AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XII

MODEL EXAMINATION PAPER 2020

Mathematics Paper II

Time: 2 hours Marks: 60

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. **Candidate's Signature**

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2. There are NINE questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. You may use a scientific calculator if you wish.

Q.1. (Total 4 Marks)

Find the value of $\lim_{x\to 3} \frac{\left(x^2-9\right)^2}{2x^2-12x+18}$.

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

Q.2. (Total 12 Marks)

a.

i. If $y = e^{x+\tan x}$, then show that $\frac{dy}{dx} = 2y + y\tan^2 x$. (3 Marks)

ii. For $y = \left(\sqrt{3x-2}\right)\left(\sqrt{3x+2}\right)$, show that $\frac{dy}{dx} = \frac{9x}{\sqrt{9x^2-4}}$. (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

b. For $y = \ln\left(\frac{\sqrt{x\cos x}}{\sin x}\right)$,

i. find $\frac{d^2y}{dx^2}$. (5 Marks)

ii. Hence, find the value of $\frac{d^2y}{dx^2}$ at $x = \frac{\pi}{4}$. (1 Mark)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

c. A water tank is cylindrical in shape. If the volume of the tank is 64 m^3, then what would be the radius and height of the water tank to have the least surface area?
Write you answer to two decimal places.

(**Note**: The volume of a cylinder is $V = \pi r^2 h$ and surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$,

where r is the radius of the cylinder and h is the height of a cylinder.) (6 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

Q.3. (Total 12 Marks)

a.

i. Convert $\frac{3x}{(x^2-4)(x^2+4)}$ into its partial fractions. (4 Marks)

ii. Hence, find $\int \frac{3x}{(x^2-4)(x^2+4)}\,dx$. (2 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

b. Evaluate the following indefinite integrals.

i. $\int \left(4x^4 - 12x^2 + 9\right)^{\frac{3}{2}} x\,dx.$ (3 Marks)

ii. $\int \frac{x}{(x+b)^2}\,dx.$ (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

c. Evaluate the following definite integrals by showing all the necessary steps. Give you answer in terms of *a*. (6 Marks)

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{a + a\cos x}$$

Q.4. (Total 5 Marks)

a. Convert the equation $x - \sqrt{3}\, y = 1$ into symmetric form. (3 Marks)

b. Find the equation of straight line in normal form which passes through the point $(0, -1)$ and whose gradient (slope) is 1. (2 Marks)

Q.5. (Total 5 Marks)

A burger shop sells chicken and beef burgers. The profit on chicken burger and beef burger is Rs 12 and Rs 10 respectively.

Due to existing cooking facilities, it cannot cook

- more than 200 chicken burgers.
- more than 250 beef burgers.
- and sell more than 400 burgers altogether.

For the given linear programming problem,

i. state the constraints. (1 Mark)

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ii. state the profit function. (1 Mark)

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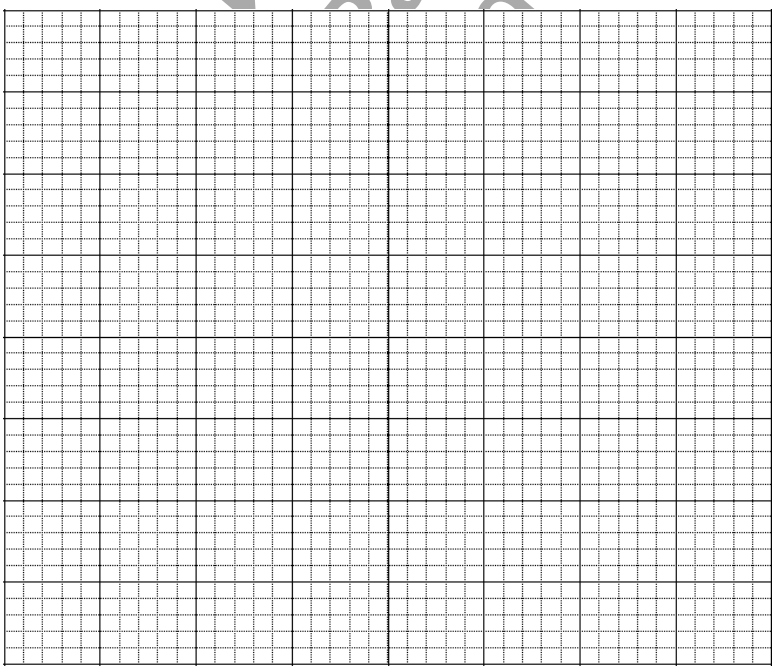
iii. find the corner points by drawing constraints on the given graph. (3 Marks)



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Q.6. (Total 4 Marks)

Find the equations of tangent and normal to the circle $7x^2 + 7y^2 - 28x + 42y - 84 = 0$ at $(-3, -3)$.

Q.7. (Total 10 Marks)

a. An ellipse with centre $(0,0)$ has vertices $(\pm 5,0)$ and foci $(\pm 4,0)$. Find

i. the equation of the ellipse. (2 Marks)

ii. the eccentricity. (1 Mark)

iii. the equation of directrices. (1 Mark)

iv. the length of minor axis. (1 Mark)

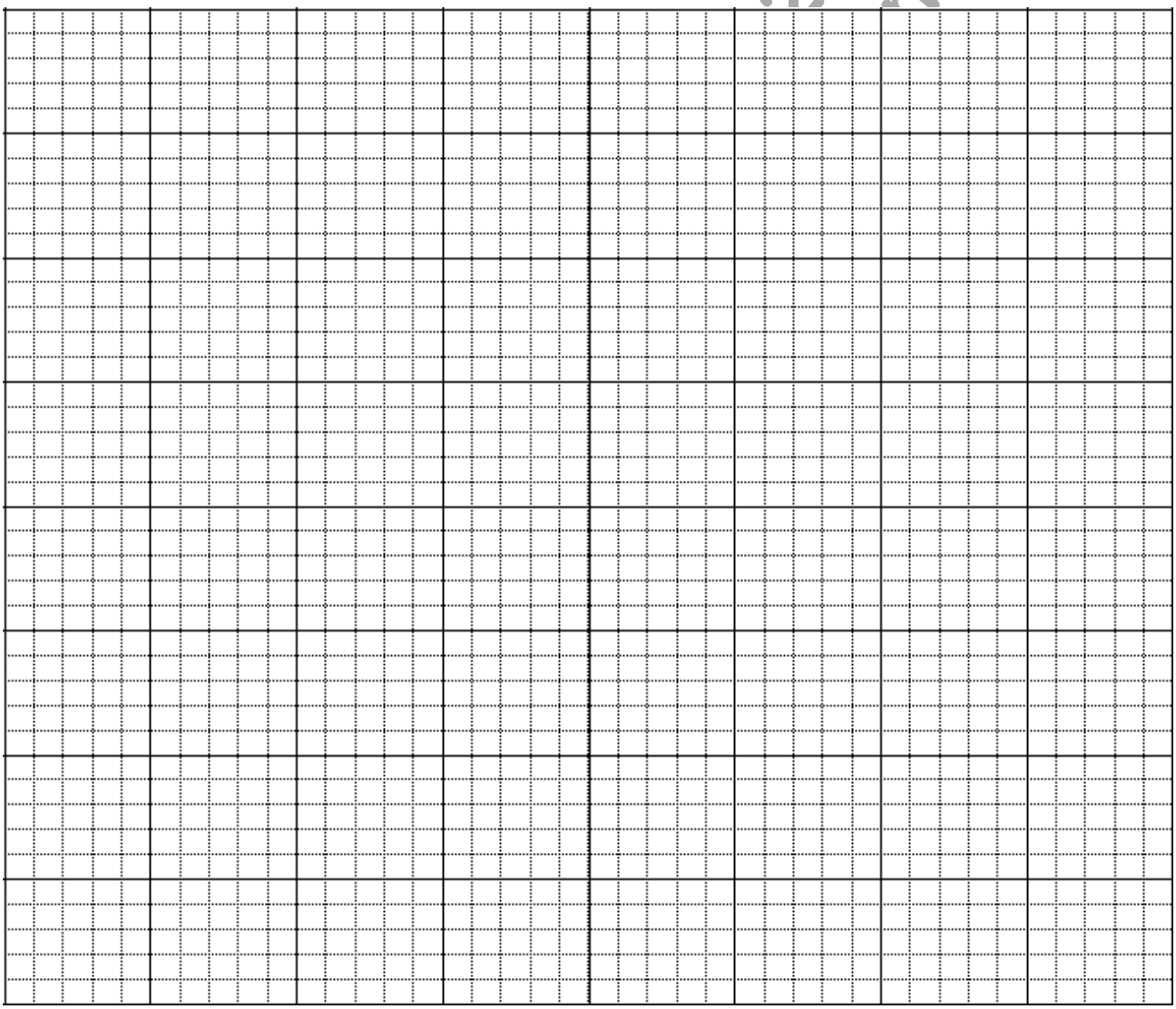
b. The vertices of a hyperbola are $(15, 1)$ and $(-1, 1)$. If the length of its conjugate axis is 12 units, then find the equation of the hyperbola in standard form. (5 Marks)

Q.8. (Total 4 Marks)

A triangle *ABC* located on *xy* - coordinate a*x*es has vertices *A* (0, 3), *B* (– 3, 0) and *C* (3, – 3). The a*x*is is translated through a distance of – 3 units while there is no change in the *y* - a*x*is.

i. Find the new vertices of the triangle *ABC* with respect to the translated a*x*is. (3 Marks)

ii. Using the given graph paper, draw the triangle *ABC* with its new coordinates. (1 Mark)



(ATTEMPT EITHER PART a OR PART b OF Q.9.)

Q.9. (Total 4 Marks)

a. Three points *P*, *Q* and *R* form a straight line such that $\vec{PQ} = \vec{PR}$ and $\vec{OQ} = k(\mathbf{a} - \mathbf{b})$.

i. Find the position vector of *R* in terms of *k*, **a** and **b**. (2 Marks)

ii. Hence, show that the unit vector in the direction of *R* is expressed as $\frac{1}{|\mathbf{a} - \mathbf{b}|}(\mathbf{a} - \mathbf{b})$. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.9.)

b. It is given that $\underline{p} \times \underline{q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 3 \\ 2 & -2 & m \end{vmatrix}$, $\underline{r} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\underline{q} \bullet (\underline{r} \times \underline{p}) = 10$.

i. Find the value of m (3 Marks)

ii. State the value of $\underline{p} \bullet (\underline{p} \times \underline{q})$. Also support your answer with a valid reason. (1 Mark)

END OF PAPER

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