AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI

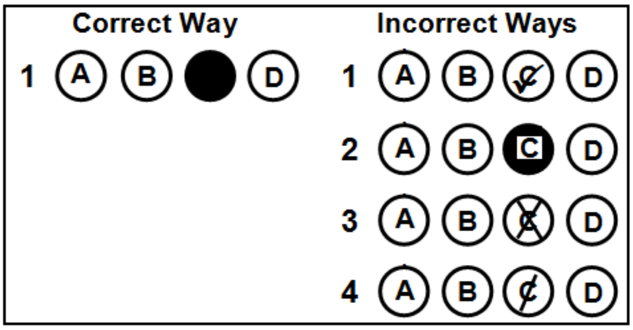
MODEL EXAMINATION PAPER 2023 AND ONWARDS

Mathematics Paper I

Time: 1 hour 30 minutes Marks: 50

INSTRUCTIONS

1. Read each question carefully.
2. Answer the questions on the separate answer sheet provided. DO NOT write your answers on the question paper.
3. There are 100 answer numbers on the answer sheet. Use answer numbers 1 to 50 only.
4. In each question there are four choices A, B, C, D. Choose ONE. On the answer grid black out the circle for your choice with a pencil as shown below.



Candidate's Signature

5. If you want to change your answer, ERASE the first answer completely with a rubber, before blacking out a new circle.
6. DO NOT write anything in the answer grid. The computer only records what is in the circles.
7. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.
8. You may use a scientific calculator if you wish.

Aga Khan University Examination Board

List of Formulae for Mathematics XI

Note:

- All symbols used in the formulae have their usual meaning.
- The same formulae will be provided in the annual and re-sit examinations.

Complex Numbers

$|z| = \sqrt{a^2 + b^2}$

Matrices and Determinants

$A_{ij} = (-1)^{i+j} M_{ij}$ $\qquad AdjA = (A_{ij})^t$ $\qquad A^{-1} = \dfrac{1}{|A|} AdjA$

Sequence & Series and Miscellaneous Series

$a_n = a_1 + (n-1)d$ $\qquad A = \dfrac{a+b}{2}$ $\qquad S_n = \dfrac{n}{2}\left(2a_1 + (n-1)d\right)$

$a_n = a_1 r^{n-1}$ $\qquad G = \pm\sqrt{ab}$ $\qquad H = \dfrac{2ab}{a+b}$

$S_n = \dfrac{a_1(1-r^n)}{1-r}$, if $|r|<1$ $\qquad S_n = \dfrac{a_1(r^n-1)}{r-1}$, if $|r|>1$ $\qquad S_\infty = \dfrac{a_1}{1-r}$, where $|r|<1$

$\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$ $\qquad \sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$ $\qquad \sum_{k=1}^{n} k^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

Permutations, Combinations and Probability

${}^nP_r = \dfrac{n!}{(n-r)!}$ $\qquad {}^nC_r = \dfrac{n!}{(n-r)!r!}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ $\qquad P(A|B) = \dfrac{P(A\cap B)}{P(B)}$ $\qquad P(A\cap B) = P(A)\times P(B)$

Binomial Theorem and Mathematical Induction

$$(a+x)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}x + \binom{n}{2}a^{n-2}x^2 + \binom{n}{3}a^{n-3}x^3 + \ldots + \binom{n}{n-1}a^1x^{n-1} + x^n$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots$$

$$T_{r+1} = \binom{n}{r}a^{n-r}x^r$$

Quadratic Equation

$x^2 - Sx + P = 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\qquad D = b^2 - 4ac$

Introduction to Trigonometry and Trigonometric Identities

$l = r\theta$ $\qquad \sin^2\theta + \cos^2\theta = 1$ $\qquad 1+\tan^2\theta = \sec^2\theta$

$1+\cot^2\theta = \operatorname{cosec}^2\theta$ $\qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\qquad \tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$ $\qquad \tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

$\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ $\qquad \sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ $\qquad \tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

$\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ $\qquad a^2 = b^2 + c^2 - 2bc\cos\alpha$ $\qquad \dfrac{a-b}{a+b} = \dfrac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}$

$\cos P - \cos Q = -2\sin\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$ $\qquad \sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$ $\qquad \sin P + \sin Q = 2\sin\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\qquad \tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$

Application of Trigonometry

$\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ac\sin\beta = \dfrac{1}{2}ab\sin\gamma$ $\qquad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma} = \dfrac{b^2\sin\alpha\sin\gamma}{2\sin\beta} = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$ $\qquad R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$

$r_1 = \dfrac{\Delta}{s-a}$, $r_2 = \dfrac{\Delta}{s-b}$ and $r_3 = \dfrac{\Delta}{s-c}$ $\qquad r = \dfrac{\Delta}{s}$ $\qquad R = \dfrac{abc}{4\Delta}$

Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations

$\sin^{-1}A \pm \sin^{-1}B = \sin^{-1}\left(A\sqrt{1-B^2} \pm B\sqrt{1-A^2}\right)$ $\qquad \cos^{-1}A \pm \cos^{-1}B = \cos^{-1}\left(AB \mp \sqrt{(1-A^2)(1-B^2)}\right)$

$\tan^{-1}A \pm \tan^{-1}B = \tan^{-1}\left(\dfrac{A \pm B}{1 \mp AB}\right)$

1. The real part of the complex number $\frac{1}{i^2}+5$ is equal to

   A. -6
   B. -4
   C. 4
   D. 6

2. The complex conjugate of $\left(2-i^2\right)^2$ is

   A. -9
   B. -6
   C. 3
   D. 9

3. On simplification of $\frac{5-4i-(4-3i)}{i}$, we get

   A. $1+i$
   B. $1-i$
   C. $-1-i$
   D. $-1+i$

4. The factorised form of the expression x^4+y^4 is

   A. $\left(x^2+iy^2\right)\left(x^2+iy^2\right)$
   B. $\left(x^2-iy^2\right)^2+2ix^2y^2$
   C. $\left(x^2-iy^2\right)\left(x^2+iy^2\right)$
   D. $\left(x^2+iy^2\right)^2-2ix^2y^2$

5. The square matrix $\left[a_{ij}\right]_{3\times 3}$ will be an upper triangular matrix if it satisfies the condition

   (**Note**: *i* and *j* represents the row and column numbers respectively.)

   A. $a_{ij}=0$ for all $j<i$.
   B. $a_{ij}=1$ for all $j<i$.
   C. $a_{ij}=0$ for all $j>i$.
   D. $a_{ij}=1$ for all $j>i$.

6. The product of the matrices $\begin{bmatrix} -i \\ i^2 \end{bmatrix}$ and $\begin{bmatrix} i \\ 1 \end{bmatrix}$ is

A. $[0]$.
B. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.
C. $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$.
D. not possible.

7. The determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ will be

A. -1
B. 0
C. 1
D. 2

8. The determinant $\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix}$ can also be written in another form as

(**Note**: *k* is a constant.)

A. $\begin{vmatrix} m_{11} + k & m_{12} \\ m_{21} + k & m_{22} \end{vmatrix}$.
B. $\begin{vmatrix} m_{11} \times k\,m_{12} & m_{12} \\ m_{21} \times k\,m_{11} & m_{22} \end{vmatrix}$.
C. $\begin{vmatrix} m_{11} \div k\,m_{21} & m_{12} \\ m_{21} \div k\,m_{11} & m_{22} \end{vmatrix}$.
D. $\begin{vmatrix} m_{11} & m_{12} - k\,m_{11} \\ m_{21} & m_{22} - k\,m_{21} \end{vmatrix}$.

9. If $M = \begin{bmatrix} 0 & x & 3 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is a singular matrix, then the value of *x* will be

A. -3
B. -1
C. 1
D. 0

10. The adjoint of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is

A. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

C. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

D. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

11. For the matrix $\begin{bmatrix} 2 & 2 & -7 \\ 1 & 1 & 3 \\ -x & y & z \end{bmatrix}$, the cofactor of the element $-x$ will be

A. -13
B. -1
C. 1
D. 13

12. If the determinant of the matrix $\begin{bmatrix} x & 0 & 1 \\ 1 & 4 & 1 \\ 0 & -2 & 2 \end{bmatrix}$ is 3, then the value of x will be

A. $-\frac{1}{2}$.

B. $-\frac{1}{10}$.

C. $\frac{1}{10}$.

D. $\frac{1}{2}$.

13. If the n^{th} term of a sequence is $(-2)^{\frac{n}{2}}$, then $(2n)^{\text{th}}$ term of the sequence will be

A. $(-2)^n$.
B. $(-2)^{2n}$.
C. $(-2)^{\frac{n}{4}}$.
D. $(-2)^{4n}$.

14. The sum of the 5th and 6th terms of an arithmetic sequence is 20. If the common difference is 2, then the first term of the sequence will be

A. -2
B. -1
C. 1
D. 2

15. If T_n of an arithmetic sequence is $2n - 1$, then T_n of the associated harmonic sequence will be

A. $\frac{2}{n-1}$
B. $\frac{2}{n}-1$
C. $\frac{1}{2n-1}$
D. $\frac{1}{2n}-1$

16. The harmonic mean between the two terms, x and y is

A. $\frac{xy}{x+y}$.
B. $\frac{xy}{x-y}$.
C. $\frac{2xy}{x+y}$.
D. $\frac{2xy}{x-y}$.

17. The arithmetic mean between two numbers is *c*. If one number is *a*, then the other number will be

A. $\frac{a+c}{2}$.
B. $\frac{c-a}{2}$.
C. $2a-c$.
D. $2c-a$.

18. The ratio of 3rd term to 5th term, i.e. $a_3 : a_5$, of a geometric sequence is $1:4$. The common ratio of the geometric sequence will be

A. $\frac{1}{2}$
B. $\pm\frac{1}{2}$
C. 2
D. ± 2

19. If the geometric mean between 3 and *b* is $\frac{9}{2}$, then the value of *b* will be

A 6
B. 9
C. $\frac{27}{2}$
D. $\frac{27}{4}$

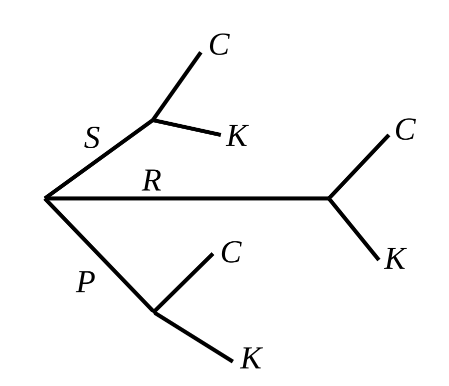
20. The first term of an infinite geometric sequence is $-\frac{1}{2}$. If the common ratio is $-\frac{1}{2}$, then sum of the infinite geometric series will be

A. $-\frac{1}{2}$
B. $-\frac{1}{3}$
C. -1
D. 1

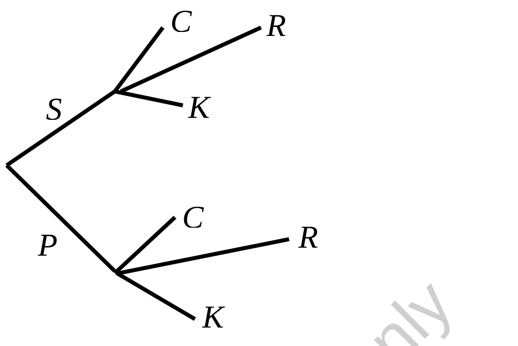
21. If $\left(\frac{p}{2}\right)! = 6$, then the value of p is equal to

A. 12!
B. 6!
C. 12
D. 6

22. A man goes to a bakery where he finds *Samosas* (S), Vegetable rolls (R) and Potato Cutlets (P). They are served with *Chatni* (C) or *Ketchup* (K). The tree diagram which illustrates the given situation for all the possible combinations will be



A



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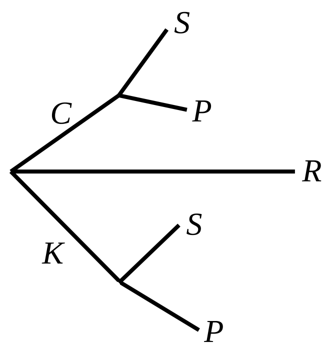

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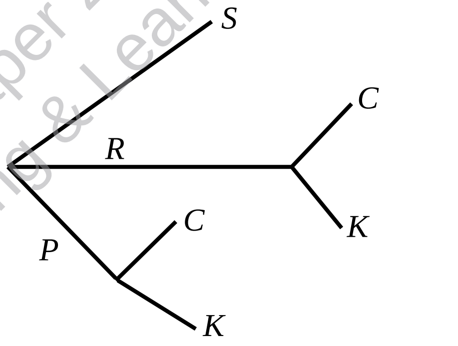


D

23. 13 applicants applied for 4 positions in a firm. The job titles are manager, computer programmer, accountant and specialist. The possible ways of selection of the applicants are

A. 52
B. 715
C. 17,160
D. 28, 561

24. The number of ways a teacher can select 6 students from a class of 30 students to create a math's club is

A. 180
B. 593,775
C. 427,518,000
D. 729,000,000

25. A bag contains 6 red, 5 yellow, 8 white, 6 black and 10 blue balls. A ball is drawn at random without replacement from the bag. The process is repeated two times. What is the probability that both the balls drawn are blue?

A. $\frac{10}{35}\times\frac{9}{34}$.
B. $\frac{10}{35}\times\frac{9}{35}$.
C. $\frac{10}{35}\times\frac{10}{34}$.
D. $\frac{10}{35}\times\frac{10}{35}$.

26. A fair coin is tossed three times. The probability of getting at least TWO heads is

A $\frac{3}{8}$.
B. $\frac{4}{8}$.
C. $\frac{3}{6}$.
D. $\frac{4}{6}$.

27. With reference to the principle of mathematical induction, the TRUE statement for $m = 1$ is

A. $5+6\left(5^{m+1}\right)$ is divisible by 3
B. $5+7\left(5^{m+1}\right)$ is divisible by 3
C. $5+6\left(5^{m-1}\right)$ is divisible by 5
D. $5+7\left(5^{m-1}\right)$ is divisible by 5

28. In the expansion of $\left(x^2+\frac{1}{x}\right)^6$, the condition required to obtain the constant term will be

(**Formula**: $T_{r+1}={}^nC_r a^{n-r}b^r$, where symbols have their usual meanings.)

A. $12-r=r$.
B. $12-2r=r$.
C. $12+r=-r$.
D. $12+2r=-r$.

29. If $(1-x)^{-5} = 1 + 5x + bx^2 + \ldots$, then the value of b will be

A. -15
B. -10
C. 10
D. 15

30. Which of the following conditions is valid for the convergence of $(1-x)^{\frac{1}{4}}$?

A. $|x| < 4$
B. $|x| > 4$
C. $|x| < \frac{1}{4}$
D. $|x| > \frac{1}{4}$

31. The solution set of the quadratic equation $2x^2 + \frac{1}{2} = 0$ is

A. $\{-i,\ i\}$.
B. $\left\{0,\ \frac{i}{2}\right\}$.
C. $\left\{\frac{i}{4},\ -\frac{i}{4}\right\}$.
D. $\left\{\frac{i}{2},\ -\frac{i}{2}\right\}$.

32. The nature of the roots of the equation $x^2 - 2bx + b^2 = 0$, where $b \in Z$, is

A. real and equal.
B. real and unequal.
C. complex and equal.
D. complex and unequal.

33. A polynomial $P(x)$ of degree 4 is divided by $x-3$ and the remainder is -11 as shown in the given synthetic division.

3	1	0	-10	-2	4
		3	9	-3	-15
	?	?	-1	-5	-11

The quotient of the given division of the polynomial will be

A. $-x-5$
B. $-x^2-5x$
C. x^3+3x^2-x-5
D. $x^4+3x^3-x^2-5x$

34. If α and β are the roots of the equation $ax^2-bx=1$, then the value of $(\alpha-\beta)^2-(\alpha+\beta)^2$ will be

A. $-\frac{1}{a}$.
B. $-4\left(\frac{1}{a}\right)$.
C. $\frac{1}{a}$.
D. $4\left(\frac{1}{a}\right)$.

35. If $x^2-y^2=k^2$ and $y=a$, then the value of x, in terms k and a, will be

A. $\pm(k^2+a)$.
B. $\pm(k^2+a^2)$.
C. $\pm\sqrt{k^2+a}$.
D. $\pm\sqrt{k^2+a^2}$.

36. For the equation $a^2x^2+bx+64=0$, the sum roots is equal to the product of the roots. The value of b will be

A. -64
B. -8
C. 8
D. 64

37. In degrees, 3π rad can be expressed as

A. 0.16°
B. 9.42°
C. 180°
D. 540°

38. The positive square root of $\left[4\left(1+\dfrac{1}{\tan^2\beta}\right)\right]$ will be equal to

A. $2\cot\beta$.
B. $2\tan\beta$.
C. $2\sec\beta$.
D. $2\mathrm{cosec}\beta$.

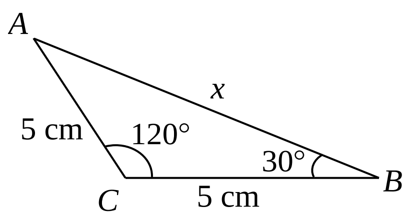
39. The terminal ray of angle 2360° lies in the

A. I[st] quadrant.
B. II[nd] quadrant.
C. III[rd] quadrant.
D. IV[th] quadrant.

40. $\dfrac{1-\sin^2\theta}{1+\tan^2\theta}$ equals to

A. 1
B. $\cot^2\theta$
C. $\sec^4\theta$
D. $\cos^4\theta$

41. In the given triangle, the value of x will be

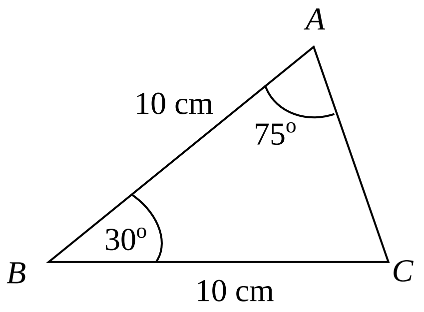


NOT TO SCALE

A. 5 cm.
B. $\dfrac{5\sqrt{3}}{2}$ cm.
C. $10\sqrt{3}$ cm.
D. $5\sqrt{3}$ cm.

42. The area of the given triangle *ABC* is equal to

NOT TO SCALE



A. $50 \times \sin 30^{\circ}$

B. $100 \times \sin 30^{\circ}$

C. $100 \times \left(\sin 75^{\circ}\right)^2$

D. $200 \times \left(\sin 75^{\circ}\right)^2$

43. The sides of a triangle *ABC* are 6 cm, 5 cm and 7 cm. If its area is $6\sqrt{6}\ \text{cm}^2$ then the in-radius of the circle associated to the triangle *ABC* is calculated to be

A. $\frac{2\sqrt{6}}{3}$ cm.

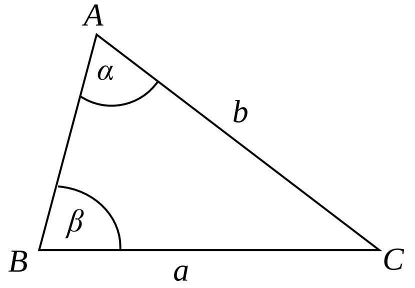
B. $\frac{3}{2\sqrt{6}}$ cm.

C. $2\sqrt{6}$ cm.

D. $\frac{1}{2\sqrt{6}}$ cm.

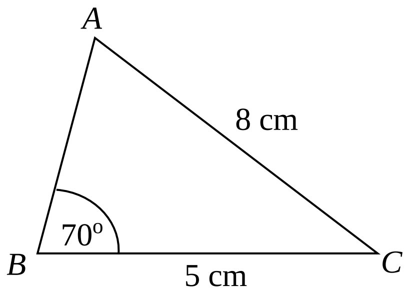
44. In the given triangle *ABC*, if $b = 10\,\text{cm}$, $\sin\alpha = 0.39$ and $\sin\beta = 0.99$, then a is approximately equal to

NOT TO SCALE



A. 3.94 cm.
B. 3.86 cm.
C. 4.86 cm.
D. 25.38 cm.

45. In the given triangle, $\angle A$ is equal to



NOT TO SCALE

A. 25.96°
B. 30.96°
C. 35.96°
D. 40.96°

46. If two sides of a right angled triangle are equal to a cm, then the measurement of the third side will be

A. $2a$ cm.
B. $\sqrt{2}a$ cm.
C. $\sqrt{2}a^2$ cm.
D. $2a^2$ cm.

47. The range of $\dfrac{\sin\alpha}{\cos\alpha}$ is the set of all

A. real numbers.
B. real numbers except 1
C. real numbers except – 1
D. real numbers between –1 and 1

48. The period of $\operatorname{cosec}\left(\dfrac{4\theta}{5}+60^{\circ}\right)$ is

A. $\dfrac{5\pi}{4}$.
B. $\dfrac{5\pi}{2}$.
C. $\dfrac{4\pi}{5}$.
D. $\dfrac{8\pi}{5}$.

49. A trigonometric equation is given as $\cot x = p.$ If p is undefined, then the value of x is

(**Note**: p is a constant.)

A. 0
B. $\frac{\pi}{2}$
C. $\frac{\pi}{4}$
D. also undefined

50. Which of the given functions is an even function?

A. $\cos^2\theta\sin\theta$.
B. $\sin\theta\tan\theta$.
C. $\cos\theta\tan\theta$.
D. $\sin\theta\cot^2\theta$.

END OF PAPER

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