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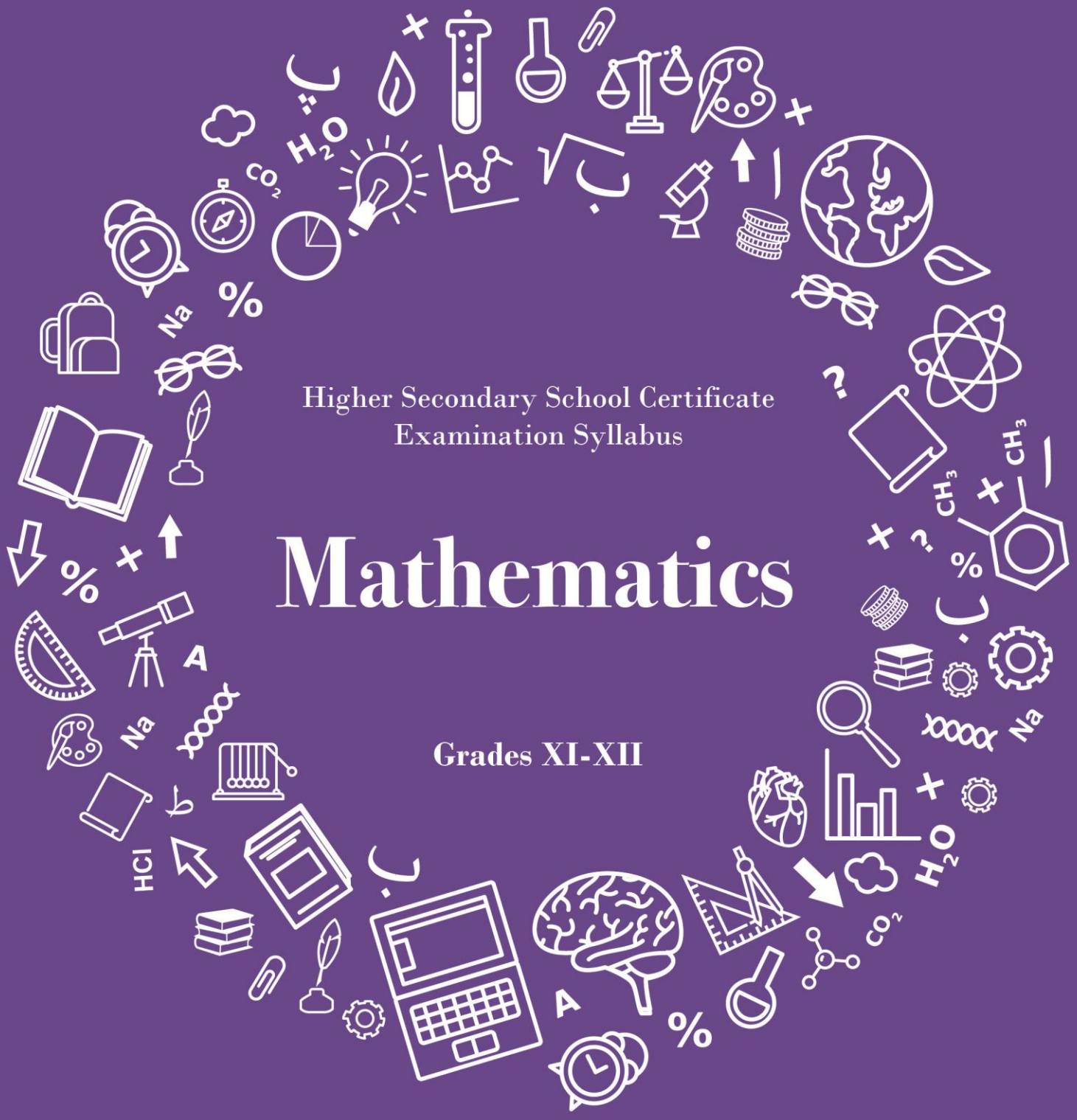

Higher Secondary School Certificate
Examination Syllabus

Mathematics

Grades XI-XII

(based on National Curriculum 2006)

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Higher Secondary School Certificate Examination Syllabus

MATHEMATICS

GRADES XI-XII

This syllabus will be examined in both May and September Examination sessions from May 2019 for Grade XI and May 2020 for Grade XII

For queries and feedback

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Preface

Established in 2002 through Ordinance CXIV, Aga Khan University Examination Board (AKU-EB) is Pakistan's first private autonomous examination body for secondary (SSC) and higher secondary (HSSC) school certifications. Its vision is to be a model of excellence and innovation in education in Pakistan and the developing world.

One of the ways in which AKU-EB achieves its vision is by developing syllabi which inculcates conceptual thinking and higher order learning based on the National Curriculum. AKU-EB revises its syllabi every 4 years so that they continue to meet the needs of students, teachers and examiners.

The aims of the current syllabus review of SSC and HSSC in 2016 were to:

- Ensure continued compatibility with the goals of the National Curriculum of Pakistan.
- Review the content for inclusion of new knowledge and deletion of obsolete knowledge**.**
- Review the content for clarity and relevance as per the changing needs of students, teachers and examiners.
- Enhance and strengthen continuation and progression of content both within and across grades IX - XII (SCC and HSSC).
- Ensure the readiness of students for higher education.

During this syllabus review, the needs of all the stakeholders were identified through a needs-assessment survey. Students and teachers of AKU-EB affiliated schools from across Pakistan participated in the survey. Thereafter, a revision panel, which consisted of examiners, schools teachers of affiliated and non-affiliated schools, teacher trainers and university academics, reviewed and revised the syllabus following a planned, meticulous and standardised syllabi review process.

This year, AKU-EB took the initiative of introducing a 'Concept Map' for each syllabus which represents links among the key concepts of the syllabus. These have been designed to improve students' interest in the subject, facilitate conceptual thinking and make the learning and teaching experience more memorable.

The syllabus is organised into topics and subtopics. Each subtopic is further divided into achievable student learning outcomes (SLOs). The SLOs of the cognitive domain are each assigned a cognitive level on which they have to be achieved. These cognitive levels are 'knowledge', 'understanding' and 'application', the latter also including other higher order skills. This is followed by the Exam Specification which gives clear guidance about the weightage of each topic and how the syllabus will be assessed.

The development of the revised syllabus have been made possible by the creativity and relentless hard work of Curriculum and Examination Development unit and the constant support provided by all the other units of AKU-EB. We are particularly thankful to Dr Sohail Qureshi for his very useful feedback on revising the syllabus review process, to Dr Naveed Yousuf for his continued guidance and support throughout the syllabus revision process and to Raabia Hirani for leading the syllabi revision. We are also thankful to all the students and teachers who took part in the needs-assessment survey and to the principals of AKU-EB affiliated schools who made this endeavour possible by facilitating and encouraging their teachers to be a part of the survey and the syllabus revision panel.

With your support and collective hard work, AKU-EB has been able to take the necessary steps to ensure effective implementation of the National Curriculum of Pakistan through this syllabus. We are confident that this syllabus will continue to provide the support that is needed by students to progress to the next level of education and we wish all the best to students and their teachers in implementing this syllabus.

Dr Shehzad Jeeva
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Introduction to AKU-EB Syllabi

1. Aga Khan University Examination Board (AKU-EB) has a mandate by Ordinance CXIV of 2002 'to test the attainment of the objectives of the national curriculum, for the purpose of enhancing student learning, and to do all such things that may be considered appropriate for the improvement of education in respect to teaching and learning, institutional effectiveness and all things ancillary and incidental thereto'.

2. The AKU-EB syllabi are an important tool in the achievement of this mandate. These syllabi are based on the National Curriculum of Pakistan 2006 and the National Scheme of Studies 2006 – 2007. The syllabi bring together all those cognitive outcomes of the National Curriculum statement which can be reliably and validly assessed. Moreover, the syllabi aim to achieve the pedagogically desirable objectives of the National Curriculum which encourage 'observation, creativity and other higher order thinking skills', better meeting the needs of the students of the twenty-first century.

3. The syllabi guide the students, teachers, parents and other stakeholders regarding the topics that will be taught and examined in each grade (IX, X, XI and XII). In each syllabus document, the content progresses from simple to complex, thereby, facilitating a gradual, conceptual learning of the content.

4. The topics of the syllabi are grouped into themes derived from the national curriculum. The connection between various themes and topics is highlighted in the '**concept map**' provided at the beginning of each syllabus. This ensures that students begin to understand the interconnectedness of knowledge, learn conceptually and think critically.

5. The topics of the syllabi are divided into subtopics and **student learning outcomes (SLOs)**. The subtopics and the SLOs define the depth and the breadth at which each topic will be taught, learnt and examined. The syllabi complement the national curriculum by providing enabling SLOs where needed to scaffold student learning.

6. Each SLO starts with an achievable and assessable **command word** such as describe, relate, evaluate, etc. The purpose of the command words is to direct the attention of teachers and students to specific tasks that the students are expected to undertake in the course of their studies. The examination questions are framed using the same command words or their connotations to elicit evidence of these competencies in students' responses.

7. The SLOs are classified under three **cognitive levels**: knowledge (K), understanding (U) and application and other higher order skills (A) for effective planning during teaching and learning and deriving multiple choice questions (MCQs) and constructed response questions (CRQs) and extended response questions (ERQs) on a rational basis from the subject syllabi, ensuring that the intentions of the national curriculum are also met during examinations.

8. By focusing on the achievement of the SLOs, these syllabi aim to counter the culture of rote memorisation as the preferred method of examination preparation. While suggesting relevant, locally available textbooks for achieving these outcomes, AKU-EB recommends that teachers and students use multiple teaching and learning resources for achieving these outcomes.

9. The syllabi follow a uniform layout for all subjects to make them easier for students and teachers to follow. They act as a bridge between students, teachers and assessment specialists by providing a common framework of student learning outcomes and **exam specifications**.

10. On the whole, the AKU-EB syllabi for Higher Secondary School Certificate (HSSC) progressively help the students to achieve the benchmarks of the national curriculum and hone in them conceptual understanding, critical thinking and problem solving skills, thereby preparing them for professional and higher education.

Aims/ Objectives of the National Curriculum (2006) [1]

The following themes permeate the National Curriculum for Mathematics.

- The curriculum is designed to help students build the solid conceptual foundation in Mathematics that will enable them to apply their knowledge skilfully and further their learning successfully.
- The curriculum emphasises on the geometrical concepts that enable the students to think logically, reason systematically and conjecture astutely.
- The curriculum stresses graphics that enable the students to visualise and interpret mathematical expressions correctly rather to manipulate them 'blindly'.
- The curriculum recognises the benefits that current technologies can bring to the learning and doing mathematics. It, therefore, integrates the use of appropriate technologies to enhance learning in an ever increasingly information-rich world.

National Curriculum for Mathematics is comprised of five standards. The competencies are intentionally kept broad as to allow flexibility to the teachers in accordance with their students. These five standards are:

i. Number and Operations
ii. Algebra
iii. Measurements and Geometry
iv. Information Handling
v. Reasoning and Logical Thinking

[1] Government of Pakistan (2006), Page 2, *National Curriculum for Mathematics Grades I – XII, Islamabad,* Ministry of Education (Curriculum Wing)

Subject Rationale of AKU-EB Mathematics

What will you learn in AKU-EB Mathematics?

Mathematics is not only the language of science, engineering and technology but also of Economics, Psychology and many other fields of study. Mathematics teaches the core skills required to function in today's world.

Most school going students understand the use of basic math in daily life. What they fail to understand is why they should learn advanced mathematics since they cannot see how concepts like algebra, matrices, imaginary numbers, and calculus can help them later on in life.

What you are actually supposed to be learning in mathematics class is the art of problem reduction by systematic and critical thinking, i.e. starting with a problem and reducing it to a simpler problem in a way that it becomes easy to solve. Any real life problem can be accessed and solved through similar mathematical/ logical thinking approach.

The current National Curriculum of Pakistan covers a wide array of topics that provide a deep conceptual understanding of Mathematics. The AKU-EB syllabus of Mathematics has enhanced it further by making conceptual connections between topics and improving the logical flow of concepts.

The AKU-EB Mathematics syllabus focuses on mathematical skills and logical thinking to help students develop their mathematical skills and understanding. It helps improve students' ability to apply their content knowledge in new and unexpected situations, rather than on rote learning. This is significantly evident in application of theorems where students are not required to reproduce theorems, but to apply them.

Where will it take you?

The AKUEB syllabus of Mathematics will provide conceptual basis for higher studies in many subjects. For those who pursue mathematics in higher studies, wide career opportunities are available such as:

- Actuary
- Banker
- Architect
- Musician
- Fashion Designer
- Pharmacologist
- Physical Scientist
- Astronomer, astrologist and navigational scientist
- Graphic designer (Creating 2D and 3D animations)

How to approach the syllabus?

The concept map of the syllabus gives an overview of the entire syllabus. The topics and the student learning outcomes (SLOs) guide regarding the details about what has to be achieved. And finally, the exam specification guides regarding what will be expected in the examination.

What is the concept map telling you?

BLOSSMING FLOWER OF MATHEMATICS

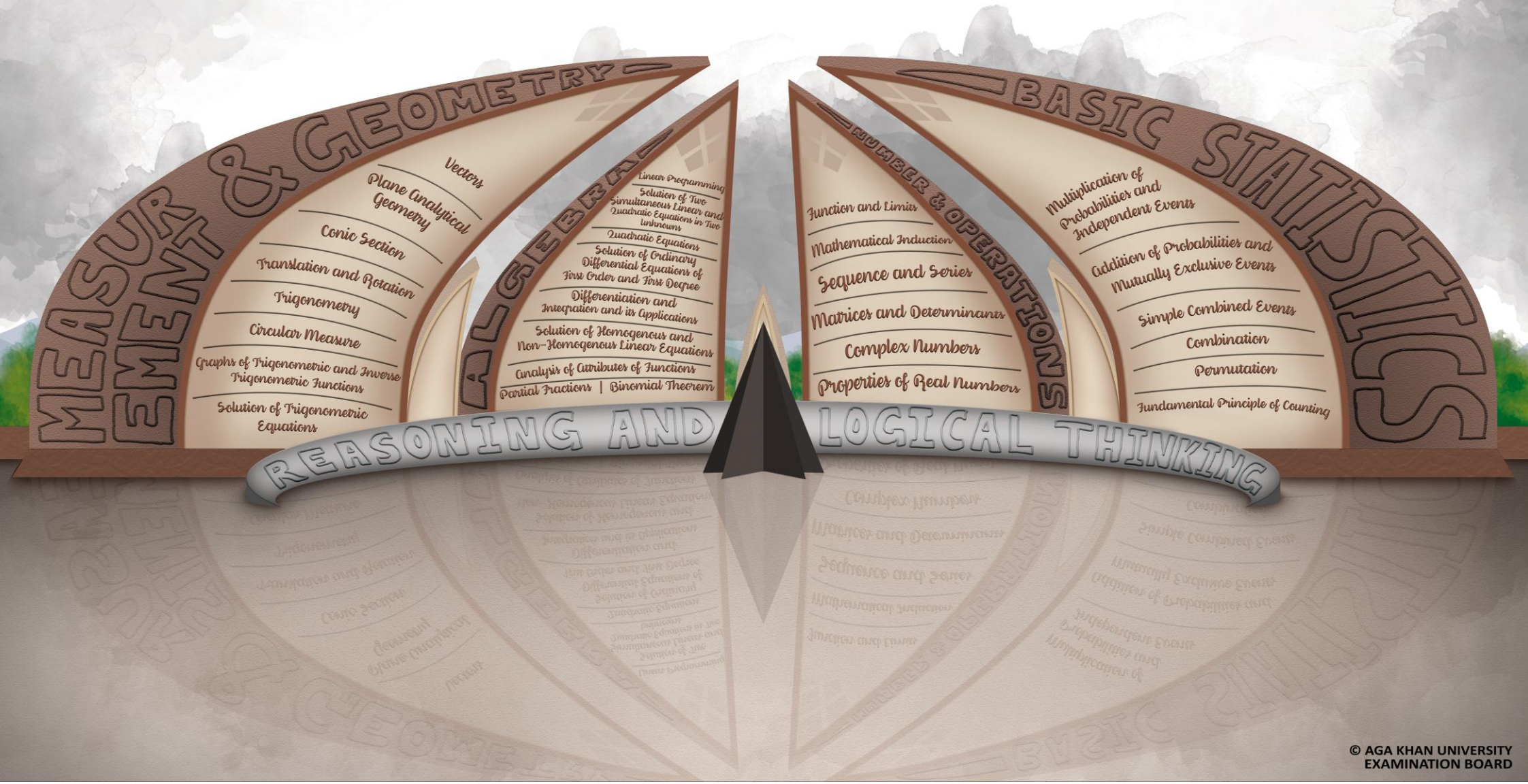
How many of us know that when we praise the magnificent Pakistan Monument[2], we are actually appreciating mathematics at work? Do you notice the curvatures and symmetry of the structure? Perhaps, when one realises that mathematics is essentially the study of patterns, its connection with architecture becomes clearer. Hence, it has been used here to map the concepts of AKU-EB HSSC Mathematics.

The metallic crescent that runs along the structure represents **Reasoning and Logical Thinking** which is the basic competency needed for mathematics. The four petals represent the four core standards of mathematics. The inner walls of the petals depict the progression of concepts (from the base to the tip). The three small petals show the conceptual linkage between all the competencies. Hence, the blooming flower represents the progression of topics in mathematics. The heart (centre) of the monument is a pentagram, i.e., a five-pointed star formed by drawing a continuous line in five straight segments, which represents excellence in principles of the nature/ universe. The destiny of every mathematician!

[2] The Pakistan Monument is located in Islamabad and was designed by Arif Masoud and constructed by Ministry of Culture, Pakistan. The original structure has a specific cultural interpretation and significance.

MATHEMATICS

CONCEPT MAP - HSSC I & II



Student Learning Outcomes of AKU-EB HSSC Mathematics Syllabus

Part I (Grade XI)

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level[3]		
				K	U	A
1. Complex Numbers		Students should be able to:				
	1.1 Complex Numbers	1.1.1	describe the complex number z represented by an expression of the form $z = a + ib$ into the form (a, b) and vice versa, where a and b are real numbers and $i = \sqrt{-1}$;		*	
		1.1.2	mention a as real part and b as imaginary part of $z = a + ib$;	*		
		1.1.3	apply the condition of equality of complex numbers to solve problems;			*
		1.1.4	apply four basic operations (addition, subtraction, multiplication and division) on complex numbers;			*
		1.1.5	state $\overline{z} = a - ib$, the complex conjugate of $z = a + ib$;	*		
		1.1.6	calculate $\lvert z \rvert = \sqrt{a^2 + b^2}$, the absolute value or modulus of a complex number $z = a + ib$;			*
	1.2 Properties of Complex Numbers	1.2.1	describe the properties of complex numbers (commutative, associative and distributive with respect to addition and multiplication);		*	
		1.2.2	find the additive inverse and multiplicative inverse of a complex number;			*

[3] K = Knowledge, U = Understanding, A = Application and other higher-order cognitive skills CA = Class Activity

Topics and Sub-topics		Student Learning Outcomes	Cognitive Level K	U	A
		Students should be able to:			
	1.2.3	prove the following properties of complex numbers: a. $\lvert z\rvert = \lvert -z\rvert = \lvert\bar{z}\rvert = \lvert -\bar{z}\rvert$ b. $\bar{\bar{z}} = z,\ z\bar{z} = \lvert z\rvert^2, \overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$ c. $\overline{z_1 z_2} = \bar{z}_1\,\bar{z}_2, \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2},\ z_2 \neq 0$			*
	1.2.4	solve problems related to SLO 1.2.3;			*
	1.2.5	find the real and imaginary parts of the following types of complex numbers: a. $(x+iy)^n$ b. $\left(\frac{x_1+iy_1}{x_2+iy_2}\right)^n;\ x_2+iy_2 \neq 0$ where $n = \pm 1$ and $n = \pm 2$;			*
1.3 Solution of Equations	1.3.1	solve the simultaneous linear equations with complex coefficients;			*
	1.3.2	factorise the polynomial $P(x)$, for example: a. $x^2+y^2 = (x+iy)(x-iy)$ b. $x^3-3x^2+x+5 = (x+1)(x-2-i)(x-2+i)$;			*
	1.3.3	solve quadratic equation $pz^2+qz+r=0;\ p \neq 0$ by completing square method, where p, q, r are real numbers and z is a complex number.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
2. Matrices and Determinants		Students should be able to:				
	2.1 Matrices	2.1.1	define: a. a matrix and its notation b. order of a matrix c. equality of two matrices;	*		
		2.1.2	describe row matrix, column matrix, square matrix, rectangular matrix, zero/ null matrix, identity matrix, scalar matrix and diagonal matrix;		*	
		2.1.3	describe upper triangular matrix, lower triangular matrix, transpose of a matrix, symmetric matrix and skew-symmetric matrix;		*	
	2.2 Algebra of Matrices	2.2.1	apply scalar multiplication, addition and subtraction of matrices;			*
		2.2.2	apply multiplication of two or more matrices having real and complex entries;			*
		2.2.3	verify that the commutative property in matrices a. holds under addition b. does not hold under multiplication, in general;			*
		2.2.4	verify that $(AB)^t = B^t A^t$ (**Note:** For matrices up to order 3 by 3);			*
	2.3 Determinants and Inverse Matrices	2.3.1	find the determinant of a square matrix;			*
		2.3.2	find the minor and cofactor of elements of a square matrix of order 3 by 3;			*
		2.3.3	define singular and non-singular matrices;	*		
		2.3.4	solve problems related to singular and non-singular matrices;			*
		2.3.5	find the adjoint of a square matrix of order 3 by 3;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		2.3.6	find the inverse of a square matrix by using adjoint method;			*
		2.3.7	verify that $(AB)^{-1} = B^{-1}A^{-1}$ (**Note:** For matrices of order 3 by 3);			*
2.4	Properties of Determinants	2.4.1	explain the properties of determinants;		*	
		2.4.2	evaluate the determinant without expansion (using properties of determinants);			*
2.5	System of Linear Equations	2.5.1	distinguish between homogeneous and non-homogeneous linear equations in two and three unknowns;		*	
		2.5.2	solve a system of three homogeneous linear equations in three unknowns;			*
		2.5.3	solve a system of 3 by 3 non-homogeneous linear equations using: a. matrix inversion method b. Cramer's rule.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
3. Sequence and Series		Students should be able to:				
3.1	Sequence (progression)	3.1.1	describe sequence (progression) and related terms;		*	
		3.1.2	find the general term when a sequence is given;			*
		3.1.3	find a particular term of a sequence using the general term;			*
3.2	Arithmetic Sequence	3.2.1	describe an arithmetic sequence;		*	
		3.2.2	derive the formula of n^{th} or general term of an arithmetic sequence;			*
		3.2.3	solve problems involving arithmetic sequence;			*
3.3	Arithmetic Mean	3.3.1	find the arithmetic mean between two numbers;			*
		3.3.2	find n arithmetic means between two numbers;			*
3.4	Arithmetic Series	3.4.1	describe arithmetic series;		*	
		3.4.2	derive the formula of sum of an arithmetic series up to n terms;			*
		3.4.3	solve problems involving arithmetic series;			*
3.5	Geometric Sequence	3.5.1	describe geometric sequence;		*	
		3.5.2	derive the formula of n^{th} or general term of a geometric sequence;			*
		3.5.3	solve problems involving geometric sequence;			*
3.6	Geometric Mean	3.6.1	find the geometric mean between two numbers;			*
		3.6.2	find 'n' geometric means between two numbers;			*
3.7	Geometric Series	3.7.1	describe geometric series;		*	
		3.7.2	find the sum to n terms of a geometric series;			*
		3.7.3	find the sum of an infinite geometric series;			*
		3.7.4	convert the recurring decimal into an equivalent common fraction;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		3.7.5	solve problems involving geometric series;			*
3.8	Harmonic Sequence	3.8.1	describe harmonic sequence;		*	
		3.8.2	find the n^{th} term of harmonic sequence;			*
		3.8.3	solve problems involving harmonic sequence;			*
3.9	Harmonic Mean	3.9.1	find harmonic mean between two numbers;			*
		3.9.2	find *n* harmonic means between two numbers;			*
		3.9.3	find the relationship between arithmetic, geometric and harmonic means.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
4. Miscellaneous Series		Students should be able to:				
	4.1 Evaluation of $\sum n, \sum n^2$ and $\sum n^3$	4.1.1	identify sigma notation ($\sum$) used to give a concise expression for a sum of the values of a variable;		*	
		4.1.2	find the sum of the: a. first n natural numbers ($\sum n$) b. squares of the first n natural numbers ($\sum n^2$) c. cubes of the first n natural numbers ($\sum n^3$);			*
		4.1.3	solve problems involving ($\sum n$),($\sum n^2$),($\sum n^3$).			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
5. Permutation, Combination and Probability		Students should be able to:				
5.1	Factorial of a Natural Number	5.1.1	explain the concept of the product of the first n natural numbers as $n!$ (Kramp's factorial) and the fact that $0! = 1$;		*	
5.2	Counting Techniques (Fundamental Principle of Counting, Permutation and Combination)	5.2.1	apply the fundamental principle of counting in different situations;			*
		5.2.2	construct tree diagram using the fundamental principle of counting;			*
		5.2.3	explain the meaning of permutation (${}^{n}P_{r}$) of n different objects taken r at a time;		*	
		5.2.4	solve problems related to ${}^{n}P_{r}$;			*
		5.2.5	find the arrangement of different objects around a circle;			*
		5.2.6	explain the meaning of combination (${}^{n}C_{r}$) of n different objects taken r at a time;		*	

Topics and Sub-topics		Student Learning Outcomes	Cognitive Level		
			K	U	A
		Students should be able to:			
	5.2.7	prove that, for any positive integers n and r, where $n > r$: a. $\binom{n}{n} = \binom{n}{0} = 1$ b. $\binom{n}{r} = \binom{n}{n-r}, \binom{n}{1} = \binom{n}{n-1} = n$ c. $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$;			*
	5.2.8	solve problems involving combination;			*
5.3 Probability	5.3.1	describe the following terms: a. statistical experiment b. sample space and event c. mutually exclusive and mutually inclusive (non-exclusive) events d. equally likely events e. dependent and independent events f. simple and compound events;		*	

Topics and Sub-topics	Student Learning Outcomes		Cognitive Level		
			K	U	A
	Students should be able to:				
	5.3.2	apply the formula to find the probability of an event E, i.e. $P(E)=\frac{n(E)}{n(S)}, 0 \le P(E) \le 1$;			*
	5.3.3	find probability in simple cases;			*
	5.3.4	find the probability of the occurrence of an event using Venn diagram, tree diagram and probability tree diagram (with and without replacement);			*
	5.3.5	define the conditional probability;	*		
	5.3.6	describe the law of addition of probability $P(A \cup B)=P(A)+P(B)-P(A \cap B)$, where A and B are two non-exclusive events;		*	
	5.3.7	describe that $P(A \cup B)=P(A)+P(B)$, where A and B are mutually exclusive events;		*	
	5.3.8	describe the law of multiplication of probability $P(A \cap B)=P(A)\times P(B \mid A)$ or $P(A \cap B)=P(B)\times P(A \mid B)$, where $P(B \mid A)$ and $P(A \mid B)$ are conditional probabilities and A and B are dependent events;		*	
	5.3.9	describe the law of multiplication of probability $P(A \cap B)=P(A)\times P(B)$, where A and B are independent events;		*	
	5.3.10	solve problems related to SLOs from 5.3.3 to 5.3.9.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
6. Mathematical Induction and Binomial Theorem		Students should be able to:				
6.1	Mathematical Induction	6.1.1	describe the principle of mathematical induction;		*	
		6.1.2	prove the statements, identities and formulae using the principle of mathematical induction; (**Note:** Questions involving inequalities are not included for example: $n^2 > n + 3$ for integral values of $n \geq 3$);			*
6.2	Binomial Theorem	6.2.1	find the expansion of $(x+y)^n$ using Pascal's triangle for positive integral values of $n \leq 5$;			*
		6.2.2	derive binomial theorem for positive integral index;			*
		6.2.3	expand $(x+y)^n$, where $x+y$ is any binomial expression, using binomial theorem and find its general term;			*
		6.2.4	find the specified term in the expansion of $(x+y)^n$;			*
6.3	Binomial Series	6.3.1	expand $(x+y)^n$ where n is a positive integer and extend this result for all rational values of n;			*
		6.3.2	expand $(x+y)^n$ in ascending powers of x where n is a negative integer or a fraction;			*
		6.3.3	explain the convergence of $(x+y)^n$ for $\lvert x \rvert < 1$ where n is a rational number;		*	
		6.3.4	determine the approximate values of the binomial expansions having indices as fractions.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
7. Quadratic Equations		Students should be able to:				
7.1	Solution of Quadratic Equation (Revision of the work done in previous classes)	7.1.1 7.1.2	describe quadratic equation in standard form; solve a quadratic equation in one variable by: a. factorisation method b. completing square method c. using quadratic formula;		*	*
7.2	Solution of Equation Reducible to Quadratic Equation in one Variable	7.2.1	solve equations reducible to quadratic equation in one variable of the following forms: a. $ax^{2n}+bx^{n}+c=0,\ \ a\neq 0$ b. $(x+a)(x+b)(x+c)(x+d)=k,$ where $a+b=c+d$ c. exponential equations (e.g. $a^{2x}-b.a^{x+2}+k=0$) d. reciprocal equations (e.g. $ax^4+bx^3+cx^2+bx+a=0$) e. i. $l(ax^2+bx)+m\sqrt{ax^2+bx+c}=0$ ii. $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$; (check extraneous roots if any by substitution) (**Note:** a, b, c and d are real numbers);			*
7.3	Nature of the Roots of a Quadratic Equation	7.3.1 7.3.2	define discriminant (b^2-4ac) of the quadratic equation $ax^2+bx+c=0;\ a\neq 0$; determine the nature of roots of a given quadratic equation;	*		*
7.4	Cube and Fourth Roots of Unity and their Properties	7.4.1	identify complex cube roots of unity, i.e. ω and ω^2;		*	

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		7.4.2	find the cube roots of unity and other numbers, e.g. $\pm 8, \pm 27$, etc.;			*
		7.4.3	prove the properties of cube roots of unity;		*	
		7.4.4	solve problems related to properties of cube roots of unity;			*
		7.4.5	find fourth roots of unity and other numbers, e.g. 16, 81, etc. ;			*
		7.4.6	describe the properties of fourth roots of unity;		*	
7.5	Roots and Coefficient of a Quadratic Equation	7.5.1	find the relationship between the roots and the coefficient of a quadratic equation;			*
		7.5.2	find the sum and product of the roots of a given quadratic equation without solving it;			*
		7.5.3	solve problems based on the sum and product of roots;			*
		7.5.4	find the value(s) of unknown(s) involved in a given quadratic equation when a. sum of roots is equal to the product of roots b. sum of the squares of roots is equal to a given number c. roots differ by a given number d. roots satisfy a given relation (e.g. the relation $2\alpha+5\beta=7$ and $\alpha=\beta$, where α and β are the roots of given equation) e. both sum and product of roots are equal to a given number.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
7.6	Formation of Quadratic Equation	7.6.1	describe the formula: $x^2 - (\text{sum of roots})\,x + (\text{product of roots}) = 0$ to find a quadratic equation from the given roots;		*	
		7.6.2	find a quadratic equation whose roots are given;			*
		7.6.3	find a quadratic equation whose roots, for example, are: a. $2\alpha+1,\ 2\beta+1$ b. α^2, β^2 c. $\frac{1}{\alpha}, \frac{1}{\beta}$ d. $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ e. $\alpha+\beta, \frac{1}{\alpha}+\frac{1}{\beta}$ f. α^3, β^3, etc.; where α and β are the roots of a given quadratic equation;			*
7.7	Synthetic Division	7.7.1	solve problems related to remainder and factor theorem;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		7.7.2	apply synthetic division to: a. find the quotient and remainder when a given polynomial is divided by linear polynomial b. find the value(s) of unknown(s) if the factors of a polynomial are given c. solve a cubic equation if one root of the equation is given. d. solve a bi-quadratic (quartic) equation if two of the real roots of the equation are given;			*
7.8	Simultaneous Equations	7.8.1	solve system of two equations in two variables when: a. one equation is linear and the other is quadratic b. both the equations are quadratic;			*
7.9	Applications of Quadratic Equations	7.9.1	solve word problems related to quadratic equations.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
8. Introduction to Trigonometry and Trigonometric Identities		Students should be able to:				
8.1	Trigonometric Ratios	8.1.1	describe quadrants and quadrantal angles;		*	
		8.1.2	identify signs of trigonometric ratios in different quadrants;		*	
		8.1.3	find the values of remaining trigonometric ratios if one of the trigonometric ratios is given;			*
8.2	Trigonometric Identities	8.2.1	prove the following fundamental trigonometric identities a. $\sin^2\theta+\cos^2\theta=1$ b. $1+\tan^2\theta=\sec^2\theta$ c. $1+\cot^2\theta=\operatorname{cosec}^2\theta$;		*	
		8.2.2	prove different trigonometric relations using trigonometric identities;			*
8.3	Fundamental Law of Trigonometry	8.3.1	find the distance between two points using distance formula;			*
		8.3.2	prove the fundamental law of trigonometry by using distance formula $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$;		*	
		8.3.3	prove the following using fundamental law of trigonometry: a. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ b. $\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$ c. $\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\pm\tan\alpha\tan\beta}$;		*	
		8.3.4	solve problems related to fundamental law of trigonometry and its deductions;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
8.4	Trigonometric Ratios of Allied Angles	8.4.1	define allied angles;	*		
		8.4.2	derive trigonometric ratios of allied angles using fundamental law and its deductions;			*
		8.4.3	convert $a\sin\theta \pm b\cos\theta$ in the form $r\sin(\theta \pm \phi)$ where $a = r\cos\phi$ and $b = r\sin\phi$;			*
8.5	Double, Half and Triple Angle Identities	8.5.1	derive double angle, half angle and triple angle identities from fundamental law and its deductions;			*
		8.5.2	prove different trigonometric relations using identities mentioned in SLO 8.5.1;			*
8.6	Sum, Difference and Product of Sine and Cosine	8.6.1	convert the product of sine and cosine into sum or difference and vice versa;			*
		8.6.2	apply the sum or difference (of sine and cosine) as products (of sine and cosine) and vice versa.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
9.	**Application of Trigonometry**	Students should be able to:				
9.1	Solution of Triangles	9.1.1	solve a right-angled triangle when length of: a. two sides are given b. one side and one angle are given;			*
		9.1.2	describe oblique triangle;		*	
		9.1.3	prove the: a. law of cosines b. law of sines c. law of tangents;		*	
		9.1.4	deduce half angle formulae by using laws mentioned in SLO 9.1.3;		*	
		9.1.5	solve problems related to SLOs from 9.1.1 to 9.1.4;			*
9.2	Area of Triangles	9.2.1	derive the formulae for the area of a triangle when length of a. two sides and their included angle are given b. one side and two angles are given c. three sides are given (Heron's formula);			*
		9.2.2	apply the above formulae to find the area of a triangle;			*
9.3	Circles Connected with Triangle	9.3.1	illustrate circum-circle, in-circle and escribed-circle;			*
		9.3.2	derive the formula for: a. circum-radius b. in-radius c. escribed-radii;			*
		9.3.3	find the circum-radius, in-radius and escribed radii using the formulae mentioned in SLO 9.3.2;			*
		9.3.4	deduce different identities using the formulae mentioned in SLO 9.2.1 and 9.3.2.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
10. Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations		Students should be able to:				
	10.1 Period of Trigonometric Functions	10.1.1	find the domain and range of the trigonometric functions;			*
		10.1.2	distinguish between even and odd trigonometric functions;		*	
		10.1.3	describe the period of trigonometric functions;		*	
		10.1.4	find the period of trigonometric functions by definition;			*
		10.1.5	find the maximum and minimum values (amplitude) of a given function of the following types: a. $a+b\sin\theta$, b. $a+b\cos\theta$, c. $a+b\sin(c\theta+d)$, d. $a+b\cos(c\theta+d)$, where *a*, *b*, *c* and *d* are real numbers;			*
	10.2 Graphs of Trigonometric Functions	10.2.1	discuss the shapes of the graphs of sine, cosine and tangent function;		*	
		10.2.2	draw the graphs of the six basic trigonometric functions within the domain -2π to 2π;			*
		10.2.3	discuss the graphical behaviour of even and odd trigonometric functions;		*	
		10.2.4	sketch the trigonometric functions of the form $a\sin b\theta$, $a\cos b\theta$ and $a\tan b\theta$, where *a* and *b* are integers;			*
		10.2.5	discuss the effects of periodicity on the graphs of trigonometric function;		*	

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		10.2.6	draw the graphs of trigonometric functions of the form $a\sin b\theta$, $a\cos b\theta$ and $a\tan b\theta$, where a and b are real numbers;			*
		10.2.7	find period, minimum and maximum (amplitude) values using graph of trigonometric functions;			*
		10.2.8	describe the following properties: a. periodic property $\sin(\theta \pm 2\pi) = \sin\theta$ b. odd property $\sin(-\theta) = -\sin\theta$ c. translation property $\sin(\theta - \pi) = -\sin\theta$ and $\sin(\pi - \theta) = \sin\theta$;		*	
10.3	Inverse Trigonometric Functions	10.3.1	describe principal trigonometric functions and inverse trigonometric functions;		*	
		10.3.2	find domain and range of principal trigonometric function and inverse trigonometric function;			*
		10.3.3	draw the graph of inverse trigonometric functions;			*
		10.3.4	prove addition and subtraction formulae of inverse trigonometric functions;			*
		10.3.5	verify related identities using addition and subtraction formulae of inverse trigonometric functions;			*
10.4	Solution of General Trigonometric Equations	10.4.1	solve trigonometric equations of the type $\sin\theta = k$, $\cos\theta = k$ and $\tan\theta = k$ where k is constant, using periodic, even, odd and translation properties;			*
		10.4.2	find general solution of a trigonometric equation and discard extraneous roots taking into account the period of the trigonometric function;			*
		10.4.3	find the solution set of trigonometric equation graphically.			*

Part II (Grade XII)

Topics and Subtopics		Student Learning Outcomes		Cognitive Level		
				K	U	A
11. MAPLE		Students should be able to:				
11.1	Introduction	11.1.1	explain MAPLE integrated development environment;			CA
		11.1.2	identify basic MAPLE commands;		CA	
		11.1.3	apply MAPLE as a calculator;			CA
		11.1.4	use online MAPLE help;		CA	
11.2	Polynomials	11.2.1	apply MAPLE commands to: a. find the factors of a polynomial b. expand an expression c. simplify an expression d. simplify a rational expression e. find the value of an expression by substituting value;			CA
11.3	Graphics	11.3.1	apply MAPLE commands to: a. plot a two-dimensional graph b. demonstrate domain and range of a plot c. sketch parametric equations d. use plotting options;			CA
11.4	Matrices	11.4.1	identify matrix and vector entry arrangement through MAPLE;		CA	
		11.4.2	apply matrix operations;			CA
		11.4.3	find the inverse and transpose of a matrix.			CA

NOTE:
MAPLE exercises can be used as **Classroom Activities (CA)**.
MAPLE is given zero weightage in examinations as per the National Curriculum Wing (Ministry of Education) of Pakistan.

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
12. Functions and Limits		Students should be able to:				
12.1	Functions	12.1.1	describe the concept and notation of a function, its domain, codomain and range;		*	
		12.1.2	find the value of a function for given values of dependent and independent variables;			*
12.2	Inverse Functions	12.2.1	describe inverse of a function and its notation;		*	
		12.2.2	find the inverse of a function and its domain and range;			*
12.3	Graph of Functions	12.3.1	distinguish between linear, quadratic and square root functions;		*	
		12.3.2	draw the graph of modulus function of the form $y = \|x\|$ and $y = c \times \|x \pm a\| \pm b$, where a and b are rational numbers;			*
		12.3.3	find the domain and range of a function through graph;			*
12.4	Composition of Functions	12.4.1	describe the composition of functions and notation used for composition of functions;		*	
		12.4.2	find the composition of two functions;			*
		12.4.3	find the corresponding values of composite functions for given values of a variable;			*
12.5	Inverse of Composition of Functions	12.5.1	describe the inverse function of a composite function;		*	
		12.5.2	find inverse of a composite function;			*

Topics and Sub-topics		Student Learning Outcomes	Cognitive Level		
			K	U	A
		Students should be able to:			
12.6 Types of Functions	12.6.1	state algebraic, trigonometric, inverse trigonometric, exponential, logarithmic and hyperbolic functions;	*		
	12.6.2	distinguish between a. explicit and implicit functions b. even and odd functions;		*	
	12.6.3	describe parametric representation of functions;		*	
12.7 Graphical Representations	12.7.1	interpret the graph/ sketch of: a. explicitly defined functions like $y = f(x)$ where $f(x) = e^x$, $f(x) = a^x$, $f(x) = \log_a x$, $f(x) = \log_e x$ or $\ln x$; b. implicitly defined relations such as $x^2 + y^2 = a^2$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (vertical line test) c. parametric equations of functions such as $x = at^2$, $y = 2at$, $x = a\sec\theta$, $y = b\tan\theta$ d. piecewise functions, for example $y = \begin{cases} x & \text{when} \quad 0 \le x < 1 \\ x-1 & \text{when} \quad 1 \le x \le 2 \end{cases}$;			*
	12.7.2	apply MAPLE graphic commands for two-dimensional plot of: a. an expression (or a function) b. parametric form of a function c. implicit function, by restricting domain and range;			CA

Topics and Sub-topics		Student Learning Outcomes	Cognitive Level K	U	A
	Students should be able to:				
	12.7.3	apply MAPLE package plots for plotting different types of functions;			CA
12.8 Limit of a Function	12.8.1	describe the following terms: a. open interval b. closed interval c. half open and half closed intervals;		*	
	12.8.2	represent the following on a number line: a. open interval b. closed interval c. half open and half closed intervals;		*	
	12.8.3	describe the phrases: a. x tends to zero $(x \to 0)$ b. x tends to a $(x \to a)$ c. x tends to infinity $(x \to \infty)$;		*	
	12.8.4	describe the limit of a sequence;		*	
	12.8.5	find the limit of a sequence whose n^{th} term is given;			*
	12.8.6	describe the limit of a function;		*	
	12.8.7	state the theorems of limits for sum, difference, power, product and quotient of functions;	*		
	12.8.8	apply theorems to find the limit of a function;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
12.9	Important Limits	12.9.1	evaluate the limits of functions of the following forms: a. $\frac{x^n - a^n}{x-a}, \frac{x-a}{\sqrt{x}-\sqrt{a}}$ when $(x \to a)$ b. $\left(1+\frac{1}{x}\right)^x$ when $(x \to \infty)$ c. $(1+x)^{\frac{1}{x}}, \frac{\sqrt{x+a}-\sqrt{a}}{x}, \frac{a^x-1}{x}, \frac{(1+x)^n-1}{x}$, and $\frac{\sin x}{x}$ when $(x \to 0)$;			*
		12.9.2	evaluate limits of different algebraic, exponential, logarithmic and trigonometric functions;			*
		12.9.3	apply MAPLE command *limit* to evaluate the limit of a function;			CA
12.10	Continuous and Discontinuous Functions	12.10.1	illustrate left hand and right hand limits with examples to decide the existence and non-existence of limit of a function;			*
		12.10.2	describe the continuity of a function at a point and in an interval;		*	
		12.10.3	find the continuity and discontinuity of a function at a point and in an interval;			*
		12.10.4	apply MAPLE command *iscont* to test continuity of a function at a point and in a given interval.			CA

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
13. Differentiation		Students should be able to:				
	13.1 Derivative of a Function	13.1.1	distinguish between independent and dependent variables;		*	
		13.1.2	find approximate value of corresponding change in the dependent variable when independent variable is increased or decreased;			*
		13.1.3	explain the concept of rate of change;		*	
		13.1.4	distinguish between the average rate of change and the instantaneous rate of change;		*	
		13.1.5	solve problems based on the average rate of change and the instantaneous rate of change;			*
		13.1.6	describe the derivative or differential coefficient of a function as an instantaneous rate of change of dependent variable with respect to independent variable and its notation for derivatives;		*	
		13.1.7	find the derivative of $y = x^n$, where $n \in Z$ (the set of integers) and $y = (ax+b)^n$, where $n = \frac{p}{q}$ and p, q are integers such that $q \neq 0$, by definition or by ab-initio or from first principles;			*
		13.1.8	find the derivative of algebraic functions by using direct method (power rule);			*
	13.2 Theorems on Differentiation and their Applications	13.2.1	find the derivative of: a. any constant multiple of a function b. sum (or difference) of two functions c. product of two functions d. quotient of two functions;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
13.3	Chain Rule	13.3.1	prove chain rule e.g. $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$ when $y = f(u), u = g(x)$ and $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$;			*
		13.3.2	solve problems related to chain rule;			*
		13.3.3	apply the chain rule to show that $\frac{d}{dx}\left[f(x)\right]^n = n\left[f(x)\right]^{n-1}\frac{d}{dx}f(x)$;			*
		13.3.4	find the derivative of implicit functions;			*
13.4	Differentiation of Trigonometric and Inverse Trigonometric Functions	13.4.1	find the derivative of trigonometric functions by first principles;			*
		13.4.2	find the derivative of trigonometric functions using direct method;			*
		13.4.3	find the derivative of inverse trigonometric functions using formulae;			*
13.5	Differentiation of Exponential and Logarithmic Functions	13.5.1	find the derivative of e^x and a^x by first principles ;			*
		13.5.2	find the derivative of lnx and $\log_a x$ from first principles;			*
		13.5.3	find the derivative of exponential and logarithmic functions by using direct method;			*
		13.5.4	find the derivative of algebraic expressions involving product, quotient and power by using laws of logarithm;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
13.6	Differentiation of Hyperbolic and Inverse Hyperbolic Functions	13.6.1	find the derivative of: a. hyperbolic functions (sinh x, cosh x, tanh x, cosech x, sech x and coth x); b. inverse hyperbolic functions $\sinh^{-1} x, \cosh^{-1} x, \tanh^{-1} x, \text{cosech}^{-1} x, \text{sech}^{-1} x$, and $\coth^{-1} x$);			*
		13.6.2	apply MAPLE command *diff* to differentiate a function.			CA

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
14. Higher Order Derivatives and its Applications		Students should be able to:				
	14.1 Higher Order Derivatives	14.1.1	find the higher order derivatives of algebraic, trigonometric, exponential and logarithmic functions;			*
		14.1.2	find the second derivative of implicit, inverse trigonometric and parametric functions;			*
		14.1.3	apply MAPLE command *diff* repeatedly to find higher order derivative of a function;			CA
	14.2 Maclaurin's and Taylor's Expansions	14.2.1	state Maclaurin's and Taylor's theorems;	*		
		14.2.2	apply these theorems to expand sin$x$, cos$x$, tan$x$, a^x, e^x, $\log_a(1+x)$ and $\ln(1+x)$;			*
		14.2.3	apply MAPLE command *taylor* to find Taylor's expansion for a given function;			CA
	14.3 Application of Derivatives	14.3.1	find the angle of intersection of the two curves;			*
		14.3.2	find the equation of tangent and normal to the curve at a given point;			*
		14.3.3	interpret geometrical interpretation of derivative;			*
		14.3.4	find the point on a curve where the tangent is parallel to the given line;			*
		14.3.5	describe the second derivative rule to find the extreme values of a function at a point;		*	
		14.3.6	apply second derivative rule to examine a given function for extreme values;			*
		14.3.7	solve word problems related to extreme values;			*
		14.3.8	apply MAPLE command *maximize* and *minimize* to compute the maximum and minimum value of a function.			CA

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
15. Partial Fractions		Students should be able to:				
15.1	Proper and Improper Rational Fractions	15.1.1	distinguish between proper and improper rational fractions;		*	
15.2	Resolution of Fractions into Partial Fractions	15.2.1	explain the meaning of partial fractions;		*	*
		15.2.2	convert $\frac{P(x)}{Q(x)}$ into partial fractions when denominator $Q(x)$ has: a. non repeated linear factors b. repeated linear factors c. non repeated irreducible quadratic factors d. repeated irreducible quadratic factors e. combination of cases a. b. c. and d.			

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
16. Integration		Students should be able to:				
	16.1 Introduction	16.1.1	describe: a. the concept of the integral as an accumulator (continuous sum) b. integration as inverse process of differentiation c. the reason of using constant of integration;		*	
		16.1.2	find the indefinite integrals to relate simple standard integrals formula with standard differentiation formulae;			*
	16.2 Rules of Integration	16.2.1	describe the following rules of integration: a. $\int \frac{d}{dx}[f(x)]dx = \frac{d}{dx}\left[\int f(x)dx\right] = f(x)+c$ where c is the constant of integration b. the integral of the product of a constant and a function is the product of the constant and the integral of the function c. the integral of the sum of a finite number of functions is equal to the sum of their integrals;		*	
		16.2.2	prove the results for the following integrals by using the standard differentiation formulae : a. $\int [f(x)]^n f'(x)dx,$ b. $\int \frac{f'(x)}{f(x)}dx,$ c. $\int e^{ax}[af(x)+f'(x)]dx$			*
		16.2.3	evaluate indefinite integrals;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
16.3	Integration by Substitution	16.3.1	explain the method of integration by substitution;		*	
		16.3.2	evaluate indefinite integrals using appropriate substitutions;			*
		16.3.3	apply the method of substitution to evaluate the integrals of the following types: a. $\int \frac{dx}{a^2 - x^2}$, $\int \sqrt{a^2 - x^2}\,dx$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$, b. $\int \frac{dx}{a^2 + x^2}$, $\int \sqrt{a^2 + x^2}\,dx$, $\int \frac{dx}{\sqrt{x^2 + a^2}}$, c. $\int \frac{dx}{x^2 - a^2}$, $\int \sqrt{x^2 - a^2}\,dx$, $\int \frac{dx}{\sqrt{x^2 - a^2}}$;			*
16.4	Integration by Parts	16.4.1	write the formula of integration by parts;	*		
		16.4.2	apply the method of integration by parts to evaluate the integrals of the following types: $\int \sqrt{a^2 - x^2}\,dx$, $\int \sqrt{a^2 + x^2}\,dx$, $\int \sqrt{x^2 - a^2}\,dx$;			*
		16.4.3	evaluate integrals using integration by parts;			*
16.5	Integration using Partial Fractions	16.5.1	find $\int \frac{f(x)}{g(x)}dx$ using partial fractions where $f(x)$ and $g(x)$ are algebraic functions such that $g(x) \neq 0$.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
16.6	Definite Integrals	16.6.1	define definite integral;	*		
		16.6.2	describe the fundamental theorem of integral calculus to recognise the following basic properties: a. $\int_a^a f(x)dx = 0,$ b. $\int_a^b f(x)dx = -\int_b^a f(x)dx,$ c. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad a < c < b,$ d. $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)\,dx \text{ when } f(-x) = f(x) \\ 0 \text{ when } f(-x) = -f(x) \end{cases}$;		*	
		16.6.3	evaluate definite integrals;			*
		16.6.4	describe the definite integral as the area under the curve with respect to *x*-axis;		*	
		16.6.5	calculate the area under the curve using definite integrals;			*
16.7	Differential Equations	16.7.1	explain the concept of ordinary differential equation (ODE), order and degree of differential equation;		*	
		16.7.2	solve differential equations of first order and first degree by separating the variables;			*
		16.7.3	solve word problems related to differentiation and integration, e.g. finding displacement from velocity, etc;			*
		16.7.4	use MAPLE command *int* to evaluate definite and indefinite integrals.			CA

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
17. Plane Analytic Geometry (Straight Line)		Students should be able to:				
17.1	Division of a Line Segment	17.1.1	derive distance formula between two points given in Cartesian plane;			*
		17.1.2	calculate the distance between two points given in Cartesian plane using distance formula;			*
		17.1.3	calculate the coordinates of a point that divides the line segment in a given ratio (internally and externally);			*
		17.1.4	solve related problems using distance formula and ratio formula;			*
		17.1.5	prove that the medians and angle bisectors of a triangle are concurrent;			*
17.2	Slope of a Straight Line	17.2.1	define the slope of a line;	*		
		17.2.2	derive the formula for the slope of a line passing through two points;			*
		17.2.3	find the slope of a line passing through two points;			*
		17.2.4	describe the conditions when two straight lines with given slopes are: a. parallel to each other b. perpendicular to each other;		*	
		17.2.5	solve problems related to SLO 17.2.4;			*
17.3	Equation of a Straight Line Parallel to Coordinate Axes	17.3.1	find the equation of a straight line parallel to: a. y-axis and at a distance of a unit from it b. x-axis and at a distance of a unit from it where $a \in R$;			*
17.4	Standard Form of Equation of a Straight Line	17.4.1	describe intercepts of a straight line;		*	

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		17.4.2	describe equation of a straight line in: a. slope-intercept form b. point-slope form c. two-point form d. intercept form e. normal form f. symmetric form;		*	
		17.4.3	convert the general form of the equation of a straight line into the forms mentioned in SLO 17.4.2;			*
		17.4.4	find the equation of straight line when different situtations are given;			*
17.5	Distance of a Point From a Line	17.5.1	determine the position of a point with respect to a line (i.e. above or below the line);			*
		17.5.2	calculate the perpendicular distance from a point to the given straight line;			*
		17.5.3	find the distance between two parallel lines;			*
17.6	Angle Between Lines	17.6.1	find the angle between two coplanar intersecting straight lines;			*
		17.6.2	find the equation of family of lines passing through the point of intersection of two given lines;			*
		17.6.3	calculate angles of the triangle when the slopes of the sides are given;			*
17.7	Concurrency of Straight Lines	17.7.1	find the condition of concurrency of three straight lines;			*
		17.7.2	find the equation of median, altitude and right bisector of a triangle;			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
		Students should be able to:				
		17.7.3	prove that the following are concurrent in a triangle: a. right bisectors b. medians c. altitudes;			*
17.8	Area of a Triangular Region	17.8.1	find the area of a triangular region and hence quadrilateral region whose vertices are given;			*
17.9	Homogenous Equations	17.9.1	describe homogeneous linear equations in two variables;		*	
		17.9.2	find the equation of the pair of lines represented jointly by the 2^{nd} degree homogeneous equation $ax^2+2hxy+by^2=0$;			*
		17.9.3	discuss the nature of the lines of $ax^2+2hxy+by^2=0$ by using the discriminant $D=2\sqrt{h^2-ab}$;		*	
		17.9.4	find the acute angle between a pair of straight lines passing through the origin.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
18. Linear Programming		Students should be able to:				
18.1	Introduction	18.1.1	describe the terms used in the linear programming;		*	
		18.1.2	describe linear programming as a planning of allocation of limited resources to obtain an optimal result;		*	
18.2	Linear Inequalities	18.2.1	explain the concept of algebraic solutions of linear inequalities in one variable and represent them on number line;		*	
		18.2.2	interpret graphically the linear inequalities in two variables;			*
		18.2.3	determine graphically the region bounded by, at most 3 simultaneous linear inequalities of non-negative variables and shade the region bounded by them;			*
18.3	Feasible Region	18.3.1	describe: a. linear programming problem b. objective function c. problem constraints d. decision variables;		*	
		18.3.2	illustrate graphically the feasible region (or solution space) of a linear programing problem;			*
		18.3.3	find the feasible region of linear programing problems;			*

Topics and Sub-topics	Student Learning Outcomes		Cognitive Level		
			K	U	A
	Students should be able to:				
18.4 Optimal Solution	18.4.1	describe the optimal solution of a linear programing problem;		*	
	18.4.2	find the optimal solution through the following systematic procedure: a. establish the mathematical formulation of linear programing problems b. construct a graph c. identify the feasible region d. locate the solution points e. evaluate the objective function f. select the optimal solution g. verify the optimal solution by actually substituting values of variables from the feasible region;			*
	18.4.3	solve simple linear programing problems.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
19. Circles		Students should be able to				
19.1	Introduction	19.1.1	describe conics and demonstrate members of its family, i.e. circle, parabola, ellipse and hyperbola;		*	
19.2	Equation of Circle	19.2.1	describe the concept of circle and its related terms;		*	
		19.2.2	derive the equation of a circle in standard form, i.e. $(x-h)^2+(y-k)^2=r^2$;			*
		19.2.3	describe the general equation of a circle $x^2+y^2+2gx+2fy+c=0$;		*	
		19.2.4	find the centre and radius by using the general and standard form of equation of a circle;			*
		19.2.5	sketch a circle when its elements are given;			*
		19.2.6	find the equation of a circle passing through: a. three non-collinear points b. two points and having its centre on a given line c. two points and equation of tangent at one of these points is known d. two points and touching a given line etc.;			*
		19.2.7	solve problems related to circles;			*
19.3	Tangents and Normals	19.3.1	find the condition when a line intersects a circle;			*
		19.3.2	find the condition when a line touches a circle;			*
		19.3.3	find the equation of a tangent to a circle in the slope-intercept form;			*
		19.3.4	find the equations of a tangent and a normal to a circle a. at a point b. which is parallel to a line c. which is perpendicular to a line;			*

Topics and Sub-topics	Student Learning Outcomes		Cognitive Level		
			K	U	A
	Students should be able to:				
	19.3.5	calculate the length of tangent to a circle from a given external point;			*
	19.3.6	determine whether the point lies inside, on or outside the circle;			*
	19.3.7	prove that the two tangents drawn to a circle from an external point are equal in length.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
20. Parabola		Students should be able to:				
20.1	Introduction	20.1.1	describe parabola and its elements (i.e. focus, directrix, eccentricity, vertex, axis, focal chord and latus rectum);		*	
20.2	Equation of a Parabola	20.2.1	derive the general form and standard form of an equation of a parabola;			*
		20.2.2	find the elements of a parabola;			*
		20.2.3	sketch a parabola with given elements;			*
		20.2.4	find the equation of a parabola with its given elements;			*
20.3	Tangents and Normals	20.3.1	find the condition when a line is tangent to a parabola at a point;			*
		20.3.2	find the equation of tangent and a normal to a parabola: a. at a point b. which is parallel to a given line c. which is perpendicular to a given line;			*
20.4	Uses of Parabola in Daily Life	20.4.1	explain the use of a parabola in daily life (i.e. suspension bridges, projectile etc);		*	
		20.4.2	solve problems related to parabola.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
21. Ellipse		Students should be able to:				
21.1	Introduction	21.1.1	describe ellipse and its elements (i.e. centre, foci, vertices, co-vertices, directrices, major and minor axes, eccentricity, focal chord and latera recta);		*	
		21.1.2	explain the concept that circle is a special case of an ellipse;		*	
21.2	Equation of an Ellipse	21.2.1	derive the standard form of equation of an ellipse;			*
		21.2.2	find the equation of an ellipse with its given elements;			*
		21.2.3	find the elements of ellipse by converting a given equation to the standard form;			*
		21.2.4	sketch an ellipse with given elements;			*
		21.2.5	solve problems related to ellipse;			*
21.3	Tangents and Normals	21.3.1	find the points of intersection of an ellipse with a line and the condition of tangency;			*
		21.3.2	find the equation of tangent and normal to an ellipse: a. at a point b. which is parallel to a given line c. which is perpendicular to a given line.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
22. Hyperbola		Students should be able to:				
22.1	Introduction	22.1.1	describe hyperbola and its elements (i.e. centre, foci, vertices, directrices, transverse and conjugate axes, eccentricity, focal chord, asymptotes and latera recta);		*	
22.2	Equation of Hyperbola	22.2.1	derive the standard form of equation of a hyperbola;			*
		22.2.2	find the equation of a hyperbola with its given elements;			*
		22.2.3	find the elements of hyperbola by converting a given equation into the standard form;			*
		22.2.4	sketch a hyperbola when its elements are given;			*
		22.2.5	solve problems related to hyperbola;			*
22.3	Tangents and Normals	22.3.1	find the points of intersection of a hyperbola with a line, including the condition of tangency;			*
		22.3.2	find the equation of tangent and normal to a hyperbola: a. at a point b. which is parallel to a given line c. which is perpendicular to a given line.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
23. Translation and Rotation		Students should be able to:				
23.1	Translation and Rotation of Axes	23.1.1	describe translation and rotation of axes with examples;		*	
		23.1.2	find the equations of transformation for: a. translation of axes b. rotation of axes;			*
		23.1.3	find the transformed equation by using translation or rotation of axes;			*
		23.1.4	find the new origin and new axes referred to old origin and old axes;			*
		23.1.5	find the angle through which the axes is rotated about the origin so that the product term *xy* is removed from the transformed equation.			*

Topics and Sub-topics		Student Learning Outcomes		Cognitive Level		
				K	U	A
24. Vectors		Students should be able to:				
24.1	Vectors in a Plane	24.1.1	describe scalar and vector quantities with examples;		*	
		24.1.2	describe magnitude of a vector, equal vectors, negative of a vector, unit vector, zero/ null vector, position vector, parallel vectors;		*	
		24.1.3	represent a vector geometrically;		*	
		24.1.4	apply concepts of multiplication of a vector by a scalar and addition and subtraction of two vectors geometrically;			*
		24.1.5	represent a vector in a Cartesian plane by describing the fundamental unit vectors ***i*** and ***j***.		*	
		24.1.6	find unit vector in the direction of a given vector;			*
		24.1.7	find the position vector of a point which divides the line segment joining two points in a given ratio;			*
		24.1.8	solve problems related to position vector;			
24.2	Vectors in Space	24.2.1	describe rectangular coordinates in space;		*	
		24.2.2	represent a vector in space by using the fundamental unit vectors ***i*** , ***j*** and ***k***;		*	
		24.2.3	find the distance between two points in space;			*
		24.2.4	apply concepts of multiplication of a vector by a scalar and addition and subtraction of two vectors in three dimensions;			*
		24.2.5	find magnitude of a vector;			*
		24.2.6	find unit vector in the direction of a given vector;			*
		24.2.7	prove: a. commutative law for vector addition b. associative law for vector addition;			*
24.3	Dot or Scalar Product	24.3.1	describe dot or scalar product of two vectors;		*	
		24.3.2	describe the properties of dot or scalar product;		*	

Topics and Sub-topics	Student Learning Outcomes		Cognitive Level		
			K	U	A
	Students should be able to:				
	24.3.3	prove that: $\boldsymbol{i}.\boldsymbol{i} = \boldsymbol{j}.\boldsymbol{j} = \boldsymbol{k}.\boldsymbol{k} = 1$ $\boldsymbol{i}.\boldsymbol{j} = \boldsymbol{j}.\boldsymbol{k} = \boldsymbol{k}.\boldsymbol{i} = 0$;		*	
	24.3.4	find dot product of two vectors;			*
	24.3.5	solve the problems based on the condition of orthogonality of two vectors;			*
	24.3.6	describe direction cosines of a vector;		*	
	24.3.7	find direction cosines of a vector;			*
	24.3.8	prove that the sum of the squares of direction cosines is unity;			*
	24.3.9	apply dot product to find the angle between two vectors;			*
	24.3.10	find the projection of a vector along another vector;			*
	24.3.11	solve problems related to SLOs 24.3.4, 24.3.7 and 24.3.9 ;			*
24.4 Cross or Vector Product	24.4.1	describe cross or vector product of two vectors with its geometrical illustration;		*	
	24.4.2	describe the properties of cross or vector product;		*	
	24.4.3	prove that: a. $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{j} \times \boldsymbol{j} = \boldsymbol{k} \times \boldsymbol{k} = 0$ b. $\boldsymbol{i} \times \boldsymbol{j} = -\boldsymbol{j} \times \boldsymbol{i} = \boldsymbol{k}$ c. $\boldsymbol{j} \times \boldsymbol{k} = -\boldsymbol{k} \times \boldsymbol{j} = \boldsymbol{i}$ d. $\boldsymbol{k} \times \boldsymbol{i} = -\boldsymbol{i} \times \boldsymbol{k} = \boldsymbol{j}$;		*	
	24.4.4	find cross product of two vectors;			*

Topics and Sub-topics		Student Learning Outcomes	Cognitive Level		
			K	U	A
	Students should be able to:				
	24.4.5	prove that the magnitude of $\boldsymbol{A} \times \boldsymbol{B}$ represents the area of a parallelogram with adjacent sides $\boldsymbol{A}$ and $\boldsymbol{B}$;			*
	24.4.6	find condition of parallel vectors of two non-zero vectors;		*	
	24.4.7	solve problems using condition of parallel vectors of two non-zero vectors;			*
	24.4.8	prove $\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$ with the help of given vectors;		*	
	24.4.9	find angle between two vectors using cross product;			*
	24.4.10	solve problems related to SLOs 24.4.4 and 24.4.9;			*
24.5 Scalar Triple Product	24.5.1	describe scalar triple product of vectors;		*	
	24.5.2	find scalar triple product of vectors;			*
	24.5.3	prove that: a. $\boldsymbol{i}.\boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{j}.\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{k}.\boldsymbol{i} \times \boldsymbol{j} = 1$ b. $\boldsymbol{i}.\boldsymbol{k} \times \boldsymbol{j} = \boldsymbol{j}.\boldsymbol{i} \times \boldsymbol{k} = \boldsymbol{k}.\boldsymbol{j} \times \boldsymbol{i} = -1$;		*	
	24.5.4	prove that dot and cross are inter-changeable in scalar triple product;		*	
	24.5.5	solve problems related to SLOs 24.5.2 and 24.5.4;			*
	24.5.6	find the volume of a a. parallelepiped b. tetrahedron; determined by three given vectors;			*
	24.5.7	find the condition for co-planarity of three vectors;		*	
	24.5.8	solve problems using the condition of co-planarity of three vectors.			*

Scheme of Assessment

Grade XI

Table 1: Number of Student Learning Outcomes by Cognitive Level

Topic No.	Topics	No. of Sub-topics	SLOs			Total
			K	U	A	
1.	Complex Numbers	3	2	2	10	14
2.	Matrices and Determinants	5	2	4	13	19
3.	Sequence and Series	9	0	6	21	27
4	Miscellaneous Series	1	0	1	2	3
5.	Permutation, Combination and Probability	3	1	8	10	19
6.	Mathematical Induction and Binomial Theorem	3	0	2	8	10
7.	Quadratic Equations	9	1	5	16	22
8.	Introduction to Trigonometry and Trigonometric Identities	6	1	5	10	16
9.	Application of Trigonometry	3	0	3	8	11
10.	Graphs of Trigonometric Functions, Inverse Trigonometric Functions and Solution of Trigonometric Equations	4	0	7	14	21
	Total	**46**	**7**	**43**	**112**	**162**
	Percentage		**4**	**27**	**69**	**100**

Table 2: Exam Specifications

Topic No.	Topics	Marks Distribution	Total Marks
1.	Complex Number	MCQs 4 @ 1 Mark CRQ 1 @ 4 Marks	8
2.	Matrices and Determinants	MCQs 6 @ 1 Mark CRQ 1 @ 7 Marks	13
3.	Sequence and Series	MCQs 6 @ 1 Mark CRQ 1 @ 5 Marks CRQ 1 @ 4 Marks	15
4.	Miscellaneous Series		
5.	Permutation Combination and Probability	MCQs 4 @ 1 Mark CRQ 1 @ 6 Marks	10
6.	Mathematical Induction and Binomial Theorem	MCQs 3 @ 1 Mark CRQ 1 @ 7 Marks	10
7.	Quadratic Equations	MCQs 6 @ 1 Mark *CRQs 2 @ 7 Marks Choose any ONE from TWO	13
8.	Introduction to Trigonometry and their Identities	MCQs 7 @ 1 Mark **CRQs 3 @ 7 Marks Choose any TWO from THREE	21
9.	Application of Trigonometry		
10.	Graph of Trigonometric and Inverse Trigonometric Functions and Solution of Trigonometric Equations	MCQs 4 @ 1 Mark CRQ 1 @ 6 Marks	10
	Total	**MCQs** 40 / **CRQs** 60	**100**

* There will be TWO questions and the Students will be required to attempt any ONE by making a choice out of the TWO.

** There will be THREE questions and the Students will be required to attempt any TWO by making a choice out of the THREE.

Grade XII

Table 3: Number of Student Learning Outcomes by Cognitive Level

Topic No.	Topics	No. of Sub-topics	SLOs			Total
			K	U	A	
11.	MAPLE	4	0	0	0	0
12.	Functions and Limits	10	2	13	14	29
13.	Differentiation	6	0	4	17	21
14.	Higher Order Derivatives and its Applications	3	1	1	9	11
15.	Partial Fractions	2	0	2	1	3
16.	Integration	7	2	6	12	20
17.	Plane Analytical Geometry (Straight Line)	9	1	5	23	29
18.	Linear Programming	4	0	5	6	11
19.	Circles	3	0	3	12	15
20.	Parabola	4	0	2	7	9
21.	Ellipse	3	0	2	7	9
22.	Hyperbola	3	0	1	7	8
23.	Translation and Rotation	1	0	1	4	5
24.	Vectors	5	0	12	32	44
	Total	**65**	**6**	**57**	**151**	**214**
	Percentage		**3**	**27**	**70**	**100**

Table 4: Exam Specifications

Topic No.	Topics	Marks Distribution	Total Marks
11.	MAPLE	-	-
12.	Functions and Limits	MCQs 4 @ 1 Mark CRQ 1 @ 4 Marks	8
13.	Differentiation	MCQs 6 @ 1 Mark **CRQs 3 @ 6 Marks Choose any TWO from THREE	18
14.	Higher Order Derivative and Application		
15.	Partial Fractions	MCQs 6 @ 1 Mark **CRQs 3 @ 6 Marks Choose any TWO from THREE	18
16.	Integration		
17.	Plane Analytical Geometry (Straight Line)	MCQs 7 @ 1 Mark CRQ 1 @ 5 Marks	12
18.	Linear Programming	MCQs 1 @ 1 Mark CRQ 1 @ 5 Marks	6
19.	Circles	MCQs 3 @ 1 Mark CRQ 1 @ 4 Marks	7
20.	Parabola	MCQs 6 @ 1 Mark CRQs 2 @ 5 Marks	16
21.	Ellipse		
22.	Hyperbola		
23.	Translation and Rotation	MCQs 2 @ 1 Mark CRQ 1 @ 4 Marks	6
24.	Vectors	MCQs 5 @ 1 Mark *CRQs 2 @ 4 Marks Choose any ONE from TWO	9
	Total	**MCQs** 40 / **CRQs** 60	**100**

* There will be TWO questions and the Students will be required to attempt any ONE by making a choice out of the TWO.

** There will be THREE questions and the Students will be required to attempt any TWO by making a choice out of the THREE.

- Tables 1 and 3 indicate the number and nature of SLOs in each topic in grades XI and XII respectively. This will serve as a guide in the construction of the examination paper. It also indicates that more emphasis has been given to the Understanding (27% in XI and 27% in XII), Application and higher order skills (69% in XI and 70% in XII) to discourage rote memorization. Tables 1 and 3, however, do not translate directly into marks.

- There will be two examinations, one at the end of grade XI and one at the end of grade XII.

- In each grade, the theory paper will be in two parts: paper I and paper II. Both papers will be of duration of 3 hours.

- Paper I theory will consist of 40 compulsory, multiple choice questions. These **questions** will involve four response options.

- Paper II theory will carry 60 marks and consist of a number of compulsory, constructed response questions. There will be no choice among the topics in constructed response questions but it may be within the topic.

- All constructed response questions will be in a booklet which will also serve as an answer script.

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