AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XII

MODEL EXAMINATION PAPER 2023 AND ONWARDS

Mathematics Paper II

Time: 1 hour 30 minutes   Marks: 50

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign if it is accurate.

I agree that this is my name and school. **Candidate's Signature**

RUBRIC

2. There are NINE questions. Answer ALL questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use black pointer to write your answers. DO NOT write your answers in pencil.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. A formulae list is provided on page 2 and 3. You may refer to it during the paper, if you wish.

6. You may use a scientific calculator if you wish.

Aga Khan University Examination Board
List of Formulae for Mathematics XII

Note:

- All symbols used in the formulae have their usual meaning.
- The same formulae will be provided in the annual and re-sit examinations.

Functions and Limits

$\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ $\qquad$ $\lim\limits_{x \to 0} \dfrac{\sqrt{x+a} - \sqrt{a}}{x} = \dfrac{1}{2\sqrt{a}}$ $\qquad$ $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$

$\lim\limits_{x \to 0} (1+x)^{\frac{1}{x}} = e$ $\qquad$ $\lim\limits_{x \to +\infty} \left(1 + \dfrac{1}{x}\right)^x = e$ $\qquad$ $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a$

Differentiation

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+h) - f(x)}{h}$ $\qquad$ $\dfrac{d}{dx}(\sec x) = \sec x \tan x$ $\qquad$ $\dfrac{d}{dx} x^n = nx^{n-1}$ $\qquad$ $\dfrac{d}{dx}(\sin x) = \cos x$

$\dfrac{d}{dx}(\cos x) = -\sin x$ $\qquad$ $\dfrac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ $\qquad$ $\dfrac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$\dfrac{d}{dx}\left[\sin^{-1} x\right] = \dfrac{1}{\sqrt{1-x^2}}, \quad x \in (-1, 1)$ $\qquad$ $\dfrac{d}{dx}\left[\cos^{-1} x\right] = -\dfrac{1}{\sqrt{1-x^2}}, \ x \in (-1, 1)$ $\qquad$ $\dfrac{d}{dx}\left[\tan^{-1} x\right] = \dfrac{1}{x^2+1}, \quad x \in R$ $\qquad$ $\dfrac{d}{dx}\left[\sec^{-1} x\right] = \dfrac{1}{|x|\sqrt{x^2-1}}, \quad x \in [-1, 1]'$

$\dfrac{d}{dx}\left[\operatorname{cosec}^{-1} x\right] = -\dfrac{1}{|x|\sqrt{x^2-1}}, \ x \in [-1, 1]'$ $\qquad$ $\dfrac{d}{dx}\left[\cot^{-1} x\right] = -\dfrac{1}{1+x^2}, \ x \in R$ $\qquad$ $\dfrac{d}{dx}(a^x) = a^x \ln a$ $\qquad$ $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x} \times \dfrac{1}{\ln a}$

$\dfrac{d}{dx}(\sinh x) = \cosh x$ $\qquad$ $\dfrac{d}{dx}(\cosh x) = \sinh x$ $\qquad$ $\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\dfrac{d}{dx}(\operatorname{cosech} x) = -\coth x \operatorname{cosech} x$ $\qquad$ $\dfrac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$ $\qquad$ $\dfrac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

$\dfrac{d}{dx}\left[f(x) \times g(x)\right] = f(x)\dfrac{d}{dx}\left[g(x)\right] + g(x)\dfrac{d}{dx}\left[f(x)\right]$ $\qquad$ $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)\dfrac{d}{dx}\left[f(x)\right] - f(x)\dfrac{d}{dx}\left[g(x)\right]}{\left[g(x)\right]^2}$

Maclaurin Series $f(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \dfrac{f'''(0)}{3!}x^3 + \ldots + \dfrac{f^n(0)}{n!}x^n + \ldots$

Taylor's Series $f(x) = f(a) + f'(a)(x-a) + \dfrac{f''(a)}{2!}(x-a)^2 + \dfrac{f'''(a)}{3!}(x-a)^3 + \dfrac{f^{iv}(a)}{4!}(x-a)^4 + \ldots + \dfrac{f^n(a)}{n!}(x-a)^n + \ldots$

Integration

$\int f'(x)dx = f(x) + c$ $\qquad$ $\int [f(x)]^n f'(x)dx = \dfrac{[f(x)]^{n+1}}{n+1} + c, \quad (n \neq -1)$ $\qquad$ $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$

$\int \dfrac{f'(x)}{f(x)}dx = \ln f(x) + c$ $\qquad$ $\int \operatorname{cosec} x \, dx = \ln|\operatorname{cosec} x - \cot x| + c$ $\qquad$ $\int \sec x dx = \ln|\sec x + \tan x| + c$

Plane Analytical Geometry (Straight Line)

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ $\qquad$ $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ $\qquad$ $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Point of internal division

$$\left(\frac{k_1x_2+k_2x_1}{k_1+k_2}, \frac{k_1y_2+k_2y_1}{k_1+k_2}\right)$$

Point of external division

$$\left(\frac{k_1x_2-k_2x_1}{k_1-k_2}, \frac{k_1y_2-k_2y_1}{k_1-k_2}\right)$$

$$\frac{x-x_1}{\cos\alpha}=\frac{y-y_1}{\sin\alpha}=r \text{ (say)}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$x\cos\alpha+y\sin\alpha=p$$

$$y=mx+c$$

$$y-y_1=m(x-x_1)$$

$$\theta=\tan^{-1}\left[\frac{m_2-m_1}{1+m_1m_2}\right]$$

$$\theta=\tan^{-1}\left[\frac{2\sqrt{h^2-ab}}{a+b}\right]$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Circles

$$(x-h)^2+(y-k)^2=r^2$$

$$x^2+y^2+2gx+2fy+c=0$$

Equation of normal to a circle

$$(y-y_1)(x_1+g)=(x-x_1)(y_1+f)$$

Equation of tangent to a circle

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

Length of tangent to a circle from a point (x_1, y_1), $l=\sqrt{x_1{}^2+y_1{}^2+2gx_1+2fy_1+c}$

Parabola

$$x^2=4ay \qquad (x-h)^2=4a(y-k) \qquad y^2=4ax \qquad (y-k)^2=4a(x-h)$$

Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a>b \qquad b^2=a^2(1-e^2) \qquad c=ae \qquad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1,\ a>b$$

Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad b^2=a^2(e^2-1) \qquad c=ae \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Translation and Rotation

$$x=X+h,\ y=Y+k \qquad X=x\cos\theta+y\sin\theta,\ Y=y\cos\theta-x\sin\theta \qquad \tan 2\theta=\frac{2h}{a-b}$$

Vectors

$$\underline{u}.\underline{v}=|\underline{u}||\underline{v}|\cos\theta$$

Area of a triangle $=\frac{1}{2}|\underline{u}\times\underline{v}|$

$$\underline{u}\times\underline{v}=\left(|\underline{u}||\underline{v}|\sin\theta\right)\hat{\underline{n}}$$

$$\underline{r}=\frac{q\underline{a}+p\underline{b}}{p+q}$$

$$\left|\overrightarrow{P_1P_2}\right|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$\frac{1}{6}(\underline{u}\times\underline{v}).\underline{w}=\frac{1}{6}[\underline{u}\ \underline{v}\ \underline{w}]$$

$$\underline{u}.(\underline{v}\times\underline{w})=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Some Trigonometric Identities

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos P+\cos Q=2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\cos P-\cos Q=-2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\sin P+\sin Q=2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P-\sin Q=2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

PLEASE TURN OVER THE PAGE

Q.1. (Total 3 Marks)

Evaluate $\lim_{x \to 3} \frac{(x^2 - 9)^2}{x^2 - 6x + 9}$.

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

Q.2. (Total 10 Marks)

a.

i. If $y = e^{x+\tan x}$, then show that $\dfrac{dy}{dx} = 2y + y\tan^2 x.$ (3 Marks)

ii. For $y = \sqrt{9x^2 - 4}$, show that $\dfrac{dy}{dx} = \dfrac{9x}{\sqrt{9x^2 - 4}}$. (2 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

b. For $y = \ln\left(\dfrac{\sqrt{x}\cos x}{\sin x}\right)$, find $\dfrac{d^2y}{dx^2}$. (5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.2.)

c. A water tank is cylindrical in shape. If the volume of the tank is 64 m^3, then what would be the radius and height of the water tank to have the least surface area?
Write you answer to two decimal places.

(**Note**: The volume of a cylinder is $V = \pi r^2 h$, surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$ and $h = \frac{64}{\pi r^2}$, where r is the radius of the cylinder and h is the height of the cylinder.)

(5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

Q.3. (Total 10 Marks)

a. Evaluate $\int \frac{xdx}{\sqrt{x^2+9}}$ by substituting $x = 3\tan\theta$. (5 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

b. Evaluate the following indefinite integrals.

i. $\int \left\{ \left(2x^2 - 3\right)^2 \right\}^{\frac{3}{2}} x \, dx$ (2 Marks)

ii. $\int \frac{x}{(x+b)^2} dx$ (3 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.3.)

c. Evaluate the following definite integrals by showing all the necessary steps. (5 Marks)

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{1+\cos x}$$

Q.4. (Total 4 Marks)

i. Convert the equation $x-\sqrt{3}\,y=1$ into the symmetric form. (3 Marks)

ii. Find the equation of a straight line which passes through the point $(0,\,-1)$ and whose gradient (slope) is 1. (1 Mark)

Q.5. (Total 4 Marks)

A burger shop sells chicken and beef burgers. The profit on the chicken burger and the beef burger is Rs 12 and Rs 10 respectively.

Due to existing cooking facilities, it cannot cook

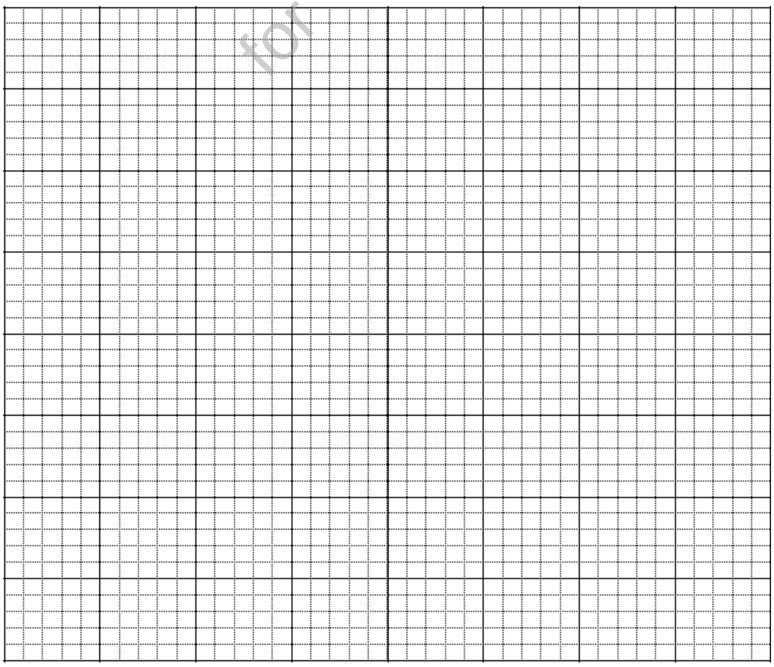
- more than 200 chicken burgers.
- more than 250 beef burgers.
- and sell more than 400 burgers altogether.

For the given linear programming problem,

i. state the constraints. (1 Mark)

ii. state the profit function. (1 Mark)

iii. find any two of the corner points by drawing constraints on the given graph. (2 Marks)



Q.6. (Total 4 Marks)

Find the centre of the circle passing through the points (0, 0) and (3, – 2) and having centre at the line $3x + y = 10$.

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

Q.7. (Total 8 Marks)

a. Find the focus and the vertex of the parabola $(x-7)^2 = 28(y+1)$. (4 Marks)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

b. An ellipse with centre $(0,\ 0)$ has vertices $(\pm 5,\ 0)$ and foci $(\pm 4,\ 0)$. Find

i. the equation of the ellipse. (2 Marks)

ii. the eccentricity of the ellipse. (1 Mark)

iii. the equations of the directrices of the ellipse. (1 Mark)

(ATTEMPT ANY TWO PARTS FROM a, b AND c OF Q.7.)

c. The vertices of a hyperbola are $(15, 1)$ and $(-1, 1)$. If the length of its conjugate axis is 12 units, then find the equation of the hyperbola in standard form. (4 Marks)

Q.8. (Total 4 Marks)

A triangle *ABC*, located on the Cartesian plane, has vertices *A* (0, 3), *B* (– 3, 0) and *C* (3, – 3). The a*x*is is translated through a distance of – 3 units while there is no change in the *y*-a*x*is.

i. Find the new vertices of the triangle *ABC* with respect to the translated a*x*is. (3 Marks)

ii. Using the given graph paper, draw the triangle *ABC* with its new vertices. (1 Mark)

(ATTEMPT EITHER PART a OR PART b OF Q.9.)

Q.9. (Total 3 Marks)

a. Three points P, Q and R form a straight line such that $\overrightarrow{PQ} = \overrightarrow{PR}$ and $\overrightarrow{OQ} = k(\mathbf{a} - \mathbf{b})$.

i. Find the position vector of R in terms of k, $\mathbf{a}$ and $\mathbf{b}$. (2 Marks)

ii. Hence, find the unit vector in the direction of R. (1 Mark)

(ATTEMPT EITHER PART a OR PART b OF Q.9.)

b. It is given that $\underset{\sim}{p} \times \underset{\sim}{q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -1 & 3 \\ 2 & -2 & m \end{vmatrix}$, $\underset{\sim}{r} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\underset{\sim}{q} \bullet \left(\underset{\sim}{r} \times \underset{\sim}{p}\right) = 10$. Find the value of m.

(3 Marks)

END OF PAPER

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