X q_1 \underline{0} 0111b)$
$\vdash (X0 q_2 y 1 b)$
$\vdash (\underline{X} q_2 0 y_1 b)$
$\vdash (X q_0 \underline{0} y 1 b)$
$\vdash (XX q_1 \underline{y} 1 b)$
$\vdash (XX y q_1 \underline{1} b)$
$\vdash (XXy q_2 y b)$
$\vdash (X\underline{X} q_2 y y b)$
$\vdash (XX q_0 \underline{y} y b)$
$\vdash (XXy q_3 y b)$
$\vdash (XXyy q_3 \underline{b})$
$\vdash (XXyy b q_4 \underline{b})$

String is accepted

	0	1	X	Y	b
$\rightarrow q_0$	q_1, X, R	–	–	q_3, Y, R	–
q_1	$q_1, 0, R$	q_2, y, L	–	q_1, Y, R	–
q_2	$q_2, 0, L$	–	q_0, X, R	$q_2 \mid y \mid 2$	–
q_3	–	–	–	$q_3 \mid y \mid R$	q_4, b, R
$* q_4$	–	–	–	–	–

instantaneous description for the String 000111

$q_0 000111b$

$\vdash (xq_1 00111b)$

$\vdash (x0q_1 0111b)$

$\vdash (x00q_1 111b)$

$\vdash (x00q_2 y11b)$

$\vdash (xx0q_2 yy1b)$

$\vdash (xxq_2 0yy1b)$

$\vdash (xxq_0 0yy1b)$

$\vdash (xxxq_1 yy1b)$

$\vdash (xxxyq_1 y1b)$

$\vdash (xxxyyq_1 1b)$

$\vdash (xxxyyq_2 yb)$

$\vdash (xxxyq_2 yyb)$

$\vdash (xxxq_2 yyyb)$

$\vdash (xxxq_0 yyyb)$

$\vdash (xq_2 00y11b)$

$\vdash (xq_0 00y11b)$

$\vdash (xxq_1 0y11b)$

$\vdash (xx0q_1 y11b)$

$\vdash (xx0yq_1 11b)$

$\vdash (xxxyq_3 yyb)$

$\vdash (xxxyyq_3 yb)$

$\vdash (xxxyyyq_3 b)$

$\vdash (xxxyyybq_4 b)$